Journal of Industrial and Systems Engineering
Vol. 1, No. 3, pp 218-234
Fall 2007

Quality Loss Function – A Common Methodology for Three Cases

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ABSTRACT

The quality loss function developed by Genichi Taguchi considers three cases, nominal-the-best, smaller-the-better, and larger-the-better. The methodology used to deal with the larger-the-better case is slightly different than the other two cases. This research employs a term called target-mean ratio to propose a common formula for all three cases to bring about similarity among them. The target-mean ratio can take different values representing all three cases to bring consistency and simplicity in the model. In addition, it eliminates the assumption of target performance at an infinite level and brings the model closer to reality. Characteristics such as efficiency, coefficient of performance, and percent non-defective are presently not larger-the-better characteristics due to the assumption of target performance at infinity and the subsequent necessary derivation of the formulae. These characteristics can also be brought under the category of the larger-the-better characteristics. An example of the efficiency of prime movers is discussed to illustrate that the efficiency can also be considered as a larger-the-better characteristic. A second example is presented to suggest the subtle differences between both methodologies.

Keywords: Networked quality loss, Signal-to-noise ratio, Target-mean ratio.

1. INTRODUCTION

The following paragraph is taken from Taguchi's Quality Engineering Handbook (Taguchi et al., 2004):

"The larger-the-better characteristic should be nonnegative, and its most desirable value is infinity. Even if the larger the better, a maximum of nonnegative heat efficiency, yield, or nondefective product rate is merely 1 (100%); therefore, they are not larger the better characteristics. On the other hand, amplification rate, power, strength, and yield amount are larger-the-better characteristics because they do not have target values and their larger values are desirable."

Two types of performance characteristics are discussed in the paragraph above. First, the characteristics that have a maximum possible target of 100% are not larger-the-better (LTB) characteristics. Second, the characteristics that have infinity as the target value do not actually have a target value and are LTB characteristics. The LTB methodology requires infinity as the target.

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It is plausible to have a target value as infinity, in which case a target value is not assigned. The purpose of this paper is to explore why characteristics are not considered LTB when the target is known, as in the case of efficiency, yield, or non-defective. It seems that due to the assumption of infinity as the target, some of the characteristics are presently not LTB characteristics. The subsequent derivation of the formulae also supports the theory that there are some characteristics that do not fall under the category of LTB characteristics.

Taguchi's loss function approximates loss based on two reasons: (1) the variation, denoted by standard deviation, of performance about some mean and (2) the mean performance away from the target, denoted by the distance by which the mean performance is away from the target. In the case of smaller-the-better (STB) and nominal-the-best (NTB), both have clearly shown to affect the mean-squared deviation (MSD) and, in turn, quality loss. In Taguchi's existing approach for LTB, however, it is not clearly shown how variation of the performance affects the MSD or quality loss.

Maghsoodloo (1991) showed that MSD for LTB cases could be approximated using Equation (1). The MSD given for the STB and NTB cases is exact while it is approximate for LTB case.

$$MSD = \frac{1}{\bar{y}^2}\left[1 + \frac{3\sigma^2}{\bar{y}^2}\right] \tag{1}$$

Where,

$\bar{y}$ = mean performance, and
σ = standard deviation of performance.

A better approximation can be given by Equation (1a):

$$MSD = \frac{1}{\bar{y}^2}\left[1 + \frac{3\sigma^2}{\bar{y}^2} - \frac{4\hat{\mu}_3}{\bar{y}^3} + \frac{5\hat{\mu}_4}{\bar{y}^4}\right] \tag{1a}$$

Where the third central moment is given by Equation (1b):

$$\hat{\mu}_3 = \frac{1}{n}\sum\left(y_i - \bar{y}\right)^3 \tag{1b}$$

In which y_i represents the performance value of each product in the lot. And the fourth central moment is:

$$\hat{\mu}_4 = \frac{1}{n}\sum\left(y_i - \bar{y}\right)^4 \tag{1c}$$

Equations 1 and 1a for the LTB case clearly show how variation of the performance affects the MSD or quality loss.

We have just started a discussion on the idea of infinite target and the problem associated with the target at infinity. In the next section, we discuss the importance of the quadratic loss function in quality engineering. Then we deliberate if a target value of infinity is needed. In the theory section, relevant formulae are derived to incorporate the idea of some finite target in place of infinity. Two examples have been given to explain the methodology. Towards the end we suggest appropriate target-mean ratios for the LTB characteristic.

Quadratic Quality Loss Function And Signal-to-Noise Ratio

Robust design is achieved by applying a three-step decision making process:

1. Define the objective;
2. Define the feasible options; and
3. Select the feasible option that best meets the objective.

The best criterion to measure robust design is the signal-to-noise (S/N) ratio. Maximum robustness means minimum quality loss and maximum customer satisfaction. The S/N ratio recognizes and measures deviation from the nominal value and integrates the information into one metric (Taguchi et al., 1999). It is very important to define the measure of the quality loss and then incorporate the same into the design.

Several performance characteristics exist and it is important to distinguish between these when evaluating quality. Therefore, a different S/N ratio is needed for each performance characteristic. The nominal value is the best performance characteristic value for a given parameter. NTB should be used whenever possible because this allows the two-step optimization. The S/N ratio measures deviation from the nominal value allowing for subsequent adjustment.

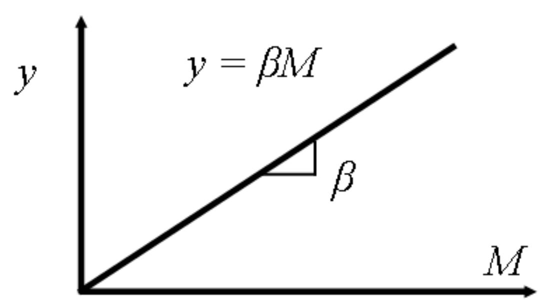

Figure 1: Response and Signal Relationship

A large S/N ratio means lower standard deviation. In the case of dynamic signals, e.g., steering wheel or brake pedal application, a series of dynamic S/N ratios exist. A signal factor is a control factor chosen that can modify output response in a linearly proportional way. For example, for a steering wheel, the turn angle of the steering wheel is the signal factor that adjusts the radius of curvature for vehicle motion as the output response. Similarly for a brake system, the brake pedal pressure is the signal factor which regulates the braking distance as the output response. Thus, an equation that measures the robustness of a system can be obtained. The objective for achieving a robust design is to have the highest S/N ratio (i.e., the smallest standard deviation or variation). Figure 1 shows a simple linear relationship between response y and signal factor M. A linear relationship between output or response and input or signal is the most desirable relationship for dynamic systems (Phadke, 1989; Fowlkes and Creveling, 1995).

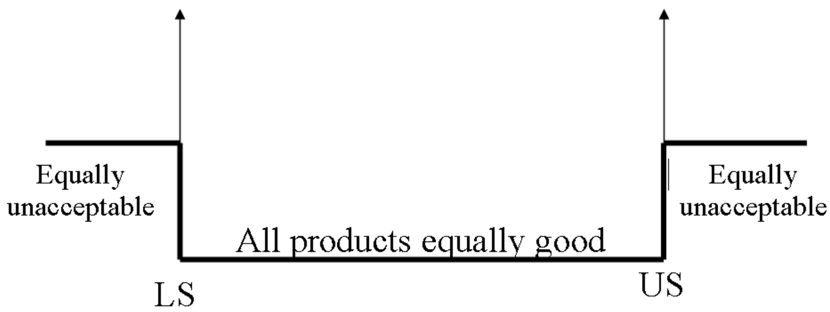

Figure 2: Goal Post Approach

In process parameter design, trials on several units of production are conducted, whereas in the product parameter design, the design is tested under different customer conditions. The main objective is satisfying the customer and not just meeting specifications. According to Figure 2, all of the parts within the specification limits are acceptable which suggests zero defects. Figure 3 implies that quality loss is incurred if parts are outside specification limits; the same way quality loss is incurred even if the parts are near the specification limits although within the limits. Optimum performance is achieved when variation is low and mean of performance is close to target. After understanding the customer's expectations, it is necessary to learn about the tools required to address these parameters.

From the customer viewpoint, there is no difference between products that their specifications are just inside or just outside the specification limits. Taguchi developed his quality loss function to convert customer satisfaction into a monetary value so that a manufacturer could estimate the loss to the company as a result of customer dissatisfaction.

The idea is to deliver a performance near the target (customer preference) which maximizes customer satisfaction value, thus, overriding the specification limits. Depending on the quality characteristics, this satisfaction level can be of three types, LTB, STB or NTB. When it is desirable to deliver a performance near the target the case is termed as NTB. In the cases of LTB these values need to be higher than and away from a certain threshold value. In the cases of STB these values need to be lower than and away from a certain limiting value.

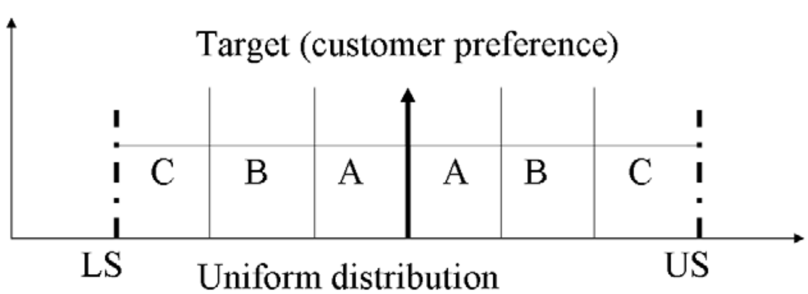

Figure 3: Product Performance Levels

It is important to understand the relationship of performance away from the target to quality loss. Products with smaller variation have smaller quality loss. The quality loss function essentially translates the qualitative terms, which affect the consumer, into quantitative terms such as monetary values. Depending on the situation, the quality loss function takes three forms:

1. Nominal-the-best (NTB) - the nominal value is best because it is the one that satisfies the customer’s need. The characteristic value away on either side of the target is undesirable, such as air pressure in vehicle tires or location of gauges on the instrument panel.

2. Smaller-the-better (STB) – a smaller value is better and higher values are undesirable, such as vehicle emissions or fuel consumption (dollar per distance).

3. Larger-the-better (LTB) – a larger value is better and smaller values are undesirable, such as gas mileage (distance per gallon).

Matching Performance to Intent

Quality can be divided into two types, i.e., design quality and production quality. As such, quality can be improved by improving design quality and production quality. This strategy basically minimizes the design loss and production loss in order to minimize quality loss. Design quality is reflected in product properties that are manifested when using the product. Design loss means off-target performance due to variable conditions. Robust design is designing and manufacturing a product that enhances customer satisfaction (Taguchi et al., 1999). Design intent should match the customer's requirements and production should match the design intent. All products that perform off-target produce a cost that is unnecessary. Elements of design quality include the following:

- Robust performance - Optimize the nominal values to achieve performance:
- Eliminate mistakes - Review the robust design: and
- Correct precision levels - Strike a balance between the manufacturing costs and precise tolerances.

In the development of new products, the most important step is early robust optimization which provides on-target performance. It is best to have a world-class quality system in hand that satisfies the customer.

For LTB characteristics, the intent is not to achieve an infinitely large performance value. That’s why other than the three cases of LTB, STB, and NTB, it is necessary to suggest two other cases that are “LTB to a point” and “STB to a point.” In many cases, higher values of performance characteristics than a certain value would add little value to the quality of the product. For example, additional tensile strength of a tire rubber may not help improve the quality of tire. What is required is wear-resistance at higher operating speeds and temperatures. Infinity is not in customer’s perspective. For example, a customer wants the cargo space in his vehicle to be LTB but only to a point. This characteristic (cargo space) requires tradeoffs with size, mass, fuel consumption and maneuverability for parking. The LTB case assumes that the larger the value of a parameter the better. In Taguchi’s loss function when the performance value approaches infinity the loss approaches zero. It is thus obvious that it does not represent reality. The target should never be infinity because it is unachievable. One view can be that infinity as the target in this case is notional and is so assumed as to facilitate mathematical derivation. However, there is no need for it to only be notional. Therefore, the formulation needs to be improved. The mean-squared deviation (MSD) for LTB is given in Equation (2):

$$MSD = \frac{1}{n}\sum_{i=1}^{n}\left(1/y_i\right)^2 \tag{2}$$

The quality loss for LTB is given in Equation (3):

$$L(y) = k\left[\frac{1}{n}\sum_{i=1}^{n}\left(1/y_i\right)^2\right] \tag{3}$$

In the formulae above, the MSD and quality loss cannot be zero regardless of how large the performance value is. It suggests that infinity is only sought after for achieving zero quality loss. Infinity is unachievable and impractical. Since “LTB to a point” is of interest it is appropriate to look for a better formulation that can use a target value rather than assuming it as infinite. The formula thus derived represents a closer to realistic situation.

This paper proposes that quality loss characteristics be comparable with one another in these three cases. In the model proposed, the case of NTB can be visualized with its target shifting from zero to a large value to generate all three cases. In the derivation, when the target shifts to zero, the case becomes STB. When the target is near the mean performance value, it becomes NTB. And when the target shifts to greater than the mean performance value, the case becomes LTB. For the case to be LTB, it is not necessary that the target shifts to infinity. By introducing this methodology, an attempt was made to streamline the mathematical as well as the practical aspects of the quality loss function.

This paper discusses how to define a target value in the case of LTB characteristics. However, it is worth mentioning here that the target should be difficult to achieve but not impossible. The achievability of the target might depend upon a technology change, innovation, material, or process.

Theory

Let us start with the NTB case. The derivation of Taguchi’s loss function expressed here is given in parts in Fowlkes and Creveling, 1995 (Also see Taguchi et al., 1989; Venkateswaren, 2003). If *y* is an observed value of a given parameter, and *m* is the target, then the loss function *L(y)* is given as follows:

$$\begin{aligned} L(y) &= L(m + y - m) \\ &= L(m) + \frac{L'(m)}{1!}(y-m) + \frac{L''(m)}{2!}(y-m)^2 + \ldots\ldots \end{aligned} \tag{4}$$

When $y = m$, quality loss and $L(y) = 0$, then $L(m) = 0$ Also, because *L(y)* attains its minimum value at this point, $L'(m) = 0$. If we neglect the higher order terms, Equations (5) and (6) are obtained:

$$L(y) = \frac{L''(m)}{2!}(y-m)^2 \tag{5}$$

$$L(y) = k(y-m)^2 \tag{6}$$

Δ_0 is defined as the point of intolerance as shown in Figure 4. It is the deviation from the target that causes an average customer to take an action. It is assumed that the corresponding monetary loss caused due to a defective component is A_0. A_0 is also defined as the cost of a corrective action. When the deviation of performance from the target of a product is Δ_0 and the corresponding loss is A_0, then for NTB and STB, $k = A_0 / \Delta_0^2$ (Phadke, 1989; Fowlkes and Creveling, 1995; Taguchi et al.,1999; and Taguchi et al., 2004;). Taguchi's loss function for the NTB and STB takes the form as in Equation (7).

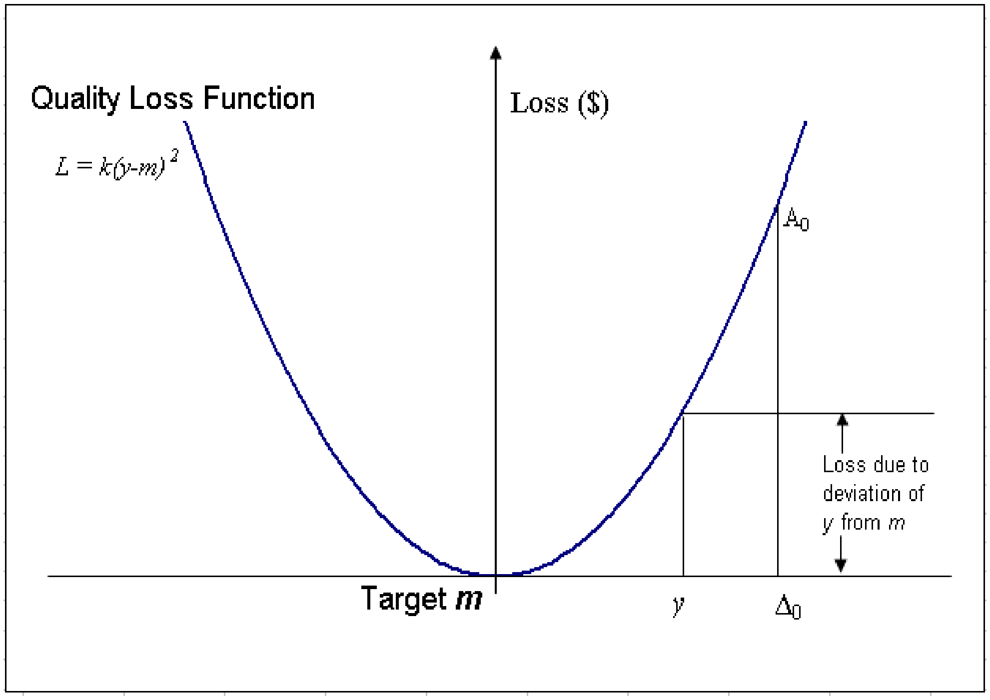

Figure 4: Loss Due to Off-Target Performance

$$L(y) = \frac{A_0}{\Delta_0^{\ 2}}(y-m)^2 \tag{7}$$

For the LTB, however, the quality loss function takes the form as in Equation (8) (Fowlkes and Creveling, 1995; Taguchi et al.,1999; and Taguchi et al., 2004).

$$L(y) = A_0 \Delta_0^{\ 2}\left(\frac{1}{y_i}\right)^2 \tag{8}$$

The term Δ_0 is taken to the numerator instead of the denominator which introduces an inconsistency among the NTB, STB, and LTB methods. Therefore, Δ_0 needs to be taken to the denominator instead of the numerator in order to bring back the consistency among all three cases.

The term $(y\text{-}m)^2$ is called the mean-squared deviation (MSD). For a group of n products, if the performance readings are $y_i = y_1, y_2, y_{3, \ldots}, y_n$, then the MSD for this group of n products can be derived as follows resulting in Equation (9):

$$
\begin{aligned}
MSD &= \frac{1}{n}\left[\left(y_1-m\right)^2+\left(y_2-m\right)^2+\ldots+\left(y_n-m\right)^2\right]\\
&=\frac{1}{n}\sum_{i=1}^{n}\left(y_i-m\right)^2\\
&=\frac{1}{n}\sum_{i=1}^{n}\left(y_i^2-2y_i m+m^2\right)\\
&=\frac{1}{n}\sum_{i=1}^{n}\left(y_i\right)^2+\frac{1}{n}\sum_{i=1}^{n}\left(-2y_i m+m^2\right)\\
&=\frac{1}{n}\sum_{i=1}^{n}\left(y_i\right)^2-2\overline{y}m+m^2\\
&=\frac{1}{n}\sum_{i=1}^{n}\left(y_i\right)^2-\overline{y}^2+\overline{y}^2-2\overline{y}m+m^2\\
&=\frac{1}{n}\sum_{i=1}^{n}\left(y_i\right)^2-2\overline{y}^2+\overline{y}^2+\left(\overline{y}-m\right)^2\\
&=\frac{1}{n}\sum_{i=1}^{n}\left(y_i\right)^2-\frac{2\overline{y}}{n}\sum_{i=1}^{n}y_i+\frac{1}{n}\sum_{i=1}^{n}\overline{y}^2+\left(\overline{y}-m\right)^2\\
&=\frac{1}{n}\sum_{i=1}^{n}\left(y_i\right)^2-\frac{1}{n}\sum_{i=1}^{n}2\overline{y}y_i+\frac{1}{n}\sum_{i=1}^{n}\overline{y}^2+\left(\overline{y}-m\right)^2\\
&=\frac{1}{n}\sum_{i=1}^{n}\left(y_i-\overline{y}\right)^2+\left(\overline{y}-m\right)^2
\end{aligned}
$$

$$MSD=\sigma^2+\left(\overline{y}-m\right)^2 \tag{9}$$

One technique to define the target is to set it equal to α (a ratio, referred to as target-mean ratio) multiplied by the mean performance. The advantage of doing this is that as the performance improves, the target also changes. Therefore, the target is not constant; rather it might change. By setting α equal to a ratio of the target and mean performance, Equation (10) is obtained.

$$\alpha=\frac{m}{\overline{y}};or;m=\alpha.\overline{y}$$

$$MSD=\sigma^2+\left(\overline{y}-\alpha.\overline{y}\right)^2$$

Or

$$MSD=\sigma^2+\overline{y}^2\left(1-\alpha\right)^2 \tag{10}$$

This equation can be used interchangeably to encompass all three cases.

Smaller-the-Better (STB)

As shown in Figure 5, in the case of STB, the target is zero. Therefore, by setting, *m* = 0 in Equation (10), Equation (11) is obtained. In this case, α is set at zero.

$$MSD = \sigma^2 + \bar{y}^2 \quad (11)$$

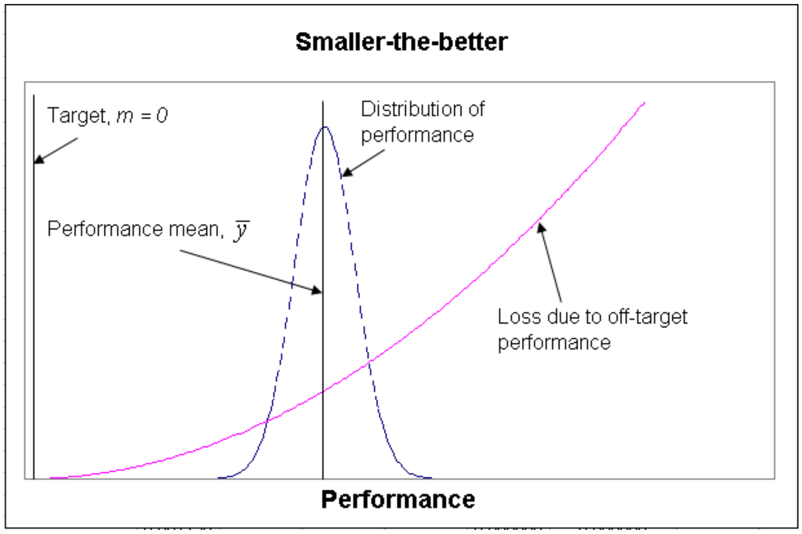

Figure 5: Smaller-the-Better

Nominal-the-Best (NTB) – Performance on Target

By setting, $m = \bar{y}$ for the NTB approach, where the value of α is set at one, we obtain Equation (12):

$$MSD = \sigma^2 \quad (12)$$

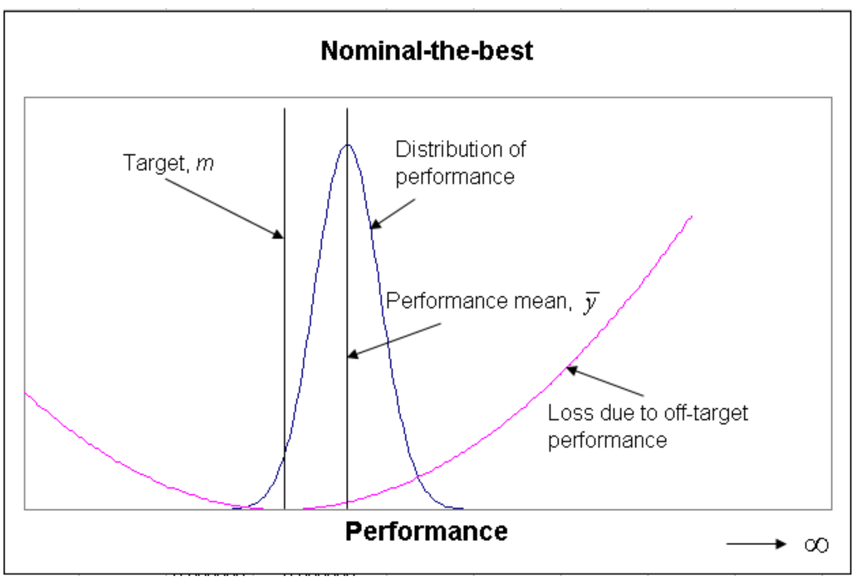

Figure 6: Nominal-the-Best

Nominal-the-Best (NTB) – Performance not on Target

When the performance is not on target formulas (13) or (14) can be used, refer to Figure 6.

$$MSD = \sigma^2 + (\bar{y} - m)^2 \quad (13)$$

By setting $m = \alpha\bar{y}$ in Equation 13:

$$MSD = \sigma^2 + \bar{y}^2(1-\alpha)^2$$

We obtain Equation (14):

$$MSD = \sigma^2 + \bar{y}^2(\alpha - 1)^2 \quad (14)$$

Larger-the-Better (LTB)

As shown in Figure 7, in the case of LTB, α needs to be significantly greater than one but not necessarily a large number or infinity.

For example, when α = 1.5, we have:

$$MSD = \sigma^2 + 0.25\bar{y}^2 \quad (15)$$

Equation (15) represents the LTB case with the target being equal to 1.5 times the mean performance.

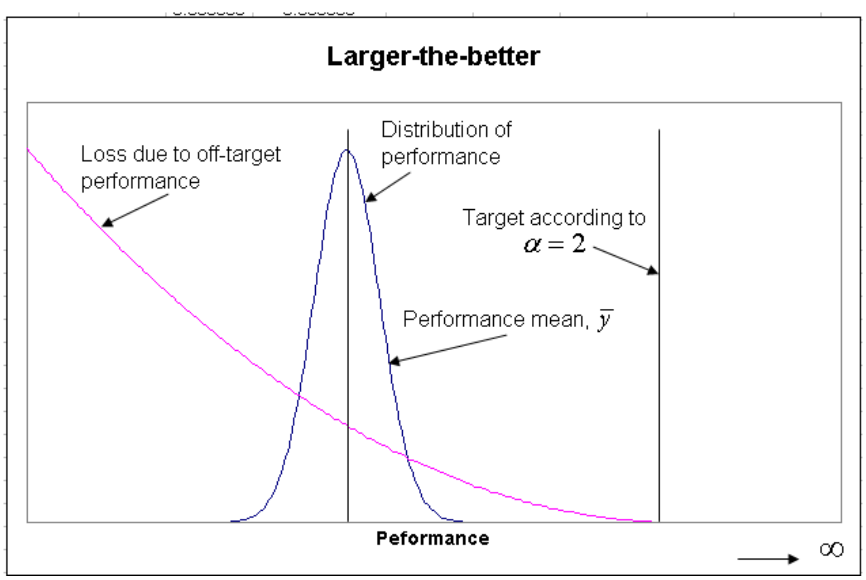

Figure 7: Larger-the-Better

When α = 2, we obtain Equation (16):

$$MSD = \sigma^2 + \bar{y}^2 \quad (16)$$

This is the LTB case with the target double the mean performance. This particular situation in LTB is analogous or equivalent to the STB case. The only difference is that the target is not zero but is placed equidistant from the mean on the right hand side. The target also is not assumed to be infinity or very large, though the case is LTB. The advantage of using this approach is that one can achieve comparable results in all the three cases. The following case studies verify the results.

2. CASE STUDIES

Let us consider a case of prime movers efficiency and illustrate how the new methodology can be used to assess the quality loss associated with the low efficiency, which according to Taguchi is not a LTB characteristic. However, this paper has considered the efficiency to be a LTB characteristic because we want it to be higher.

Example 1: Efficiency of engine / electric motor

Internal combustion engines were developed to serve as a primary source of power for automobiles, ships, airplanes, and many other mobile and stationary applications. Consider a manufacturer engaged in the manufacturing of internal combustion engines and simultaneously using engines and other prime movers for operating machines in a factory. The manufacturer has been manufacturing engines from the time of invention when the efficiency of engine was as low as 4%. Therefore, the manufacturer has gone through almost all possible phases of improvement in engine efficiency. The efficiencies of commonly used motor are given in Table 1, which also has an entry for the efficiency corresponding to an ideal motor. The values of engine efficiency given in Table 1 have been taken from Ferreira, 2005 shows the evolution of engine efficiency over a period of nearly a century. Engine manufacturing and their use can be seen as continuous improvement in the quality of the product and its production.

Table1: Evolution of Engine/Motor Efficiency

Machine	Engine	Engine	Engine	Engine	Engine	Engine	Hydrated alcohol Engine	Most efficient Engine	Motor	Motor	Energy efficient motor	Ideal motor
Year	1902	1923	1935	1958	1975	2000	2000	2005	1937	2005	2005	2007
Efficiency	0.04	0.07	0.10	0.20	0.28	0.32	0.38	0.52	0.76	0.90	0.96	1.00

Suppose there are eight sets of engines and four sets of electric motors. Because of their year of development and manufacture, each set of machines operates at an efficiency level that is different from the other sets. The mean efficiency levels for these twelve sets are 4%, 7%, 10%, 20%, 28%, 32%, 38%, 52%, 76%, 90%, 96% and 100%. It is also given that at 50% efficiency, the estimated loss is $10.

Table 2 shows the calculated values of α and MSD in terms of standard deviation and mean efficiency. It is evident from Table 2 that the quality loss depends on the mean efficiency and standard deviation of the performance. It is also depicted how much quality loss can be attributed to variation and how much to the distance between the performance mean and the target. With this

methodology, the root cause can be identified and by reducing the effect of root cause the quality loss can be reduced. In contrast, the LTB case in Taguchi's methodology does not show whether the variation or performance away from target causes a quality loss.

Table 2: Target-mean ratio and MSD

Efficiency, $\bar{y}$	A	*MSD*
4%	25	$\sigma^2 + 576\,\bar{y}^2$
7%	14.29	$\sigma^2 + 176.6241\,\bar{y}^2$
10%	10	$\sigma^2 + 81\,\bar{y}^2$
20%	5.00	$\sigma^2 + 16\,\bar{y}^2$
28%	3.57	$\sigma^2 + 6.6049\,\bar{y}^2$
32%	3.13	$\sigma^2 + 4.5369\,\bar{y}^2$
38%	2.63	$\sigma^2 + 2.6569\,\bar{y}^2$
52%	1.92	$\sigma^2 + 0.8464\,\bar{y}^2$
76%	1.32	$\sigma^2 + 0.1024\,\bar{y}^2$
90%	1.11	$\sigma^2 + 0.0121\,\bar{y}^2$
96%	1.04	$\sigma^2 + 0.0016\,\bar{y}^2$
100%	1.00	$\sigma^2 + 0.0000\,\bar{y}^2$

The value of k equates to $10/0.5^2$ = \$40. The associated loss in dollars in each case can be computed by multiplying MSD with k once MSD for each efficiency level is known. A comparison between the computations of quality loss using the new method and the Taguchi method is provided in Table

Table 3: Comparison of Quality Loss

k_{STB}	Mean efficiency $\bar{y}$	Target-mean ratio	Standard deviation	MSD-Variance	MSD-Bias	MSD-New	Quality loss (\$) - New method	$\frac{1}{\bar{y}}$	k_{LTB}	MSD-Taguchi	Quality loss (\$) - Taguchi method
40	0.04	25.00	0.01	0.0001	0.922	0.922	36.86	25.00	2.5	625.00	1562.50
40	0.07	14.29	0.01	0.0001	0.865	0.865	34.60	14.29	2.5	204.08	510.20
40	0.10	10.00	0.01	0.0001	0.810	0.810	32.40	10.00	2.5	100.00	250.00
40	0.20	5.00	0.01	0.0001	0.640	0.640	25.60	5.00	2.5	25.00	62.50
40	0.28	3.57	0.01	0.0001	0.518	0.519	20.74	3.57	2.5	12.76	31.89
40	0.32	3.13	0.01	0.0001	0.462	0.463	18.50	3.13	2.5	9.77	24.41
40	0.38	2.63	0.01	0.0001	0.384	0.385	15.38	2.63	2.5	6.93	17.31
40	0.52	1.92	0.01	0.0001	0.230	0.231	9.22	1.92	2.5	3.70	9.25
40	0.76	1.32	0.01	0.0001	0.058	0.058	2.31	1.32	2.5	1.73	4.33
40	0.90	1.11	0.01	0.0001	0.010	0.010	0.40	1.11	2.5	1.23	3.09
40	0.96	1.04	0.01	0.0001	0.002	0.002	0.07	1.04	2.5	1.09	2.71
40	1.00	1.00	0.00	0	0.000	0.000	0.00	1.00	2.5	1.00	2.50

3. Standard deviation for each efficiency level is assumed to be small and constant at 0.01 except at the ideal efficiency of 100%, where it would not make sense to have variation and still have mean efficiency at 100%. Figure 8 shows two graphs of the mean efficiency versus quality loss using the new methodology and the Taguchi methodology.

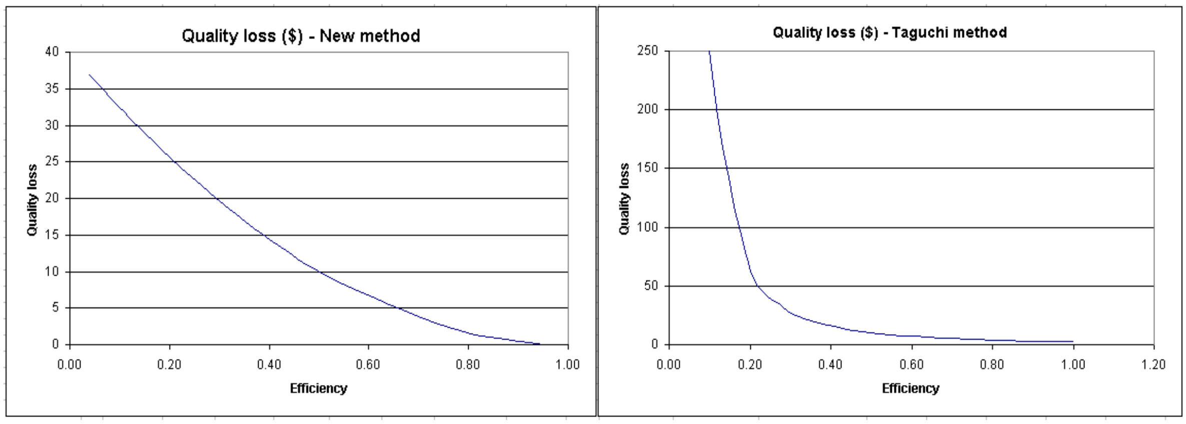

Figure 8: Comparison of Quality Losses

The results from Table 3 were also used to plot these two graphs. It is observed from the computation as well as from the graphs that the quality loss is zero if the maximum possible efficiency is achieved when the new methodology is employed. In comparison, at the ideal situation, the Taguchi methodology delivers a small quality loss. Moreover, the new methodology gives a finite loss when the efficiency approaches zero. On the other hand, the Taguchi methodology suggests that the loss is infinite at zero efficiency. The Taguchi method does not produce realistic results at both the ideal boundary conditions. The proposed methodology, on the other hand, produces realistic results at all increments including the ideal boundary conditions.

Example 2: Thermal conductivity of material

In the second example, a characteristic was selected which can be STB in certain situations and LTB in certain other situations. Among many such characteristics a simple and more common property, thermal conductivity of a material, the data observed is given Table 4, was selected to illustrate the example. A numerical problem is considered as a STB case and solved using the Taguchi method. The results achieved are given in the column designated as A* in Table 5. The same STB numerical problem is then converted into a LTB numerical problem. Next, the LTB problem is solved using the Taguchi approach and the results are presented in the column designated as B* in Table 5. The LTB problem is also solved using the proposed approach and the results are shown in the column C* in Table 5. The results in columns A*, B*, and C* are then compared to appreciate the difference between Taguchi methodology for LTB and proposed methodology for LTB and the equality or correspondence between Taguchi methodology for STB and proposed methodology for LTB.

Smaller-the-Better Numerical Problem

It is assumed that stainless steel is used as an insulator because of its strength and formability. Twenty sample pieces of stainless steel are drawn from a production system for inspection with regard to heat/thermal conductivity. The following readings are observed as shown in Table 4. The unit used for thermal conductivity is ($W \cdot m^{-1} \cdot K^{-1}$).

Thermal conductivity is required to be within (or no more than) 15 $W \cdot m^{-1} \cdot K^{-1}$. Because of heat dissipation, the estimated quality loss is $2. The quality loss due to the production is then calculated. The STB problem is solved using Taguchi's approach and the results can be seen in

Table 4: Thermal Conductivity Readings

13.97	14.04	14.09	13.99	14.00	14.06	13.92	13.93	14.01	14.03
15.12	15.09	13.93	13.98	14.02	14.05	14.08	13.98	14.00	14.11

Column A* of Table 5. Table 5 also contains results obtained using LTB problem also.

Larger-the-Better Numerical Problem

Using the same thermal conductivity readings, the quality loss due to the production is then calculated. The value 15 is chosen here to keep the data similar between STB and LTB. The LTB

Table 5: Comparison of Results

STB	LTB		LTB	LTB	LTB	LTB	LTB
Taguchi Method	Taguchi Method		New Method				
			α=1.5	α=2	α=2.5	α=4	α=5
y_i	y_i	$1/y_i^2$	y_i	y_i	y_i	y_i	y_i
13.97	13.97	0.00512	13.97	13.97	13.97	13.97	13.97
14.04	14.04	0.00507	14.04	14.04	14.04	14.04	14.04
14.09	14.09	0.00504	14.09	14.09	14.09	14.09	14.09
13.99	13.99	0.00511	13.99	13.99	13.99	13.99	13.99
14.00	14.00	0.00510	14.00	14.00	14.00	14.00	14.00
14.06	14.06	0.00506	14.06	14.06	14.06	14.06	14.06
13.92	13.92	0.00516	13.92	13.92	13.92	13.92	13.92
13.93	13.93	0.00515	13.93	13.93	13.93	13.93	13.93
14.01	14.01	0.00509	14.01	14.01	14.01	14.01	14.01
14.03	14.03	0.00508	14.03	14.03	14.03	14.03	14.03
15.12	15.12	0.00437	15.12	15.12	15.12	15.12	15.12
15.09	15.09	0.00439	15.09	15.09	15.09	15.09	15.09
13.93	13.93	0.00515	13.93	13.93	13.93	13.93	13.93
13.98	13.98	0.00512	13.98	13.98	13.98	13.98	13.98
14.02	14.02	0.00509	14.02	14.02	14.02	14.02	14.02
14.05	14.05	0.00507	14.05	14.05	14.05	14.05	14.05
14.08	14.08	0.00504	14.08	14.08	14.08	14.08	14.08
13.98	13.98	0.00512	13.98	13.98	13.98	13.98	13.98
14.00	14.00	0.00510	14.00	14.00	14.00	14.00	14.00
14.11	14.11	0.00502	14.11	14.11	14.11	14.11	14.11
14.12	← Mean →	0.00502	14.12	14.12	14.12	14.12	14.12
0.3323	← SD →	0.0002	0.3323	0.3323	0.3323	0.3323	0.3323
2	← A_0 →	2	2	2	2	2	2
15	← Δ_0 →	15	15	15	15	15	15
0.008889	← k →	450	0.008889	0.008889	0.008889	0.008889	0.008889
199.4848	← MSD →	0.0051	49.9540	199.4848	448.7028	1794.4800	3190.1008
1.77	← QL ($) →	2.31	0.44	1.77	3.99	15.95	28.36
A*		B*		C*			

problem is solved using Taguchi's approach where the results are shown in Column B* of Table 5. The same LTB problem is also solved using the new approach where the results are provided in Column C* of Table 5.

Comparison of Results

When the STB problem is solved using Taguchi's approach, the quality loss computed is $1.77 per piece. After the same problem is converted to LTB, the quality loss computed using the Taguchi's approach changes to $2.31 per piece which is 30.5% higher than the previous value. This signifies inadequate consistency in Taguchi's method for STB and LTB because the formulae are different. When the converted LTB problem is solved using the new methodology, choosing $\alpha = 2$, the quality loss is restored to its original value of $1.77 per piece.

Let us see how α affects the quality loss. Five values of α have been considered: 1.5, 2, 2.5, 4, and 5. The quality loss increases somewhat quadratically with α. Figure 9, shown in the next section, illustrates the relationship between the target-mean ratio, α, and the quality loss.

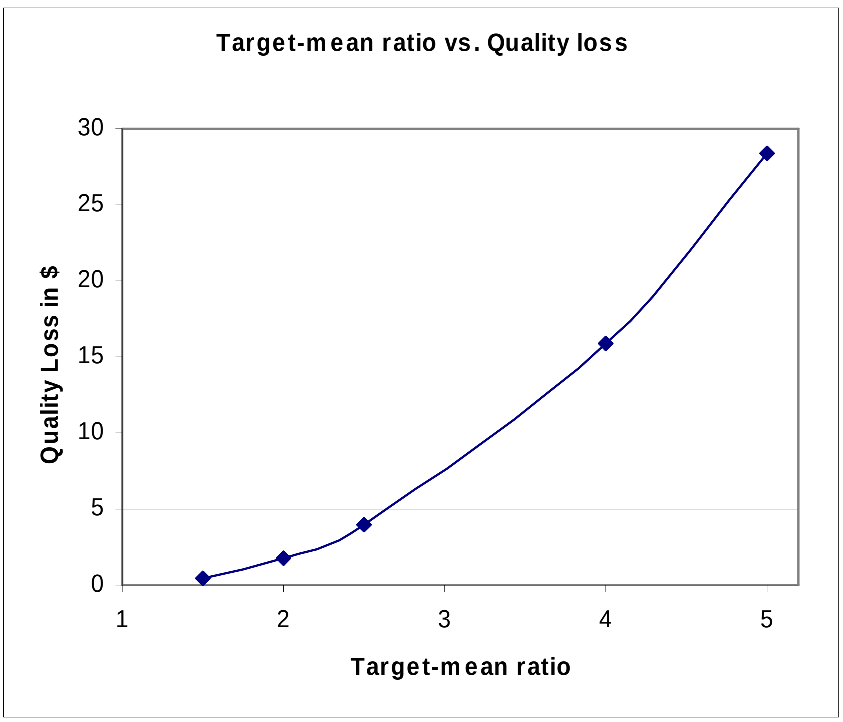

Figure 9: Alpha versus Quality Loss

Value of Target-Mean Ratio

Target-mean ratio eliminates the assumption of target at infinity for LTB case and can be used as a tool to bring about similarity among all three cases. A similar term scaling ratio is used in robust engineering after the variation is minimized using signal-to-noise ratio to modify the adjustment factor in order to put the system performance on target. The value of α can take different values depending on the type of case. Two types of LTB characteristics exist: (1) LTB characteristics which have a naturally available target, i.e., efficiency and coefficient of performance (COP) characteristics, and (2) characteristics which have no obvious-target such as strength of material.

There are two types of LTB performance parameters which have a natural target. The first type includes those parameters which have an ideal limit of performance such as efficiency, coefficient of performance, and percent nondefective as opposed to percent defective. Here the target value is naturally available and any target value, even theoretically, more than 100% for efficiency is not possible. Similarly for COP, a given maximum value cannot be exceeded. In such cases, it is recommended to set the target at its ideal limit. For example, if the present performance of a machine is at 33.33% and the ideal limit is 100%, then the target is set at 100%. In this case, the value of α equates to 3. Similarly, if the present performance of a machine is at 80% and the ideal limit is 100%, then the target is set at 100%. In this case the value of α equates to 1.25.

For no obvious-target characteristics, the target can be set at α equal to a specific value. In this case, it is theoretically possible to assume a certain higher target value. It is worthwhile to mention that

the target should be difficult to achieve but not impossible. Whether the target can really be achieved might depend upon technology change, innovation, material type, process, or conditions, etc. The target-mean ratio α needs to be significantly greater than 1 but does not need to be infinity. Since the loss is equivalent to that for STB at α=2 it is recommended that the target be set according to $\alpha=2$. It can easily be visualized that the target is placed equidistant from the mean on the other side as the mean is away from zero. The target also need not be assumed as infinity though the case is LTB. The examples discussed above verify the results.

3. CONCLUSION AND FUTURE RESEARCH

This paper attempts to present a similarity among all three cases of quality loss function by employing the target-mean ratio and proposing a common formula for all three cases. It is shown that the target-mean ratio can take different values to represent all three cases. The new method brings about uniformity with regard to the methodology among all three cases of STB, NTB, and LTB. It leads to consistent results and, because of this consistency the results can be compared easily. Also, it is easy to compute the quality loss in the case of LTB using the proposed method. The proposed method eliminates the need to assume the target value as infinity.

The proposed methodology is especially suitable for characteristics such as efficiency, coefficient of performance, and percent non-defective among many other characteristics. The same model can be converted into any of the three cases, i.e., STB, NTB, or LTB. It permits a target to be assumed for LTB cases so that improvement can be measured with reference to the target. With this approach, there is a possibility that the quality loss may become zero for a certain case when the target is reached. In contrast, with Taguchi's approach, this is only possible if the performance characteristic reaches an infinite value.

The proposed methodology also permits most characteristics that could not be designated as LTB characteristics to be brought under the LTB category. In the case of characteristics where the target value is naturally available it is recommended to set the target at an ideal limit. Finally, in the case of parameters which have no obvious ideal performance limit, unlike efficiency, it is recommended to set the target at two times the present mean performance, i.e., α may be set equal to 2. By introducing the new methodology an attempt was made to streamline the mathematical aspect as well as the practical aspect of the quality loss function.

The methodology suggested using the target-mean ratio to determine the target for LTB characteristics. More research should be done on selecting the appropriate target-mean ratio for different types of LTB characteristics such as coefficient of performance, efficiency, percent defective, and strength of a material. Further research is also needed to study the implications of this methodology on the concept of signal-to-noise ratio and optimization of systems using signal-to-noise ratio. Also further research may be conducted as to how the signal-to-noise ratio for operating window is affected due to new methodology. Another possibility is to extend quality loss function thus obtained for LTB characteristics to multivariate cases wherein other types (e.g., STB and NTB) are also part of quality loss function.

Acknowledgement

We are grateful to the University of Missouri-Rolla in particular Engineering Management and Systems Engineering for providing opportunity and funding for this research. We also thank our colleagues in the Design Engineering Center for their involvement in discussion on the thought of finite target in place of infinity.

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