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One-for-One Period Policy and its Optimal Solution

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ABSTRACT

In this paper we introduce the optimal solution for a simple and yet practical inventory policy with the important characteristic which eliminates the uncertainty in demand for suppliers. In this new policy which is different from the classical inventory policies, the time interval between any two consecutive orders is fixed and the quantity of each order is one. Assuming the fixed ordering costs are negligible, lead times are constant, and demand forms a Poisson process, we use queuing theory concepts to derive the long-run average total inventory costs, consisting of holding and shortage costs in terms of the average inventory. We show that the total cost rate has the important property of being entirely free of the lead time. We prove that the average total cost rate is a convex function and thus has a unique solution. We, then derive the relation for the optimal value of the time interval between any two consecutive orders. Finally we present a numerical example to compare the performance of this new policy with the classical one-for-one ordering policy. The provided example intends to re-examine the optimality of (*s*, *S*) policy in continuous review inventory models as well to establish the fact that even for the case where demand forms a Poisson process the optimality does not hold.

Keywords: Inventory control, One for one period policy, (*s*, *S*) Policy, Queuing system, Poisson demand

1. INTRODUCTION

Two fundamental questions that must be answered in controlling the inventory of any item are *when* and *how much* should be ordered for replenishment. The answers of these questions determine the *inventory control policy*. In the face of uncertainty it is more complicated to determine the inventory control policy. The most frequent uncertain factor is the stochastic nature of customer demand.

Most of the classical policies for controlling the inventory are derived from the (*s*, *S*) policy and are classified as periodic or continuous review policies. The distinction involves the frequency with which the inventory level must be observed (reviewed) in order to implement the policy. A periodic-policy is one in which the inventory level is observed only at equally spaced points in time (The time interval between such two points is called a "period." If the length of the period is *T*, then (*s*, *S*) policy is substituted by the (*s*, *S*, *T*) policy. If the maximum stock level in (*s* ,*S*, *T*) policy equals to the reorder point, then another classical policy called periodic review (*R*, *T*) is formed, where *R*=*S* is the maximum inventory level.

A continuous-review policy requires knowledge of the inventory level at all times. If the customer demand is one unit, the ordering size (*Q*) is always equal *S*-*s*. Therefore, the (*s*, *S*) policy changes into the well known continuous review (*r*, *Q*) policy, where *r*=*s* is the reorder point. If the ordering

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cost is negligible in the (r, Q) policy, then the policy is called one-for-one, (S-1, S), or base stock policy (Hadley and Whitin, 1963, and Love, 1979).

Hadley and Whitin (1963), Johnson and Montgomery (1974), Love (1979). Zipkin (2000), and Nahmias (2005), among others, present the methods to find the optimal or near optimal solution to minimize the inventory costs at a single stocking point with stochastic demand, based on the continuous review (r,Q), periodic review (R,T), and one-for-one polices.

Due to its widespread acceptance, one can find an abundance of work dealing with (s, S) policy in the literature (See, for example, Beckman, 1961; Sherbrooke, 1968; Nahmias, 1976; Sahin, 1979, 1982, 1990; Kruse, 1980, 1981 Archibald, 1981; Grave, 1985; Moinzadeh and Lee, 1986; Sivlazian, 1974; Zipkin, 1986a, 1986b; Axsäter , 1990, 1993, 2003; Lee and Moinzadeh, 1987a, 1987b Zheng, 1992 ; Federgruen and Zheng, 1992; Axsäter et al., 1994; Nahmias and Smith, 1994; Matta and Sinha, 1995; Forsberg, 1995, 1996, 1997; Axsäter and Zhang, 1999; Ganeshan, 1999; Andersson and Melchiors, 2001; Moinzadeh, 2002; Seo et al, 2002; Marklund, 2002; Ghalebsaz et al., 2004).

While it is well known that for the case of stochastic demand, there is no inventory control policy in which both the order size and the order interval are constant with an optimal solution, in this paper, we introduce the optimal solution for a new policy which is different from the classical inventory policies used in the literature of inventory and production control systems. This new policy is to order 1 unit at each fixed time period T. We call this new policy *(1, T)* or *one for one period ordering policy*. In this policy *the time interval between two consecutive orders and the value of the order size are both constant.*

In what follows we discuss the advantages of this policy. Then, assuming that the fixed ordering cost is zero or negligible, demand process is a Poisson process, and the replenishment lead time is constant, we derive the long-run average total cost, the total holding and shortage costs, per unit time in terms of the average inventory for the lost sales case. To obtain the total cost rate we use some concepts from queueing theory. We prove that this total cost is a convex function in terms of the average inventory and has a unique optimal solution.

Further, we obtain the optimal value of T, the time interval between two consecutive orders, which minimizes the long-run average total cost. Finally, by a numerical example we compare the long-run average total cost of this policy with the total cost rate of the base stock, one-for-one ordering, policy for the lost sales case. The provided example intends to re-examine the optimality of (s, S) policy in continuous review inventory models in order to establish the fact that even for the case where demand forms a Poisson process the optimality does not hold.

The optimality of (s,S) policy has been addressed in the literature of inventory control by a number of scholars (Scarf, 1960, Iglehart,1963, and Sahin 1990). The optimality of the (s, S) policy was first investigated by Schultz (1989) (Moinzadeh, 2001). He considered an (S-1, S) inventory system with an arbitrarily chosen distribution for the inter-demand times and fixed lead time. In order to simplify the analysis, he assumes that orders can also be expedited and received immediately at a premium cost. Furthermore, he only considers a special case of (S-1, S) inventory policies with S=1; that is, a maximum of one unit of inventory is allowed at any point in time. Schultz (1989) shows that by delaying the placing of orders until after demand occurs, one can achieve a lower expected total cost rate in such systems. Although Schultz (1989) examines the concept of order delays as a policy parameter, he fails to explicitly model the relationship between the delay and stocking policy parameters and only optimizes the delay in this somewhat simplified model (recall that the maximum inventory level, *S*, is pre-set at unity). It is believed that in many situations the assumption that orders can be expedited and received instantaneously does not hold and setting the value of *S*=1 will lead to inferior policies (Moinzadeh, 2001). Thus the joint determination of the optimal inventory policy and the delay becomes critical.

Later, Moinzadeh (2001) revisited the optimality of (*s*, *S*) policies in continuous review inventory models where demand forms a renewal process. He explains why when orders are placed at demand epochs, (*s*, *S*) policies are not optimal in general. By introducing a delay in order placement as a policy parameter, he proposes a new policy to establish this fact. He investigates the optimality of the (*s, S*) policy for Gaussian distribution for the inter-demand times to show the merits of his proposed policy. When the inter-demand times follow a general distribution he develops an efficient heuristic to compute the policy parameters. He shows that under his proposed policy when inter-demand times are exponential the optimal delay time becomes zero. That is, for Poisson demand his proposed policy reduces to the (*s, S*) policy. Based on the results contained in his work one may conclude that if the demand follows a Poisson process then the optimality of (*s, S*) policy holds true.

The example presented in this paper re-examines the optimality of (*s, S*) policy in hope of achieving the goal that the optimality of (*s, S*) policy in continuous review inventory models does not hold even for the case where demand follows a Poisson process.

2. ADVANTAGES OF *(1, T)* POLICY

When the demand is uncertain, in the classical inventory policies at least one of the two values, the order quantity and the time interval between any two consecutive orders, has an uncertain nature. For example, consider the continuous review (*r*, *Q*) and the periodic review (*R*, *T*) policies which are two usual policies employed by practitioners when demand is uncertain. When (*r*, *Q*) policy is used, the order quantity, *Q*, is fixed but the time interval between any two consecutive orders has an uncertain nature. For the case of (*R*, *T*) policy, the time interval between any two consecutive orders, *T*, is fixed whereas the value of the order quantity is uncertain. Therefore, the orders which constitute the demands for the supplier will have an uncertain nature. Hence, the uncertainty spreads into the supply chain.

For the new *one for one period* policy, discussed in this paper an order for one unit of item is placed in a pre-determined time interval. Hence, both the order size and the order interval are constant. Therefore, this policy prevents expanding the demand uncertainty for the supplier. That is, the demand for the supplier is deterministic, one unit every *T* units of time.

The advantages of this policy, which eliminates uncertainty in demand and leads to a uniform and deterministic demand for the supplier, are:

1- The safety stock in supplier is eliminated. (cost reduction)
2- Shortage cost in supplier due to uncertainty in demand is eliminated.
3- Information exchange cost for supplier due to the elimination of uncertainty of its demand is eliminated.
4- Inventory control and production planning in supplier are simplified.
5- This policy is very easy to apply.

3. COST EVALUATION

Using this policy in which an order constantly is placed for one unit of product in a pre-determined time interval, we derive the total inventory cost rate, consisting of holding and shortage costs in terms of the average on hand inventory. The objective is to determine the optimal time interval between any two consecutive orders.

Assumptions:

1- Unsatisfied demand will be lost.
2- The lead time for an order is constant.
3- The fixed ordering cost is zero or negligible.

Notation:

μ	The demand rate.
π:	Cost of a lost sale.
h	Rate of holding cost.
T	Time interval between any two consecutive orders.
I	Average inventory on hand for the (1, T) policy.
H	Average holding cost per unit time.
Π	Average shortage cost per unit time.
K_T	Average total cost rate, for the (1, T) policy.
K_S	Average total cost rate, for the (S-1, S) policy.
L	Lead time.

3.1. Methodology

To obtain the average inventory level, one can resort to some concepts of queuing theory. To do this, consider the arrival units of product to the system as the arrival process to a queuing system, the inter-demand times to the system as the service times of these units, and inventory on hand as the number of units in the queuing system. Hence, the inventory problem can be interpreted as a *D/M/*1 (Haji et al, 2006), a single channel queuing system in which the inter-arrival times are constant, equal to T, and the service times have exponential distribution with mean $1/\mu$. Thus, the arrival rate of units to the system is $\lambda = 1/T$ and the service rate is μ.

Let P_0 denote the proportion of time that the number in system is zero, then for a single channel queue

$$P_0 = 1 - \rho$$

Where ρ is the ratio of the arrival rate $\lambda = 1/T$ to the service rate μ, i. e.,

$$\rho = \frac{\lambda}{\mu} = \frac{1}{T\mu} \tag{1}$$

Also, let I denote the long-run average number of units in system, then for the queuing system *G/M/*1: we can write (Ross, 1993):

$$I = \frac{\rho}{1-\beta} \tag{2}$$

Where β is the solution of the following equation:

$$\beta = \int_0^\infty e^{-\mu t(1-\beta)} dG(t) \qquad (3)$$

It can be shown that if the mean of *G* is greater than the mean service time $1/\mu$, then there is a unique value of β satisfying Equation (3) which is between 0 and 1. The exact value of β usually can be obtained by numerical methods.

The system under consideration is a *D/M/*1 queuing system in which the inter-arrival time of units of product to the queueing system is constant and is equal to *T*. Thus $G(T) = 1$, and we can write (3) as

$$\beta = e^{-T\mu(1-\beta)} = e^{-\frac{\mu}{\lambda}(1-\beta)} \qquad (4)$$

Or

$$\beta = e^{-\frac{(1-\beta)}{\rho}} \qquad (5)$$

Since ρ is independent of the lead time, *L*, so is β as defined in (4) and (5) which in turn leads to independence of *I* in (2) from lead time.

From (2) we can write

$$\beta = e^{-\frac{1}{I}} \qquad (6)$$

The average holding cost per unit time is:

$$H = hI \qquad (7)$$

Since P_0 is the proportion of time that the system is out of stock-. Thus the proportion of demand that is lost is μP_0 and the average lost sale cost per unit time is:

$$\Pi = \pi\mu P_0 = \pi\mu(1-\rho) \qquad (8)$$

Or from (2) and (6) we can write (8) as:

$$\Pi = \pi\mu\left[1 - I(1 - e^{-\frac{1}{I}})\right] \qquad (9)$$

Thus, the total cost rate is:

$$K_T = H + \Pi \qquad (10)$$

Or from (7) and (9)

$$K_T = hI + \pi\mu\left[1 - I(1 - e^{-\frac{1}{I}})\right] \tag{11}$$

As mentioned before, since *I* is independent of the value of the lead time, one can easily conclude that the total cost rate for the (1, *T*) policy is independent of the lead time.

We now prove the following theorem for function K_T as specified in (11).

Theorem K_T is a convex function

Proof: The first derivative of *K* with respect to *I* is:

$$\frac{dK_T}{dI} = h + \pi\mu\left[-1(1 - e^{-\frac{1}{I}}) - I(-\frac{1}{I^2}e^{-\frac{1}{I}})\right]$$

Or

$$\frac{dK_T}{dI} = h - \pi\mu + \pi\mu\left[e^{-\frac{1}{I}} + \frac{1}{I}e^{-\frac{1}{I}}\right] \tag{12}$$

The second derivative of K with respect to *I* is:

$$\frac{d^2K_T}{dI^2} = \pi\mu\left[\frac{1}{I^2}e^{-\frac{1}{I}} - \frac{1}{I^2}e^{-\frac{1}{I}} + \frac{1}{I}(\frac{1}{I^2}e^{-\frac{1}{I}})\right]$$

Or equivalently

$$\frac{d^2K_T}{dI^2} = \pi\mu\frac{1}{I^3}e^{-\frac{1}{I}} \tag{13}$$

It is clear from (13) that the second derivative of *K* with respect to *I* is positive for all values of $I > 0$. This means that *K* is a convex function and thus it has a unique solution which proves the theorem.

The optimal *I*, *I**, is the solution of

$$\frac{dK_T}{dI} = h - \pi\mu + \pi\mu\left[e^{-\frac{1}{I^*}} + \frac{1}{I^*}e^{-\frac{1}{I^*}}\right] = 0$$

Or

$$e^{-\frac{1}{I^*}} + \frac{1}{I^*}e^{-\frac{1}{I^*}} = (\pi\mu - h)/\pi\mu$$

To find the optimal value of the time interval between two consecutive orders, T^*, note that from (1), (2), and (6) we can write

$$\frac{1}{\mu T^*} = I^*(1 - e^{-\frac{1}{I^*}})$$

Thus, the optimal T, T^*, can be obtained from the following relation

$$T^* = \frac{1}{\mu I^*(1-e^{-\frac{1}{I^*}})} \tag{14}$$

4. NUMERICAL RESULTS

This section is devoted to comparison between the one-for-one period that is, the (1, T) policy with the one-for-one policy in terms of total cost rate.

We study the effect of varying the values of L and π/h on the total cost rates for both policies. The adopted values for the parameters are:

$$h = 1; \ \mu = 1, 2, 3;$$

$$\pi / h = 0.1, 0.5, 1(1)10;$$

$$L = 1(1)10\ (5)\ 20\ (10)\ 60.$$

The total cost rate for, (S-1,S), is as follows (Tijms, 1986):

$$K_S = h\left[S - L\mu\left(1-((L\mu)^j/S!)\Big/\sum_{x=0}^{S}(L\mu)^j\big/j!\right)\right] + \pi\mu((L\mu)^j/S!)\Big/\sum_{x=0}^{S}(L\mu)^j\big/j! \tag{15}$$

Where L stands for the lead time.

It is obvious from (15) that K_S depends on the lead time but K_T is entirely independent from the lead time as mentioned before.

Table 1 presents the outcome of comparing the total cost rates for the (1, T) and (S-1, S) policies. In this table we consider the values 0.1, 0.5, 1, 2, 3, and 4 for π / h. Additional values for π / h are used in Table 2. The value of μ in Table 1 and Table 2 is assumed to be 1 while in Tables 3 and 4 μ is 2.

The upper section of Table 1 presents the optimal values of T (cycle times), T^*, the optimal values of inventory on hand, I^*, and the optimal values of total cost rates, K^*_T, for the (1, T) policy for different values of π / h. The lower section of this table displays the optimal values of S as well as the optimal values of the total cost rates, K^*_S, for the (S-1, S) policy for different values of L and π / h. As the contents of Table 1 indicate the performance of (1, T) and (S-1, S) policies are exactly the same for all values of L as long as π / h does not exceed 1 (obviously $S \geq 0$).

When $\pi / h \geq 2$, application of (S-1, S) policy amounts to a lower total cost rate provided that, in the worst case considered, $L \leq 8$. Each row of this table also provides the total cost rate differentials, $\Delta K = K_S - K_T$, for various values of L. As can be seen the $Max\ \Delta K$ occurs at the largest values of L and the largest values of π / h, i.e., $Max\ \Delta K$ = 5.0302 – 4.1246 = 0.9056 corresponding to L = 60 and π / h = 10. For the values of parameters used in this example, the maximum percent change in total cost increase due to application of (S-1, S) policy instead of (1, T) policy is more than 21 percent.

Table 1. Cost comparisons between (1, T) and (S-1, S) policies for μ = 1

T^*	∞	∞	∞	2.0636	1.7095	1.5565
I^*	0.0000	0.0000	0.0000	0.5958	0.8412	1.0403
$K(T^*)$	0.1000	0.5000	1.0000	1.6266	2.0863	2.4704
π / h	0.1	0.5	1	2	3	4

L	S*	K_S^*	S*	K_S^*	S`*	K_S^*	S*	K_S^*	S*	K_S^*	S*	K_S^*
1	0	0.1000	0	0.5000	0,1	1.0000	1	1.5000	2	1.8000	2	2.0000
	ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.0000		ΔK= -0.1266		ΔK= -0.2863		ΔK= -0.4704	
2	0	0.1000	0	0.5000	0,1	1.0000	2	1.6000	2	2.0000	3	2.2632
	ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.0000		ΔK= -0.0266		ΔK= -0.0863		ΔK= -0.2072	
3	0	0.1000	0	0.5000	0,1	1.0000	2	1.6471	3	2.0769	3	2.4231
	ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.0204		ΔK= -0.0093		ΔK= -0.0473	
4	0	0.1000	0	0.5000	0,1	1.0000	2	1.6923	3	2.1549	4	2.4854
	ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.0657		ΔK= 0.0687		ΔK= 0.0151	
5	0	0.1000	0	0.5000	0,1	1.0000	3	1.7076	4	2.1867	5	2.5638
	ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.0810		ΔK= 0.1005		ΔK= 0.0934	
6	0	0.1000	0	0.5000	0,1	1.0000	3	1.7213	4	2.2261	5	2.6040
	ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.0947		ΔK= 0.1398		ΔK= 0.1336	
7	0	0.1000	0	0.5000	0,1	1.0000	3	1.7379	5	2.2472	6	2.6446
	ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.1113		ΔK= 0.1609		ΔK= 0.1742	
8	0	0.1000	0	0.5000	0,1	1.0000	4	1.7464	5	2.2691	6	2.6770
	ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.1197		ΔK= 0.1828		ΔK= 0.2066	
9	0	0.1000	0	0.5000	0,1	1.0000	4	1.7519	6	2.2862	7	2.7006
	ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.1253		ΔK= 0.1999		ΔK= 0.2302	
10	0	0.1000	0	0.5000	0,1	1.0000	4	1.7600	6	2.2987	7	2.7266
	ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.1333		ΔK= 0.2124		ΔK= 0.2562	
15	0	0.1000	0	0.5000	0,1	1.0000	6	1.7799	8	2.3466	10	2.7965
	ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.1532		ΔK= 0.2603		ΔK= 0.3261	
20	0	0.1000	0	0.5000	0,1	1.0000	7	1.7912	11	2.3728	13	2.8412
	ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.1646		ΔK= 0.2865		ΔK= 0.3708	
30	0	0.1000	0	0.5000	0,1	1.0000	10	1.8028	15	2.3991	18	2.8866
	ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.1761		ΔK= 0.3128		ΔK= 0.4163	
40	0	0.1000	0	0.5000	0,1	1.0000	13	1.8088	19	2.4140	23	2.9118
	ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.1822		ΔK= 0.3277		ΔK= 0.4415	
50	0	0.1000	0	0.5000	0,1	1.0000	16	1.8126	23	2.4235	28	2.9279
	ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.1859		ΔK= 0.3372		ΔK= 0.4575	
60	0	0.1000	0	0.5000	0,1	1.0000	19	1.8151	28	2.4300	33	2.9389
	ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.1885		ΔK= 0.3437		ΔK= 0.4685	

Table 1. (continued) Cost comparisons between (1, T) and (S-1, S) policies for $\mu = 1$

T^*	1.4682	1.4097	1.3675	1.3354	1.3101	1.2894
I^*	1.2130	1.3679	1.5096	1.6410	1.7641	1.8804
$K(T^*)$	2.8075	3.1116	3.3908	3.6505	3.8942	4.1246
π / h	5	6	7	8	9	10

L	S^*	K_S^*	S^*	K_S^*	S^*	K_S^*	S^*	K_S^*	S^*	K_S^*	S^*	K_S^*
1	2	2.2000	2	2.4000	3	2.5000	3	2.5625	3	2.6250	3	2.6875
	ΔK= -0.6075		ΔK= -0.7116		ΔK= -0.8908		ΔK= -1.0880		ΔK= -1.2692		ΔK= -1.4371	
2	3	2.4737	3	2.6842	4	2.8571	4	2.9524	4	3.0476	4	3.1429
	ΔK= -0.3338		ΔK= -0.4274		ΔK= -0.5337		ΔK= -0.6981		ΔK= -0.8466		ΔK= -0.9817	
3	4	2.6489	4	2.8550	4	3.0611	5	3.2106	5	3.3207	5	3.4307
	ΔK= -0.1586		ΔK= -0.2566		ΔK= -0.3298		ΔK= -0.4399		ΔK= -0.5736		ΔK= -0.6939	
4	5	2.7916	5	2.9907	5	3.1897	5	3.3888	6	3.5231	6	3.6403
	ΔK= -0.0159		ΔK= -0.1209		ΔK= -0.2011		ΔK= -0.2617		ΔK= -0.3711		ΔK= -0.4843	
5	5	2.8487	6	3.1103	6	3.3022	6	3.4940	6	3.6859	7	3.8078
	ΔK= 0.0412		ΔK= -0.0013		ΔK= -0.0887		ΔK= -0.1565		ΔK= -0.2084		ΔK= -0.3168	
6	6	2.9141	6	3.1791	7	3.4057	7	3.5908	7	3.7758	8	3.9500
	ΔK= 0.1067		ΔK= 0.0675		ΔK= 0.0149		ΔK= -0.0597		ΔK= -0.1184		ΔK= -0.1746	
7	6	2.9760	7	3.2353	7	3.4842	8	3.6823	8	3.8611	8	4.0400
	ΔK= 0.1685		ΔK= 0.1238		ΔK= 0.0934		ΔK= 0.0318		ΔK= -0.0331		ΔK= -0.0846	
8	7	3.0061	8	3.2980	8	3.5336	8	3.7691	9	3.9434	9	4.1165
	ΔK= 0.1987		ΔK= 0.1864		ΔK= 0.1427		ΔK= 0.1186		ΔK= 0.0492		ΔK= -0.0081	
9	8	3.0482	8	3.3374	9	3.5888	9	3.8131	10	4.0233	10	4.1913
	ΔK= 0.2407		ΔK= 0.2258		ΔK= 0.1980		ΔK= 0.1626		ΔK= 0.1291		ΔK= 0.0667	
10	8	3.0748	9	3.3713	9	3.6445	10	3.8625	10	4.0771	11	4.2646
	ΔK= 0.2673		ΔK= 0.2597		ΔK= 0.2537		ΔK= 0.2120		ΔK= 0.1828		ΔK= 0.1400	
15	11	3.1758	12	3.5021	13	3.7909	13	4.0541	14	4.2796	14	4.4996
	ΔK= 0.3683		ΔK= 0.3906		ΔK= 0.4000		ΔK= 0.4036		ΔK= 0.3854		ΔK= 0.3750	
20	14	3.2349	15	3.5799	16	3.8849	17	4.1600	17	4.4157	18	4.6378
	ΔK= 0.4275		ΔK= 0.4683		ΔK= 0.4941		ΔK= 0.5095		ΔK= 0.5215		ΔK= 0.5132	
30	20	3.3030	21	3.6685	23	3.9977	24	4.2940	24	4.5649	25	4.8130
	ΔK= 0.4955		ΔK= 0.5569		ΔK= 0.6069		ΔK= 0.6435		ΔK= 0.6707		ΔK= 0.6884	
40	25	3.3400	27	3.7179	29	4.0573	30	4.3667	31	4.6516	32	4.9150
	ΔK= 0.5325		ΔK= 0.6063		ΔK= 0.6665		ΔK= 0.7162		ΔK= 0.7574		ΔK= 0.7904	
50	31	3.3633	33	3.7496	35	4.0977	37	4.4163	38	4.7093	39	4.9823
	ΔK= 0.5558		ΔK= 0.6380		ΔK= 0.7069		0.7658		ΔK= 0.8151		ΔK= 0.8577	
60	36	3.3802	39	3.7717	41	4.1267	43	4.4509	45	4.7509	46	5.0302
	ΔK= 0.5728		ΔK= 0.6602		ΔK= 0.7359		ΔK= 0.8004		ΔK= 0.8566		ΔK= 0.9056	

Table 2. Cost comparisons between (1, T) and (S-1, S) policies for μ = 2

T^*	∞		∞		1.0318		0.7782		0.7048		0.6677	
I^*	0.0000		0.0000		0.5958		1.0403		1.3679		1.6410	
$K(T^*)$	0.2000		1.0000		1.6266		2.4704		3.1116		3.6505	
π / h	0.1		0.5		1		2		3		4	
L	S*	K_S^*	S*	K_S^*	S`*	K_S^*	S*	K_S^*	S*	K_S^*	S*	K_S^*
1	0	0.2000	0,1	1.0000	2	1.6000	3	2.2632	4	2.6842	4	2.9524
	ΔK= 0.0000		ΔK= 0.0000		ΔK= -0.0266		ΔK= -0.2072		ΔK= -0.4274		ΔK= -0.6981	
2	0	0.2000	0,1	1.0000	2	1.6923	4	2.4854	5	2.9907	5	3.3888
	ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.0657		ΔK= 0.0151		ΔK= -0.1209		ΔK= -0.2617	
3	0	0.2000	0,1	1.0000	3	1.7213	5	2.6040	6	3.1791	7	3.5908
	ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.0947		ΔK= 0.1336		ΔK= 0.0675		ΔK= -0.0597	
4	0	0.2000	0,1	1.0000	4	1.7464	6	2.6770	8	3.2980	8	3.7691
	ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.1197		ΔK= 0.2066		ΔK= 0.1864		ΔK= 0.1186	
5	0	0.2000	0,1	1.0000	4	1.7600	7	2.7266	9	3.3713	10	3.8625
	ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.1333		ΔK= 0.2562		ΔK= 0.2597		ΔK= 0.2120	
6	0	0.2000	0,1	1.0000	5	1.7689	8	2.7625	10	3.4347	11	3.9553
	ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.1423		ΔK= 0.2921		ΔK= 0.3231		ΔK= 0.3048	
7	0	0.2000	0,1	1.0000	5	1.7788	9	2.7898	11	3.4881	13	4.0205
	ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.1522		ΔK= 0.3194		ΔK= 0.3766		ΔK= 0.3700	
8	0	0.2000	0,1	1.0000	8	1.7824	11	2.8109	13	3.5228	14	4.0741
	ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.1557		ΔK= 0.3406		ΔK= 0.4112		ΔK= 0.4236	
9	0	0.2000	0,1	1.0000	7	1.7876	12	2.8274	14	3.5525	15	4.1270
	ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.1609		ΔK= 0.3570		ΔK= 0.4410		ΔK= 0.4765	
10	0	0.2000	0,1	1.0000	7	1.7912	13	2.8412	15	3.5799	17	4.1600
	ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.1646		ΔK= 0.3708		ΔK= 0.4683		ΔK= 0.5095	
15	0	0.2000	0,1	1.0000	10	1.8028	18	2.8866	21	3.6685	24	4.2940
	ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.1761		ΔK= 0.4163		ΔK= 0.5569		ΔK= 0.6435	
20	0	0.2000	0,1	1.0000	13	1.8088	23	2.9118	27	3.7179	30	4.3667
	0.0000		ΔK= 0.0000		ΔK= 0.1822		ΔK= 0.4415		ΔK= 0.6063		ΔK= 0.7162	
30	0	0.2000	0,1	1.0000	19	1.8151	33	2.9389	39	3.7717	43	4.4509
	ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.1885		ΔK= 0.4685		ΔK= 0.6602		ΔK= 0.8004	
40	0	0.2000	0,1	1.0000	25	1.8183	43	2.9533	51	3.8007	56	4.4968
	ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.1917		ΔK= 0.4829		ΔK= 0.6891		ΔK= 0.8463	
50	0	0.2000	0,1	1.0000	31	1.8203	53	2.9621	63	3.8189	69	4.5258
	ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.1937		ΔK= 0.4918		ΔK= 0.7073		ΔK= 0.8753	
60	0	0.2000	0,1	1.0000	37	1.8217	63	2.9682	75	3.8314	82	4.5459
	ΔK= 0.0000		ΔK= 0.0000		ΔK= 0.1950		ΔK= 0.4978		ΔK= 0.7198		ΔK= 0.8954	

Table 2. (continued) Cost comparisons between (1, T) and (S-1, S) policies for μ = 2

T^*	0.6447	0.6287	0.6169	0.6076	0.6002	0.5941
I^*	1.8804	2.0961	2.2941	2.4782	2.6508	2.8140
$K(T^*)$	4.1246	4.5529	4.9464	5.3126	5.6565	5.9816

π / h	5	6	7	8	9	10

L	S*	K_S^*	S*	K_S^*	S*	K_S^*	S*	K_S^*	S*	K_S^*	S*	K_S^*
1	4	3.1429	4	3.3333	4	3.5238	5	3.6606	5	3.7339	5	3.8073
	ΔK= -0.9817		ΔK= -1.2195		ΔK= -1.4226		ΔK= -1.6521		ΔK= -1.9225		ΔK= -2.1743	
2	6	3.6403	6	3.8746	6	4.1089	7	4.2550	7	4.3805	7	4.5060
	ΔK= -0.4843		ΔK= -0.6783		ΔK= -0.8375		ΔK= -1.0577		ΔK= -1.2760		ΔK= -1.4756	
3	8	3.9500	8	4.1938	8	4.4375	9	4.6532	9	4.8035	9	4.9538
	ΔK= -0.1746		ΔK= -0.3591		ΔK= -0.5089		ΔK= -0.6594		ΔK= -0.8530		ΔK= -1.0278	
4	9	4.1165	10	4.4332	10	4.6765	10	4.9199	11	5.1135	11	5.2761
	ΔK= -0.0081		ΔK= -0.1196		ΔK= -0.2699		ΔK= -0.3928		ΔK= -0.5430		ΔK= -0.7055	
5	11	4.2646	11	4.5911	12	4.8737	12	5.1132	12	5.3527	13	5.5302
	ΔK= 0.1400		ΔK= 0.0383		ΔK= -0.0727		ΔK= -0.1994		ΔK= -0.3038		ΔK= -0.4514	
6	12	4.3685	13	4.7176	13	5.0274	14	5.2819	14	5.5163	15	5.7433
	ΔK= 0.2439		ΔK= 0.1648		ΔK= 0.0810		ΔK= -0.0308		ΔK= -0.1402		ΔK= -0.2383	
7	14	4.4593	14	4.8309	15	5.1381	15	5.4336	16	5.6642	16	5.8932
	ΔK= 0.3347		ΔK= 0.2780		ΔK= 0.1916		ΔK= 0.1210		ΔK= 0.0078		ΔK= -0.0884	
8	15	4.5269	16	4.9086	17	5.2488	17	5.5321	18	5.8017	18	6.0253
	ΔK= 0.4023		ΔK= 0.3558		ΔK= 0.3024		ΔK= 0.2194		ΔK= 0.1452		ΔK= 0.0437	
9	17	4.5921	17	4.9915	18	5.3271	19	5.6317	19	5.9041	20	6.1501
	ΔK= 0.4675		ΔK= 0.4387		ΔK= 0.3806		ΔK= 0.3190		ΔK= 0.2476		ΔK= 0.1685	
10	18	4.6378	19	5.0451	20	5.4023	20	5.7201	21	5.9946	21	6.2574
	0.5132		ΔK= 0.4922		ΔK= 0.4559		ΔK= 0.4075		ΔK= 0.3381		ΔK= 0.2758	
15	25	4.8130	26	5.2660	27	5.6624	28	6.0128	29	6.3288	30	6.6230
	ΔK= 0.6884		ΔK= 0.7131		ΔK= 0.7159		ΔK= 0.7002		ΔK= 0.6724		ΔK= 0.6414	
20	32	4.9150	34	5.3914	35	5.8124	36	6.1914	37	6.5352	38	6.8514
	ΔK= 0.7904		ΔK= 0.8385		ΔK= 0.8660		ΔK= 0.8788		ΔK= 0.8787		ΔK= 0.8698	
30	46	5.0302	48	5.5391	50	5.9928	51	6.4064	53	6.7813	54	7.1280
	ΔK= 0.9056		ΔK= 0.9863		ΔK= 1.0463		ΔK= 1.0937		ΔK= 1.1248		ΔK= 1.1464	
40	60	5.0943	63	5.6221	65	6.0955	67	6.5270	68	6.9232	70	7.2909
	ΔK= 0.9697		ΔK= 1.0692		ΔK= 1.1490		ΔK= 1.2143		ΔK= 1.2667		ΔK= 1.3093	
50	74	5.1354	77	5.6752	80	6.1623	82	6.6066	84	7.0166	85	7.3976
	ΔK= 1.0108		ΔK= 1.1224		ΔK= 1.2159		ΔK= 1.2939		ΔK= 1.3601		ΔK= 1.4160	
60	87	5.1641	91	5.7130	94	6.2095	97	6.6631	99	7.0829	101	7.4735
	ΔK= 1.0395		ΔK= 1.1601		ΔK= 1.2630		ΔK= 1.3504		ΔK= 1.4264		ΔK= 1.4919	

For the case $\mu = 2$, Table 2 provides a similar set of results for the same range of values for L and π / h for which the maximum percent change in total cost increase due to application of (S-1, S) policy instead of (1, T) policy is better than 24 percent.

Figure 1 depicts the behavior of ΔK in terms of the percent change as a function of L, π / h, and $\mu = 1$.

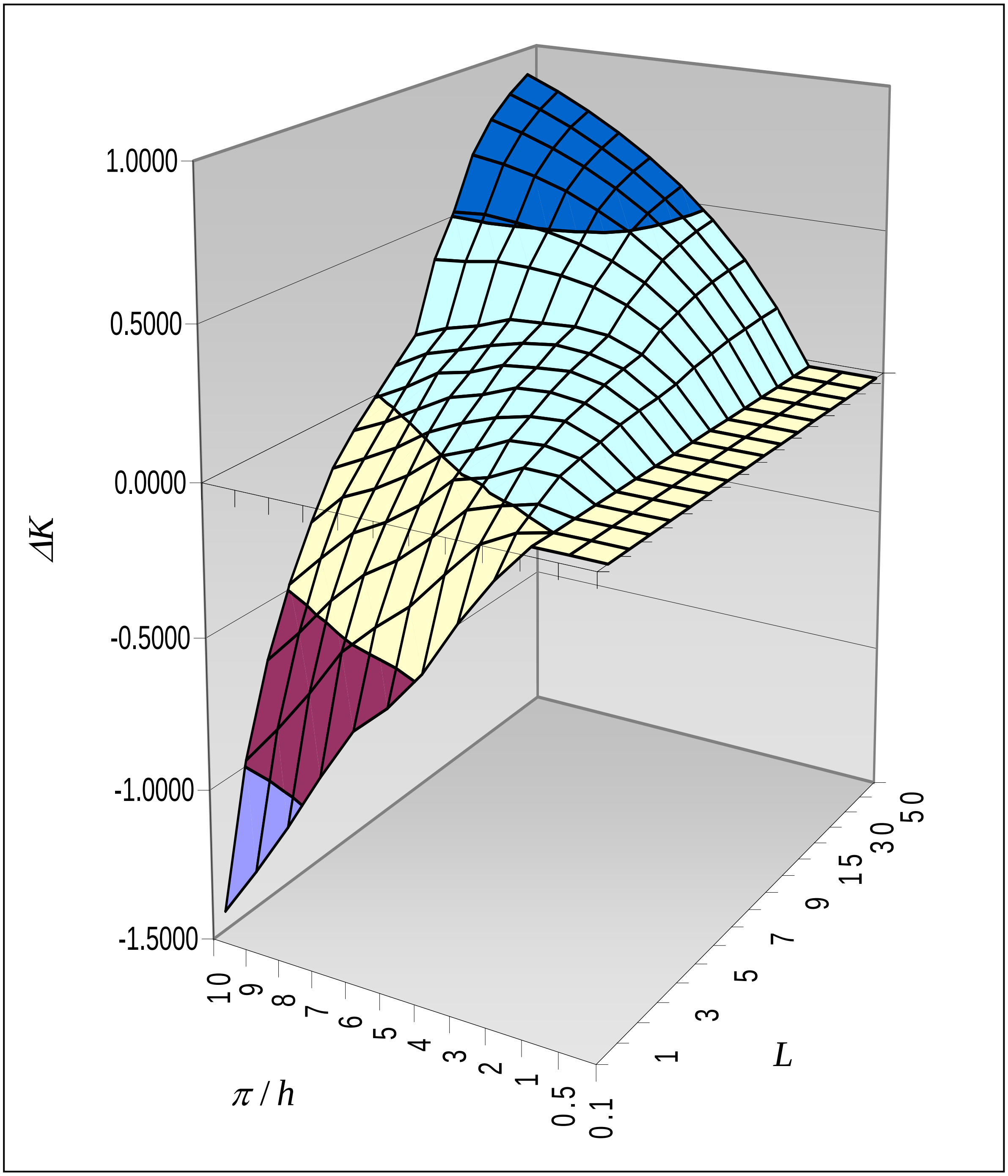

Figure 1. The value of ΔK for $\mu = 1$ as a function of L and π / h

Figures 2 and 3 display the variations in percent change in ΔK for $\mu = 2$ and 3 with the same values of L, and π / h.

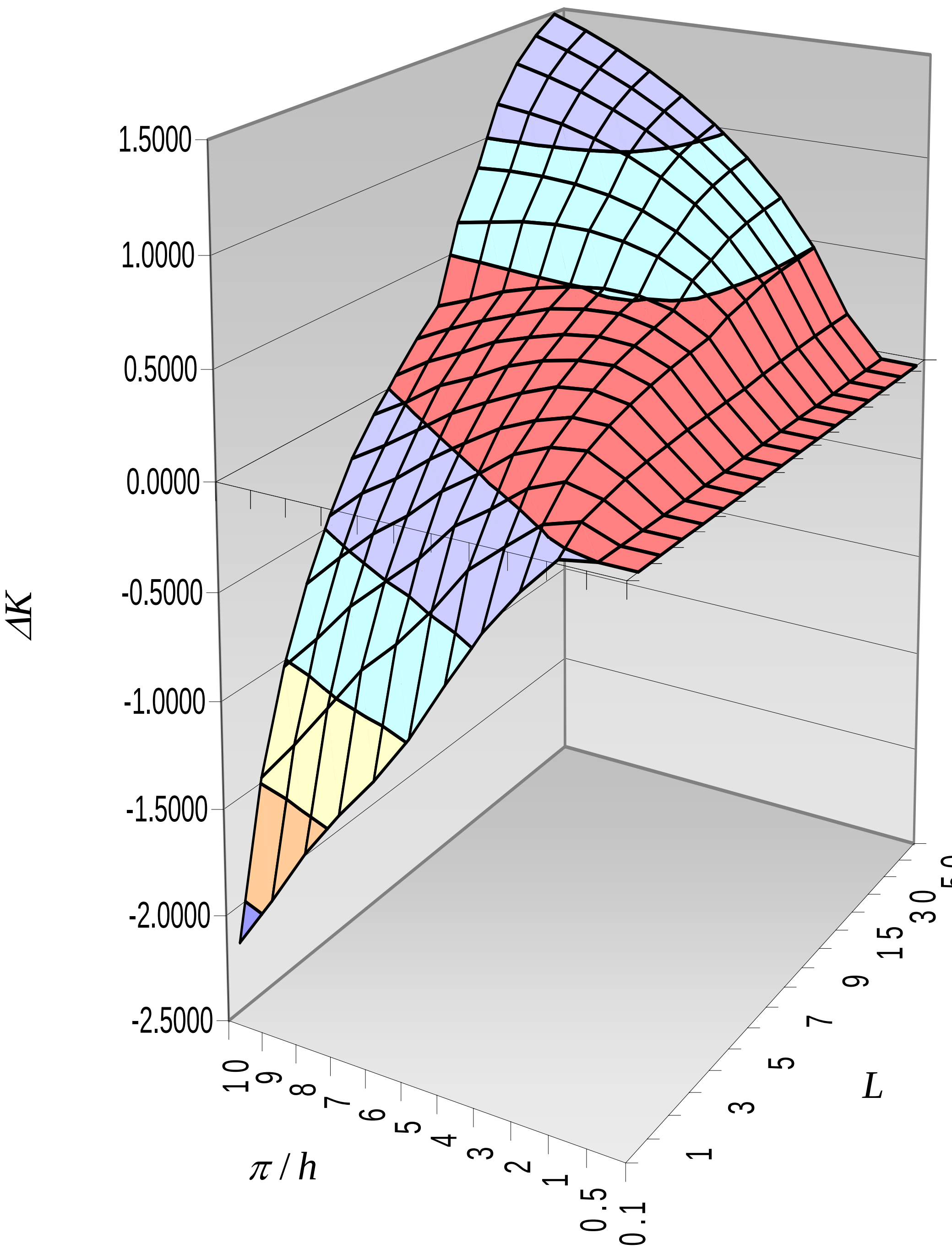

Figure 2. The value of ΔK for $\mu = 2$ as a function of L and π / h

As can be seen, the increase in percent change in ΔK becomes more pronounced as μ increases from 1 to 3.

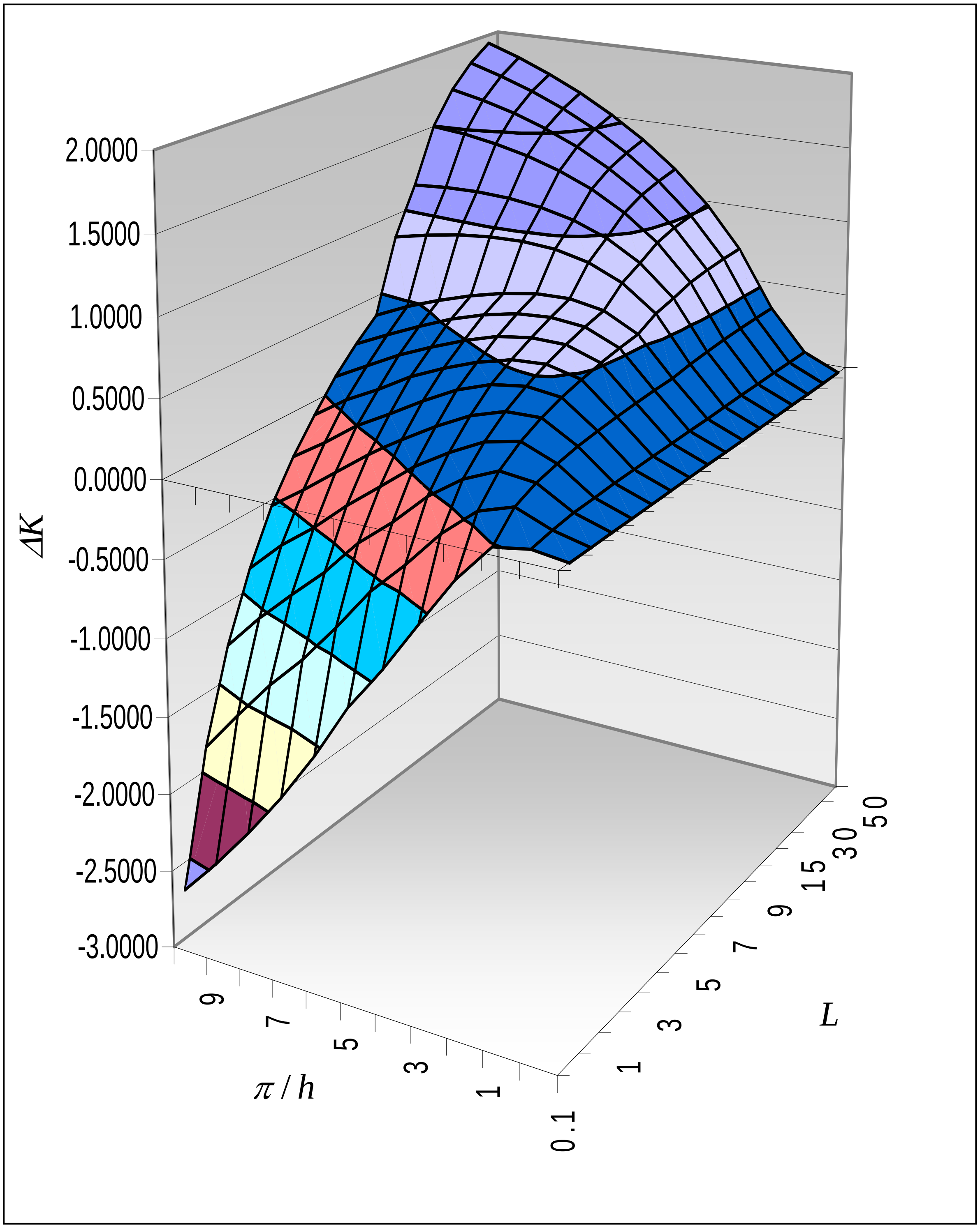

Figure 3. The value of ΔK for $\mu = 3$ as a function of L and π / h

5. CONCLUSION

While it is well known that for the case of stochastic demand, there is no inventory control policy in which both the order size and the order interval are constant with an optimal solution, in this paper, we introduced the optimal solution for a new policy which is different from the classical inventory policies. An important characteristic of this policy is that it eliminates the uncertainty in demand for suppliers. This new policy is to order 1 unit at each fixed time period T. We called this new policy *one for one period ordering* or *(1, T) policy*. In this policy *the time interval between two consecutive orders and the value of the order size are both constant.*

We assumed that the fixed ordering cost is zero or negligible, demand process is a Poisson process, and the replenishment lead time is constant. Then we derived the long-run average total cost, consisting of holding and shortage costs, per unit time for the lost sales case in terms of the average inventory level. To obtain this total cost rate we used some concepts from queueing theory and proved that it is a convex function in terms of the average inventory on hand. The one-for-one period, or (1, T) policy enjoys several advantages one of which is the very significant property that establishes the fact that its total cost rate is free of the lead time length.

Further, we obtained the optimal value of T, the time interval between two consecutive orders, which minimizes the total expected cost. Finally, by a numerical example we compared the total cost rate of this policy with the total cost rate of the one-for-one ordering or (S-1 , S) policy for the lost sales case. This example also served to re-examine the optimality of (s, S) policy in continuous review inventory models and established the fact that even for the case where demand forms a Poisson process the optimality of (s, S) policy does not hold.

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