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Monitoring and Diagnosing Multistage Processes: A Review of Cause Selecting Control Charts

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ABSTRACT

A review of the literature on cause selecting charts (CSCs) in multistage processes is given, with a concentration on developments which have occurred since 1993. Model based control charts and multiple cause selecting charts (MCSCs) are reviewed. Several articles based on normally and non-normally distributed outgoing quality characteristics are analyzed and important issues such as economic design, autocorrelated processes and adaptive design parameters of cause selecting charts are discussed. The results reveal that cause selecting charts outperform traditional Shewhart charts for individuals when the process steps are dependent, in view of the relationship between input and output quality characteristics. A new method for modeling and simulating a multistage process is proposed which can prove to be more reasonable in real practice. Finally, various directions for future research are given.

Keywords: Multistage processes, Cause selecting control chart, Economic design, Autocorrelation, Adaptive design parameters, Generalized linear model.

1. INTRODUCTION

Many industrial products are produced by several different process steps and not just one process step. For instance, Figure 1 depicts a production line with component parts processed through several operations before they are finally assembled. In each step, one or more quality characteristics of interest may be monitored according to their operation sequences. It implies that in multistage processes consisting of serial value-added manufacturing operations, a parameter shift in any process variable may affect some or all of the measures in the downstream stages but none of the measures preceding it. This property of multistage processes is referred to as the cascade property (Hawkins, 1993, Shu and Tsung, 2003). While monitoring a multistage process, there are some important issues to be addressed. The first issue involves prompt detection of the out-of-control conditions. Providing an appropriate remedy to this problem requires that the dependency structure among process quality characteristics, which are grouped into successive stages of the process to be determined. Additionally, in case of out-of-control settings, the stages and corresponding subsets of process variables responsible for the out-of-control conditions must be discovered. To this end, consider using a Shewhart or any of the univariate control charts such as

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Cumulative Sum (CUSUM) or Exponentially Weighted Moving Average (EWMA) charts at the individual stage in a multistage process. If the stages of the process are independent, then this is a meaningful procedure. However, as in most processes, the steps are inter-dependent and thus the signals on the aforementioned charts are misleading. Therefore, it is difficult to interpret the correct process state. An alternative approach is to use a multivariate control chart such as a Hotelling T^2 control chart, to monitor dependent process steps. There are, however, some drawbacks to using the Hotelling T^2 chart. When it signals that a process is out-of-control, it does not indicate which stage of the process has caused the problem and for this purpose, further diagnostic detective work is needed. In addition, the process quality characteristics are assumed to be multivariate normal random variables (Wade and Woodall, 1993). However, this assumption is not realistic in some processes. A different approach to this problem is the cause selecting control chart (CSC), proposed by Zhang (1980, 1982, 1984, 1985a, 1985b, 1989a, 1989b, 1992). The CSC is constructed for an outgoing quality characteristic only after it has been adjusted for the effect of incoming quality characteristic. The advantage of this approach is that once an out-of-control signal is given, it is easy to determine the peculiar stage responsible for the out-of-control condition. Therefore, it is more practical and beneficial for monitoring, diagnosing and analyzing multistage processes by taking into account the cascade property. The purpose of this paper is to review the articles dealing with the monitoring of multistage processes using CSCs. The overview and origin of model based control charts are discussed and several important aspects of using these kinds of control charts are studied. A more sensible simulation approach is proposed and finally, research areas of interest for future exploration are analyzed and discussed.
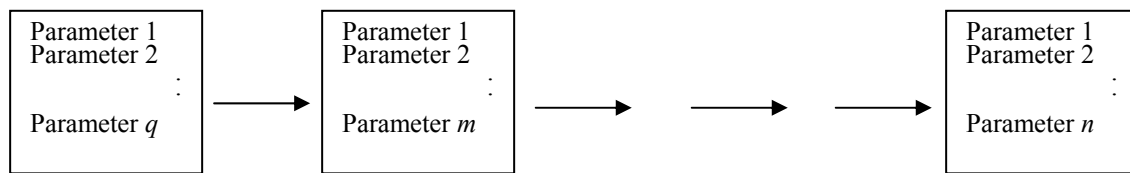

Figure1. A multistage process

2. OVERVIEW OF CAUSE SELECTING CHART (CSC) AND MULTIPLE CAUSE SELECTING CHART (MCSC)

In a production system, assume that there are two dependent steps in a process. Denote *X* to represent the quality variable for the first step which follows normal distribution and *Y* the quality variable for the second step which follows normal distribution given *X*. A random sample (X_i, Y_i) is taken at the end of the second process. The first step in constructing the CSC is to establish a relationship between *X* and *Y* using either linear or non-linear models. Among them, the simple linear regression model is one of the most useful models that takes the form:

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i \qquad i = 1, 2, ..., n \tag{1}$$

where β_0 and β_1 are constants and $\varepsilon_i \sim \mathrm{NID}(0, \sigma_\varepsilon^2)$. In practice the true relationship between *X* and *Y* is usually unknown and the parameters must be estimated from the initial set of *n* observations. To monitor the second step, the specific quality of the second step is determined by adjusting the effect of *X* on *Y*; that is, the specific quality is presented by cause selecting values z_i:

$$z_i = Y_i - \hat{Y}_i \tag{2}$$

where $\hat{Y}_i$ is the fitted value of Y_i given X_i. If the method of least squares is implemented to establish the fitted model, then $\bar{z} = 0$. Zhang (1984) suggests using Moving Range as the estimate of σ. Accordingly, the control limits will be:

$$UCL = 3\frac{\overline{MR}}{d_2} = 2.66\overline{MR} \ , \ LCL = -3\frac{\overline{MR}}{d_2} = -2.66\overline{MR} \tag{3}$$

where $d_2 = 1.128$. In addition, one may employ the square root of the mean square error as the estimate of σ. Thus:

$$UCL = 3\sqrt{MSE} = 3\sqrt{\frac{SSE}{n-2}} = 3\sqrt{\frac{\sum_{i=1}^{n} Z_i^2}{n-2}} \ , \ LCL = -3\sqrt{MSE} = -3\sqrt{\frac{SSE}{n-2}} = -3\sqrt{\frac{\sum_{i=1}^{n} Z_i^2}{n-2}} \tag{4}$$

However, Wade and Woodall (1993) proposed a modification to Zhang's CSC by using prediction limits as the control limits for monitoring the predicted values. Therefore:

$$UCL = t(\frac{\alpha}{2}, n-2)K \ , \ LCL = -t(\frac{\alpha}{2}, n-2)K \tag{5}$$

where, $K = \sqrt{MSE(1 + \frac{1}{n} + \frac{(X_0 - \bar{X})^2}{\sum_{i=1}^{n}(X_i - \bar{X})^2})}$.

and $t(\alpha/2, \upsilon)$ is the upper $\alpha/2$ quantile of Student's t distribution with υ degrees of freedom. They strongly recommended using the X chart for the variable in the first stage with the CSC for the variable in the second stage to diagnose a two-stage process because it more adequately reflects the incoming product quality. This results in four possible combinations. The outcomes with their interpretations are given in Table 1.

To review the basic design procedures of the MCSC, consider the quality characteristic of interest Y which is dependent on "m" quality characteristics denoted by a vector **X**. Needless to say that the characteristics in **X** are all inputs for Y. The first step is again to establish the relationship between input measures **X** and output measure Y. Using multiple regression as one of the most commonly used models yields:

$$Y_i = \beta_0 + \beta_1 X_{i1} + \cdots + \beta_m X_{im} + \varepsilon_i \quad i = 1, 2, ..., n \tag{6}$$

where $\varepsilon_i \sim \text{NID}(0, \sigma_\varepsilon^2)$, is a random error. The rest of the procedure is similar to that of constructing control limits for CSC. Note that the cascade property should also be taken into consideration. It is of paramount importance that both CSC and MCSC must be constructed for the variable based on other variables that are not affected by its mean shift (Hawkins, 1993).

Table1. Decision rules

Case	X chart	Z chart	Interpretation
1	Signal	Signal	Both subprocesses out of control
2	Signal	No signal	Previous subprocess out of control
3	No signal	Signal	Current subprocess out of control
4	No signal	No signal	Both subprocesses in control

3. ORIGIN OF THE REGRESSION AND MODEL BASED CONTROL CHARTS

The earliest reference to regression control charts in the quality control literature appeared in Dipaola (1945). The chart described was for the simultaneous control of two variables and might better have been labeled as a correlation control chart. This type of control problem was later treated more extensively (see, Jackson, 1956 and Weis, 1957). However, many authors claim that the model based control strategy was originated in Mandel (1969), in which Mandel combined the conventional control chart and regression analysis in order to control a varying rather than a constant mean.
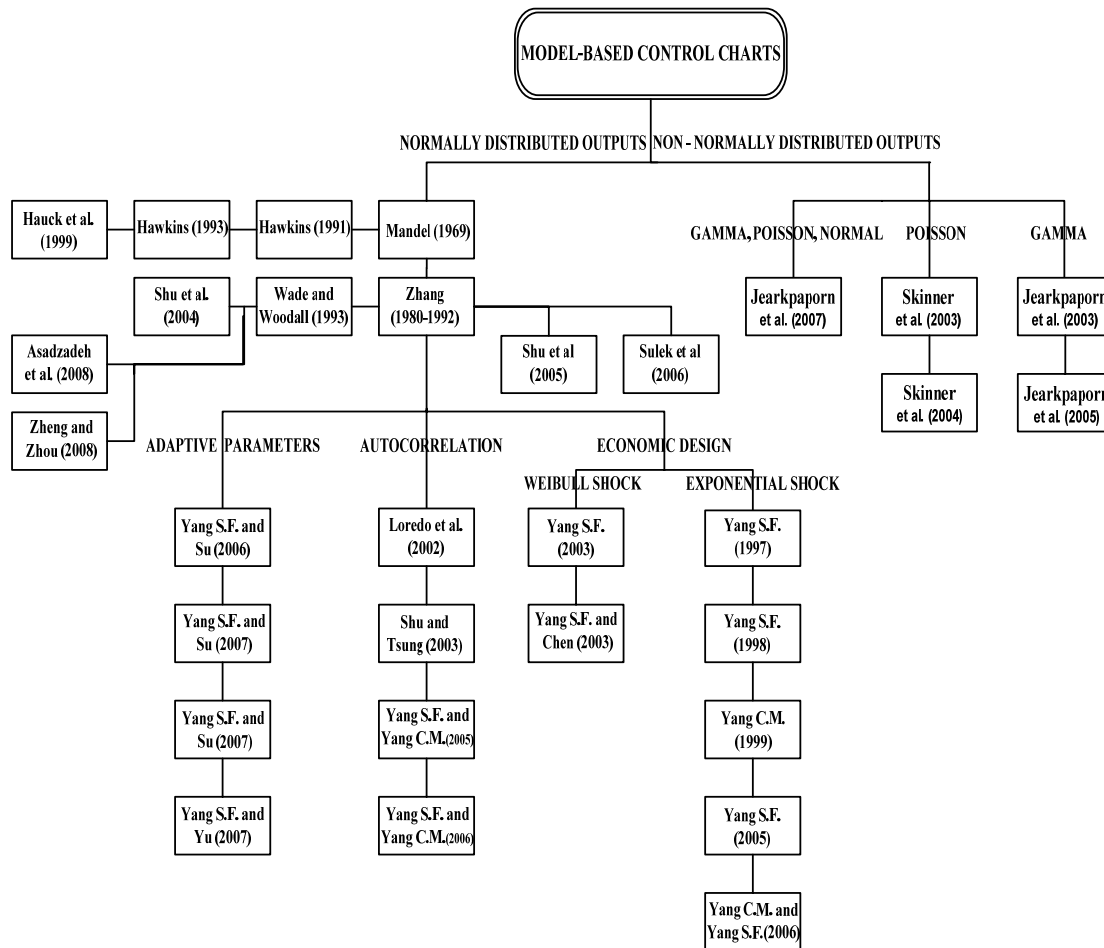

Figure 2. Schematic representation of categorized papers using model-based control charts in multistage processes

Model based monitoring procedures, are beneficial especially when there exists a cascade property that should be justified. Due to the importance of the model based control charts and great strides in studying multistage monitoring, we have classified the relevant papers into two major categories:

1) Outgoing quality characteristics of interest follow normal distribution.

2) Outgoing quality characteristics of interest follow non-normal distribution.

Figure 2 shows the schematic representation of a large variety of research efforts made in the area of multistage processes, using model based control charts, specifically CSCs. The next two sections deal widely with the review and analysis of these papers.

4. MODEL BASED CONTROL CHARTS WHEN THE OUTPUT VARIABLE IS NORMAL

As stated, Zhang (1984) proposed the CSC which is in fact similar to the regression control chart of Mandel in terms of charting a variable after the observations have been adjusted for the effect of an outside covariate. The CSC was implemented in the automobile industry which revealed the high efficiency of this type of control chart in comparison with the common Shewhart control charts for a given output quality characteristic. Wade and Woodall (1993) reviewed the basic principles of the CSC in the simple case of a two-step process and gave an example to illustrate its use. They also examined the relationship between the CSC and the Multivariate T^2 chart. In their opinion, the CSC has some advantages over the T^2 chart. They modified the CSC by employing prediction limits as the control limits for monitoring the cause selecting values. Results of simulation implied that prediction limits perform slightly better than Zhang's limits in detecting shifts in $E(Y|x)$ resulting from a change in β_0, the Y intercept of the true regression line. Moreover, it was illustrated that the false alarm rate of the prediction limits does not change significantly when shifts in $E(X)$ occur. This is what we really seek, since such shifts are associated with the previous subprocess and the CSC must effectively remove the effect of previous subprocesses from the current subprocess. It is strongly recommended that prediction limits be used in CSCs in case least squares regression determines the relationship between the variables.

A new type of model-based regression adjustment procedure called model-void was introduced by Hawkins (1991). The model-void procedure involves a mechanism where some or all of the quality characteristics are related, but a shift in one of the characteristics does not affect the others. It is suggested regressing X_j on all other variables X_i where $j \neq i$ and **X** denotes the quality characteristics of interest. Shewhart and Cumulative Sum (CUSUM) control charts based on residuals from the regression of each variable on all others, were used to monitor the process, which gave better performance than existing techniques. Hawkins claims that as the correlation coefficient among quality characteristics increases, the performance of model-based and Hotelling T^2 control charts will be superior to simple Shewhart charts.

Another model-based regression adjustment called model-fixed was proposed by Hawkins (1993). Model-fixed is appropriate to be applied to a process that has a natural ordering where a shift in any characteristic will affect some or all of the measures following it. Hawkins believes that each variable should be controlled using regression adjustment by those other variables that are not affected by its shifts from control, otherwise, the correct regression adjustment has not been achieved and the diagnostic procedure will be misleading. Using appropriate regression adjustment procedure for each variable in a process instead of using multivariate process control is preferred

and recommended especially when the correct relationship between the variables is taken into account.

Hauck *et al.* (1999) carried out the subsequent work based on the model-fixed procedure of Hawkins (1993). In the former model, it is assumed that there exists only one output quality variable in each step. However, in the extended model there are *P* total variables partitioned into *K* groups with P_i denoting the number of variables within groups so that $\sum_{i=1}^{K} P_i = P$. Hauck *et al.* propose a control statistic *U* defined as:

$$\mathbf{U}_i = \mathbf{Y}_i - \Sigma_{21} \Sigma_{11}^{-1} \mathbf{X}_i \tag{7}$$

where *i* is the step index, $\mathbf{Y}_i$ is a vector of output variables of size P_i and $\mathbf{X}_i$ is a vector of the input variables at stage *i*. The matrix $\Sigma_{21} = \text{cov}(\mathbf{Y}_i, \mathbf{X}_i)$ is the covariance matrix of $\mathbf{Y}_i$ and $\mathbf{X}_i$ and Σ_{11} is the covariance matrix of $\mathbf{X}_i$. It has been shown that using an individual Shewhart chart for each $\mathbf{U}_i$ statistic or multivariate process control for the $\mathbf{U}_i$ statistic at each step instead of using multivariate process control for the outgoing quality characteristics at each step ($\mathbf{Y}_i$), can be encouraging provided that the process knowledge and correlation between variables at each step are considered.

Multiple cause selecting chart (MCSC), taking the parameter uncertainties into account, was first discussed in Shu et al. (2004). Model estimation errors can cause the actual false alarm rate to differ from what is required. Therefore, a reasonable precaution would be to use control limits that are wider than normally used if the model had been known. It was shown that the variance of residuals is $\text{Var}(Z_f \mid \mathbf{x}_f) = \sigma^2(1 + \mathbf{x}_f'(X'X)^{-1}\mathbf{x}_f)$ for a given vector of input variables $\mathbf{x}_f$. Evidently, the variance of residuals has been decomposed into two components: the inherent variation due to the random error and the additional variation introduced by parameter uncertainties. Shu *et al.* claim that prediction limits can account for parameter uncertainties and thus must be implemented. The limits were developed using two types of procedures: the least squares estimation and principal component regression in the presence of multicollinearity. Note that the reason for attacking multicollinearity is the inflation of the variance of the regression coefficients. The simulation results showed that the prediction limits are quite effective in terms of maintaining a desired false alarm rate while significantly distinguishing between the incoming and outgoing qualities.

Shu *et al.* (2005) investigated the effect of parameter estimation errors on the performance of CSCs. It was stated that estimating β_0 and β_1 increases the false alarm rate of CSC, independent of whether they are under or overestimated. If a shift of $a\sigma$ occurs in $E(Y \mid x)$ and a shift of $b\sigma_x$ occurs in $E(X)$ and the relationship $\left| a\sigma + (\beta_0 - \hat{\beta}_0) \right| > a\sigma$ holds, then the signal probability of the CSC is larger than when the β_0 is known and if it does not hold, it will be less. On the other hand, if $(\beta_1 - \hat{\beta}_1)(\mu_x + b\sigma_x) > 0$, a Shewhart CSC with estimated β_1 tends to signal more quickly than when β_1 is known, otherwise it will signal less. Subsequently, simulation was conducted to study the overall effect of parameter estimation on the performance of CSC. The results confirmed that the in-control ARL of Shewhart CSC is larger than 370, about 525. On the other hand, Wade and Woodall (1993) showed that in-control false alarm rate of CSC is about 0.006. These outcomes are contradictory since we expect to have the in-control ARL of less than 370. In addition, the

parameter estimation results in the dependence of the residuals and so the ARL should be smaller than usual. However, there is no one-to-one mapping between the two performance measurements, ARL and α. This problem can be justified by concentrating on the distribution of the ARL. It revealed that the distribution is highly skewed to the right, causing the average ARL to be larger than usual. In general, it has been mentioned that parameter estimation leads to the degradation in the performance of CSCs. To attack this problem, it is strongly recommended that prediction limits be used and a large sample of data be collected to ensure that the estimates are sufficiently close to their true values.

Sulek *et al.* (2006) examined the CSC as a methodology to monitor and identify potential problem areas in a cascade service process. It confirms that the CSC can be used not only in firms but also in service operations, since many service organizations are implementing six-sigma programs and therefore the need to monitor processes has increased. The analysis demonstrated that using the traditional Shewhart chart instead of the CSC can be misleading and results in erroneous information in cascade processes.

Asadzadeh et.al (2008) proposed a robust monitoring procedure in multistage processes with dependent steps where outliers are present in historical dataset. Two robust CSCs based on M-estimators (Huber and Tukey bisquare functions) were established. Simulations were carried out to investigate the performance of control charts under various shifts. The results indicated that the robust schemes are superior to the non-robust CSC in detecting sustained shifts in either the intercept or the slope. In addition, it is noteworthy that the Bisquare based CSC is more effective than the Huber based CSC by completely ignoring the presence of outliers. Therefore, using Bisquare based CSC makes the two-stage trial control limit procedure dispensable.

Although monitoring the mean response is the primary objective with variance being a nuisance parameter, it is interesting to control the variance at each step of the process. Zeng and Zhou (2008) studied the properties of a regression-adjustment-based method in the monitoring of variation propagation in multistage processes. *S* chart was employed for the purpose of detecting variance changes instead of mean shifts. Moreover, the impacts of measurement errors and regressor selection on the monitoring scheme were investigated. They claim that the diagnostic capability of the monitoring procedure will degrade in the presence of measurement errors. On the other hand, two procedures, namely all-adjustment and subset-adjustment were introduced in order to verify the influence of regressor selection. It was found that the choice between these procedures depends upon the process characteristics and preference about the false alarm rate and misdetection rate. If the former is important, then the subset-adjustment should be seriously considered, whereas if the latter is critical, the all-adjustment procedure should be used.

There are various distinct areas in quality control known as economic design, autocorrelation and adaptive design parameters for control charts. As a result, classification and investigation of the papers in each of these areas are given in the following subsections.

4.1. Design of Cause Selecting Control Charts

The use of a control chart requires that the analyst selects a sample size, a sampling frequency and the control limits for the chart. Selection of these three parameters is usually called the design of the control chart. Control charts can be designed with respect to economic and statistical criteria. The most popular assumption in modeling the economic and statistical designs is that the time between the occurrences of assignable causes follows exponential distribution. However, other distributions

such as Weibull may be employed as well especially when the failure rate is increasing. Therefore, we will discuss exponential and Weibull shock models herein.

Exponential Shock Model

The first research on the economic design of CSC in a two-step process can be found in Yang S.F. (1997). The method of designing the economic CSC and individual X chart simultaneously with two assignable causes occurring independently at each step was proposed. Yang assumes the same search and repair time and cost for both assignable causes to simplify the model. Hence, it is worthwhile to assign different times and costs for each assignable cause to achieve a better and more complete model. The cost model derived by the renewal theory approach entails studying the possible states at the end of the first sampling and testing. The grid search method was applied to obtain the optimum design parameters. It has been shown that the proposed model is effective for simple two-dependent process steps.

A renewal equation approach was proposed to derive the multiple assignable-cause cost model for a system with two dependent subprocesses (Yang C.M., 1999). It is assumed that double assignable causes would occur in the current subprocess and a single assignable cause in the previous one. Thus, the economic model has been improved in comparison to the former model with regard to taking the presence of more assignable causes into account. Having the same cost and time to search and repair any special cause is once again the deficiency of the proposed model. The algorithm used to obtain optimum design parameters is the simple grid search method. An example of its application in the bank industry proves that these control charts may also be used to control service organizations with dependent subprocesses.

Yang S.F. (1998) proposed a model using the renewal theory approach for multiple dependent subprocesses. The process consists of $K+1$ subprocesses. Therefore, it necessitates using MCSCs. It is assumed that an assignable cause can occur in each step. Consequently, there are $K+1$ assignable causes. The deficiencies of the proposed model are: 1) The same cost and time to search and repair the assignable causes for all K subprocesses are contemplated. 2) The quality cost per hour while the process is disturbed by any j assignable causes is equal, while in practice each assignable cause may induce various quality costs to the system. The simple grid search method has been again implemented to find the optimum solution. Although the calculations are more sophisticated, the aforementioned model may be considered as an improvement over the previous models due to adopting MCSC.

Overadjustment to processes may result in shifts in process mean, ultimately affecting the quality of the products. This happens when a control chart gives a false alarm. The problem has been dealt with by Yang S.F. (2005). A Markov chain approach was used to derive the economic-adjustment model and to determine the design parameters so that the average long-term cost of the process was minimized. It is assumed that an assignable cause might occur in the first or the second subprocess but not in both. However, this assumption is not realistic in practice. Yang strongly believes that the expression for the economic-adjustment model is easier to obtain through the proposed approach rather than by those of others.

Another economic adjustment model was developed for the joint design of $\bar{X} - S^2$ control charts and $\bar{e} - S_e^2$ CSCs in order to control both means and variances of the two dependent process steps (Yang C.M. and Yang S.F., 2006). It is worth mentioning that this is the only research concerning the economic design while monitoring both the mean and variance of a process. One failure

mechanism may occur in each subprocess, which shifts the mean and variance of the quality variable. However, an assignable cause does not necessarily shift the mean and variance simultaneously and this assumption was made for simplicity. Also receiving a false alarm in each step will lead to overadjustment and subsequently a shift in the mean and the variance. The cost and time to search and repair have been considered the same for both assignable causes that is not satisfactory. A Markov chain was extended to derive the economic-adjustment model and a simple grid search method was used to yield the optimum design parameters.

Weibull Shock Model

Yang S.F. (2003) considered the economic statistical design of a two-step process with an assignable cause, which obeys the Weibull shock model and has an increasing failure rate. In this case, the time between the occurrences of assignable causes follows the Weibull distribution as below:

$$f(t) = \lambda \theta t^{\theta-1} \exp\{-\lambda t^{\theta}\} \qquad t > 0, \theta \geq 1, \lambda > 0 \tag{8}$$

To avoid the difficulty of estimating cost parameters, the asymmetric quadratic loss function was used. The renewal theorem approach was again extended to construct the cost model by defining six states at the end of the first sampling, given that an assignable cause might occur in only one of the two subprocesses with a special probability. In addition, due to poor statistical properties of control charts based on optimal economic designs, some statistical constraints have been introduced to overcome that problem. According to Yang S.F. (2003), the optimum values have both economic and statistical properties and therefore are of great importance. Results of sensitivity analysis revealed that increasing Weibull parameters leads to reduction in control limit coefficients (k) and sampling intervals (h) but results in an increase in the expected cost per unit time.

Yang S.F. and Chen Y.C. (2003) extended the previous research to present the economic statistical model for two-dependent process steps with two failure mechanisms. There are two assignable causes occurring independently at each step which is the only superiority of the proposed model over the former one. The asymmetric quadratic loss function has been implemented and eight states have been defined at the end of the first sampling to make use of the renewal theorem. It is recommended that statistical constraints be added to the model in order to tackle the problem of the weak statistical property of economic design.

4.2. A Process with Autocorrelated Observations

The observations from the process output are always assumed independent when using a control chart to monitor a process. However, for many processes including the chemical and pharmaceutical ones, observations are autocorrelated. This subsection provides a review of several procedures proposed to remedy the autocorrelation issue.

A method for monitoring dependent processes based on regression adjustment in the presence of both autocorrelation and cross-correlation was presented in a paper by Loredo et al. (2002). The first-order autoregressive model has been implemented to explain autocorrelation in a special variable. Each output variable is regressed on the input variables and residuals are obtained from the model. Monte Carlo simulations, performed to compare the performance of residual based and observation-based control charts, indicate the strong negative effect of autocorrelation on the ARL performance of observation-based control charts. Traditional Shewhart CSC, EWMA CSC and

CUSUM CSC, were used to investigate the out-of-control ARL performance by shifting the mean of one of the input variables. Investigation results demonstrate that the CUSUM and EWMA outperform the Shewhart charts for residual based control charts when the process mean has small shifts, and the performance of CUSUM is slightly better than EWMA in detecting small shifts. However, they made a great mistake. They assumed the cascade property in the process and used the model based control chart proposed by Hawkins (1993) and Zhang (1984). But it has been proved that the CSC does not detect shifts in the previous subprocess variables (Wade and Woodall, 1993). The authors overlooked this point which is in fact the special property of the CSC.

Shu and Tsung (2003) investigated the effect of autocorrelated data in the current stage of a multistage process. In their opinion, when data are collected at a high frequency, the disturbance (random error) to the current process step is more likely to be autocorrelated. Consequently, the output variable is autocorrelated. An autoregressive integrated moving average (ARIMA) model has been used to describe the autocorrelation. Uncorrelated residuals are obtained by inverting the process with an inverse filter. Due to the time-varying property of the mean shift in the residuals, which is referred to as fault signature, the Cumulative Score (CUSCORE) chart and the Triggered Cuscore chart have been used to take advantage of the dynamic information in the fault signature. It has been demonstrated that the Triggered Cuscore chart outperforms the Cuscore chart and the residual-based CUSUM chart.

The problem of monitoring a two-step process in which the observations X in the first step can be modeled as an AR(1) model and observations in the second step Y can be modeled as a transfer function of X, has been referred to in Yang S.F. and Yang C.M. (2006). The AR(1) model used, can be shown as:

$$X_t = (1-\phi)\xi_X + \phi X_{t-1} + a_t \qquad t = 1, 2, \dots \tag{9}$$

where ϕ is the AR parameter satisfying $\phi < 1$ and a_t s are assumed to be independent normal random variables with mean zero and variance σ_a^2. They point out that the expectation of a residual, after a shift occurs, is a decreasing function of time. Hence, the rate of a true alarm on residuals control chart in the first step is the highest for the sample immediately after the shift. In addition, the transfer function to express the relationship between X, Y is:

$$Y_t = C_Y + V_0 X_t + V_1 X_{t-1} + N_t \qquad t = 1, 2, \dots \tag{10}$$

where C_Y is a constant and N_t s are independent normal random variables with mean zero and variance σ_N^2. It has been mentioned that the expectation of residuals can be valued only at the time of the first sampling after the mean shift. The performance of the proposed control charts was measured using the rate of false or true alarm. The proposed control charts showed better performance than using the two individual X and Y control charts and the Hotelling T^2 control chart for a two-stage process in the presence of autocorrelated observations.

The performance of control charts and other statistical process control tools could be seriously affected when the process measurement includes the errors due to the measurement instrument (Yang S.F. and Yang C.M. 2005). The existence of measurement errors has been highlighted in the literature. However, the influence of these errors on the multistage process monitoring has not been explored. Yang and Yang modeled a two-step process with autocorrelated observations, including

measurement errors. The observations X on the first step can be modeled as ARMA(1,1) model which is actually the AR(1) model with measurement errors:

$$X_t = (1-\phi)\xi_X + \phi X_{t-1} + \gamma_t - \theta\gamma_{t-1} \quad t = 1,2,... \tag{11}$$

where θ is the MA parameter ϕ is AR parameter and γ_t s are normal and independent random variables with mean zero. In their opinion, the detection ability of the chart in the first step is the lowest after a failure mechanism occurs but increases and converges to a constant over time. This is contradictory to the fact that the expectation of a residual is a decreasing function of the time after the shift. A transfer function used to relate the two dependent variables in the two steps is:

$$Y_t = C_Y + V_0 X_t + V_1 X_{t-1} + N_t + \varepsilon_{Y_t} \quad t = 1,2,... \tag{12}$$

where N_t are independent normal random variables with mean zero and ε_{Y_t} are independent normal random variables with mean zero and variance $\sigma_{Y_t}^2$. The detection ability of the proposed control chart under diverse variations of measurement errors was evaluated. It indicated that imprecise measurement can seriously deplete the ability of the proposed control charts to detect process disturbances quickly.

4.3. Adaptive Cause Selecting Control Charts

Traditional SPC techniques usually employ samples of fixed size taken at a fixed sampling interval and chart them in a control chart with fixed control limits. However, the performance of control charts for detecting small to moderate shifts can be improved by varying one or some of the parameters mentioned above. A control chart with such characteristics is called an adaptive control chart.

Yang S. F. and Su (2006) constructed variable sample size (VSS) CSC in a two-step dependent process. Warning limits are obtained by equalizing the in-control average sample size for fixed sample size (FSS) and VSS schemes. To facilitate the computation, both the warning limits and control limits of the Shewhart $\bar{X}$ control chart and CSC have been considered the same, implying $k_{\bar{X}} = k_{\bar{e}} = k$ and $w_{\bar{X}} = w_{\bar{e}} = w$, which can be contemplated as one of the deficiencies of the model. The performance of the proposed VSS control charts is measured by the adjusted average time to signal (AATS) using a Markov chain approach:

$$AATS = ATC - \frac{1}{\lambda_1 + \lambda_2} \tag{13}$$

AATS is the mean time when the process remains out of control and ATC is the average time from the start of the process until one of the charts generates the first signal. Because each of the assignable causes occurring at each step follows exponential distribution with parameter λ_i, therefore the distribution of the occurrence of the first assignable cause will also be exponential with parameter $\lambda_1 + \lambda_2$. According to the results gained, the proposed VSS scheme improves the performance of the FSS scheme in detecting small shifts ($\delta_1 \leq 1$, $\delta_2 \leq 1$) in the mean of each subprocess.

Another adaptive control chart with variable sampling interval (VSI) for a two-step process was proposed in Yang S. F. and Su (2007a). The AATS derived by a Markov chain approach, has been implemented again to measure the performance of the proposed VSI control charts. The warning and control limits for both the Shewhart X chart and the CSC are assumed equal to simplify the model. It is demonstrated that using VSI control charts can decrease the AATS under small and moderate shifts ($\delta_1 \leq 1.5$, $\delta_2 \leq 1.5$) in each process step in comparison to the FSI control charts.

The aforementioned papers were explored to make use of both VSS and VSI properties which is deemed to be more fruitful (Yang S. F. and Su, 2007b). An adaptive sample size and sampling interval (ASSI) control chart for dependent process steps was constructed. Equal warning and control limits are assumed for both control charts. AATS is computed using Markov chain with the aid of a transition probability matrix. The results confirm that using proposed control charts with VSSI is preferred due to the rapid detection of small shifts ($\delta_1 \leq 0.75$, $\delta_2 \leq 0.75$) in the mean of each process step.

Yang S. F. and Yu (2007) extended the research by Yang S. F. and Su (2007a) to a case with incorrect adjustment. EWMA control charts are used instead of Shewhart control charts owing to the faster detection properties under small shifts. Although some research has been done using VSI EWMA control charts, this is the first paper dealing with VSI EWMA control charts in multistage processes. Not only the warning and control limits have been considered the same, but also the EWMA parameter (λ) has been assumed to take the same value for both control charts in two process steps. Comparing the AATS illustrates that the VSI EWMA control charts outperform the FSI   EWMA control charts in detecting small and moderate shifts of size $0.5 \leq \delta_1 \leq 2$ and $0.5 \leq \delta_2 \leq 2$.

5. MODEL BASED CONTROL CHARTS WHEN THE OUTPUT VARIABLE IS NON-NORMAL

Although all the previous methods are effective, those papers are based on the assumption that the monitored quality characteristics are normally distributed. Generally, quality characteristics do not always follow this assumption. Therefore, in the case of violation, a modification of the model-based control strategy using a generalized linear modeling (GLM) technique is needed. GLMs extend the regression framework to situations where the distribution of the response is a member of the exponential family. The GLM relates the mean of the response variable (μ), to the linear combination of the input variables, through a link function (g):

$$\mathbf{X}\boldsymbol{\beta} = g(\mu)$$

$$\mu = E(Y) = g^{-1}(\mathbf{X}\boldsymbol{\beta}) \tag{14}$$

An iterative procedure is used to obtain maximum likelihood estimates of the regression coefficients. The rest of this paper is focused on the use of GLM techniques in multistage process monitoring and diagnosing.

Monitoring multiple gamma-distributed response variables was studied by Jearkpaporn et al. (2003). The procedure is based on the deviance residual, which is proved to be a likelihood ratio statistic for detecting a mean shift due to a change in the scale parameter while the shape parameter is assumed to be unchanged:

$$r = sign \left[2\left\{ -\ln\left(\frac{y_i}{\hat{\mu}_{y_i}}\right) - 1 + \frac{y_i}{\hat{\mu}_{y_i}} \right\} \right]^{\frac{1}{2}} \tag{15}$$

where y_i is a new observed value and $\hat{\mu}_{y_i}$ is the fitted value from a fitted GLM. The "sign" function takes on the value of +1 if $y_i \geq \hat{\mu}_{y_i}$, and -1 otherwise. The distribution of the deviance residuals is approximately normal and as a result can be easily plotted on a traditional Shewhart-type control chart. Average run length (ARL) was used to compare the detection ability of the deviance residual control chart to the individual control chart under an additive shift and a multiplicative shift as a change in any regression coefficients. A univariate case with one output and a bivariate case with two correlated output variables were studied. It is claimed that the greatest improvement in ARL performance at 97% when an additive shift occurs and the least improvement at 22% when a multiplicative shift occurs, can be achieved in a univariate case. The results in a bivariate case also confirm that the individual control charts are inferior to the deviance residual based control charts. However, there is a drawback to the proposed model which is worthy of attention. The deviance residual is a likelihood statistic only if a mean shift due to a change in the scale parameter is assumed, not a shape parameter.

Skinner et al. (2003) discussed the monitoring of the Poisson distributed output variable. A log link was used as a link function to express the relationship between input variables and response mean. Deviance residuals have been obtained from the generalized likelihood ratio (GLR) test and a control chart based on these residuals has been established. ARL performance of the deviance residual control chart and 'C' chart was studied under three scenarios: univariate, bivariate with equal means and bivariate with unequal means. The results show that the deviance residual from GLM outperforms univariate and bivariate 'C' charts. Although detecting multiplicative shifts is slightly more difficult than additive shifts in the mean, the authors strongly recommend using the deviance residual chart due to its quick detection.

Count data modeled by Poisson distribution, frequently exhibit more variation than is expected. This problem has been referred to as overdispersion (Skinner et al., 2004). Overdispersion is often a result of a varying input parameter. It has been discussed that the effectiveness of the GLM-based procedure depends on the amount of overdispersion and the type of the shift. The best performance is obtained when shift is additive and the counts are overdispersed. This is not surprising since the GLM effectively removes the impact that input variables have on response variables, thus eliminating the overdispersion from the control statistics in CSCs. Although a common adjustment of using historical variance rather than theoretical variance can be made to the 'C' chart, Skinner *et al*. suggest using the GLM-based chart in detecting a change in a process when the process has input parameters that cause overdispersion.

The main assumption in the papers discussed so far in this section is that historical data used to establish the GLM do not contain outliers. However, historical data often contain observations that reflect process upsets and have a substantial impact on the initial model as they pull the regression fit in their direction (Jearkpaporn et al., 2005). To dampen the detrimental effect of outliers, a robust GLM was used in monitoring the multistage process. Their robust fitting technique uses the Huber's M-estimator approach. The deviance residuals are computed and plotted on a Shewhart control chart for individuals. Moreover, due to the presence of outliers, control limits should be calculated robustly as well. The median absolute deviation (MAD) from the sample median proved to be the best estimate for the standard deviation and thus was used to construct robust control limits. In

addition, control limits were constructed using the two-step iterative trial control limit procedure described in Montgomery (2001). In their opinion, it is prudent to use a robust GLM with a two-stage MAD scale estimate to detect a change in mean when the historical data contain unusual observations.

In some multistage processes, there may exist combinations of normal and non-normal response variables. The procedure for monitoring such processes with three stages was presented in Jearkpaporn et al. (2007). Deviance residuals have been obtained from the GLM model and plotted on control charts. The traditional Shewhart control chart for each response variable and the Hotelling T^2 chart based on the U statistic proposed by Hauck et al. (1999), were also employed to compare the performance of control charts. Three scenarios were defined to assess the detection power of the mentioned control charts. In the first scenario with a gamma-distributed response at each of the three steps, the deviance residual based control chart outperforms the other control charts in terms of ARL. They claim that the poor performance of the Hotelling T^2 control chart is due to the sensitivity of the U statistic to normality assumption. Consequently, when the normality assumption is violated, the Hotelling T^2 chart performs very similarly to the individuals control charts. The simulation results for the second scenario with two correlated gamma distributed responses are the same as the first scenario. However, in the third scenario with three responses with normal, gamma and Poisson distributions, the deviance residual control chart performs better than the others, except in the case where the shift occurs in the normally distributed response where T^2 chart performs as well as the deviance residual-based control chart. The deviance residual based control chart is strongly recommended for monitoring multistage processes with a mixture of normally and non-normally distributed responses.

6. A NEW APPROACH IN SIMULATION

Wade and Woodall (1993) introduced an approach to model and simulate a multistage process. They generated a bivariate normal sample of size *n* with specified mean and covariance matrix to relate the two quality variables in two dependent steps. This procedure has been widely adopted in the literature (see, for example, Shu et al., 2004 and Shu et al., 2005). The problem arises when sensitivity analysis is conducted to investigate the effect of correlation on the detection power of the CSCs. Because the linear relationship exists between *X* and *Y*, and the slope and the intercept of the true regression line contain the correlation parameter (ρ), the regression line differs drastically. Therefore, it seems as if one analyzes the effect of the correlation on different processes with various relationships between the quality characteristics; not the same process with a fixed regression line, but different correlation coefficients. To elaborate on the simulation approach, consider:

$$\mu_X = 210, \mu_Y = 201, \sigma_X = 1, \sigma_Y = 1, \rho = 0.1, 0.5, 0.9$$

Hence, the true regression lines will be:

$$\rho = 0.1 \to Y = 0.1X + 189, \quad \rho = 0.5 \to Y = 0.5X + 96, \quad \rho = 0.9 \to Y = 0.9X + 12$$

Because:

$$E(Y \mid x) = \mu_Y + \rho\left(\frac{\sigma_Y}{\sigma_X}\right)(x - \mu_X) \tag{16}$$

As a result, the standard deviation of the outgoing quality characteristic remains unchanged whereas the regression line and the standard deviation of error terms change to meet the specified correlation. It is noteworthy that the standard deviation of the error term alters slightly according to the formula below:

$$\sigma_{\varepsilon} = \sqrt{\sigma_Y^{\,2}(1-\rho^{\,2})} \tag{17}$$

On the other hand, we suggest using another method to perform the simulation that is more realistic. The proposed method is closer to real situations since there exists a fixed linear relationship between quality characteristics in multistage processes. Consequently, even assessing the effects of correlation on the performance of CSCs should not vary the parameters of the regression line. In the proposed approach, *X* is generated in advance (which is exactly what happens in multistage processes) and it will be transformed by a fixed regression line to yield *Y*. The difference between the two approaches is that in the latter model, one must not specify the standard deviation of *Y* beforehand and in order to achieve the desirable correlation coefficient, the standard deviation of error terms in the regression line should be changed. For instance, consider the regression line $Y = 0.9X + 12$. Apparently, reaching a small correlation coefficient between quality characteristics entails increasing the variation of points around the regression line which leads to much wider control limits as against the previous approach. The comparison of these two approaches is given in Table 2.

Assume, it is desirable to investigate the detection power of CSCs using the first approach while a shift of size δ has changed the *Y* intercept of the true regression line. Therefore,

$$\beta_0' = \beta_0 + \delta = \left(\mu_Y - \rho(\frac{\sigma_Y}{\sigma_X})(\mu_X) \right) + \delta$$

This conveys that overlooking the parameter estimation errors, the mean of residuals will shift by the amount δ. Meanwhile, the control limits are calculated through $\pm 3\sqrt{\sigma_Y^{\,2}(1-\rho^{\,2})}$.

Moreover, shifting the intercept (β_0) in the proposed approach by the aforementioned amount (δ) will not yield the same results. To clarify and support our claim, the simulation is performed based on Wade and Woodall (1993) and the proposed approach, in order to determine if one method of establishing control limits is better in detecting shifts in $E(Y\,|\,x)$ from a change in β_0. The simulation is repeated 1000 times and the prediction limits were calculated using an overall false-alarm rate of 0.006 so that the in-control false alarm rate is approximately the same as for Zhang's limits. Table 3 shows the results of both Zhang's limits and the prediction limits.

One can easily observe that the prediction limits perform slightly better than Zhang's limits. However, comparing the alarm rates obtained from implementing different approaches while $\rho = 0.1$ or $\rho = 0.5$, indicates distinct outcomes. The results are not extraordinary, since in the proposed simulation procedure, the standard deviation of regression line is very large compared to the previous approach and the established control limits will be too wide ($\pm 3\sqrt{\sigma_{\varepsilon}^{\,2}}$). Therefore, the shift of size δ can not induce the same effect. In order to equalize the results, a shift of size δ' may be applied in such a way that the equation (18) is satisfied:

Table 2. The schematic comparison of the two approaches for simulating a two-step process

	Previous Approach	Proposed Approach
μ_X	✓	✓
μ_Y	✓	$\beta_1 \mu_X + \beta_0$
σ_X	✓	✓
σ_Y	✓	$\sqrt{\beta_1^2 \sigma_X^2 + \sigma_\varepsilon^2}$
ρ	✓	$\dfrac{\beta_1 \sigma_X^2}{\sqrt{\sigma_X^2 (\beta_1^2 \sigma_X^2 + \sigma_\varepsilon^2)}}$
σ_ε	$\sqrt{\sigma_Y^2 (1-\rho^2)}$	✓
β_1	$\rho(\dfrac{\sigma_Y}{\sigma_X})$	✓
β_0	$\mu_Y - \rho(\dfrac{\sigma_Y}{\sigma_X})(\mu_X)$	✓

Note: ✓ indicates that the parameter should be specified prior to the simulation

$$\frac{\delta}{\sqrt{\sigma_Y^2 (1-\rho^2)}} = \frac{\delta'}{\sqrt{\sigma_\varepsilon^2}} \tag{18}$$

Defining σ_ε to reach the same correlation coefficient in the proposed approach and shifting the intercept of true regression line by δ' will lead to similar gains. Suppose, for instance, shifts are made in units of Y standard deviation ($k \sigma_Y$) while applying the first approach. Subsequently, employing the foregoing formula gives:

$$\delta' = \frac{\delta}{\sqrt{\sigma_Y^2 (1-\rho^2)}} \sqrt{\sigma_\varepsilon^2} = \frac{k \sigma_Y}{\sqrt{\sigma_Y^2 (1-\rho^2)}} \sqrt{\sigma_\varepsilon^2} = k \sqrt{\frac{\sigma_\varepsilon^2}{1 - (\frac{\beta_1 \sigma_X^2}{\sqrt{\sigma_X^2 (\beta_1^2 \sigma_X^2 + \sigma_\varepsilon^2)}})^2}} =$$

$$k \sqrt{\beta_1^2 \sigma_X^2 + \sigma_\varepsilon^2}$$

which are in fact the units of Y standard deviation, achieved from the proposed approach.

Due to the facts that the proposed policy does not require the parameters of the outgoing quality characteristic, and that the true regression line remains fixed, it is strongly recommended that one performs a simulation of a multistage process in this way, which can prove to be more reasonable compared to the conventional approach.

Table3. Alarm rates for shifts in $E(Y \mid x)$ for *n*=50 using the two approaches

	Previous Approach				Proposed Approach			
	Prediction Limits		Zhang's Limits		Prediction Limits		Zhang's Limits	
δ	Overall Rate	Std error	Overall Rate	Std error	Overall Rate	Std error	Overall Rate	Std error
	correlation=0.9							
0	0.0064	0.00020	0.0064	0.00024	0.0064	0.00020	0.0064	0.00024
0.5	0.0482	0.00103	0.046	0.00121	0.0482	0.00103	0.046	0.00121
1	0.2809	0.00320	0.2708	0.00383	0.2809	0.00320	0.2708	0.00383
1.5	0.6912	0.00334	0.6674	0.00429	0.6912	0.00334	0.6674	0.00429
2	0.9441	0.00111	0.9314	0.00165	0.9441	0.00111	0.9314	0.00165
2.5	0.9962	0.00014	0.9945	0.00024	0.9962	0.00014	0.9945	0.00024
3	0.9999	0.00001	0.9998	0.00002	0.9999	0.00001	0.9998	0.00002
	correlation=0.5							
0	0.0064	0.00018	0.0065	0.00025	0.0062	0.00018	0.0064	0.00024
0.5	0.0148	0.00038	0.0147	0.00047	0.0082	0.00022	0.0081	0.00028
1	0.0504	0.00106	0.0496	0.00128	0.0174	0.00047	0.0173	0.00056
1.5	0.1375	0.00210	0.1322	0.00253	0.0332	0.00076	0.0324	0.00090
2	0.2846	0.00324	0.2664	0.00381	0.0635	0.00130	0.0616	0.00152
2.5	0.4975	0.00380	0.4712	0.00459	0.1098	0.00193	0.104	0.00225
3	0.7005	0.00331	0.6782	0.00421	0.1782	0.00262	0.1726	0.00310
	correlation=0.1							
0	0.0064	0.00020	0.0064	0.00025	0.0063	0.00019	0.0063	0.00024
0.5	0.0123	0.00034	0.012	0.00042	0.0062	0.00019	0.0061	0.00024
1	0.0354	0.00078	0.0346	0.00093	0.0063	0.00020	0.0064	0.00024
1.5	0.0949	0.00167	0.0927	0.00200	0.0069	0.00021	0.0072	0.00027
2	0.2022	0.00263	0.1924	0.00319	0.0072	0.00022	0.0074	0.00029
2.5	0.3564	0.00354	0.333	0.00426	0.0083	0.00024	0.0082	0.00030
3	0.5412	0.00409	0.5226	0.00499	0.0088	0.00025	0.0085	0.00030

7. DIRECTIONS FOR FUTURE RESEARCH

There are many areas in the field of multistage processes and CSCs worthy of continued research efforts. We briefly discuss some important and interesting issues hereafter. There continues to be a need for research in monitoring the multiple-input, multiple-output case using prediction limits to account for parameter uncertainties. In this case, it is necessary to simultaneously monitor the output measures adjusted for the effects of input measures. The performance comparison of the Hotelling T^2 chart using different diagnostic procedures with individual residual based control charts would be of considerable interest and practical value.

The work of Sulek et al. (2006) can be extended to the modeling of service processes as multistage systems with multiple performance indicators at each stage. Furthermore, developing a model with *n* (*n*>2) subprocesses can be done. Examining service processes where the statistical relationship between successive stages is nonlinear would prove interesting as well, since no work has been done using nonlinear relationship in CSCs.

The economic design of CSC with normally distributed outputs for a two-step process was discussed. However, the proposed methods can be extended to the case of multiple special causes

occurring in both the current and previous subprocesses. Another area for future research involves the economic modeling of a CSC subject to adaptive sampling time and/or sampling size.

An approach to controlling autocorrelated observations with CSC in multistage processes can be usefully extended to study other control charts for detecting small shifts like EWMA, CUSUM, or charts for attributes. An adaptive process control for correlated observations with ARMA model under two dependent process steps may be considered as another potentially useful procedure to detect shifts more quickly.

Adaptive control charts have received considerable attention from the outset. The VSS, VSI and VSSI CSCs have been studied but no research has been done for AP (adaptive parameter) CSC in which all three parameters of sample size, sampling interval and control limits vary due to the position of the previous sample statistic. Extension of the previous models discussed in this paper, in order to study adaptive EWMA or CUSUM CSC in multistage processes with multiple assignable causes would also be of considerable interest. In addition, assuming equal warning and control limits for the Shewhart chart and the CSC in a two-step process is not realistic. Therefore, much research is needed indeed to violate these assumptions, made for simplicity, and develop a more practical model.

Although the effects of unusual observations in historical data have been studied in normal and gamma distributed responses, no work has been done when the distribution of the output variable follows other members of the exponential family. Moreover, multistage process monitoring in the presence of both *X* and *Y* space outliers warrants future research since in real practice all variables are prone to hold outliers.

Another point worth mentioning is that the detrimental influences of measurement errors in regression-adjusted methods can be optimally reduced if the ordinary least squares regression is replaced with an Error-In-Variables modelling technique. It is expected to obtain consistent estimates of regression parameters and thus the impact of measurement error is minimized.

A large variety of the literature considers regression-adjustment-based method in order to monitor the mean of quality characteristics. However, its properties in the monitoring of variability are not comprehensively studied and this issue has been neglected to a large extent. Hence, another area of possible research might be to see how the variation is passed on through the process especially when quality variables follow distributions in the exponential family.

The concentration of the literature review given in this paper was on the methods which adopt statistical models, mainly the linear regression models, for describing multistage processes. However, it is essential to incorporate engineering knowledge in multistage modeling and analysis for more effective process control. Resent research on multistage process has adopted engineering models in a linear state space model structure based on physical laws and engineering knowledge that characterize the quality information. Xiang and Tsung (2008) used the state space model to describe a complex multistage monitoring problem. They assumed that fault merely occurs in one stage and applied group exponentially weighted moving average (GEWMA) charts to the one-step forecast errors of the model of each stage. In addition, the EM algorithm was employed to obtain maximum likelihood estimates of the model parameters since the assumption of known parameters value may be not valid in real practice. A bibliography and comprehensive references of the engineering-based state space model have been provided by Tsung et.al (2008). Therefore, further research is needed to address the newly developed method for monitoring manufacturing and

service operations. For instance, how to determine the transition matrix and variances in state space model in service processes has remained an open field that needs additional exploration.

8. CONCLUSION

This paper has reviewed the state of the art control chart entitled cause selecting chart (CSC) for monitoring multistage processes when stages are dependent. Cause selecting charts and multiple cause selecting charts were overviewed and classification of the papers was considered according to the distribution of the outgoing quality characteristics. In the first classification with normally distributed outgoing quality characteristics, crucial issues such as economic design, autocorrelation and adaptive parameters were discussed and analyzed. It has been proved that CSCs can effectively distinguish a special cause occurs in the process and thus provides diagnostic information regarding which subprocess is out of control. Therefore, it is much preferred to Shewhart and Hotelling T^2 control charts and highly recommended. In the second classification, with non-normally distributed outgoing quality characteristics, generalized linear model (GLM) was implemented to relate gamma and/or Poisson-distributed variables to incoming variables and problems such as overdispersion and unusual observations with their remedial measures were completely addressed. Different aspects and applications of CSCs were discussed and a new approach for performing the simulation in a multistage process was proposed by considering a fixed regression line and changing the standard deviation of the error terms to attain a desirable correlation coefficient. Since it is more realistic than the traditional simulation method, it is strongly recommended that the proposed approach is implemented in simulation. Finally, diverse areas for future research were given which can lead to growing interest in multistage quality control as one of the most crucial areas in statistical process control.

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