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# An improved approach to find and rank BCC-efficient DMUs in data envelopment analysis (DEA)

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**Abstract**

Recently, a mixed integer data envelopment analysis (DEA) model has been proposed to find the most BCC-efficient (or the best) decision making unit(DMU) by Toloo (2012).This paper shows that the model may be infeasible in some cases, and when the model is feasible, it may fail to identify the most efficient DMU, correctly. We develop an improved model to find the most BCC-efficient DMU that removes the mentioned drawbacks. Also, an algorithm is proposed to find and rank other most BCC-efficient DMUs, when there exist more than one BCC-efficient DMUs. The capability and usefulness of the proposed model are indicated, using a real data set of nineteen facility layout designs (FLDs) and twelve flexible manufacturing systems (FMSs).

**Keywords:** data envelopment analysis (DEA), most BCC-efficient DMU, mixed integer DEA models, ranking, facility layout design

## 1- Introduction

Data envelopment analysis (DEA), introduced by Charnes, Cooper, and Rhodes (1978), is a linear programming for the assessment of relative efficiency of a set of decision making units (DMUs).The DMUs usually use a set of resources,referred to as input indices, and transform them into a set of outcomes, referred to as output indices. The CCR model is developed for constant returns to scale of DMUs. Banker, Charnes, and Cooper (1984), promoted it to variable returns to scale.

DEA effectively divides DMUs into two groups: efficient DMUs and inefficient DMUs. The efficiency score of efficient DMUs is equal to one and the efficiency score of inefficient DMUs is less than one. It should be noted that the efficient DMUs do not necessarily have the equivalent performance in real practices. In the practical applications, it is necessary to rank all DMUs, or find the most efficient DMU. For this purpose, different approaches have been proposed. Cross efficiency (Liu and Peng, 2008), super efficiency (Andersen and Petersen, 1993), imposing restrictions on the weights and using a common set of weights (Sexton et al., 1986; Allen, 1997), are some examples of these approaches.

Mentioned approaches solve at leastone linear DEA model for each DMU to find the most efficient DMUs. However, some researchers proposed various methods to determine the most efficient DMU by solving just one model, (Karsak and Ahiska, 2005;Amin and Toloo, 2007; Amin, 2009; Toloo and Nalchigar, 2009; Foroughi, 2011;Toloo, 2012; Wang and Jiang, 2012; Foroughi, 2013; Toloo, 2014a; Toloo, 2014b; Toloo, 2014c; Toloo and Ertayb, 2014; Toloo, 2015).

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In a recent paper, Toloo, (2012), proposed a new integrated mixed integer programming – data envelopment analysis (MIP–DEA) model to find the most BCC-efficient DMU.We show that this model may lead to infeasibility in some cases. We also, show that when the model is feasible, the DMU reported as the most BCC-efficient, may be wrong (see example 3 in the section 4).The aims of the current paper are to develop new models to find the most BCC-efficient DMUs and eliminate the mentioned drawbacks. In addition, we propose a new algorithm to find and rank the other efficient DMUs. The remainder of the paper is organized as follows: in section 2, the proposed model by Toloo (2012), is presented and the infeasibility problem of the model has been shown by an example. Section 3, presents an improved integrated DEA modeland a new algorithm to find and rank BCC-efficient DMUs. Numerical examples and conclusion are given in section 4 and 5, respectively.

## 2- Infeasibility problem of Toloo's Model

Recently, Toloo, (2012), proposed a new MIP–DEA model to find the most BCC-efficient DMU by a common set of optimal weights, as follows:

$$
\begin{aligned}
& M^* = Min \quad d_{\max} \\
& s.t. \quad d_{\max} - d_j \geq 0; \quad j = 1,\dots,k \\
& \quad \sum_{i=1}^{n} v_i x_{ij} \leq 1; \quad j = 1,\dots,k \\
& \quad \sum_{r=1}^{m} u_r y_{rj} - u_0 - \sum_{i=1}^{n} v_i x_{ij} + d_j = 0; \quad j = 1,\dots,k \\
& \quad \sum_{j=1}^{k} \theta_j = k-1; \\
& \quad d_j \leq M\theta_j; \quad j = 1,\dots,k \\
& \quad \theta_j \leq N d_j; \quad j = 1,\dots,k \\
& \quad d_j \geq 0; \quad j = 1,\dots,k \\
& \quad \theta_j \in \{0,1\}; \quad j = 1,\dots,k \\
& \quad u_r \geq \varepsilon^*; \quad r = 1,2,\dots,m \\
& \quad v_i \geq \varepsilon^*; \quad i = 1,2,\dots,n \\
& \quad u_0 \textit{ is free in sign,}
\end{aligned}
\tag{1}
$$

Where$M$ and $N$ are large enough numbers. $x_{ij}$ is the amount of $i_{th}$ input consumed by $DMU_j$, $y_{rj}$ is the amount of $r_{th}$ output produced by $DMU_j$ ($j = 1,2,\dots,k; r = 1,2,\dots,m \,\&\, i = 1,2,\dots,n$). $d_j$ is the deviation of $DMU_j$ from the BCC-efficiency. $d_{\max}$ is maximum inefficiency that should be minimized. $\varepsilon^*$ is the maximum non-Archimedean epsilon. He proposed the following linear programming (LP) to determine $\varepsilon^*$ :

$$
\begin{aligned}
&\varepsilon^* = \max \quad \varepsilon \\
&s.t. \\
&\quad \sum_{i=1}^{n} v_i x_{ij} \le 1; \quad j = 1,\dots,k \\
&\quad \sum_{r=1}^{m} u_r y_{rj} - u_0 - \sum_{i=1}^{n} v_i x_{ij} \le 0; \;\; j = 1,\dots,k \\
&\quad u_r - \varepsilon \ge 0; \quad r = 1,2,\dots,m \\
&\quad v_i - \varepsilon \ge 0; \quad i = 1,2,\dots,n \\
&\quad u_0 \textit{ is free in sign},
\end{aligned}
\tag{2}
$$

Toloo and Nalchigar, (2009),proved that model (2) is equivalent to the following model (3):

$$
\begin{aligned}
&\varepsilon^* = \max \quad \varepsilon \\
&s.t. \\
&\quad \sum_{i=1}^{n} v_i x_{ij} \le 1; \quad j = 1,\dots,k \\
&\quad v_i - \varepsilon \ge 0; \quad i = 1,2,\dots,n
\end{aligned}
\tag{3}
$$

Toloo, (2012),showed that the optimal value of model (3) is equal to $\varepsilon^* = \dfrac{1}{\max\{\sum_{i=1}^{n} x_{ij} : j = 1,2,\dots,k\}}$.

He also proved that model (1), reports only one single BCC-efficient DMU with the common set of optimal weights. In the other words, at the optimal solution of model (1), $d_p = 0$ for only one $p \in \{1,2,\dots k\}$ and $d_j \ne 0, \forall j \ne p$ .It should be noted that, in model (1),$DMU_p$ is BCC-efficient if in the optimal solution of the model, $d_p^* = 0$. The following example shows the case of infeasibility of model (1).

**Example 1**: Consider 3 DMUs each uses two inputs to produce one output.

**Table 1**: Data for 3 DMUs

| DMU No. | $I_1$ | $I_2$ | $O_1$ |
|---|---|---|---|
| 1 | 1 | 1 | 2 |
| 2 | 2 | 1 | 4 |
| 3 | 1 | 2 | 4 |

For this example, the maximum non-Archimedean is $\varepsilon^* = \dfrac{1}{\max\{1+1, 2+1, 2+1\}} = \dfrac{1}{3}$.

Now, by considering the value of $\varepsilon^* = \dfrac{1}{3}$, we show that model (1) is infeasible.From the constraints of this model for the data presented in table 1,we have:

$v_1 + v_2 \le 1 \quad A$
$2v_1 + v_2 \le 1 \quad B$
$v_1 + 2v_2 \le 1 \quad C$
$2u_1 - u_0 - v_1 - v_2 + d_1 = 0 \quad D$
$4u_1 - u_0 - 2v_1 - v_2 + d_2 = 0 \quad E$
$4u_1 - u_0 - v_1 - 2v_2 + d_3 = 0 \quad F$

From the other constraints of model (1), we have $u_1, v_1, v_2 \ge \frac{1}{3}$ & $d_j \ge 0,\ j = 1,2,3$ .Thus:

$$v_1, v_2 \ge \frac{1}{3} \Rightarrow 2v_1 + v_2 \ge 1 \ \&\ v_1 + 2v_2 \ge 1 \quad G$$

Hence, fromthe constraints of*B*, *C*and*G*we should have:

$$2v_1 + v_2 = 1 \ \&\ v_1 + 2v_2 = 1 \Rightarrow 3v_1 + 3v_2 = 2 \Rightarrow v_1 + v_2 = \frac{2}{3} \xrightarrow{v_1, v_2 \ge \frac{1}{3}} v_1 = v_2 = \frac{1}{3}$$

Now, by considering the constraints of *E* and *F*we have*:*

$(E) + (-F) = d_2 - d_3 = 0 \Rightarrow d_2 = d_3$

In this example for only one $p \in \{1,2,3\}$ we have $d_p = 0$ , and for the others we have $d_j > 0, \forall j \ne p$ , (see theorem 4 in Toloo, 2012). So, we should have $d_1 = 0 \ \&\ d_2, d_3 > 0$ . Fromthe constraint *D*, we have $u_0 = 2u_1 - \frac{2}{3} \ge 0$ , (note that $u_1 \ge \frac{1}{3}$ ).Now by considering $(E) - (2 \times D)$ ,we should have:

$$u_0 + d_2 + \frac{1}{3} = 0 \xrightarrow{d_2 > 0} u_0 < 0$$

This is inconsistent with $u_0 \ge 0$ .So, model (1) is infeasible for this example.

## 3- Developed improved model

It was shown that the proposed model of Toloo, (2012), may be infeasible. The purpose of model (1) is finding a set of weights such that only one DMU has the largest efficiency score corresponding to those weights. Hence, the model may be infeasible when there are more than one BCC-efficient DMU. In this section, an integrated DEA model proposed which is always feasible and can determine a single most BCC-efficient DMU, when such a DMU exists, and otherwise proposes a set of BCC-efficient DMUs as the most BCC-efficient. The model is formulated as follows.

$$
\begin{aligned}
& M^* = Max \quad d_{\max} \\
& s.t. \quad d_{\max} - \beta_j \le 0; \quad j = 1,...,k \\
& \quad \sum_{i=1}^{n} v_i x_{ij} \le 1; \quad j = 1,...,k \\
& \quad \sum_{r=1}^{m} u_r y_{rj} - u_0 - \sum_{i=1}^{n} v_i x_{ij} + d_j - \beta_j = 0; \; j = 1,...,k \\
& \quad \sum_{j=1}^{k} d_j = k - 1 \\
& \quad 0 \le \beta_j \le 1, \quad d_j \in \{0,1\}; \quad j = 1,...,k \\
& \quad u_r \ge \varepsilon_1^*; \quad r = 1,2,...,m \\
& \quad u_r \ge \varepsilon_1^*; \quad r = 1,2,...,m \\
& \quad u_0 \; \textit{is free in sign,}
\end{aligned} \tag{4}
$$

Where $d_j$ is a binary variable and $\beta_j$ is considered because $d_j$ is discrete. $d_j - \beta_j$ is the deviation of $DMU_j$ from the BCC-efficiency and $d_{\max}$ is maximum inefficiency that should be minimized. Note that maximizing the minimum values of $\beta_j$ is equivalent to minimizing the maximum values of $d_j - \beta_j, \forall j$. So this model minimizes the maximum inefficiency.The optimal non-Archimedean epsilon is obtained by solvingthe following model (5).

$$
\begin{aligned}
& \varepsilon_1^* = \max \quad \varepsilon \\
& s.t. \quad \sum_{i=1}^{n} v_i x_{ij} \le 1; \quad j = 1,...,k \\
& \quad \sum_{r=1}^{m} u_r y_{rj} - u_0 - \sum_{i=1}^{n} v_i x_{ij} + d_j - \beta_j = 0; \; j = 1,...,k \\
& \quad \sum_{j=1}^{k} d_j = k - 1 \\
& \quad 0 \le \beta_j \le 1, \quad d_j \in \{0,1\}; \quad j = 1,...,k \\
& \quad u_r - \varepsilon \ge 0; \quad r = 1,2,...,m \\
& \quad v_i - \varepsilon \ge 0; \quad i = 1,2,...,n \\
& \quad u_0 \; \textit{is free in sign,}
\end{aligned} \tag{5}
$$

The lemmas 1 and 2 prove that model (4) and model (5) are always feasible.

**Lemma 1**: model (5) is always feasible.

Proof: let $\varepsilon = u_0 = v_i = u_r = d_1 = \beta_1 = 0, \forall i, r$ & $d_j = \beta_j = 1, \forall j \ne 1$. Clearly, $(v, u, d, \beta, u_0, \varepsilon)$ is a feasible solution of model (5), where

$v = (v_1, v_2, ..., v_n)$, $u = (u_1, u_2, ..., u_m)$, $d = (d_1, d_2, ..., d_k)$, $\beta = (\beta_1, \beta_2, ..., \beta_k)$

**Lemma 2**: model (4) is always feasible.

Proof:Suppose $(\varepsilon_1^*, v^*, u^*, d^*, \beta^*, u_0^*)$ tobe an optimal solution of model (5), in this case $(d_{\max}, v, u, d, \beta, u_0) = (\min\{\beta_1^*, \beta_2^*, ..., \beta_k^*\}, v^*, u^*, d^*, \beta^*, u_0^*)$ is a feasible solution of model (4).

Theorem 1, proves that model (4) finds a single most BCC-efficient DMU.

**Theorem 1**: solving model (4) gives a single BCC-efficient DMU.

Proof: suppose $(d^*_{\max}, v^*, u^*, d^*, \beta^*, u^*_0)$ to be an optimal solution of model (4), by considering the constraint $\sum_{j=1}^{k} d_j = k-1$, there is only one $p \in \{1,2,...,k\}$ such that $d^*_p = 0$ and $d^*_j = 1, \forall j \neq p$ . According to the third type constraints of model (4) we have:

$$\sum_{r=1}^{m} u^*_r y_{rp} - u^*_0 - \sum_{i=1}^{n} v^*_i x_{ip} = \beta^*_p \geq 0 \quad \Rightarrow \quad \frac{\sum_{r=1}^{m} u^*_r y_{rp} - u^*_0}{\sum_{i=1}^{n} v^*_i x_{ip}} \geq 1$$

We also show that at the above optimal solution, the efficiency score of other DMUs are less than or equal to one and this completes the proof. For $DMU_j$ $(\forall j \neq p)$, we have:

$$d^*_j = 1 \,\&\, 0 \leq \beta^*_j \leq 1, \forall j \neq p \quad \Rightarrow d^*_j - \beta^*_j \geq 0 \Rightarrow \sum_{r=1}^{m} u^*_r y_{rj} - u^*_0 - \sum_{i=1}^{n} v^*_i x_{ij} \leq 0$$

$$\Rightarrow \frac{\sum_{r=1}^{m} u^*_r y_{rj} - u^*_0}{\sum_{i=1}^{n} v^*_i x_{ij}} \leq 1$$

The above expression shows that at the optimal weights the other DMUs have the efficiency score less than or equal to one. It should be noted that if $\beta^*_p \geq M^* > 0$ then $DMU_p$ is super efficient, since the efficiency score of $DMU_p$ is larger than one while the efficiency score of other DMUs is less than or equal to one. Note that theorem 1 does not say that there are only one BCC-efficient DMU but reports one most BCC-efficient DMU form a set of BCC-efficient DMUs.

## 3-1- Finding and Ranking other efficient DMUs

To rank efficient DMUs in DEA, different approaches have been developed by researchers. Hosseinzadeh Lotfi et al., (2013), reviewed the ranking method in DEA and divided them into seven groups. The readers can refer to this paper for further discussion on ranking approaches. In this section, a new algorithm is presented to find and rank BCC-efficient DMUs. This algorithm is as follows:

**Step 0:** Solve model (4), the DMU with the maximum $d_{\max}$ is the most BCC-efficient DMU, say $DMU_p$,let $T = \{p\}$.

**Step 1:** Add the constraint $d_j = 1, \ \forall j \in T$ to model (4) and resolve it.

**Step 2:** If the model is infeasible there is no other most BCC-efficient DMU exist and *T* shows the set of most BCC-efficient DMUs, else suppose that at the optimal solution $d^*_l = 0$.

**Step 3:** Let $T = T \cup \{l\}$ and go to step 1.

In this algorithm the BCC-efficient DMUs have been found, one by one until the model with additional constraints will be infeasible. So, in the result of using this algorithm the decision maker could find and rank all the most BCC-efficient DMUs. Indeed, DMUs are ranked based on the optimal values of $d^*_{\max}$ that represents the maximum distance that $DMU_p$ can have from the best DMUs in *T*.

The proposed algorithm is necessary to show that the determined DMU by the model (4) is the best DMU. Also, this algorithm capable us to find the other BCC-efficient DMUs with the same efficiency score, similar to the example 2. The existing ranking approaches cannot be used with the model (4) to find and rank BCC-efficient DMUs. For example, the proposed method by Andersen and Petersen, (1993) to rank the efficient DMUs should be used with the traditional BCC-DEA model.

## 4-Numerical examples

In this section, two numerical examples are presented to shows the capability and usefulness of the proposed methodology of the paper.

**Example 2**: In this example, we apply model (4) to find the most BCC-efficient DMU in example 1.In section 2, it was shown that model (1) is infeasible for this example. Solving model (5) for data presented in table 1gives $\varepsilon_1^* = 0.333$. The following table 2 shows the results of the proposed algorithm step by step for data presented in table 1.

**Table 2**. results of the proposed algorithm for example 2

<table>
<tr><td rowspan="4">First iteration</td><td>Step 0</td><td>Eff. $DMU_1$ = 0.1670, Eff. $DMU_2$= 0.9999,Eff. $DMU_3$= 1.0004<br>$d^*_{\max} = 0.5\times10^{-3}$ , $d_3^* = 0$ T={3}</td></tr>
<tr><td>Step 1</td><td>Add the constraint $d_3$=1 to model (4)and resolve it</td></tr>
<tr><td>Step 2</td><td>Eff. $DMU_1$= 0.5011, Eff. $DMU_2$= 1.0004, Eff. $DMU_3$= 0.9999<br>$d^*_{\max} = 0.5\times10^{-3}$ , $d_2^* = 0$</td></tr>
<tr><td>Step 3</td><td>T={2 , 3}</td></tr>
<tr><td>Second iteration</td><td>Steps 1, 2</td><td>Model (4) is infeasible with the additional restrictions of $d_2+d_3=2$</td></tr>
</table>

In the step 0, solving model (4) with $\varepsilon_1^* = 0.333$, leads to $d^*_{\max} = \beta_3^* = 0.5\times10^{-3}$ and $d_3^* = 0$. This solution implies that $DMU_3$ is most BCC-efficient DMU. Solving model (4) with additional restriction of $d_3 = 1$, results to $d^*_{\max} = \beta_2^* = 0.5\times10^{-3}$ and $d_2^* = 0$. So $DMU_2$ is also most BCC-efficient. Solving model (4) with an additional restriction of $d_2 = 1 \& d_3 = 1$ or $d_3 + d_2 = 2$, leads to infeasibility. Therefore, model (4) implies that both $DMU_2$ and $DMU_3$ are BCC-efficient.

It may be wrong to search just one efficient DMU as a single most efficient DMU, because in fact there are may be different DMUs in a set of DMUs as the most efficient. The above example shown this fact and results both DMUs 2 and 3 as the most efficient. Also,it can be shown that other ranking methods as Andersen and Petersen, (1993), result to the same score for DMU 2 and 3 and could not discriminate between $DMU_2$ and $DMU_3$. We found that the only way to determine just one DMU as most BCC-efficient DMU is restricting the feasible region using weight restrictions.

**Example 3:** This numerical example, contains a real dataof nineteen facility layout designs (FLDs) that be shown in table 3, originally provided by Ertay et al., (2006) and used in Toloo, (2012). Each FLD (or DMU) has two inputs: cost and adjacency score and four outputs: flexibility, hand-carry utility, quality and shape ration.

Model (5) is applied for data presented in Table 3 and leads to $\varepsilon_1^* = 26\times10^{-6}$. To find the most BCC-efficient FLD solving model (4) gives $d_{\max}^* = \beta_{10}^* = 0.6319$ and $d_{10}^* = 0$. Solving model (4) with an additional restriction of $d_{10} = 1$ gives $d_{\max} = \beta_{12} = 0.2187$ and $d_{12} = 0$ .Decreasing the optimal value of $d_{\max}^*$ emphases that $FLD_{10}$ is the most BCC-efficient DMU, and the $DMU_{12}$ is the second most BCC-efficient DMU.It should be noted that the proposed models of Wang and Jiang, (2012), and Foroughi, (2011), to find the most BCC-efficient DMU, selects $FLD_{10}$ as the most BCC-efficient DMU.The result of applying the proposed algorithm to find and rank the other most BCC-efficient FLDs, summarized in table 4.Note that, model (1) selects $DMU_{14}$ as the most BCC-efficient FLD. As it can be seen from table 4, $FLD_{14}$ is one of the most BCC-efficient DMUs and it is not the only most BCC-efficient DMU. This implies that model (1) is unable to find the most efficient DMU, correctly.

**Table 3**. Inputs and outputs of 19 FLDs

| DMU No. | DEA inputs | | DEA outputs | | | |
|---|---|---|---|---|---|---|
| | Cost ($) | Adjacency score | Shape rate | Flexibility | Quality | Hand-carry utility |
| 1 | 20309.56 | 6405 | 0.4697 | 0.0113 | 0.041 | 30.89 |
| 2 | 20411.22 | 5393 | 0.438 | 0.0337 | 0.0484 | 31.34 |
| 3 | 20280.28 | 5294 | 0.4392 | 0.0308 | 0.0653 | 30.26 |
| 4 | 20053.20 | 4450 | 0.3776 | 0.0245 | 0.0638 | 28.03 |
| 5 | 19998.75 | 4370 | 0.3526 | 0.0856 | 0.0484 | 25.43 |
| 6 | 20193.68 | 4393 | 0.3674 | 0.0717 | 0.0361 | 29.11 |
| 7 | 19779.73 | 2862 | 0.2854 | 0.0245 | 0.0846 | 25.29 |
| 8 | 19831.00 | 5473 | 0.4398 | 0.0113 | 0.0125 | 24.80 |
| 9 | 19608.43 | 5161 | 0.2868 | 0.0674 | 0.0724 | 24.45 |
| 10 | 20038.10 | 6078 | 0.6624 | 0.0856 | 0.0653 | 26.45 |
| 11 | 20330.68 | 4516 | 0.3437 | 0.0856 | 0.0638 | 29.46 |
| 12 | 20155.09 | 3702 | 0.3526 | 0.0856 | 0.0846 | 28.07 |
| 13 | 19641.86 | 5726 | 0.269 | 0.0337 | 0.0361 | 24.58 |
| 14 | 20575.67 | 4639 | 0.3441 | 0.0856 | 0.0638 | 32.20 |
| 15 | 20687.50 | 5646 | 0.4326 | 0.0337 | 0.0452 | 33.21 |
| 16 | 20779.75 | 5507 | 0.3312 | 0.0856 | 0.0653 | 33.60 |
| 17 | 19853.38 | 3912 | 0.2847 | 0.0245 | 0.0638 | 31.29 |
| 18 | 19853.38 | 5974 | 0.4398 | 0.0337 | 0.0179 | 25.12 |
| 19 | 20355.00 | 17402 | 0.4421 | 0.0856 | 0.0217 | 30.02 |

**Table 4**: Ranking of efficient FLDs

| Efficient FLDNo. | Ranking | $d_{\max}^*$ |
|---|---|---|
| 10 | 1 | 0.6319 |

| | | |
|---|---|---|
| 12 | 2 | 0.2187 |
| 15 | 3 | 0.1385 |
| 16 | 4 | 0.1179 |
| 7 | 5 | 0.0324 |
| 17 | 6 | 0.0294 |
| 14 | 7 | 0.0143 |

**Example 4:** The data of this example, twelve flexible manufacturing systems (FMSs), are taken from Wang and Jiang, (2012) that are presented in table 5. The goal is finding the most BCC-efficient FMSs.
**Input 1:** Annual operating and depreciation cost measured inunits of $100,000,
**Input 2:** Floor space requirements of each specific system measuredin thousands of square feet,
**Output 1:** Improvements in qualitative benefits,
**Output 2:** Work in process (WIP),
**Output 3:** Average number of tardy jobs,
**Output 4:** Average yield
For this example, solving the model (3) gives $\varepsilon^* = 0.040258$. Model (1) is infeasible with the $\varepsilon^*$. Indeed, by using the Toloo (2012) model, we are unable to find the best FMS. Now,we apply the proposed model in this paper to find the most BCC-efficient FMS. Appling the model (5) for the data presented in table 5 implies $\varepsilon_1^* = 0.01781876$. Model (4) with the $\varepsilon_1^*$ identifies $DMU_4$ as the most BCC-efficient FMS. This is also the selection made byWang and Jiang, (2012).

**Table 5**. Inputs and outputs of 12FMSs

| DMU No. | DEA inputs | | DEA outputs | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 1 | 2 | 3 | 4 |
| 1 | 17.02 | 5 | 42 | 45.3 | 14.2 | 30.1 |
| 2 | 16.46 | 4.5 | 39 | 40.1 | 13 | 29.8 |
| 3 | 11.76 | 6 | 26 | 39.6 | 13.8 | 24.5 |
| 4 | 10.52 | 4 | 22 | 36 | 11.3 | 25 |
| 5 | 9.5 | 3.8 | 21 | 34.2 | 12 | 20.4 |
| 6 | 4.79 | 5.4 | 10 | 20.1 | 5 | 16.5 |
| 7 | 6.21 | 6.2 | 14 | 26.5 | 7 | 19.7 |
| 8 | 11.12 | 6 | 25 | 35.9 | 9 | 24.7 |
| 9 | 3.67 | 8 | 4 | 17.4 | 0.1 | 18.1 |
| 10 | 8.93 | 7 | 16 | 34.3 | 6.5 | 20.6 |
| 11 | 17.74 | 7.1 | 43 | 45.6 | 14 | 31.1 |
| 12 | 14.85 | 6.2 | 27 | 38.7 | 13.8 | 25.4 |

## 5- Conclusion

In this paper, the drawbacks of the integratedDEA model to find the most BCC-efficient DMU introduced by Toloo, (2012), is discussed. It was shown that this model may be infeasible in some cases,and in the feasible cases, it cannot correctly find the most BCC-efficient DMU, (see example 3). To overcome the drawbacks, a new integrated DEA model presented. It was proved that the proposed model is always feasible and can find the most BCC-efficient DMUs.Also, we argued that the most BCC-efficient DMU may not be single and in some cases there are several most BCC-efficient DMUs. To find and rank all most BCC-efficient DMUs, a new algorithm proposed. The proposed approach in this paper is applied to a real data oftwelve flexible manufacturing systems (FMSs) and nineteen facility layout designs (FLDs).

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