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Predicting Customer-Expectation-Based Warranty Cost for Smaller-the-Better and Larger-the-Better Performance Characteristics

Naresh K. Sharma[1], David Drain[2], Elizabeth A. Cudney[3], Kenneth M. Ragsdell[4], and Kioumars Paryani[5*]

[1,2,3,4]Missouri University of Science and Technology, Rolla, Missouri 65409 USA
[1]nkscnc@mst.edu, [2]draind@mst.edu, [3]cudney@mst.edu, [4]ragsdell@mst.edu

[5]Lawrence Technological University, Michigan, 48075 USA
kparyani@ltu.edu

ABSTRACT

The quality loss function assumes a fixed target and only accounts for immediate issues within manufacturing facilities whereas warranty loss occurs during customer use. Based on the two independent variables, product performance and consumers' expectation, a methodology to predict the probability of customer complaint is presented in this paper. The formulation presented will serve as a basic model for predicting warranty loss for larger-the-better and smaller-the-better characteristics which is dependent on both product performance and customer expectation. As an example, warranty cost is estimated for automotive disc brakes to demonstrate the methodology for the smaller-the-better case. Another example of solar panels is considered for demonstrating the prediction of warranty loss for the larger-the-better characteristic.

***KeyWords*:** Quality loss function (QLF), Warranty loss function (WLF), Warranty probability, Warranty cost (WC), Product performance, Customer expectation, Smaller-the-better (STB), and larger-the-better (LTB).

1. INTRODUCTION

Warranty

A seller provides assurance to a buyer that a product will perform as stated or implied. This assurance is termed as warranty which works many folds as confidence to the buyer. Performance of a product not as expected by the customer causes additional cost of the product, and requires fire fighting, attention, and manpower. Still companies may lose reputation, goodwill, and market share. Warranty expenditure should be avoided or at least reduced. An occurrence of warranty cost is a loss to society as a whole. Often a problem can be traced back to design and development in contrast to manufacturing. It may be noted that warranty cost is a result of the conflict between product performance and customer expectation as depicted in Figure 1.

[*] Corresponding Author

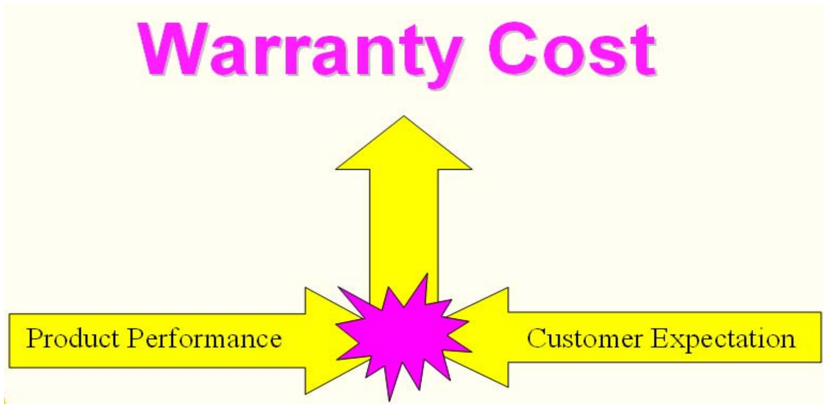

Figure 1 Product performance versus customer expectation from quality loss function to warranty loss function

The rationale behind the development of the quality loss function (QLF), proposed first by Taguchi (2004) is to direct engineers to design products and processes that perform on target. Off-target performance and variation in performance are, in general, two components of the poor performance. Quality loss is a concept applicable in a manufacturing process and it does not directly relate to customer expectation. As part of a QLF, the point of intolerance is assumed to be a point on the QLF where an ‘average customer’ is unsatisfied with the product performance and will initiate an action. The point of intolerance is where half of the customers will consider the product to be defective (El-Haik, 2005). This assumes that a constant customer expectation rate has been taken into account in the QLF.

In contrast, customer satisfaction can be considered as a statistical concept because there are a large number of customers for a given product and each customer has a different level of expectation from a product. The QLF only accounts for current issues (i.e., variations) within manufacturing facilities whereas the warranty loss manifests itself during customer use. A manufacturing facility can use the QLF to assess quality and improve performance by forcing engineer to consistently perform on target. To compete in the market it is imperative to consider what the customer wants. Therefore, one should be interested in the deviation of the product or process performance from each customer’s expectation. The deviation of the product or process performance from the customer expectation is the reason for customer complaints and warranty expenses. The objective of this paper is to develop a model which embodies customer expectation to assess warranty loss by considering customer expectation and product performance as two interdependent variables. The proposed models capture what is required in terms of customer expectation versus what happens in the hands of customers measured in terms of product performance. The rationale behind developing the Warranty Loss Function (WLF) is to relate customer expectation with the product performance in such a way as to be able to predict the probability of a warranty complaint and calculate cost estimates of warranty claims. In this paper two models have been developed; one for smaller-the-better (STB) and one for larger-the-better (LTB) characteristics. Another objective of formulating the warranty loss in conjunction with the quality loss is to equip an engineer with a basic tool to make decisions to reduce the occurrence of warranty. The QLF enables an engineer to develop and improve products and processes that perform on target with little variation. In the same way, the WLF will help an engineer to develop and improve products and processes that perform on a target with such little variation as to cause minimum warranty costs because customer expectation is taken into account. Therefore, the engineer will be able to design the variance of the product or process performance according to warranty targets.

In the design stage as well as in the manufacturing stage, the WLF can be used to estimate the warranty cost. With the aid of these models, engineers can estimate warranty cost before production of a new product begins. Therefore, engineers can draw inferences regarding probable warranty cost of a final product developed because of each component. There are two main differences between QLF and WLF. In QLF, the STB or LTB deviation of performance on one side of a fixed target is considered. On the other hand, in WLF for the STB or LTB deviation of performance on one side from customer expectation is considered. Also, QLF does not compute the probability of occurrence of an event (called quality loss) whereas WLF computes the probability of occurrence of an event (called warranty loss).

Product performance is measured in terms of certain characteristics that need to be within a certain range for STB or LTB in order to satisfy the customer. There may be a difference between expected performance and actual performance of a product. This paper presents a methodology to calculate the warranty cost where both product performance and customer expectation are considered as variables. A manufacturer must choose a particular fixed target value within a certain range for a particular product; however, some variation is inevitable. Therefore, it is imperative to consider the distribution of product performance. On the other hand, different customers have different choices. The population of customers does not have a fixed expected value for the performance of the product. Therefore, it is also necessary to consider the distribution of customer expectation. In order to estimate the warranty cost of performance away from customer expectation the quadratic loss is used. Two types of characteristic larger-the-better (LTB) or smaller-the-better (STB) cases have been considered and warranty cost is computed as an example to demonstrate the methodology.

In this paper first a literature review pertaining to warranty cost is given then the underlying theory is discussed for STB and LTB. To put the theory in practice, two examples are considered to demonstrate the computation of warranty cost using the methodology.

2. LITERATURE REVIEW

Many of the currently used warranty cost models consider aggregate costs such as average material costs, labor costs, personnel costs, and inventory/logistics management costs, etc. Supply chain managers or material managers find these models useful. However, for product engineers such models are not necessarily useful for making a product design choice. Some relevant literature is reviewed as follows.

Venkateswaren (2003) integrates Mahalanobis distance and Ragsdell's warranty cost model. Mahalanobis distance represents multiple interrelated quality characteristics. The optimization of a number of variables is considered for predicting the warranty claim to improve the estimation of warranty cost (Venkateswaren, 2003). However, the thesis does not address the relationship between the actual warranty cost incurred at the retailer-customer interface and the manufacturing performance.

Blischke and Murthy (1993) develop a strategic approach to warranty management where warranty related decisions are made in a framework encompassing the product life cycle and a business perspective which links technical and commercial issues. A warranty strategy depends on the type of product, the customer, and the overall business strategy. It also depends on a number of external factors, particularly competitors' strategies. Warranty cost must link commercial issues at the dealers and technical issues at the manufacturers. This is because technical issues (e.g., design-related issues) affect warranty cost which in turn affect business issues (e.g., pricing) both in turn affecting the product life cycle cost. This paper considers customer expectation, which is a strategic factor in warranty, as a variable.

Cooper and Ross (1988) use a two-period model to examine inter-temporal properties of product warranties. Their research explored why the typical warranty life was considerably shorter than the expected life of a product. And why warranty coverage depreciates in time. They argue that a double moral hazard problem stems from this disconnect. Buyers cannot readily observe the product quality and sellers cannot observe buyers use of the product. Warranty must balance incentives for seller quality and buyer care in product usage. It is evident from the paper that the seller quality and buyer usage or expectation both affect warranty costs.

Joseph (2004) addressed QLF for nonnegative variables. That is, Joseph derives a new set of loss functions for nonnegative variables using Taguchi's definition of quality as a basis. The proposed loss functions assume that the loss is additive and employs STB, nominal-the-better (NTB), and LTB. The new loss function is compared with the quadratic loss function and is shown to be comparable since the quadratic loss function is meant for unrestricted variables. Joseph also proposed a multivariate extension of the QLF. However, customer expectation as a variable has not been addressed which affects warranty cost.

Warranty has also been shown to signal quality in oligopolistic markets. Gal-Or (1989) illustrates that warranties can act as a perfect signal for products with intrinsic attributes that are not widely spaced or clustered. Other market mechanisms providing information about product durability include reputation, advertising, and product-specific investment. Gal-or investigates providing warranties or service contracts as a signal of product durability. Inherent durability, which depends on intrinsic characteristics, is distinguished from provided durability, which is a combination of both intrinsic characteristics and warranty terms. Warranty is a signal of quality in a perfectly competitive market since the cost of providing a warranty rises as the probability of product breakdown increases.

In a reliability context, Vintr (1999) reports a dependency among price, warranty cost and product reliability. Warranty presents an additional cost for the manufacturer. Vintr presents two optimization methods for product reliability. In the first method, the reliability requirements are specified for a product which leads to minimal costs for the manufacturer with respect to research, development, production, and warranty costs if the length of the warranty is firm. The second method presented specifies reliability requirements that will lead to the maximum possible length of warranty if the manufacturer's costs are firm. This research is focused on minimizing the warranty costs based on the length of the warranty period rather than reducing warranty costs at design conception.

In the research of Hussain and Murthy (1998), the warranty costs are shown to be dependent on product reliability and quality uncertainty. The use of redundancies or quality control techniques is shown to reduce warranty costs. Hussain and Murthy developed a model to determine a trade-off between manufacturing costs and warranty costs. From the model, the optimal redundancy and quality control strategies can be determined.

Murthy and Blischke (2000) present a strategic approach to warranty management where warranty related decisions are encompassed in the product life cycle. Warranty depends on the performance of the product and warranty terms. The product performance can be dependent on engineering design, manufacturing process design, part quality, quality control, product use, and product maintenance among others. The research introduces a strategy to incorporate decisions relating to warranty at the infancy stage of the product life cycle.

Bai and Pham (2007) have presented several system warranty cost models of a renewable risk-free policy for multi-component products. Their research utilizes system structures such as series, parallel, series-parallel, and parallel-series configurations. Warranty models depend heavily on the structure of underlying warranty policies over which companies have control. Therefore, changes in a policy may provide a different cost model. This research addresses Renewable Free Replacement Warranty (RFRW) for a pre-specified warranty period. The authors recommend future research in nonrenewable warranty policies and the time discounting effect in warranty cost.

It appears that adequate research is not available on warranty cost linking customer expectation or business issues and product performance or technical issues. This is an important area from the product life cycle standpoint because technical issues (e.g., design-related) affect warranty cost which in turn affects business issues (e.g., pricing). A customer is not likely to be aware of the distributions of customer expectation and product performance. On the other hand, a manufacturer should know the distributions of customer expectation and product performance. In this research both customer expectation and product performance have been assumed to be normally distributed around their individual means with individual variances. Therefore, the models presented are simple and use easily obtainable parameters, e.g., mean and standard deviation. These warranty loss models and a similar one for NTB characteristics provide as a basic model for computing warranty cost under the given assumptions.

3. THEORY

Potential number of complaints

The term 'potential number of complaints' denoted by N is the total number of complaints where the product performance is greater than customer expectation by less than a certain threshold value y^* for LTB characteristic and where the product performance exceeds customer expectation by all values greater than certain limiting value y^* for STB characteristic assuming that all the customers make a complaint. Tables 1 and 2 depict scenarios where a warranty complaint will and will not occur. The potential number of complaints is the total number of complaints that are likely to be made because the gap between product performance and customer expectation is more than a predetermined value y^*.

Table 1. LTB performance vs. expectation

Product performance (PP)	**Customer expectation (CE)**	**PP-CE**	**Whether Y^* = PP-CE < y^* = 0.5**	**Result**
5	6	-1	Yes	Warranty complaint
10	10	0	Yes	Warranty complaint
15	13	2	No	No warranty complaint
19	16	3	No	No warranty complaint

Warranty probability

It is assumed that there exists a fairly constant ratio, denoted as P_W, between the potential number of complaints and the total number of products of a particular production volume. It is important to note that the term P_W can be called warranty probability. Warranty probability is defined as the ratio between the potential number of complaints, N and the total number of products, T_P. This ratio is given in Equation (1).

$$P_W = \frac{N}{T_P} \tag{1}$$

Table 2. STB performance vs. expectation

Product performance (PP)	Customer expectation (CE)	PP-CE	Whether Y^* = PP-CE > y^* = -0.01	Result
0.85	0.60	0.25	Yes	Warranty complaint
0.71	0.71	0.00	Yes	Warranty complaint
0.15	0.21	-0.06	Yes	Warranty complaint
0.59	0.46	0.13	No	No warranty complaint

Given the warranty probability, the potential number of complaints, N, can be computed by multiplying the warranty probability with the amount of production or number of products, as seen in Equation (2).

$$N = P_W T_P \tag{2}$$

Actual number of complaints

The actual number of complaints is the number of complaints made by customers from among those who can potentially complain because the product performance is smaller than customer expectation by any value more than y^*. The actual number of complaints is defined as the total number of complaints that are actually lodged. It is denoted by the symbol n.

Complaint factor

The complaint factor ω is the ratio of the actual number of complaints n to the potential number of complaints N. Thus, Equation 3 depicts the relationship and Equation 4 can be used to compute the actual number of complaints.

$$\omega = \frac{n}{N} \tag{3}$$

$$n = \omega . N \tag{4}$$

In developing the theory, the distribution of a transformed characteristic, Y^*, relates customer expectation with the product performance. A value of y^* for Y^* is assumed in each case of LTB and STB. The value of y^* can be negative to positive including zero and can be viewed as a margin of safety from a warranty standpoint.

4. SMALLER-THE-BETTER

Notations for STB case

P	product performance
C	customer expectation
$Y^* = P - C$	transformed characteristic
y^*	restrictive value of Y^*
P_W	probability of warranty complaint when $Y^* > y^*$
A	dollar loss when $Y^* > y^*$
k	proportionality constant when $Y^* > y^*$
ω	complaint factor when $Y^* > y^*$

It is assumed that there will not be a complaint when the product performance is smaller than customer expectation by a certain value y^*.

Normally distributed product performance and customer expectation

This section discusses the derivation of the formula for calculating warranty probability dependent on product performance and customer expectation. Assume customer expectation, C, and product performance, P, are normally distributed as:

$$C \sim N\left(\mu_c, \sigma_c^2\right) \tag{5}$$

$$P \sim N\left(\mu_p, \sigma_p^2\right) \tag{6}$$

The assumption is made that customer expectation, C, and product performance, P, are independent of each other. Also, assume Y^* is a parameter that is a measure of customer satisfaction such that:

$$Y^* = P - C$$

Also suppose y^* is the maximum value for which the customer is still satisfied. This means that when $Y^* \le y^*$ the customer is satisfied and when $Y^* > y^*$ the customer is dissatisfied and will complain for the STB characteristic.

From statistics, $Y^* = P - C$ is distributed as:

$$Y^* = P - C \sim N\left\{\mu_P - \mu_C, \sigma_C^2 + \sigma_C^2\right\} \tag{7}$$

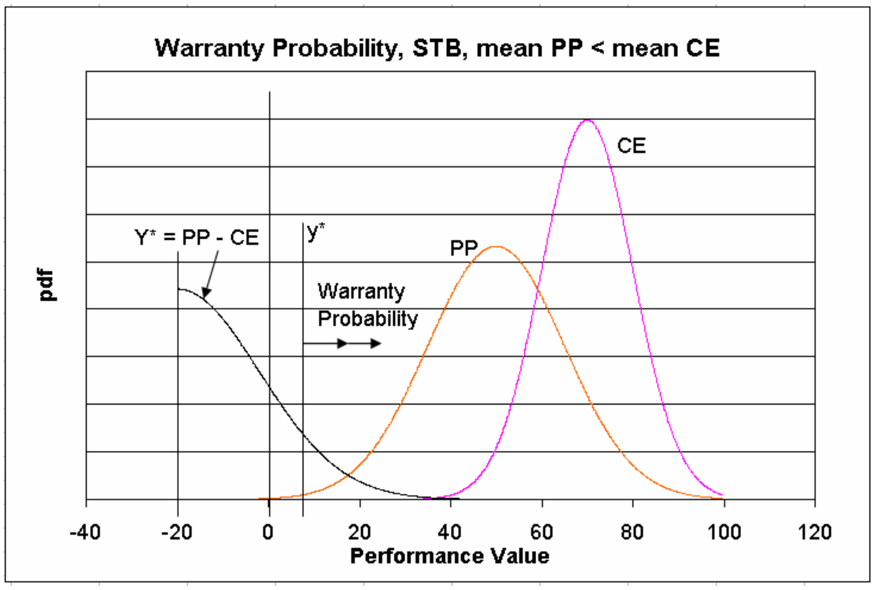

Figure 2. Warranty probability (STB) when mean product performance is smaller than mean customer expectation

It is assumed that there is a potential warranty complaint. Therefore, the probability of a potential warranty (or warranty probability) is given as:

$$P_W = P\left(Y^* > y^*\right) = \int_{y^*}^{\infty} f(t)dy = \int_{y^*}^{\infty} \frac{1}{\sqrt{2\pi}\sqrt{\sigma_P^2 + \sigma_C^2}} e^{-\frac{1}{2}\left[\frac{t-(\mu_P-\mu_c)}{\sqrt{\sigma_P^2+\sigma_C^2}}\right]^2} dt \tag{8}$$

Where, t is a dummy variable for Y^*. If Z is defined as:

$$Z = \frac{t-(\mu_P-\mu_c)}{\sqrt{\sigma_P^2+\sigma_C^2}} \tag{9}$$

then

$$P_W = P\left(Y^* > y^*\right) = P\left[Z > z\right] = 1 - P\left[Z \le z\right]$$

$$P_W = 1 - P\left[\frac{Y^* - (\mu_P - \mu_c)}{\sqrt{\sigma_P^2 + \sigma_C^2}} \le \frac{y^* - (\mu_P - \mu_c)}{\sqrt{\sigma_P^2 + \sigma_C^2}}\right] = 1 - \Phi\left(\frac{y^* - (\mu_P - \mu_c)}{\sqrt{\sigma_P^2 + \sigma_C^2}}\right) \tag{10}$$

The probability of the customer not complaining would then be:

$$P_{noW} = P\left(Y^* \le y^*\right) = P\left[Z \le z\right] = \Phi\left(\frac{y^* - (\mu_P - \mu_c)}{\sqrt{\sigma_P^2 + \sigma_C^2}}\right) \tag{11}$$

Using Equations 2 and 4, the actual number of complaints n can be calculated as follows:

$$n = \omega T_P P_W \tag{12}$$

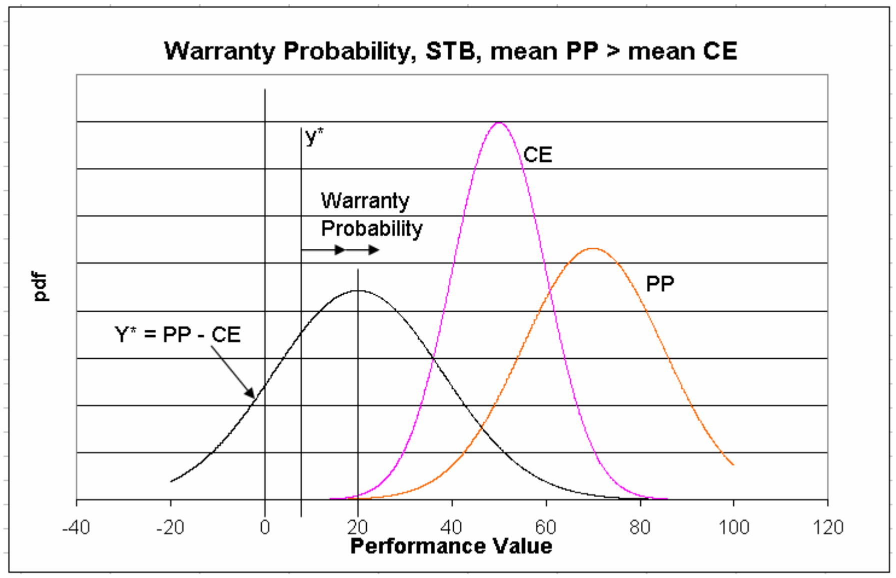

Figure 3. Warranty probability (STB) when mean product performance is greater than mean customer expectation

It is assumed that the cost of corrective action or repairs, A, is a function of the distance of the performance from y^* and the function is quadratic:

$$A = k\left(Y^* - y^*\right)^2 \tag{13}$$

where, k is a proportionality constant. Therefore, warranty cost (WC) for all the products under consideration can be estimated as:

$$WC = nA = k\left(Y^* - y^*\right)^2 \omega T_P P_W$$

$$WC = \int_{y^*}^{\infty} \frac{k\omega T_P \left(t - y^*\right)^2}{\sqrt{2\pi}\sqrt{\sigma_P^2 + \sigma_C^2}} e^{-\frac{1}{2}\left[\frac{t-(\mu_P-\mu_c)}{\sqrt{\sigma_P^2+\sigma_C^2}}\right]^2} dt$$

$$WC = k\omega T_P \int_{y^*}^{\infty} \frac{\left(t - y^*\right)^2}{\sqrt{2\pi}\sqrt{\sigma_P^2 + \sigma_C^2}} e^{-\frac{1}{2}\left[\frac{t-(\mu_P-\mu_c)}{\sqrt{\sigma_P^2+\sigma_C^2}}\right]^2} dt \tag{14}$$

It is more useful and easy to understand if the warranty cost is given per unit of product. Therefore, by setting $T_P = 1$ the unit warranty cost can be computed as shown in Equation (15).

$$WC_{unit} = k\omega \int_{y^*}^{\infty} \frac{\left(t - y^*\right)^2}{\sqrt{2\pi}\sqrt{\sigma_P^2 + \sigma_C^2}} e^{-\frac{1}{2}\left[\frac{t-(\mu_P-\mu_c)}{\sqrt{\sigma_P^2+\sigma_C^2}}\right]^2} dt \tag{15}$$

Unknown Distribution of Customer Expectation

A general formulation of the problem has been given which can be reduced to suit cases where the distribution of customer expectation is unknown but the point of warranty claim is known. For a special case where the distribution of customer expectation is unknown but is assumed by the manufacturer as a cut off point, this methodology can be used. Suppose a manufacturer decides a cut off point such that above which some warranty loss must be assigned and below which no warranty loss should be assigned. Therefore, it can be assumed that the customer expectation is normally distributed with the mean at the cut off point, μ_C, and zero variance, i.e., $\sigma_C = 0$.

The point of a warranty claim can be viewed as the customer expectation distributed with the mean at the point of warranty claim and variance as zero. With this understanding, the same formula can be used and the probability of warranty claims can be assessed. Therefore, the formula reduces to

$$WC = k\omega T_P \int_{y^*}^{\infty} \frac{\left(t - y^*\right)^2}{\sqrt{2\pi}\sqrt{\sigma_P^2}} e^{-\frac{1}{2}\left[\frac{t-(\mu_P-\mu_c)}{\sqrt{\sigma_P^2}}\right]^2} dt \tag{16}$$

STB Example

This is an example of disc brakes for medium sized vehicles. The critical to quality characteristic is axial run-out (RO) of rotors. The run-out is measured before and after mounting the wheel on the rotor. When the wheel is mounted on the rotor the RO increases which can cause a customer complaint. Ideally, the run-out should be zero. Therefore, it essentially is an STB characteristic. Based on historical data a maximum of $40\mu M$ RO can be permitted without a warranty claim. It is also observed that less than $20\mu M$ is very expensive to produce for the manufacturer and not really necessary. The customer expects and accepts variation conforming to a three sigma process. It is estimated that the mean of customer expectation, $\mu_C = 30\mu M$ and standard deviation of customer expectation is approximated as $\sigma_C = 3.33\mu M$ on the basis of the customer's request of a three sigma process.

The total number of products is $T_P = 636,000$ for which the warranty cost must be estimated. A data set of 225 pieces was simulated and recorded for the analysis. The mean of product performance, i.e. RO, is found to be $\mu_P = 30.7\mu M$ and standard deviation is $\sigma_P = 18.1\mu M$. On the basis of past experience the complaint factor is estimated to be $\omega = 0.05$. The cost of corrective action is $A_0 = \$148$ at a distance of $\Delta_0 = 50\mu M$. The point of occurrence for a warranty claim at one end is $y^* = 0\mu M$. This means that if $Y^* > 0\mu M$, potentially a warranty complaint will occur. Therefore, $k = A_0 / \Delta_0^2 = \$148/50^2 = \$0.0592$, and the estimate of warranty cost WC is:

$$WC = 0.0592 \times 0.05 \times 636000 \int_0^\infty \frac{(t-0)^2}{\sqrt{2\pi}\sqrt{18.1^2 + 3.333^2}} Exp\left[-\frac{1}{2}\left\{ \frac{t-(30.7-30.0)}{\sqrt{18.1^2+3.333^2}} \right\}^2 \right] dt = \$338,646$$

The estimate of unit warranty cost is

$$WC = 0.0592 \times 0.05 \int_0^\infty \frac{(t-0)^2}{\sqrt{2\pi}\sqrt{18.1^2 + 3.333^2}} Exp\left[-\frac{1}{2}\left\{ \frac{t-(30.7-30.0)}{\sqrt{18.1^2+3.333^2}} \right\}^2 \right] dt = \$0.53$$

Now suppose the customer does not specify any variation in their expectation then the standard deviation of customer expectation is $\sigma_C = 0\mu M$. In these circumstances, holding all other parameters the same, the estimate of warranty cost reduces to:

$$WC = 0.0592 \times 0.05 \times 636000 \int_0^\infty \frac{(t-0)^2}{\sqrt{2\pi}\sqrt{18.1^2}} Exp\left[-\frac{1}{2}\left\{ \frac{t-(30.7-30.0)}{\sqrt{18.1^2}} \right\}^2 \right] dt = \$327,870$$

Further, suppose the mean of customer expectation $\mu_C = 0\mu M$ and standard deviation is assumed to be $\sigma_C = 0\mu M$, then the estimated warranty cost increases to

$$WC = 0.0592 \times 0.05 \times 636000 \int_0^\infty \frac{(t-0)^2}{\sqrt{2\pi}\sqrt{18.1^2}} Exp\left[-\frac{1}{2}\left\{ \frac{t-(30.7-00.0)}{\sqrt{18.1^2}} \right\}^2 \right] dt = \$2,382,640$$

5. LARGER-THE-BETTER

Notations for LTB case

P	product performance
C	customer expectation
$Y^* = P - C$	transformed characteristic
y^*	restrictive value of Y^*

P_W probability of warranty complaint when $Y^* < y^*$

A dollar loss when $Y^* < y^*$

k proportionality constant when $Y^* < y^*$

ω complaint factors when $Y^* < y^*$

It is assumed that there will be no complaint when the product performance is greater than customer expectation by a specified value y^*.

PP normally distributed and CE normally distributed

This section discusses with the derivation of the formula for calculating of warranty probability dependent on product performance and customer expectation. Suppose customer expectation C and product performance P are normally distributed as

$$C \sim N\left(\mu_c, \sigma_c^2\right) \text{ and } P \sim N\left(\mu_p, \sigma_p^2\right).$$

It is assumed that the customer expectation, C, and product performance, P, are independent of each other. Also, suppose Y^* is a parameter that is a measure of customer satisfaction such that

$$Y^* = P - C$$

Furthermore, suppose y^* is a minimum value that ensures customer satisfaction, such that when $Y^* \geq y^*$ the customer is satisfied for LTB. Alternatively, when $Y^* < y^*$ the customer is dissatisfied and will complain for an LTB characteristic. The distribution of $Y^* = P - C$ is given as:

$$Y^* = P - C \sim N\left\{\mu_P - \mu_C, \sigma_C^2 + \sigma_C^2\right\}.$$

Therefore, the probability of customer complaint is given as:

$$P_W = P\left(Y^* < y^*\right) = \int_{-\infty}^{y^*} f\left(t\right) dy = \int_{-\infty}^{y^*} \frac{1}{\sqrt{2\pi}\sqrt{\sigma_P^2 + \sigma_C^2}} e^{-\frac{1}{2}\left[\frac{t - \left(\mu_P - \mu_C\right)}{\sqrt{\sigma_P^2 + \sigma_C^2}}\right]^2} dt \tag{17}$$

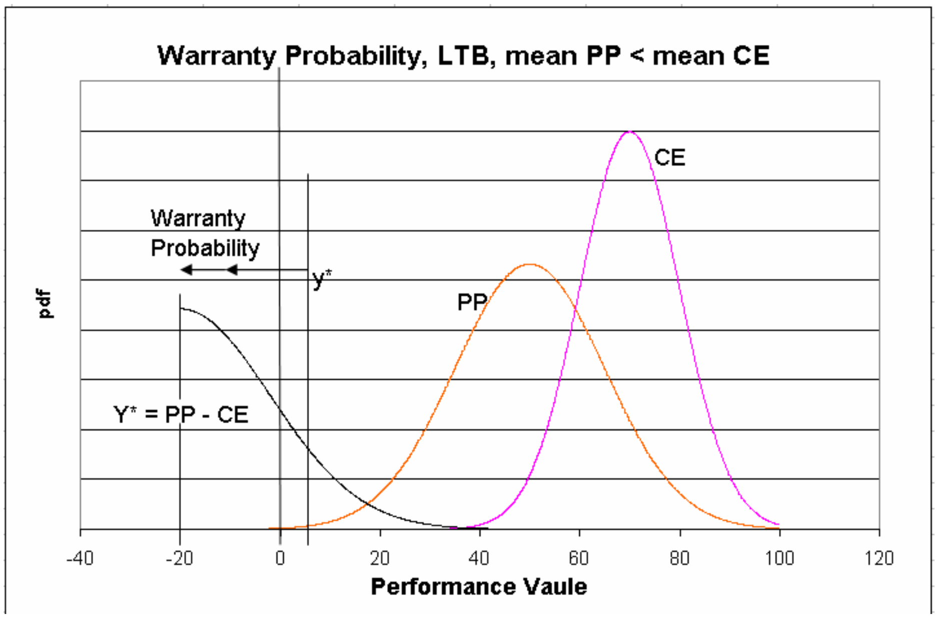

Figure 4: Warranty probability (LTB) when mean product performance is smaller than mean customer expectation

Where, t is a dummy variable for Y^*. If Z is defined as

$$Z = \frac{t-(\mu_P-\mu_c)}{\sqrt{\sigma_P^2+\sigma_C^2}}$$

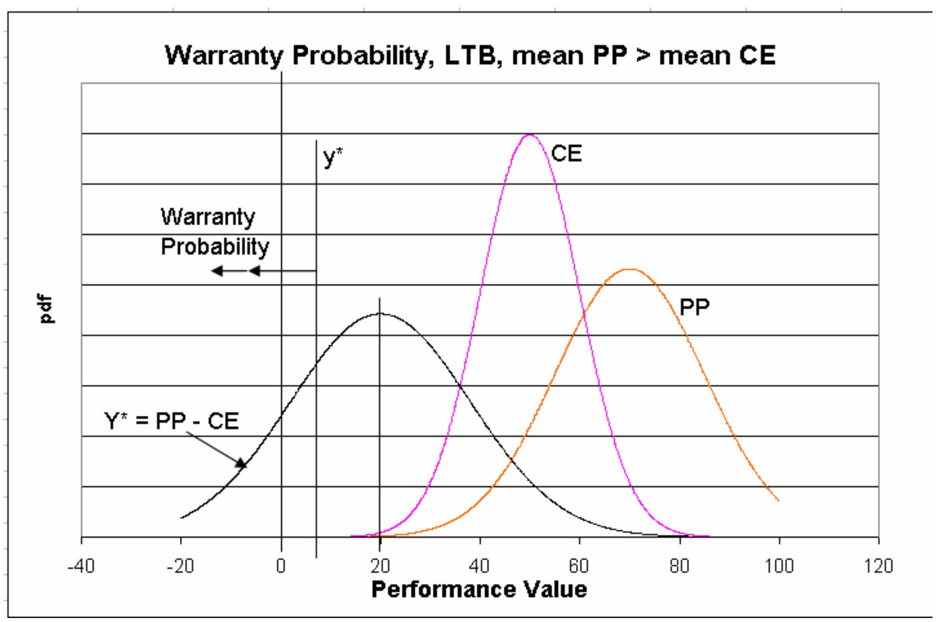

Figure 5: Warranty probability (LTB) when product mean performance is greater than mean customer expectation

The probability of a potential warranty complaint or warranty probability is given as

$$P_W = P\left(Y^* < y^*\right) = P\left[Z < z\right]$$

$$P_W = P\left[\frac{Y^* - \left(\mu_P - \mu_c\right)}{\sqrt{\sigma_P^2 + \sigma_C^2}} < \frac{y^* - \left(\mu_P - \mu_c\right)}{\sqrt{\sigma_P^2 + \sigma_C^2}}\right] = \Phi\left(\frac{y^* - \left(\mu_P - \mu_c\right)}{\sqrt{\sigma_P^2 + \sigma_C^2}}\right) \tag{18}$$

The probability of the customer not submitting a complaint would then be

$$P_{noW} = 1 - P\left(Y^* < y^*\right) = 1 - P\left[Z < z\right] = 1 - \Phi\left(\frac{y^* - \left(\mu_P - \mu_c\right)}{\sqrt{\sigma_P^2 + \sigma_C^2}}\right) \tag{19}$$

It is assumed that the cost of corrective action or repairs, A, is a function of the distance of the performance from $y*$ and the function is quadratic:

$$A = k\left(y^* - Y^*\right)^2 \tag{20}$$

where k is a proportionality constant. Therefore, using Equation (12) warranty cost (WC) for all the products in question can be estimated as:

$$WC = nA = k\left(y^* - Y^*\right)^2 \omega T_P P_W$$

$$WC = \int_{-\infty}^{y^*} \frac{k\omega T_P \left(y^* - t\right)^2}{\sqrt{2\pi}\sqrt{\sigma_P^2 + \sigma_C^2}} e^{-\frac{1}{2}\left[\frac{t - \left(\mu_P - \mu_c\right)}{\sqrt{\sigma_P^2 + \sigma_C^2}}\right]^2} dt$$

$$WC = k\omega T_P \int_{-\infty}^{y^*} \frac{\left(y^* - t\right)^2}{\sqrt{2\pi}\sqrt{\sigma_P^2 + \sigma_C^2}} e^{-\frac{1}{2}\left[\frac{t - \left(\mu_P - \mu_c\right)}{\sqrt{\sigma_P^2 + \sigma_C^2}}\right]^2} dt \tag{21}$$

Often, it is more useful and easy to understand if warranty cost is given per unit of product. Therefore, by setting $T_P = 1$ the unit warranty cost can be calculated as shown in Equation (22).

$$WC_{unit} = k\omega \int_{-\infty}^{y^*} \frac{\left(y^* - t\right)^2}{\sqrt{2\pi}\sqrt{\sigma_P^2 + \sigma_C^2}} e^{-\frac{1}{2}\left[\frac{t - \left(\mu_P - \mu_c\right)}{\sqrt{\sigma_P^2 + \sigma_C^2}}\right]^2} dt \tag{22}$$

Unknown Distribution of Customer Expectation

A general formulation of the problem given above can be reduced to fit cases where the distribution of customer expectation is unknown but the point of warranty claim is known. When the distribution of customer expectation is unknown but can be assumed by the manufacturer as a cut off point, this methodology can be used. Suppose a manufacturer decides a cut off point such that below which some warranty loss needs to be assigned and above which no warranty loss should be assigned. It can, therefore, be assumed that the customer expectation is normally distributed with the mean at the cut off point μ_C and zero variance i.e., $\sigma_C = 0$.

The point of warranty claim can be viewed as customer expectation distributed with the mean at the point of warranty claim and variance as zero. With this understanding the same formula can be used and probability of warranty claims can be assessed. Therefore, the formulation reduces to

$$WC = k\omega T_P \int_{-\infty}^{y^*} \frac{\left(y^* - t\right)^2}{\sqrt{2\pi}\sqrt{\sigma_P^2}} e^{-\frac{1}{2}\left[\frac{t-(\mu_P-\mu_c)}{\sqrt{\sigma_P^2}}\right]^2} dt \tag{23}$$

LTB Example

This example relates to solar panel modules (Vintr, 1999). The grid-connected solar power plants are used in commercial PV projects by leading solar project developers. An advanced film design provided on panels is claimed to produce high energy over a wide range of varying climatic conditions with excellent low light response and temperature coefficient.

The minimum power expected is 67.5W $\pm 5\%$ for each solar panel. The characteristic is considered as LTB because higher power given by the panel does not invite customer complaint. Therefore, the mean of customer expectation is $\mu_C = 67.5W$ with a tolerance of $\pm 3.375W$. The customer also expects and accepts a variation conforming to a three sigma process. Given this information, it is approximated that the standard deviation of customer expectation is $\sigma_C = 1.125W$. The total number of products is $T_P = 50,000$.

A data set for 500 pieces was simulated and recorded for analysis. The mean of product performance is found to be $\mu_P = 69.0272W$ and standard deviation is $\sigma_P = 2.9298W$. On the basis of past experience the complaint factor is estimated to be $\omega = 0.30$. The cost of corrective action is $A_0 = \$2000$ at a distance of $\Delta_0 = 3.375W$. The point of occurrence of warranty claim at one end is $y^* = 1W$. Therefore, if $Y^* < 1W$ then potentially a warranty complaint will occur. Therefore, $k = \frac{A_0}{\Delta_0^2} = \frac{\$2000}{3.375^2} = \$175.583$

$$WC = 175.583 \times 0.3 \times 50000 \int_{-\infty}^{1} \frac{(1-t)^2}{\sqrt{2\pi}\sqrt{2.9298^2 + 1.125^2}} Exp\left[-\frac{1}{2}\left\{\frac{t-(69.0272-67.5000)}{\sqrt{2.9298^2+1.125^2}}\right\}^2\right] dt = \$9,843,120$$

Estimated unit warranty cost will be

$$WC = 175.583 \times 0.3 \int_{-\infty}^{1} \frac{(1-t)^2}{\sqrt{2\pi}\sqrt{2.9298^2 + 1.125^2}} Exp\left[-\frac{1}{2}\left\{\frac{t-(69.0272-67.5000)}{\sqrt{2.9298^2+1.125^2}}\right\}^2\right] dt = \$196.86$$

Now suppose the customer does not specify any variation in his expectation, then the standard deviation of customer expectation is $\sigma_C = 0.000W$. Under these circumstances, other parameters remain the same; the estimate of warranty cost is calculated as

$$WC = 175.583 \times 0.3 \times 50000 \int_{-\infty}^{1} \frac{(1-t)^2}{\sqrt{2\pi}\sqrt{2.9298^2}} Exp\left[-\frac{1}{2}\left\{\frac{t-(69.0272-67.5000)}{\sqrt{2.9298^2}}\right\}^2\right] dt = \$8,406,360$$

Note on results of STB and LTB examples

When the variation in customer expectation reduces, the estimate of warranty cost also reduces. This is consistent with the idea that variation in both customer expectation and product performance contribute to higher warranty expenditures. Therefore, between variation of customer expectation and variation of product performance if the target is fixed (i.e., if customer expectation is fixed) the only factor that causes warranty expenditure is the variation of product performance. On the other hand, when customer expectation has variation it causes the warranty expenditure to increase. Hypothetically, if product performance also does not have any variation, the only reason to cause warranty cost is the shift of product performance from the customer expectation. However, it was assumed in this paper that the variation in product performance is never zero. Therefore, it can be said that, if a process is designed according to a fixed target, it gives lower warranty loss than can actually occur. In the light of this, it is advisable to consider customer expectation in addition to product performance.

6. CONCLUSION

This paper presents a methodology to predict the probability of customer complaint on the basis of two independent variables: product performance and consumer’s expectation. A warranty probability function was developed to assess the probability of warranty. Further, an attempt was made to develop a warranty loss function for STB and LTB characteristics that takes into account customer expectation as a variable in addition to product performance to predict warranty expenditure. Warranty cost can be estimated from the probability of customer complaint. The formulation presented serves as a basic model for predicting warranty loss dependent on both product performance and customer expectation. The model was developed to account for the two types of characteristics, i.e., the smaller-the-better (STB) and the larger-the-better (LTB) cases independently. An example of disc brakes was provided for STB and warranty cost was estimated to demonstrate the methodology. Also, an example of solar panels was given for LTB and warranty cost was estimated to demonstrate the methodology.

FUTURE RESEARCH

The measurable and controllable product characteristics in the factory can be used to successfully predict warranty loss using the proposed models when the objective of customer satisfaction is of prime importance. Therefore, both customer expectation and product performance were considered to be normally distributed in this paper. Although the proposed approach is novel, it may be argued that because almost all characteristics are non-negative, the assumption of normal distribution is not the best choice for the formulation of the problem. The reliability measures of a product such as mean time to failure (MTTF), or mean time between failures (MTBF), etc., are important characteristic that can be used with the proposed methodology to predict warranty cost. Therefore, in future research, in place of the normal distribution, other distributions spreading from zero to infinity should be considered. Also, a different distribution for customer expectation and a different distribution for product performance may be considered in future research.

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