Journal of Industrial and Systems Engineering
Vol. 2, No. 2, pp 144-153
Summer 2008

A Mathematical Model for Cell Formation in CMS Using Sequence Data

Kaveh Fallah-Alipour[1], Ramin Shamsi[2*]

[1]Department of Industrial Engineering, Iran University of Science and Technology, P.C. 16844, Narmak, Tehran, Iran
Kaveh.fap@gmail.com

[2]young researchers club, Islamic Azad University, Tehran, Iran
raminshamsi@hotmail.com

ABSTRACT

Cell formation problem in Cellular Manufacturing System (CMS) design has derived the attention of researchers for more than three decades. However, use of sequence data for cell formation has been the least investigated area. Sequence data provides valuable information about the flow patterns of various jobs in a manufacturing system. This paper presents a new mathematical model to solve a cell formation problem based on sequence data in CMS. The objective is to minimize the total costs of inter and intra-cell movements. This model depends on the attitude of the decision maker towards the minimum utilization level of each cell in such a way that the part-machine grouping can be changed significantly. A number of examples from the literature are solved by the LINGO software package to validate and verify the proposed model. Finally, computational results are reported and analyzed.

Keywords**:** Cell formation, CMS, Intra/inter-cell movement, Mathematical model, Cell utilization

1. INTRODUCTION

Cellular manufacturing is an application of the Group Technology philosophy to designing manufacturing systems. The main idea of Group Technology is to identify similar manufacturing processes and features, where machines are grouped into machine cells based on their contribution to the production process (Mahdavi et al., 2007). One of the first problems encountered in implementing cellular manufacturing is that of cell formation. Over the last three decades, many solution methods such as mathematical programming, heuristics, optimization procedures and clustering techniques have been proposed to address the cell formation problems (Wemmerlov and Hyer, 1986; Singh, 1993; Sarker and Xu, 1998). Most of the cell formation studies have considered independence of cells, in which the number of inter-cell movements is used as an indicator for the cell formation (see for instance Askin and Subramanian, 1987; and Vakharia and Wemmerlov, 1990). In addition, various objectives, such as maximizing utilization, minimizing material handling cost, and minimizing load unbalance, have been employed in assessing the quality of the solution. These approaches can be found in Ballakur and Steudel (1987), Wei and Gaither (1990), Shafer and Rogers (1991), and Lee and Chen (1997).

[*] Corresponding Author

During the last three decades of research in cell formation, researchers have mainly used zero – one machine component incidence matrix as the input data for the problem. However, as of late efforts are being made to use other data structures such as interval data (Harhalakis et al., 1990), and ordinal data consisting of sequence of processing (see for instance, Kiang et al., 1995; Nair and Narendran, 1998; Won and Lee, 2001; and Jayaswal and Adil, 2004). Different data structures provide different set of information and enable the cell designers to use them appropriately in solving the cell formation problem. A zero – one incidence matrix offers the advantages of computational simplicity for solving the cell formation problem. However, it is not possible to address issues pertaining to machine utilization, inter-cell workload, and impact of using multiple machines and layout of machines within each identified cell. On the other hand, use of additional data pertaining to setup time, process time and production volumes enable cell designers to address these issues albeit using a much more complex solution methodology. Research in cellular manufacturing using sequence data has been mainly confined to two areas; one is to identify appropriate similarity measures for the cell formation problem and the other is to develop new algorithms for cell formation using sequence data. Selvam and Balasubramanian (1985) reported use of sequence data for cell formation using a heuristic procedure based on set covering technique to identify cells. Choobineh (1988) proposed new similarity coefficients using sequence data for cell formation. Suresh et al. (1999) utilized fuzzy neural network approach to cell formation using sequence data. Park and Suresh (2003) compared the performance of fuzzy neural network with other clustering methods.

The problem of locating bottleneck machines in a cellular manufacturing system was formulated by Wang and Sarker (2002) as a Quadratic Assignment Problem (QAP). They developed a lower bound for this problem and used a 'bubble search' heuristic for the solution to get impressive empirical results. Zolfaghari and Liang (2002) investigated the performance of simulated annealing (SA), genetic algorithm (GA) and tabu search (TS) method for the machine-parts grouping problem and concluded that SA outperforms both GA and TS and that GA is slightly superior to TS. Defersha and Chen (2006) proposed a comprehensive mathematical model for the design of CMS based on tooling requirements of parts and tooling available on machines. This model incorporated dynamic cell configuration, alternative routing, lot splitting, sequence of operations, multiple units of identical machines, machine capacity, workload balancing among cells, operation cost, cost of subcontracting part processing, tool consumption cost, setup cost, cell size limits, and machine adjacency constraints. Albadawi et al. (2005) proposed a new mathematical approach for forming manufacturing cells. The proposed approach involves two phases. In the first phase, machine cells are identified by applying factor analysis to the matrix of similarity coefficients. In the second phase, an integer-programming model is used to assign parts to the identified machine cells.

Mahdavi et al. (2007) proposed a new mathematical model for cell formation in cellular manufacturing system (CMS) based on cell utilization concept. The objective of the model is to minimize the exceptional elements (EE) and number of voids in cells to achieve a higher performance of cell utilization.

Cell utilization, inter- and intra-cell movements according to sequenced data, and different cost rates for inter- and intra-cell movements are the main themes of this paper. So, not only is the aim of this paper to present a mathematical model based on utilization for sequenced data, but also to minimize the total costs of inter- and intra-cell movements in CMS using sequence data and solving some problems from literature by Lingo 8.0 software. Due to different minimum utilization levels of each cell which are set by the decision maker, the proposed model presents different scenarios of part-machine grouping.

2. PROBLEM FORMULATION

In this section, the mathematical model based on sequence data in CMS is formulated. This proposed model deals with the minimization of the integrated inter and intra-cell movement cost function.

2.1. Indexing sets

i index for parts (i =1, 2,…, P)

j index for machines (j =1,2...,M)

k index for cells (k =1, 2,…, C)

s index for operations (s =1, 2,…,OP)

2.2. Parameters

$\gamma^{inter-cell}$: Material handling cost between cells.

γ^{intra_cell} :Material handling cost within cells.

$$a_{isj} = \begin{cases} 1 & \text{if } s^{\text{th}} \text{ operation of part } i \text{ needs machine } j, \\ 0 & \text{otherwise.} \end{cases}$$

Min_ut_k: minimum utilization of cell k.

L_k: lower bound on the number of machines in cell k.

U_k: Upper bound on the number of machines in cell k.

N_j : number of machines of type 'j' available for allotment to cells**.**

f_i: number of operations of part i.

$$r_{ij} = \begin{cases} 1 & \text{if part } i \text{ needs machine } j, \\ 0 & \text{otherwise.} \end{cases}$$

2.3. Decision Variables

$$X_{isk} = \begin{cases} 1 & \text{if } s^{th} \text{ operation of part } i \text{ is assigned to cell } k, \\ 0 & \text{otherwise.} \end{cases}$$

$$Y_{jk} = \begin{cases} 1 & \text{if machine } j \text{ is assigned to cell } k, \\ 0 & \text{otherwise.} \end{cases}$$

$$Z_{ik} = \begin{cases} 1 & \text{if part } i \text{ is assigned to cell } k, \\ 0 & \text{otherwise.} \end{cases}$$

2.4. Mathematical Model

2.4.1. Objective Function

We propose the objective function as

$$\text{Min Z} = \gamma^{\text{inter-cell}} \times \left[\sum_{i}^{P} \left(f_i - 1 \right) - \left(\sum_{i}^{P} \sum_{k}^{C} \sum_{s}^{OP} X_{isk} X_{i,s+1,k} \right) \right] + \gamma^{\text{intra-cell}} \times \left(\sum_{i}^{P} \sum_{k}^{C} \sum_{s}^{OP} X_{isk} X_{i,s+1,k} \right) \tag{1}$$

The first term of the objective function accounts for cost inclusion of parts between cells in which f_i shows the total number of operations of i^{th} part and the second term accounts for cost inclusion of parts within cells.

2.4.2. Constraints

Number of machines to be allocated to cells:

$$\sum_{j=1}^{M} Y_{jk} \geq L_k \qquad \forall k \tag{2}$$

$$\sum_{j=1}^{M} Y_{jk} \leq U_k \qquad \forall k \tag{3}$$

Each machine must be allocated at least to one cell:

$$\sum_{k=1}^{C} Y_{jk} \geq 1 \qquad \forall j \tag{4}$$

Limit for number of machines available for a given type:

$$\sum_{k=1}^{C} Y_{jk} \leq N_j \qquad \forall j \tag{5}$$

Each operation of each part must be allocated to exactly one cell:

$$\sum_{\text{k}=1}^{\text{C}} X_{isk} = 1 \qquad \forall i, s \tag{6}$$

For $a_{isj} \neq 0$, s^{th} operation of part i can be allocated to cell k when its required machine has been allocated to cell k (Y_{jk} =1):

$$\sum_{k=1}^{C} X_{isk} Y_{jk} = a_{isj} \quad \text{if} \quad a_{isj} \neq 0 \qquad \forall i, s, j \tag{7}$$

Each part must be assigned to one cell:

$$\sum_{k=1}^{C} Z_{ik} = 1 \qquad \forall i \tag{8}$$

We could specify minimum utilization of cells to be achieved as follows:

$$\sum_{i=1}^{P} \sum_{j=1}^{M} Z_{ik} Y_{jk} r_{ij} \geq \min\_ut_k \times \sum_{i=1}^{P} \sum_{j=1}^{M} Z_{ik} Y_{jk} \qquad \forall k \tag{9}$$

In the above constraint, r_{ij} is a binary parameter which indicates the relationship between i^{th} part and j^{th} machine (r_{ij} is equal to 1; if $\sum_{s=1}^{OP} a_{isj} \geq 1$ and 0 otherwise).

Constraints (10), (11) and (12) are to allocate each part to one cell, according to the maximum number of operations of that part to be performed in that cell:

$$f_m(i,k) = \sum_{s=1}^{OP} X_{isk} \qquad \forall i, k \tag{10}$$

$$f_{\max}(i) = \underset{\forall k}{Max}\{f_m(i,k)\} \qquad \forall i \tag{11}$$

$$Z_{ik} \leq \frac{f_m(i,k)}{f_{\max}(i)} \qquad \forall i, k \tag{12}$$

$$X_{isk}, Z_{ik}, Y_{jk} \in \{0,1\} \qquad \forall i, j, s, k \tag{13}$$

3. COMPUTATIONAL RESULTS

To validate and verify the proposed model, a number of test problems from the literature are solved by a branch-and-bound (B&B) approach when various values of cell utilization level are defined by the decision maker.

Table 1 shows the sequence data pertaining to the problem consisting of 5 machines and 7 parts. Tables 2 and 3 display the results for a small instance from literature (Sing and Rajamani, 1996) using Lingo software package. The problem is solved by different cell utilization levels as (0.4, 0.4) and (0.4, 1), and the results of cell utilization are shown in Tables 2 and 3 respectively.

Table 1. Sing and Rajamani (1996) – 5 x 7 machines- part matrix

		Machines				
		1	2	3	4	5
Parts	1	0	0	2	1	0
	2	1	0	2	0	0
	3	0	2	0	1	3
	4	2	0	3	0	1
	5	0	1	0	0	2
	6	0	0	0	1	2
	7	1	0	2	0	0

Table 2. The cell formation with $min\_ut_1 = 0.4$, $min\_ut_2 = 0.4$.

		Machines					
		1	3	4	2	4	5
Parts	1	0	1	1	0	0	0
	2	1	1	0	0	0	0
	4	1	1	0	0	0	1
	7	1	1	0	0	0	0
	3	0	0	0	1	1	1
	5	0	0	0	1	1	0
	6	0	0	0	0	1	1

Table 3. The cell formation with $min\_ut_1 = 0.4$, $min\_ut_2 = 1$.

		Machines				
		2	4	5	1	3
Parts	1	0	1	0	0	1
	3	1	1	1	0	0
	5	1	0	1	0	0
	6	0	1	1	0	0
	2	0	0	0	1	1
	4	0	0	1	1	1
	7	0	0	0	1	1

As it can be seen in Tables 2 and 3, part-family groups are changed to meet minimum cell utilization. In the second run (see Table 3), part-family groups prevent formations with more machines of type 4.

To validate the proposed model to solve larger problems, it is used to solve larger size of problems from the literature (Nair and Narendran, 1998) as shown in Table 4. Table 4 shows the sequence data pertaining to the problem consisting of 8 machines and 20 parts.

Table 4. Nair and Narendran (1996) – 8 x 20 machine-part matrix

		Parts																			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Machines	1	0	1	2	0	0	0	0	1	1	0	3	0	1	1	0	1	3	0	1	0
	2	0	0	1	1	0	1	4	0	0	0	0	0	0	2	0	0	0	2	0	2
	3	0	2	0	0	0	0	0	2	3	0	2	0	2	3	0	2	1	0	2	0
	4	0	0	5	2	0	2	2	0	0	2	0	0	0	0	1	0	0	1	0	1
	5	2	0	0	0	2	5	0	0	0	3	0	1	0	0	2	0	2	0	0	0
	6	1	0	0	0	1	0	0	0	2	1	0	3	0	0	0	0	0	0	0	3
	7	0	0	3	3	0	3	3	0	0	0	1	2	0	0	0	0	0	4	0	4
	8	0	0	4	4	0	4	1	0	0	0	0	0	0	0	0	0	0	3	0	5

This problem is solved by different cell utilization levels (0.2, 0.2, 0.2), (0.6, 0.6, 0.6) and (0.9, 0.9, 0.9) for cells as shown in Tables 5, 6, and 7, respectively.

Table 5. The cell formation with $min\_ut_1 = 0.2$, $min\_ut_2 = 0.2$, $min\_ut_3 = 0.2$

		Parts																			
		2	8	9	11	13	14	16	17	19	3	4	6	7	12	18	20	15	1	5	10
Machines	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	3	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
	6	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0	0	1	1	1	1	0	1	1	0	0	0	0
	4	0	0	0	0	0	0	0	0	0	1	1	1	1	0	1	1	0	0	0	0
	7	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0
	8	0	0	0	0	0	0	0	0	0	1	1	1	1	0	1	1	0	0	0	0
	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
	5	0	0	0	0	0	0	0	1	0	0	0	1	0	1	0	0	1	1	1	1
	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1
	7	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 6. The cell formation with $min\_ut_1 = 0.6$, $min\_ut_2 = 0.6$, $min\_ut_3 = 0.6$

		Parts																			
		9	11	12	16	19	2	8	13	14	17	1	3	4	5	6	7	10	15	18	20
Machines	1	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	3	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	6	1	0	1	0	0	0	0	0	0	0	1	0	0	1	0	0	1	0	0	1
	7	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1	0	0	0	0	0	1	1	1	1	1	0	1	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0	1	0	0	1	1	0	1	1	0	0	1	0
	3	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	1	1	1	1	1
	5	0	0	1	0	0	0	0	0	0	0	1	0	0	1	1	0	1	1	0	0
	7	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	1	0	0	1	1
	8	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	1	0	0	1	1

Table 7. The cell formation with $min\_ut_1 = 0.9$, $min\_ut_2 = 0.9$, $min\_ut_3 = 0.9$

		Parts																			
		2	8	9	11	13	14	16	17	19	3	4	6	7	18	20	1	5	10	12	15
Machines	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
	3	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	1	0	0	0	1	1	1	1	1	1	0	0	0	0	0
	4	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	1	0	1
	7	0	0	0	1	0	0	0	0	0	1	1	1	1	1	1	0	0	0	1	0
	8	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0
	5	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	1	1	1	1
	6	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0

4. FINDINGS

The utilization concept which is considered as the number of non-zero components of each cell divided by the entire components of that cell, is a useful yardstick to the decision maker.

Once the decision maker increases utilization of cell two from 0.4 to 1, one arrives at the solution in Table 3. The solution shows that machine type 4 is used once to satisfy utilization of that cell.

Further, in Table 4, with the availability of two machines and minimum utilization of 0.2 for each cell, the model has no problem in using maximum number of machines in each cell (the maximum number of machines, U_k , is 4 for all cells). The solution which satisfies a utilization level more than 0.2 is shown in Table 5.

With increasing utilization from 0.2 to 0.6 for all cells, new part-machine grouping is generated, but utilization level doesn't prevent the model from using multiple machines. So it can be seen that machines 1, 3, 5, and 7 are used in more than one cell (see Table 6).

To show the importance of utilization, we increased the utilization level to 0.9 for each cell, and generated the new part-machine grouping shown in Table 7. As it can be seen, the solution is forced to use one machine of each type to satisfy the utilization level of 0.9.

To summarize, in the two examples taken from literature, apart from considering inter and intra-cell movement costs of parts in a sequenced problem, the proposed model allows the decision maker to set different utilization levels to maximize the utilization of machines and part-machine similarity.

5. CONCLUSIONS

In this paper, a new mathematical model for cell formation in cellular manufacturing system (CMS) is proposed based on sequence data considering cell utilization levels. A small and a large size problems from the literature are solved by the Lingo software package for different utilization levels. According to the results, different part-machine groupings are generated for different cell utilization levels which means the decision maker enjoys the luxury of having more than one choice in forming cells. Also the decision maker is in a position to force a higher utilization of each cell, and to use the utilization concept to reduce the number of non-utilized cells.

REFERENCES

[1] Albadawi Z., Bashir H.A., Chen M. (2005), A mathematical approach for the formation of manufacturing cells; *Computer and Industrial Engineering* 48; 3-21.

[2] Askin R.G., Subramanian S.P. (1987), A cost-based heuristic for group technology configuration; *International Journal of Production Research* 25; 101–113.

[3] Ballakur A., Steudel H.J. (1987), A within-cell utilization based heuristic for designing cellular manufacturing systems; *International Journal of Production Research* 25; 639–665.

[4] Choobineh F. (1988), A framework for the design of cellular manufacturing systems; *International Journal of Production Research* 26; 1161–1172.

[5] Defersha F.M., Chen M. (2006), A comprehensive mathematical model for the design of cellular manufacturing systems; *International Journal of Production Economics*; Article in press.

[6] Harhalakis G., Nagi R., Proth J.M. (1990), An efficient heuristic in manufacturing cell formation for group technology applications; *International Journal of Production Research* 28; 185–198.

[7] Jayaswal S., Adil G.K. (2004), Efficient algorithm for cell formation with sequence data, machine replications and alternative process routings; *International Journal of Production Research* 42; 2419–2433.

[8] Kiang M.Y., Kulkarni U.R., Tam K.Y. (1995); Self-organizing map network as an interactive clustering tool – an application to group technology; *Decision Support Systems* 15; 351–374.

[9] Lee S-D., Chen Y.-L. (1997), A weighted approach for cellular manufacturing design: minimizing inter-cell movement and balancing workload among duplicate machines; *International Journal of Production Research* 35; 1125–1146.

[10] Mahdavi I., Javadi B., Fallah-Alipour K., Slomp J. (2007), designing a new mathematical model for cellular manufacturing system based on cell utilization; *Applied Mathematics and Computation* 190; 662–670.

[11] Nair G.J., Narendran T.T. (1998), CASE: A clustering algorithm for cell formation with sequence data; *International Journal of Production Research* 36; 157–179.

[12] Park S., Suresh N.C. (2003), Performance of Fuzzy ART neural network and hierarchical clustering for part–machine grouping based on operation sequences; *International Journal of Production Research* 41; 3185–3216.

[13] Sarker B.R., Xu Y. (1998), Operation sequences-based cell formation methods: a critical survey; *Production Planning and Control* 9; 771–783.

[14] Selvam R.P., Balasubramanian K.N. (1985), Algorithmic grouping of operation sequences; *Engineering Costs and Production Economics* 9; 125-134.

[15] Shafer S.M., Rogers D.F. (1991), A goal programming approach to the cell formation problem; *Journal of Operations Management* 10; 28–43.

[16] Singh N. (1993), Design of cellular manufacturing systems: an invited review; *European Journal of Operational Research* 69; 284–291.

[17] Suresh N.C., Slomp J., Kaparthi S. (1999), Sequence-dependent clustering of parts and machines: a Fuzzy ART neural network approach; *International Journal of Production Research* 37; 2793–2816.

[18] Vakharia A.J., Wemmerlov U. (1990), Designing a cellular manufacturing system: a materials flow approach based on operations sequences; *IIE Transactions* 22; 84–97.

[19] Wang S., Sarker B.R. (2002), Locating cells with bottleneck machines in cellular manufacturing systems; *International Journal of Production Research* 40; 403-424.

[20] Wemmerlov U., Hyer N.L. (1986), Procedures for the part-family/ machine group identification problem in cellular manufacturing; *Journal of Operations Management* 6(2); 125–147.

[21] Wei J.C., Gaither N. (1990), An Optimal Model for Cell Formation Decisions; *Decision Sciences* 21; 416–433.

[22] Won Y., Lee K.C. (2001), Group technology cell formation considering operation sequences and production volumes; *International Journal of Production Research* 39; 2755–2768.

[23] Zolfaghari S., Liang M. (2002), Comparative study of simulated annealing, genetic algorithms and tabu search for solving binary and comprehensive machine grouping problems; *International Journal of Production Research* 40; 2141–2158.