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# A Project Scheduling Method Based on Fuzzy Theory

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**ABSTRACT**

In this paper a new method based on fuzzy theory is developed to solve the project scheduling problem under fuzzy environment. Assuming that the duration of activities are trapezoidal fuzzy numbers (TFN), in this method we compute several project characteristics such as earliest times, latest times, and, slack times in term of TFN. In this method, we introduce a new approach which we call modified backward pass (MBP). This approach, based on a linear programming (LP) problem, removes negative and infeasible solutions which can be generated by other methods in the backward pass calculation. We drive the general form of the optimal solution of the LP problem which enables practitioners to obtain the optimal solution by a simple recursive relation without solving any LP problem. Through a numerical example, calculation steps in this method and the results are illustrated.

***Keywords:*** Project scheduling, Fuzzy theory, Modified backward pass (MBP), Trapezoidal fuzzy number (TFN), Linear programming (LP)

## 1. INTRODUCTION:

Scheduling is deemed to be one of the most fundamental and essential bases of the project management science. There are several methods for project scheduling such as CPM, PERT and GERT. Since too many drawbacks are involved in methods estimating the duration of activities, these methods lack the capability of modeling practical projects. In order to solve these problems, a number of techniques like fuzzy logic, genetic algorithm (GA) and artificial neural network can be considered.

A fundamental approach to solve these problems is applying fuzzy sets. Introducing the fuzzy set theory by Zadeh in 1965 opened promising new horizons to different scientific areas such as project scheduling. Fuzzy theory, with presuming imprecision in decision parameters and utilizing mental models of experts is an approach to adapt scheduling models into reality. To this end, several methods have been developed during the last three decades. The first method called FPERT, was proposed by Chanas and Kamburowski (1981). They presented the project completion time in the form of a fuzzy set in the time space. Gazdik (1983) developed a fuzzy network of an a priori unknown project to estimate the activity duration, and used fuzzy algebraic operators to calculate the duration of the project and its critical path. This work is called FNET. An extension of FNET

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was proposed by Nusution (1994) and Lorterapong and Moselhi (1996). Following on this, McCahon (1993), Chang et al. (1995), and, Lin and Yao (2003) presented three methodologies to calculate the fuzzy completion project time. Other researchers such as Kuchta (2001), Yao and Lin (2000), Chanas and Zielinski (2001), and, Oliveros and Robinson (2005), using fuzzy numbers, presented other methods to obtain fuzzy critical paths and critical activities and activity delay. Previous work on network scheduling using fuzzy theory provides methods for scheduling projects.

These methods, however, do not support backward pass calculations in direct manner similar to that used in the forward pass. This is mainly due to the fact that fuzzy subtraction is not proportionate to the inverse of fuzzy addition. Therefore, these methods are incapable to calculate project characteristics such as the latest times and slack times.

In this paper a new method is introduced for project scheduling in fuzzy environment. This method is developed based on a number of assumptions and definitions in the fuzzy set and project scheduling. In the fuzzy project network considered in this paper, we assume that the duration of activities are trapezoidal fuzzy numbers (TFN). The project characteristics such as fuzzy earliest times and fuzzy project completion time are calculated as TFN by forward pass. As mentioned above, backward pass in fuzzy environment fails to compute the fuzzy latest times and fuzzy slack times. Therefore, for computation of these parameters we propose a new approach which we call modified backward pass (MBP).

This approach is based on the project scheduling fundamental concepts and linear programming. In MBP, using the project scheduling concepts, fuzzy latest times and fuzzy slack times relations are transformed to linear programming (LP) problem. After that, we drive the general form of the optimal solution of the LP problem which enables practitioners to obtain the optimal solution by a simple recursive relation without solving any LP problem.

The advantage of MBP approach in comparison with the previous approaches is that it does not use the fuzzy subtraction operator in its relations. Due to, these modifications, the inherent defects discussed before in the fuzzy environment will remove. Therefore, the obtained fuzzy latest times and fuzzy slack times in the MBP approach are correct and calculated as TFNs, as well. Finally, through a numerical example, calculation steps in this approach and result are illustrated.

## 2. DEFINITIONS AND ASSUMPTIONS

In this section, some basic notions of the area of fuzzy theory that have been defined by Kaufmann and Gupta (1985) and Zimermann (1996), are introduced. Then, project network is defined as a directed and acyclic graph in fuzzy environment.

### 2.1. Definitions

**Definition1**: Let R be the space of real numbers. A fuzzy set $\tilde{A}$ is a set of ordered pairs $\{(x, \mu_{\tilde{A}}(x)) \mid x \in R\}$, where $\mu_{\tilde{A}}(x): R \to [0,1]$ and is upper semi continuous. Function $\mu_{\tilde{A}}(x)$ is called membership function of the fuzzy set.

**Definition2**: A convex fuzzy set, $\tilde{A}$, is a fuzzy set in which:

$$\forall\, x, y \in R,\ \forall \lambda \in [0,1],$$
$$\mu_{\tilde{A}}(\lambda x + (1-\lambda) y) \geq \min[\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)]$$

**Definition3:** A fuzzy set $\tilde{A}$ is called positive if its membership function is such that $\mu_{\tilde{A}}(x) = 0 \ , \forall x \leq 0$ .

**Definition4***:* Trapezoidal Fuzzy Number (TFN) is a convex fuzzy set which is defined as: $\tilde{A} = \left(x, \mu(x)\right)$ where

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x \leq a \\ \dfrac{x-a}{b-a} & a < x \leq b \\ 1 & b < x \leq c \\ \dfrac{x-d}{c-d} & c < x \leq d \\ 0 & x < d \end{cases} \tag{1}$$

For convenience, TFN represented by four real parameters $a, b, c, d \quad (a \leq b \leq c \leq d)$ will be denoted by a tetraploid $(a, b, c, d)$ (Fig.1).

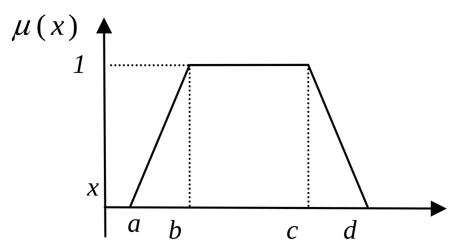


Fig.1 Trapezoidal Fuzzy Number (TFN)

**Definition5:** A Trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ is called positive TFN if: $0 \leq a \leq b \leq c \leq d$ .

## 2.2. Operation on TFNs

Chen and Hwang (1992) and Dubois and Prade (1988) have been defined a number of operations can be performed on TFNs. The following are employed operations in the development of the proposed method:
Let $\tilde{A} = (a_1, b_1, c_1, d_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2)$ be any two TFNs, then:

$$\tilde{A} \oplus \tilde{B} = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2) \tag{2}$$

$$\tilde{A} \ominus \tilde{B} = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2) \tag{3}$$

$$M\tilde{A}X(\tilde{A}, \tilde{B}) = (\ \max(a_1, a_2), \max(b_1, b_2), \max(c_1, c_2), \max(d_1, d_2)\ ) \tag{4}$$

$$M\tilde{I}N(\tilde{A}, \tilde{B}) = (\ \min(a_1, a_2), \min(b_1, b_2), \min(c_1, c_2), \min(d_1, d_2)\ ) \tag{5}$$

where $\oplus$ = fuzzy addition; $\ominus$= fuzzy subtraction; and $M\tilde{A}X$ and $M\tilde{I}N$ are fuzzy maximum and minimum respectively.

### 2.3. Fuzzy Project Network

A network $N = \langle V, A, \tilde{D} \rangle$, being a fuzzy project model, is given. $V$ is a set of nodes (events) and $A \subset V \times V$ is a set of arcs (activities). The network $N$ is a directed and acyclic graph in the fuzzy environment. The set $V = \{1, 2, ..., n\}$ is labeled in such a way that the following condition holds: $(i, j) \in A \Rightarrow i < j$. In the fuzzy environment the duration of this activity $(\tilde{D})$ is a positive TFN: $\tilde{D}_{ij} = (d_{ij}^1, d_{ij}^2, d_{ij}^3, d_{ij}^4)$. Let us denote by $P(j) = \{ i \in V \mid (i, j) \in A \}$ the set of predecessors and by $S(i) = \{ j \in V \mid (i, j) \in A \}$ the set of successors of event $i \in V$, respectively. Starting time of the fuzzy project model is a positive TFN: $\tilde{T}_S = (t_S^1, t_S^2, t_S^3, t_S^4)$.

## 3. FUZZY PROJECT SCHEDULING

Fuzzy project scheduling consists of the forward pass and modified backward pass (MBP) calculations to obtain the substantial project characteristics. In this section, for the fuzzy project network, these characteristics such as earliest times, latest times, and, slack times are obtained by carrying out the calculations as follows.

### 3.1. Fuzzy Forward Pass Calculations

The earliest times and also project completion time in a project network can be detected by forward pass. In this case, using the relations of CPM in the fuzzy environment, results in the following fuzzy forward calculations:

$$\tilde{E}_j = \left( e_j^1, e_j^2, e_j^3, e_j^4 \right) = \begin{cases} \underset{i \in P(j)}{M\tilde{A}X} \left\{ \tilde{E}_i \oplus \tilde{D}_{ij} \right\} & , P(j) \neq \phi \\ \tilde{T}_S = \left( t_S^1, t_S^2, t_S^3, t_S^4 \right) & , P(j) = \phi \end{cases} \tag{6}$$

$$E\tilde{S}_{ij} = \left( es_{ij}^1, es_{ij}^2, es_{ij}^3, es_{ij}^4 \right) = \tilde{E}_i \tag{7}$$

$$E\tilde{F}_{ij} = \left( ef_{ij}^1, ef_{ij}^2, ef_{ij}^3, ef_{ij}^4 \right) = E\tilde{S}_{ij} \oplus \tilde{D}_{ij} \tag{8}$$

$$\tilde{T}_F = \left( t_F^1, t_F^2, t_F^3, t_F^4 \right) = \underset{i \in V}{M\tilde{A}X}\ \tilde{E}_i \tag{9}$$

In the above, $\tilde{E}_j$ is the fuzzy earliest time of event *j*; $\tilde{T}_S$ is the fuzzy time of starting the project; $E\tilde{S}_{ij}$ is the fuzzy earliest starting of activity $(i, j)$; $E\tilde{F}_{ij}$ is the fuzzy earliest finishing of activity $(i, j)$ and $\tilde{T}_F$ is the fuzzy time of project completion. Based on the above equations, $\tilde{E}_j$, $E\tilde{S}_{ij}$, $E\tilde{F}_{ij}$ and $\tilde{T}_F$ can be calculated as positive TFNs.

### 3.2. Fuzzy Modified Backward Pass (MBP) Calculations

Backward pass calculations are employed to calculate the latest times in the project network. In this case, if the backward pass calculations of CPM are entirely done in the fuzzy environment, the fuzzy latest time of event $i$ $(\tilde{L}_i)$ can be written as:

$$\tilde{L}_i = \begin{cases} \underset{j\in S(i)}{M\tilde{I}N} \left\{ \tilde{L}_j \ominus \tilde{D}_{ij} \right\} & , S(i) \neq \phi \\ \\ \tilde{T}_F & , S(i) = \phi \end{cases} \tag{10}$$

As mentioned above, the fundamental manner of the backward pass is based on an inversion between addition and subtraction. In a crisp environment, the equation of A+B–B=A is always correct but the addition and subtraction are not always inverse in the fuzzy theory. It means that $\tilde{A}$ and $\tilde{B}$ do not satisfy the relation $\tilde{A} \oplus \tilde{B} \ominus \tilde{B} = \tilde{A}$. Therefore, the fuzzy backward pass in the equation above faces serious problems. For example, for a typical project data, in which $S(2) = \{3\}$, $\tilde{L}_3 = (10,20,22,30)$, and $\tilde{D}_{23} = (5,10,15,20)$, using (10), it is found that: $\tilde{L}_2 = (-10,5,12,25)$

It is clear that $\tilde{L}_2$ is a trapezoidal fuzzy number with a negative part. It depicts that the latest time of event 2 may happen in a negative time. But, the negative time is not feasible since it is not defined in the project scheduling. To avoid this problem, we propose a new approach which we call modified backward pass (MBP). Therefore, according to the concept of $\tilde{L}_i$ and using (10), the fuzzy latest time of event $i$ $(\tilde{L}_i)$ can be defined as:

$$\tilde{L}_i = \begin{cases} \underset{j\in S(i)}{M\tilde{I}N} \left\{ \tilde{X}_j \middle| \tilde{X}_j \oplus \tilde{D}_{ij} = \tilde{L}_j \right\} & , S(i) \neq \phi \\ \\ \tilde{T}_F & , S(i) = \phi \end{cases} \tag{11}$$

Using this relation leads to a full positive tetraploid $(l_i^1, l_i^2, l_i^3, l_i^4)$. Therefore, the problem due to the appearance of the negative time is removed. But in some cases, the calculated $\tilde{L}_i$ may not satisfy the definition of TFN. For example, for a typical project data, in which $S(2) = \{3\}$, $\tilde{L}_3 = (10,20,22,30)$, and $\tilde{D}_{23} = (5,10,15,20)$ using (11) is found that: $\tilde{L}_2 = (5,10,7,10)$. It can be seen that, $\tilde{L}_2$ is not a trapezoidal fuzzy number because it violates the convex conditions $(5 \leq 10 \leq 7 \leq 10)$. Therefore, the calculation should be done in such a way that $\tilde{L}_i$ becomes a trapezoidal fuzzy number. By adding the trapezoidal condition to the above relations, the following relation is obtained:

$$\tilde{L}_i = \begin{cases} \underset{j\in S(i)}{M\tilde{I}N} \left\{ M\tilde{A}X \ \tilde{X}_j \middle| \begin{matrix} \tilde{X}_j \oplus \tilde{D}_{ij} \leq \tilde{L}_j \ , \\ X_j \ is\ Positive\ TFN \end{matrix} \right\} & , S(i) \neq \phi \\ \tilde{T}_F & , S(i) = \phi \end{cases} \tag{12}$$

In the above relation, we define the relation $\leq$ for any two TFNs such as $\tilde{A} = (a^1, a^2, a^3, a^4)$ and $\tilde{B} = (b^1, b^2, b^3, b^4)$ as:

$$\tilde{A} \leq \tilde{B} \quad \approx \quad a^1 \leq b^1 \ , \ a^2 \leq b^2 \ , \ a^3 \leq b^3 \ , \ a^4 \leq b^4$$

where $S(i) \neq \phi$. Relation (12) results in the following fuzzy mathematical programming problem:

 

$$\tilde{L}_i = \underset{j\in S(i)}{M\tilde{I}N} \left\{ M\tilde{A}X \quad \tilde{X}_j = (x_j^1, x_j^2, x_j^3, x_j^4) \right\}$$

Subject to

$$\begin{aligned} &\tilde{X}_j \oplus \tilde{D}_{ij} \le \tilde{L}_j \qquad j \in S(i) \\ &0 \le x_j^1 \le x_j^2 \le x_j^3 \le x_j^4 \end{aligned} \tag{13}$$

This problem can be rearranged in a more convenient form as following:

$$\tilde{L}_i = M\tilde{I}N \quad \tilde{Y}_i = (\, y_i^1 \,,\, y_i^2 \,,\, y_i^3 \,,\, y_i^4 \,)$$

$$\begin{aligned} s.t.& \\ &x_j^1 \le y_i^1 && \forall j \in S(i) \\ &x_j^2 \le y_i^2 && \forall j \in S(i) \\ &x_j^3 \le y_i^3 && \forall j \in S(i) \\ &x_j^4 \le y_i^4 && \forall j \in S(i) \\ &x_j^1 \le l_j^1 - d_{ij}^1 && \forall j \in S(i) \\ &x_j^2 \le l_j^2 - d_{ij}^2 && \forall j \in S(i) \\ &x_j^3 \le l_j^3 - d_{ij}^3 && \forall j \in S(i) \\ &x_j^4 \le l_j^4 - d_{ij}^4 && \forall j \in S(i) \\ &x_j^3 \le x_j^4 \\ &x_j^2 \le x_j^3 \\ &x_j^1 \le x_j^2 \\ &0 \le x_j^1 \end{aligned} \tag{14}$$

By replacing the objective function with $MIN\ y_i^1 + y_i^2 + y_i^3 + y_i^4$, the problem above converts to a linear programming problem. It can be easily observed that the optimal solution of this problem, $\tilde{L}_i$ is obtained as a positive TFN using a simple recursive relation:

$$\begin{aligned} &\tilde{L}_i = (\, l_i^1 \,, l_i^2 \,, l_i^3 \,, l_i^4 \,): \\ &l_i^4 = \max\,(\, 0 \;,\; \min_{j\in S(i)} (\, l_j^4 - d_{ij}^4 \,) \;) \\ &l_i^3 = \max\,(\; 0 \;,\; \min\,(\; l_i^4 \;,\; \min_{j\in S(i)} (\, l_j^3 - d_{ij}^3 \,))) \\ &l_i^2 = \max\,(\; 0 \;,\; \min\,(\; l_i^3 \;,\; \min_{j\in S(i)} (\, l_j^2 - d_{ij}^2 \,))) \\ &l_i^1 = \max\,(\; 0 \;,\; \min\,(\; l_i^2 \;,\; \min_{j\in S(i)} (\, l_j^1 - d_{ij}^1 \,))) \end{aligned} \tag{15}$$

In the MBP, following the calculation of $\tilde{L}_i$, the fuzzy latest finishing of activities $(L\tilde{F}_{ij})$ is calculated as follows:

$$L\tilde{F}_{ij} = (\, lf_{ij}^1 \,, lf_{ij}^2 \,, lf_{ij}^3 \,, lf_{ij}^4 \,) = \tilde{L}_j \tag{16}$$

Based on the equations above, $L\tilde{F}_{ij}$, can be calculated as a positive TFN.

Another important characteristic of the backward pass is the fuzzy latest starting of activities $(L\tilde{S}_{ij})$. In order to calculate $L\tilde{S}_{ij}$, when the backward pass calculations of CPM are applied directly to fuzzy environment, the following relation is obtained:

$$L\tilde{S}_{ij} = L\tilde{F}_{ij} \ominus \tilde{D}_{ij} \tag{17}$$

In the relation above, the use of fuzzy subtraction is required. Due to the presence of fuzzy subtraction similar to that used in calculation of $\tilde{L}_i$, in some cases, the calculating of $L\tilde{S}_{ij}$ may face some obstacles. Then, by adding positive and trapezoidal conditions and also using a linear programming problem similar to (14) and (15), the following relations would be obtained:

$$\begin{aligned} & L\tilde{S}_{ij} = ( ls_{ij}^1 , ls_{ij}^2 , ls_{ij}^3 , ls_{ij}^4 ) : \\ & ls_{ij}^4 = \max ( 0 , ( lf_{ij}^4 - d_{ij}^4 ) ) \\ & ls_{ij}^3 = \max ( 0 , \min ( lf_{ij}^4 , ( lf_{ij}^3 - d_{ij}^3 ) ) ) \\ & ls_{ij}^2 = \max ( 0 , \min ( lf_{ij}^3 , ( lf_{ij}^2 - d_{ij}^2 ) ) ) \\ & ls_{ij}^1 = \max ( 0 , \min ( lf_{ij}^2 , ( lf_{ij}^1 - d_{ij}^1 ) ) ) \end{aligned} \tag{18}$$

### 3.3. Fuzzy Slack Times

One of the main characteristics in project control and planning is the slack time. There are three types of slacks for any activity, i.e. fuzzy total slack $(T\tilde{F}_{ij})$, fuzzy free slack $(F\tilde{F}_{ij})$, and, fuzzy independent slack $(I\tilde{F}_{ij})$. If classical relations of CPM are applied for the calculation of these characteristics in the fuzzy environment, we can write the following relations:

$$T\tilde{F}_{ij} = L\tilde{F}_{ij} \ominus E\tilde{F}_{ij} \tag{19}$$

$$F\tilde{F}_{ij} = \tilde{E}_j \ominus E\tilde{F}_{ij} \tag{20}$$

$$I\tilde{F}_{ij} = \tilde{E}_j \ominus \tilde{L}_i \ominus \tilde{D}_{ij} \tag{21}$$

By using the relations above, the slack times may be out of positive TFNs definition. Therefore, similar to the calculation of $\tilde{L}_i$ in modified backward pass, the following relations are proposed for slack times:

$$\begin{aligned} & T\tilde{F}_{ij} = ( tf_{ij}^1 , tf_{ij}^2 , tf_{ij}^3 , tf_{ij}^4 ); \\ & \begin{cases} tf_{ij}^4 = \max (0 , ( lf_{ij}^4 - ef_{ij}^4 )) \\ tf_{ij}^3 = \max (0 , \min (tf_{ij}^4 , (lf_{ij}^3 - ef_{ij}^3))) \\ tf_{ij}^2 = \max (0 , \min ( tf_{ij}^3 , (lf_{ij}^2 - ef_{ij}^2))) \\ tf_{ij}^1 = \max (0 , \min ( tf_{ij}^2 , (lf_{ij}^1 - ef_{ij}^1))) \end{cases} \end{aligned} \tag{22}$$

$$\widetilde{FF}_{ij} = ( ff_{ij}^1 \ , \ ff_{ij}^2 \ , \ ff_{ij}^3 \ , \ ff_{ij}^4 \ );$$

$$\begin{cases} ff_{ij}^4 = \max(0, (e_j^4 - ef_{ij}^4)) \\ ff_{ij}^3 = \max(0, \min(ff_{ij}^4, (e_j^3 - ef_{ij}^3))) \\ ff_{ij}^2 = \max(0, \min(ff_{ij}^3, (e_j^2 - ef_{ij}^2))) \\ ff_{ij}^1 = \max(0, \min(ff_{ij}^2, (e_j^1 - ef_{ij}^1))) \end{cases} \tag{23}$$

$$\widetilde{IF}_{ij} = ( if_{ij}^1 \ , if_{ij}^2 \ , if_{ij}^3 \ , if_{ij}^4 \ );$$

$$\begin{cases} if_{ij}^4 = \max(0, (e_j^4 - l_i^4 - d_{ij}^4)) \\ if_{ij}^3 = \max(0, \min(if_{ij}^4, (e_j^3 - l_i^3 - d_{ij}^3))) \\ if_{ij}^2 = \max(0, \min(if_{ij}^3, (e_j^2 - l_i^2 - d_{ij}^2))) \\ if_{ij}^1 = \max(0, \min(if_{ij}^2, (e_j^1 - l_i^1 - d_{ij}^1))) \end{cases} \tag{24}$$

## 4. NUMERICAL EXAMPLE

The network representing a structure of project is given in Fig.2

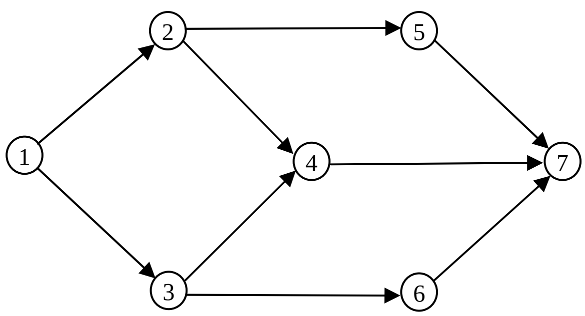


Fig.2 Project network in the numerical example

The duration of activities are positive TFNs (Table1). The fuzzy start time of this example is (0,0,0,0).

Table.1 $\tilde{D}_{ij}$ of the numerical example

| Activity(i,j) | Duration ($\tilde{D}_{ij}$) |
|---|---|
| *(1,2)* | (25,28,32,35) |
| *(1,3)* | (40,55,65,70) |
| *(2,4)* | (32,37,43,48) |
| *(3,4)* | (20,25,35,40) |
| *(2,5)* | (35,38,42,45) |
| *(3,6)* | (42,45,55,60) |
| *(4,7)* | (60,65,75,85) |
| *(5,7)* | (65,75,85,90) |
| *(6,7)* | (15,18,22,26) |

Using the previously described relations, the main fuzzy characteristics for the numerical example are obtained. These values are positive TFNs.

Table 2 represents fuzzy earliest and latest times of events by using (6) and (15).

Table.2 Calculated values of $\tilde{\tilde{E}}_i$ and $\tilde{\tilde{L}}_i$ for the numerical example

| Event *(i)* | $\tilde{\tilde{E}}_i$ | $\tilde{\tilde{L}}_i$ |
|---|---|---|
| *1* | (0,0,0,0) | (0,0,0,0) |
| *2* | (25,28,32,35) | (25,32,48,60) |
| *3* | (45,55,65,70) | (45,55,65,70) |
| *4* | (60,80,100,110) | (65,80,100,110) |
| *5* | (60,66,74,80) | (60,70,90,105) |
| *6* | (82,100,120,130) | (110,127,153,169) |
| *7* | (125,145,175,195) | (125,145,175,195) |

For example:

$P(1) = \phi \quad \Rightarrow \quad \tilde{E}_1 = (\, e_1^1 \,, e_1^2 \,, e_1^3 \,, e_1^4 \,) = \tilde{T}_S = (0,0,0,0)$

$P(2) = \{1\} \Rightarrow \quad \tilde{E}_2 = (\, e_2^1 \,, e_2^2 \,, e_2^3 \,, e_2^4 \,) = \tilde{E}_1 \oplus \tilde{D}_{12} = (25,28,32,35)$

$S(2) = \{4,5\} \Rightarrow \tilde{L}_2 = (\, l_2^1 \,, l_2^2 \,, l_2^3 \,, l_2^4 \,) = (25,32,48,60):$

$l_2^4 = \max(0, \min((l_5^4 - d_{25}^4),(l_4^4 - d_{24}^4))) = \max(0, \min(60,62)) = 60$

$l_2^3 = \max(0, \min(l_2^4, \min((l_5^3 - d_{25}^3),(l_4^3 - d_{24}^3)))) = \max(0, \min(60, \min(48,57))) = 48$

$l_2^2 = \max(0, \min(l_2^3, \min((l_5^2 - d_{25}^2),(l_4^2 - d_{24}^2)))) = \max(0, \min(48, \min(32,43))) = 32$

$l_2^1 = \max(0, \min(l_2^2, \min((l_5^1 - d_{25}^1),(l_4^1 - d_{24}^1)))) = \max(0, \min(32, \min(25,33))) = 25$

The fuzzy time of project completion, $\tilde{\tilde{T}}_F$, is calculated using (9) as: (125,145,175,195).

$E\tilde{\tilde{S}}_{ij}$, $E\tilde{\tilde{F}}_{ij}$, $L\tilde{\tilde{S}}_{ij}$, and $L\tilde{\tilde{F}}_{ij}$ are obtained using (7), (8), (16), and (18) respectively. The results have been presented in Table 3. As an example for activity (2-4) we obtain:

Table. 3 Calculated values of $E\tilde{\tilde{S}}_{ij}$, $E\tilde{\tilde{F}}_{ij}$, $L\tilde{\tilde{S}}_{ij}$, and $L\tilde{\tilde{F}}_{ij}$ for the numerical example

| *(i,j)* | $E\tilde{\tilde{S}}_{ij}$ | $E\tilde{\tilde{F}}_{ij}$ | $L\tilde{\tilde{S}}_{ij}$ | $L\tilde{\tilde{F}}_{ij}$ |
|---|---|---|---|---|
| *(1,2)* | (0,0,0,0) | (25,28,32,35) | (0,4,16,25) | (25,32,48,60) |
| *(1,3)* | (0,0,0,0) | (40,55,65,70) | (0,0,0,0) | (45,55,65,70) |
| *(2,4)* | (25,28,32,35) | (57,65,75,83) | (33,43,57,62) | (65,80,100,110) |
| *(3,4)* | (40,55,65,70) | (60,80,100,110) | (45,55,65,70) | (65,80,100,110) |
| *(2,5)* | (25,28,32,35) | (60,66,74,80) | (25,32,48,60) | (60,70,90,105) |
| *(3,6)* | (40,55,65,70) | (82,100,120,130) | (68,82,98,109) | (110,127,153,169) |
| *(4,7)* | (60,66,74,80) | (120,145,175,195) | (65,80,100,110) | (125,145,175,195) |
| *(5,7)* | (60,66,74,80) | (125,141,159,170) | (60,70,90,105) | (125,145,175,195) |
| *(6,7)* | (82,100,120,130) | (97,118,142,156) | (110,127,153,169) | (125,145,175,195) |

$$E\tilde{S}_{24} = (es_{24}^1 , es_{24}^2 , es_{24}^3 , es_{24}^4) = \tilde{E}_2 = (25, 28, 32, 35)$$
$$E\tilde{F}_{24} = (ef_{24}^1 , ef_{24}^2 , ef_{24}^3 , ef_{24}^4) = E\tilde{S}_{24} \oplus \tilde{D}_{24} = (57, 65, 75, 83)$$
$$L\tilde{F}_{24} = ( lf_{24}^1 , lf_{24}^2 , lf_{24}^3 , lf_{24}^4 ) = \tilde{L}_4 = (65,80,100,110)$$
$$L\tilde{S}_{24} = ( ls_{24}^1 , ls_{24}^2 , ls_{24}^3 , ls_{24}^4 ) = (33, 43, 57, 62) :$$
$$ls_{24}^4 = \max ( 0 , ( lf_{24}^4 - d_{24}^4 ) ) = \max ( 0 , 62 ) = 62$$
$$ls_{24}^3 = \max ( 0 , \min ( lf_{24}^4 , ( lf_{24}^3 - d_{24}^3 ) ) ) = \max ( 0 , \min ( 62 , 57 ) ) = 57$$
$$ls_{24}^2 = \max ( 0 , \min ( lf_{24}^3 , ( lf_{24}^2 - d_{24}^2 ) ) ) = \max ( 0 , \min ( 57 , 43 ) ) = 43$$
$$ls_{24}^1 = \max ( 0 , \min ( lf_{24}^2 , ( lf_{24}^1 - d_{24}^1 ) ) ) = \max ( 0 , \min ( 43 , 33 ) ) = 33$$

Using (22)-(24) and the results presented in tables 2 and 3, the fuzzy slack times are calculated (Table 4). These values can be used for calculation of the criticality of the activities as well as determination of the critical paths.

Table. 4 Calculated values of $T\tilde{\tilde{F}}_{ij}$ , $F\tilde{\tilde{F}}_{ij}$ , and $I\tilde{\tilde{F}}_{ij}$ for the numerical example

| *(i,j)* | $T\tilde{\tilde{F}}_{ij}$ | $F\tilde{\tilde{F}}_{ij}$ | $I\tilde{\tilde{F}}_{ij}$ |
|---|---|---|---|
| *(1,2)* | (0,4,16,25) | (0,0,0,0) | (0,0,0,0) |
| *(1,3)* | (0,0,0,0) | (0,0,0,0) | (0,0,0,0) |
| *(2,4)* | (8,15,25,27) | (3,15,25,27) | (2,2,2,2) |
| *(3,4)* | (0,0,0,0) | (0,0,0,0) | (0,0,0,0) |
| *(2,5)* | (0,4,16,25) | (0,0,0,0) | (0,0,0,0) |
| *(3,6)* | (27,27,33,39) | (0,0,0,0) | (0,0,0,0) |
| *(4,7)* | (0,0,0,0) | (0,0,0,0) | (0,0,0,0) |
| *(5,7)* | (0,4,16,25) | (0,4,16,25) | (0,0,0,0) |
| *(6,7)* | (27,27,33,39) | (27,27,33,39) | (0,0,0,0) |

## 5. CONCLUSION

Previous works on network scheduling using fuzzy sets theory provides methods for scheduling projects. These methods, however, do not support the backward pass calculations in direct manner similar to that used in the forward pass. In this paper a new method based on the fuzzy theory has been developed to solve the project scheduling problem under the fuzzy environment. In this method, duration of activities are considered as positive trapezoidal fuzzy numbers. Then project characteristics such as earliest times, latest times and slack times are calculated as trapezoidal fuzzy numbers (TFNs). A major advantage of this method is to employ direct arithmetic fuzzy operations in obtaining meaningful computable results. In this method, we introduced a new approach which we called Modified Backward Pass (MBP). This approach, based on a linear programming (LP) problem, removes negative and infeasible solutions which can be generated by other methods in the backward pass calculation. We drove the general form of the optimal solution of the LP problem which enables practitioners to obtain the optimal solution by simple recursive relation without solving any LP problem. Through a numerical example, calculations involved in this method have been illustrated.

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