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Simultaneous coordination of review period and order-up-to-level in a manufacturer-retailer chain

Maryam Johari[1], Seyyed-Mahdi Hosseini-Motlagh[1*], Mohammadreza Nematollahi[1]

[1]*School of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran*

m_johari@ind.iust.ac.ir, motlagh@iust.ac.ir, nematollahi@iust.ac.ir

Abstract

In this article, the manufacturer-retailer supply chain coordination (SCC) has been developed under periodic review inventory system. In the studied model, the retailer as downstream member uses periodic review inventory policy and decides about review period (T) and order-up-to-level (R). Also, the manufacturer as upstream member faces EPQ system and determines the number of shipments from manufacturer to retailer per production run (n) as a decision variable. Firstly, the problem is investigated in decentralized and centralized models and accordingly, in order to coordinate the mentioned supply chain, a coordination scheme based on quantity discount is proposed. Maximum and minimum discounts, which are acceptable for both members, are determined. Numerical examples and sensitivity analysis indicate that the proposed coordination scheme improves the profitability and performance of both members and entire SC toward decentralized model. Moreover, the developed coordination model can share extra profit between SC members according to their bargaining power fairly.

Keywords: Manufacturer-retailer chain, Periodic review inventory system, Production, supply chain coordination, quantity discount.

1- Introduction and literature review

The supply chain needs to make decisions which led to greater profit. Two of the main replenishment decisions that affect supply chain profit are review period (T) and order-up-to-level (R). In the decentralized business environment, due to the complicated relationship among members, their own individual decisions and interests may conflict and as a result, the performance of entire SC may reduce (Masihabadi and Eshghi, 2011). Hence, coordination policies aim to encourage SC members to accept decisions which are optimal for the entire supply chain and as a result improve the SC performance. Meanwhile, applying coordination scheme is needed to coordinate SC members such as manufacturer and retailer to decide properly about production, inventory, and distribution policies and consequently satisfy their customers. Coordination mechanisms applied to improve supply chain performance by aligning different decisions throughout the SC (Govindan, Diabat and Popiuc, 2012). Various coordination mechanisms for different conditions have been extended such as coordination contracts. Among several coordination contracts, quantity discount contracts are the most common ones.

*Corresponding author
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Quantity discount contracts have been developed in supply chains with multi-period inventory systems in which supplier decreases the overstocking risk of retailer by offering a discount (Jeuland and Shugan, 1983). Recently, quantity discount contracts have been developed such as (Duan, Luo and Huo, 2010); (Liu, et al., 2014); (Li,Wang and Dai, 2016); (FerhanÇebi, 2016) to coordinate SC decisions. Moreover, other contracts have been proposed to coordinate SC such as revenue sharing contracts (Feng, Moon and Ryu, 2014; Arani, et al., 2016), buy back contracts (Hou, Zeng and Zhao, 2010; Dutta, Das and Schultma, 2016), sales rebate contracts (Wong, Qi and Leung, 2009; Chiu, Choi and Li, 2011) and so forth.

There are important decisions throughout supply chains which need to be coordinated such as replenishment (Heydari and Norouzinasab, 2016; Heydari, 2013; Boyacı and Gallego, 2002), safety stock and reorder point (Heydari, 2014a; Chaharsooghi and Heydari, 2010a; Cachon and Zipkin, 1999), order quantity (Li and Liu, 2006), protection interval (Lin, 2010), lead time (Heydari, 2014b), pricing (Mokhlesian and Zegordi, 2015), and corporate social responsibility (CSR) (Nematollahi, Hosseini-Motlagh and Heydari, 2016a;Nematollahi, Hosseini-Motlagh and Heydari, 2016b; Hsueh, 2014). For more detailed information, readers are referred to the article by (Glock, 2012) which contains a comprehensive review of coordinated inventory replenishment decisions. Hitherto, coordination models under continuous review inventory models have been widely discussed such as (Heydari and Norouzinasab, 2016); (Cobb, 2016). Conversely, the coordination model which is designed in this article aims to coordinate decisions under periodic review inventory system. Meanwhile, by applying an incentive mechanism, three decisions including (1) review period, (2) order-up-to-level, and (3) number of shipments from manufacturer to retailer per production run are coordinated simultaneously. To the best of our knowledge, there is no previous study which has been investigated coordinating these decisions under periodic review inventory system.

Periodic review inventory models are extensively applied in practice. There are wide researches in the inventory literature investigating periodic review inventory system. However, most of the periodic review inventory models have assumed a fixed length of the review periods. Several studies considered review period as decision variable similar to our work. Annadurai and Uthayakumar (2010) explored a (T, R, L) inventory model with controllable lead time in which the review period, lead time, and lost sales rate are considered as decision variables. Bijvank and Johansen (2012) proposed new models allowing constant lead times of any length when demand is compound Poisson. They considered review period as a decision variable. Soni and Joshi (2015) developed a periodic review inventory model with respect to review period, lead time, and back order rate as control variables in the fuzzy stochastic environment. Kouki and Jouini (2015) modeled a periodic review perishable inventory system. They considered review period, optimal reorder level, and optimal order quantity as decision variables in their model.

All of the above mentioned models have been investigated under single echelon inventory system. Several researchers considered periodic review inventory policy in multi echelon supply chain. Kanchana and Anulark (2006) studied an approximate periodic model for fixed-life perishable products in a two-echelon inventory distribution system. Hsu and Lee(2009) investigated the decisions of replenishment and lead-time reduction for a single manufacturer and multiple-retailer integrated periodic inventory system. Lin(2010) investigated a stochastic integrated supplier-retailer supply chain under periodic review inventory system. He explored the issue of back order price discount, protection interval, lead time, and the numbers of shipments from supplier to retailer in one production run as control variables in his model. Song, Dong and Xu, (2014) considered the optimally integrated inventory management with respect to periodic review policy in a manufacturing supply chain considering several suppliers alongside multiple kinds of uncertainties. Moreover, several studies have considered EPQ system in multi echelon SC. Kreng and Tan (2011) developed optimal replenishment decision in an Economic Production Quantity (EPQ) model under supply chain trade credit policy. Aljazzara, Jabera and Goyal(2016) investigated Joint Economic Lot Sizing Problem (JELSP) in a two stage (manufacturer–retailer) supply chain under continuous inventory system with a permissible delay in payments as a decision variable. Muniappana, Uthayakumarb and Ganesha(2015) developed production inventory models (EPQ) with back ordering and rework process. As a result, the related papers in the literature have investigated multi echelon periodic review inventory systems in integrated supply chains. Conversely, this study models a two echelon periodic review inventory system under three different decision making structures including: (1) decentralized, (2) centralized, and (3) coordinated models.

The safety factor is one of the most crucial decisions in the supply chain inventories, and particularly safety stocks, are used to cope with demand and supply uncertainties in various stages of the chain. According to Baganha and Cohen(1998), the main role of safety factor is to sustain the material flow pattern. The level of safety stock will affect the other members' decisions beside the decision maker and therefore it is needed to coordinate the safety factor throughout the supply chain. Some papers have been considered safety factor as coordinated decision similar to our work. (Chaharsooghi and Heydari, 2010b) proposed a coordination model of the joint determination of order quantity and reorder point and considered safety stock as a decision variable. Heydari (2014c) investigated the supply chain coordination in a buyer–seller chain with an order size constraint. The buyer keeps safety stock to cope with lead time demand uncertainties from customers' side. By proposing a time-based temporary price discount in each replenishment cycle, the seller tries to persuade the buyer to optimize its safety stock globally. Braglia,Castellano and Frosolini (2015) considered the management of safety stock in a coordinated single-vendor and single-buyer supply chain under continuous review and Gaussian lead-time demand. They developed a novel approach to optimize the safety stock under the investigated conditions. An exhaustive literature review on the safety factor is provided by Silver, Pyke and Peterson (1998).

In this paper, a two stage supply chain consisting of one retailer and one manufacturer is investigated. The retailer uses periodic review inventory system and decides on review period and order-up-to-level as decision variables. The manufacturer uses EPQ system and makes decision on number of shipments from manufacturer to retailer per production run as decision variable. Three models are developed: (1) none coordinated decentralized model, (2) centralized model, and (3) coordination model. Firstly, in the decentralized model, decisions are made individually by SC members. In such a case, the retailer maximizes his/her own profit and achieves optimal review period and order-up-to-level, which are locally optimal solutions from the entire SC viewpoint. Likewise, the manufacturer determines the number of shipments from manufacturer to retailer per production run. In the centralized model, one decision maker decides according to whole SC viewpoint and maximizes whole SC profit. The closed form solutions of the optimal values of decision variables under decentralized and centralized models are presented and then the concavity of the total profit functions are proved. Although the integrated decisions create greater benefit for entire SC, the retailer profit does not increase and as a result, the retailer has no motivation to accept integrated decisions. Thus, in order to encourage the retailer to accept globally optimal decisions, quantity discount contract as an incentive mechanism is considered. Under coordinated decisions, both members obtain more profit toward decentralized model. Therefore, proposed coordination scheme is enough interesting for both participants. The main contribution of the proposed coordination model is to coordinate the review period, order-up-to-level, and number of shipments from manufacturer to retailer per production run simultaneously in a manufacturer-retailer SC under periodic review inventory system.

The remainder of this paper is organized as follows. The problem is explained in section 2, in addition, mathematical models are proposed in three subsections involving decentralized model, centralized model, and eventually coordination model. The numerical examples and sensitivity analysis are prepared in section 3 and section 4 concludes the paper and discusses future research directions.

2- Problem definition and mathematical Models

In this paper, a two stage supply chain consisting of one retailer at downstream level and one manufacturer at upstream level is considered. The retailer faces stochastic demand and deterministic lead time. Demand has a normal distribution with a known mean and standard deviation. Also, shortage is considered only on the retailer side and is fully backordered. The retailer uses a periodic review inventory system for replenishing inventories. The manufacturer uses EPQ system and decides on production multiplier n to replenish its stock. When the retailer orders quantity DT, the manufacturer produces n DT at one set-up, with a finite production rate P. Each batch is dispatched to the retailer in n equally-sized shipments, where n is a positive integer (Rosenblatt and Lee, 1985).The shortage is not permitted at the upstream side.

Figure 1 depicts a sample of the inventory time plots for a retailer and manufacturer. For the retailer, there is a replenish-up-to level of R units. For the manufacturer, there is a produce-up-to level of R_m units. Given a common shipment cycle of T year, the manufacturer produces a batch of item every nT year, which is indicated the case of n=3. The manufacturer after 3th shipment cycle, starts the production of another lot $\frac{DT}{P}$ year to the next planned shipment time, as shown in Figure 1.

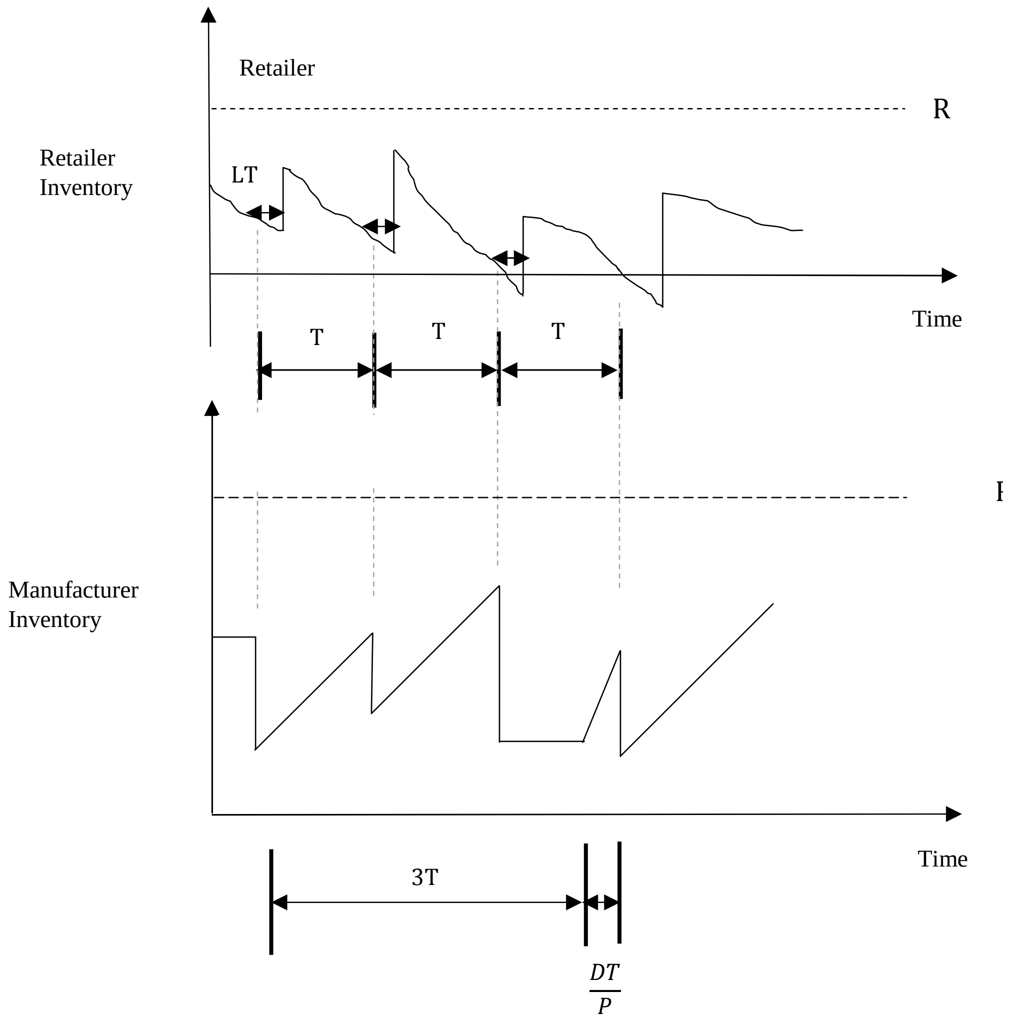

Fig. 1. Inventory levels for a retailer and manufacturer under a shipment cycle strategy.

The following notations and assumptions are used all over this paper.

2-1- Notations

Decision variables:

T: Length of a review period

R: Order-up-to-level

n =Number of shipments from manufacturer to retailer per production run, a positive integer

Parameters:

V: Manufacturer's production cost per unit

S: Manufacturer's setup cost per setup

P: Manufacturer's production rate per year

w: Unit price charged by the manufacturer to the retailer, $w > V$
D: Retailer's average demand per year
A_r: Retailer's ordering cost per order
h_r: Retailer's inventory holding cost per unit per year
h_m: Manufacturer's average inventory holding cost per unit per year
L : Length of the lead time, is given
σ: Standard deviation of the demand per unit time
X^+: Maximum value of X and 0, that is X^+= max {x,0}.
X : Protection interval (T +L) demand that has a normal distribution function with finite mean D(T +L) and standard deviation $\sigma\sqrt{T+L}$
π: Backorder charge
p : The retailing price per item (as sold by retailer to customer)
β: Bargaining power of retailer
Note: Subscripts r, m, and SC explain retailer, manufacturer, and Total SC, respectively. In addition, the superscripts de, ce and co in each variable denote decentralized, centralized and coordinated models, respectively.

2-2- Assumptions

- A two echelon SC consisting of one manufacturer and one retailer for a single product is considered and the inventory system deals with only one type item.
- The inventory level is reviewed every T units of time. A sufficient quantity is ordered up to the level R, and the ordering quantity is received after L units of time.
- The length of the lead time L does not exceed an inventory cycle time T, so that there is never more than a single order outstanding in any cycle. That is, $L \leq T$.
- The order-up-to-level R=expected demand during protection interval + safety stock (SS), and SS=k* (standard deviation of protection interval demand), which is, $R = D(T+L) + k\sigma\sqrt{T+L}$(where k is the safety factor) satisfies P(X > R) =q, (where q displays the allowed stock-out probability during the protection interval and is given).
- Fully back order shortage is considered only for the retailer (only the retailer who is connected directly to customers incurs shortage cost and upstream member does not incur any penalty for delayed orders)

2-3- Decentralized model

Under decentralized decision making model, each member decides based on its own profit. The retailer decides on review period and order-up-to-level variables. Similarly, the manufacturer decides on production multiplier n to replenish its stock. In the following, decentralized decision making models for the two SC members are analyzed.

2-3-1- Retailer model

In this part, the retailer's model is investigated without considering SC as a Total, which is called "traditional decision making". The manufacturer does not use any system to control the retailer's behavior. The retailer optimizes the order-up-to-level and review period according to its own profit function. The protection interval demand X is assumed that follows a normal distribution with mean D(T +L) and standard deviation $\sigma\sqrt{T+L}$and the order-up-to-level is $R = D(T+L) + k\sigma\sqrt{T+L}$. For the periodic review inventory model, the expected holding cost per year is $h_r\left[R - DL - \frac{DT}{2}\right]$ and the expected stock out cost is $\frac{1}{T}\pi E(X-R)^+$. Hence, the retailer profit function is equal to revenue minus sum of purchase cost, ordering cost, holding cost and stock out cost as following:

$$\pi_r(T,R) = (p-w)D - \frac{A_r}{T} - h_r\left[R - DL - \frac{DT}{2}\right] - \frac{1}{T}\pi E(X-R)^+ \qquad (1)$$

in which, the expected shortage quantity $E(X-R)^+$ at the end of the cycle can be expressed as:

$$E(X-R)^{+} = \int_{R}^{\infty}(X-R)\,f_{x}dx = \sigma\sqrt{T+L}\psi(k) > 0 \tag{2}$$

Where $\psi(k) = \varphi(k) - k[1-\Phi(k)]$ and k is safety factor, $\varphi(k)$and$\Phi(k)$denote the standard normal *p.d.f* and distribution function($d.f.$), respectively. Thus, the safety factor k can be treated as a decision variable instead of the order-up-to-level is R and therefore the retailer profit function Equation (1) can be transformed to:

$$\pi_r(T,k) = (p-w)D - \frac{A_r}{T} - h_r\left[\frac{DT}{2} + k\sigma\sqrt{T+L}\right] - \frac{1}{T}\pi\sigma\sqrt{T+L}\psi(k) \tag{3}$$

Proposition 1: *The retailer profit function is concave with respect to k for a given T.*
Proof: To prove concavity with respect to k, it is enough to show that its second order derivative with respect to k is less than zero.

The first order derivative of $\pi_r(T,k)$ with respect to k for a given T is:

$$\frac{\partial\pi_r(T,k)}{\partial k} = -h_r\sigma\sqrt{T+L} - \frac{1}{T}\pi\sigma\sqrt{T+L}(\Phi(k)-1) \tag{4}$$

The second order derivative of $\pi_r(T,k)$ with respect to k for a given T is:

$$\frac{\partial^2\pi_r(T,k)}{\partial k^2} = -\frac{1}{T}\pi\sigma\sqrt{T+L}\varphi(k) < 0 \tag{5}$$

The second-order derivative is negative, and therefore, the retailer profit function in Equation (3) is concave with respect to *k* for a given T.
By optimizing the retailer profit function $\pi_r(T,k)$with respect to k, the optimal value of *k* will be

$$1-\Phi(k) = \frac{h_r T}{\pi} \tag{6}$$

From Equation (6), it can be concluded that $T \le \frac{\pi}{h_r}$ which is an upper bound of positive T. The optimal value of (T, k) can be obtained by using a solution procedure. The following computational solution procedure is provided to obtain the optimal solution of (T, k) which is displayed by (T^*, K^*).

-Decentralized solution procedure:

Step1: Assign the lowest possible value to T
Step2: calculate k from Eq. (6), according to given T value from Step1.
Step3: calculate $\pi_r(T,k)$ from Eq. (3) according to value of T and k from step 1and 2.
Step4: If $T > \frac{\pi}{h_r}$ then terminate the solution procedure; otherwise, T=T+ε (where ε is the lowest possible value for T)and go to Step2.
Step5: A combination of T and k with the greatest expected retailer profit is optimal.

2-3-2- Manufacturer model

Under decentralized model, the manufacturer decides on production multiplier n to replenish its stock as decision variable (n) based on its profit function. The manufacturer uses EPQ policy. For each production run, the setup cost per unit time is $\frac{S}{nT}$. When the manufacturer produces the first DT units, she/he will deliver them to the retailer, after that the manufacturer will make the deliver on the average every T unit of time until inventory is exhausted.
According to Joglekar (1988), the average inventory per unit time can be calculated as
$\left\{nDT\left[\frac{DT}{P} + (n-1)T\right] - \frac{(nDT)^2}{2P} - \frac{D^2T^2}{D}[1+2+\cdots+(n-1)]\right\}\frac{D}{nDT} = \frac{DT}{2}\left[\frac{D}{P}(2-n) + (n-1)\right]$.
Thus, the manufacturer's average holding cost per unit time is$h_m\frac{DT}{2}\left[\frac{D}{P}(2-n) + (n-1)\right]$.
Hence, the manufacturer profit function, which is equal to revenue minus sum of setup cost, production cost and holding cost, is expressed by:

$$\pi_m(n) = (w-V)D - \frac{s}{nT} - h_m\frac{DT}{2}\left[\frac{D}{P}(2-n) + (n-1)\right] \tag{7}$$

Proposition 2: *The manufacturer profit function is concave with respect to n.*

Proof: To prove concavity with respect to *n*, it is sufficient to indicate that second-order derivative of $\pi_m(n)$with respect to n is less than zero. To show concavity, it is temporarily assumed that the variable *n* is a continuous variable.

The first order and second order derivatives of $\pi_m(n)$ with respect to n can be calculated as following respectively:

$$\frac{\partial \pi_m(n)}{\partial n} = \frac{s}{n^2 T} + \frac{h_m D^2 T}{2P} - \frac{h_m DT}{2} \tag{8}$$

$$\frac{\partial^2 \pi_m(n)}{\partial n^2} = -\frac{2S}{Tn^3} < 0 \tag{9}$$

The second-order derivative of $\pi_m(n)$ has a negative value. Hence, the manufacturer profit function is concave with respect to *n.*

By setting Eq. (8) equal to zero, the optimal value of ncan be calculated as

$$n = \sqrt{\frac{2PS}{T(h_m DTP - h_m D^2 T)}} \tag{10}$$

The calculated n maximizes the manufacturer profit function. Since, n is an integer variable. Hence, integer values larger and smaller than n, i.e., [n] and [n]+1, are set in the manufacturer profit function Eq. (7) and the integer that conducts more profit would be selected as optimal n in the traditional model.

2-4- Centralized model

Under centralized decision making, one decision maker aims to maximize the entire SC profit. Profit function of the entire SC is equal to the sum of SC members profit functions. Whole SC profit function per year can be formulated as:

$$\pi_{SC}(T,k,n) = \pi_r(T,k) + \pi_m(n)$$
$$= (p-w)D - \frac{A_r}{T} - h_r\left[\frac{DT}{2} + k\sigma\sqrt{T+L}\right] - \frac{1}{T}\pi\sigma\sqrt{T+L}\psi(k)$$
$$+(w-v)D - \frac{s}{nT} - h_m\frac{DT}{2}\left[\frac{D}{P}(2-n) + (n-1)\right] \tag{11}$$

Some simplifications give:

$$\pi_{SC}(T,k,n) = (p-v)D - \frac{A_r}{T} - h_r\left[\frac{DT}{2} + k\sigma\sqrt{T+L}\right] - \frac{1}{T}\pi\sigma\sqrt{T+L}\psi(k)$$
$$\pi_{SC}(T,k,n) = (p-v)D - \frac{A_r}{T} - h_r\left[\frac{DT}{2} + k\sigma\sqrt{T+L}\right] - \frac{1}{T}\pi\sigma\sqrt{T+L}\psi(k)$$
$$-\frac{s}{nT} - h_m\frac{DT}{2}\left[\frac{D}{P}(2-n) + (n-1)\right] \tag{12}$$

Proposition 3: *The supply chain profit function is concave with respect to k and n for a given T.*

Proof: To prove concavity with respect to k and n for a given T, the hessian matrix is considered to the whole SC profit function (by temporary relaxing the condition that n must be an integer into continuous variable) under given T:

$$H(\pi_{SC}) = \begin{bmatrix} \frac{\partial^2 \pi_{SC}}{\partial n^2} & \frac{\partial^2 \pi_{SC}}{\partial n\,\partial k} \\ \frac{\partial^2 \pi_{SC}}{\partial k\,\partial n} & \frac{\partial^2 \pi_{SC}}{\partial k^2} \end{bmatrix}$$

Where,

$$H_{11} = \frac{\partial^2 \pi_{SC}}{\partial n^2} = -\frac{2S}{Tn^3} < 0$$

$$\frac{\partial^2 \pi_{SC}}{\partial k\, \partial n} = \frac{\partial^2 \pi_{SC}}{\partial n\, \partial k} = 0$$
$$\frac{\partial^2 \pi_{SC}}{\partial k^2} = -\frac{1}{T}\pi\sigma\sqrt{T+L}\varphi(k) < 0$$
Where,
$$H_{22} = (-\frac{2S}{Tn^3}) * (-\frac{1}{T}\pi\sigma\sqrt{T+L}\varphi(k)) > 0$$
The first principal minor of the above Hessian matrix (H_{11}) has a negative value with respect to k and n for a given T. Also the second principal minor (H_{22}) is always positive. As observed, the Hessian matrix is negative definite, thus the SC profit function is concave with respect to k and n for a given T.

Thus, the following solution procedure is proposed to find optimal values of (T, k, n) under the centralized model.

Centralized solution procedure:

Step 1: Assign the lowest possible value to T

Step 2: Let $n = 1$ (Assign the lowest possible value to*n*).

Step 3: Calculate *k* using Equation (6).

Step 4: Calculate *n* based on obtained *k* using Equation (10).

Step 5: If two successive values of *n* are equal, then go to Step 6; otherwise, go to Step 3.

Step 6: Calculate entire SC profit function using Equation (12) according to the latest obtained T, k, and *n*.

Step 7: If $T > \frac{\pi}{h_r}$ then terminate the solution procedure; otherwise, T=T+ε (where ε is the lowest possible value for T) and go to Step 2.

Step 8: A combination of T, k, and *n* with the greatest entire SC profit function is optimal.

Therefore, according to the proposed solution procedure, the optimal values of decision variables can be calculated. Since the optimal values for decision variablesT^{**},k^{**}, and $n^{**}$are calculated to maximize entire SC profit function Eq. (12), it is clear that

$$\pi_r(dw, k_T T^*, k_k k^*) \geq \pi_r(w, T^*, k^*) \qquad (13)$$

Since T^* and k^* are calculated such that maximize the retailer profit function, hence, using T^* and k^*rather than T^{**} and k^{**}makes more profitability for the retailer. In mathematical expression is

$$\pi_r(T^*, k^*, n^*) \geq \pi_r(T^{**}, k^{**}, n^{**}) \qquad (14)$$

Each member accepts the integrated decision making policy if and only if its profitability increases. As a result, the retailer refuses to take part in the integrated decision making model. Thus, proposing an incentive scheme to convince the retailer to accept integrated decision making model is required. In the next section, an incentive scheme based on quantity discount contract is developed such that it guarantees the participation of both members in the integrated decision making policy.

2-5- Coordination model and incentive scheme

The basic aim of any coordination model is the achievement of channel coordination. In addition, the coordination model must be desirable for all SC members. In this paper, this aim is followed by applying a motivation mechanism according to a quantity discount contract. In this contract, the retailer can benefit from lower whole sale price in the purchase of the items and therefore, he/she accepts the integrated decisions on review period and order-up-to-level. In other words, under coordinated policy, if the retailer is committed to the agreed review period and order-up-to-level, then he/ she can purchase at the lower price of its purchased items to the manufacturer.

A set of operational coefficients has been defined to increase the performance of chain up to the centralized model according to the calculated optimal decision variables in the centralized structure. The coefficients for all decision variables, including T, k, and n have been defined. Regarding the values of the retailer's review period that were calculated in previous sections, for both decentralized and centralized models, the review period coefficient $d_T$for achieving channel coordination is $d_T = \frac{T^{*ce}}{T^{*de}}$. Other coordinator coefficients for the variables k and n are defined as $d_k = \frac{k^{*ce}}{k^{*de}}$, $d_n = \frac{n^{*ce}}{n^{*de}}$, respectively.

By applying these coordinator coefficients in the decentralized SC model, channel coordination will be achieved. By usingd_T, d_k on the optimal review period and safety factor, respectively, the retailer shifts the review period from T^{*de} to $T^{*ce} = d_T * T^{*de}$and the safety factor from k^{*de} to $k^{*ce} = d_k * k^{*de}$. Changes in the operational variables conduct to decrease in the retailer's profit with respect to the decentralized model. Hence, designing an incentive mechanism to share the benefits of the coordination model is remarkable. Therefore, the retailer's purchasing price from the manufacturer is decreased from w to dw. d is namely the discount factor and must be between 0 and 1.

The condition which is incentive for the retailer to shifts decisions from traditional plan toward coordinated scheme is as follows:

$$\pi_r(dw, d_T \mathrm{T}^*, d_k \mathrm{k}^*) \geq \pi_r(w, \mathrm{T}^*, \mathrm{k}^*) \tag{15}$$

A maximum allowable value for dw from the retailer point of view is extracted which is namely d_{max}

$$d_{max} = \frac{1}{wD}[wD + \frac{A_r}{T^*} - \frac{A_r}{d_T T^*} + h_r\left(\frac{DT^*}{2} + k^*\sigma\sqrt{T^*+L}\right) - h_r(\frac{Dd_T T^*}{2} + d_k k^*\sigma\sqrt{d_T T^* + L}) + \frac{\pi\sigma}{T^*}\sqrt{T^*+L}\,\psi(\mathrm{k}^*) - \frac{\pi\sigma}{d_T T^*}\sqrt{d_T T^* + L}\,\psi(d_k \mathrm{k}^*)] \tag{16}$$

Similarly, the manufacturer condition and minimum acceptable value of dw which is called $d_{min}$from the manufacturer perspective is as follows:

$$\pi_m(dw, d_n n^*) \geq \pi_m(w, n^*) \tag{17}$$

$$\mathrm{d_{min}} = \frac{1}{\mathrm{wD}}[\mathrm{wD} - \frac{\mathrm{S}}{\mathrm{n^*T^*}} + \frac{\mathrm{S}}{\mathrm{d_n n^* d_T T^*}} - \frac{\mathrm{h_m DT^*}}{2}\left(\frac{\mathrm{D}}{\mathrm{P}}(2-\mathrm{n}^*) + (\mathrm{n}^*-1)\right) + \frac{\mathrm{h_m D d_T T^*}}{2}\left(\frac{\mathrm{D}}{\mathrm{P}}(2-\mathrm{d_n n^*}) + (\mathrm{d_n n^*}-1)\right) \tag{18}$$

If the interval [$\mathrm{d_{min}}$,$\mathrm{d_{max}}$] is non-empty, then channel coordination is achievable. Each value of d within [$\mathrm{d_{min}}$,$\mathrm{d_{max}}$] creates more profit than decentralized decisions for both members. At the lower bound,$\mathrm{d_{min}}$, all coordination profits have been gained by the retailer, and at the upper bound, $\mathrm{d_{max}}$, all coordination profits have been obtained by the manufacturer. The bargaining power of the retailer is considered β against the manufacturer, therefore the bargaining power of the manufacturer will be $(1-\beta)$. A linear profit sharing mechanism based on members' bargaining powers is used to find an appropriate value for d. Thus, based on the proposed mechanism, d is determined as

$$d = \beta d_{max} + (1-\beta)d_{min} \tag{19}$$

Hence, the additional profit obtained by applying the coordination model is shared between the two members according to the calculated discount factor d.

3- Numerical Examples and Sensitivity Analysis

In this section, a set of numerical examples is conducted to investigate the performance of the proposed incentive scheme. The proposed model is examined by using three test problems. Parameters of the three test problems are shown in Table 1.

Table 1.Three investigated test problems

	Test Problem 1	Test Problem 2	Test Problem 3
A_r	500	700	900
S	300	400	500
D	600	1000	2000
h_r	25	40	50
h_m	20	30	40
L (Day)	1	0.5	0.2
p	90	120	160
w	60	70	100
π	50	60	70
P	700	1200	2300
σ	28	150	300
v	30	50	80
β	0.4	0.3	0.7

Running the model and calculating the decision variables indicate that the integrated decision making can increase SC profitability. In addition, the developed quantity discount model has the capability of coordinating the SC by sharing additional profits between SC members. The results of running model for three test problems are shown in Table 2.By comparing the SC profit in the decentralized and centralized decision making models, it can be concluded that integrated decision making can increase the SC profitability. Although centralized decision making can improve SC profitability, but decreases the retailer profitability (see Table 2). The investigated motivation mechanism could share the obtained profit between SC members based on their bargaining power. The proposed model achieves channel coordination in all three test problems. As shown in Table 2, in all test problems, the SC profitability as well as both members' profit in coordinated model is more than that in decentralized model; thus, participating in the proposed model is guaranteed. Furthermore, a set of sensitivity analysis on the two parameters σ and h_r is examined. Parameters for sensitivity analyses are taken from test problem 2.

Table 2. Running model in the decentralized, centralized and coordinated model

	Test Problem 1	Test Problem 2	Test Problem 3
Decentralized SC			
T	91.56	60.66	41.06
k	1.15	1.22	1.40
n	2.00	2.00	3.00
π_r	13,545.48	38,274.29	97,012.91
π_m	15,896.94	16,303.69	33,431.78
π_{SC}	29,442.42	54,577.98	130,444.69
Centralized SC			
T	73.06	50.01	34.26
k	1.28	1.33	1.50
n	3.00	3.00	3.00
π_r	13,447.57	38,138.81	96,790.32
π_m	16,127.85	16,629.12	33,980.12
π_{SC}	29,575.43	54,767.93	130,770.44
Coordinated SC			
T	73.06	50.01	34.26
k	1.28	1.33	1.50
n	3.00	3.00	3.00
π_r	13,598.69	38,331.27	97,240.94
π_m	15,976.74	16,436.66	33,529.51
π_{SC}	29,575.43	54,767.93	130,770.44
d_{min}	0.99359	0.99535	0.99726
d_{max}	0.99728	0.99806	0.99889
d	0.99580	0.99725	0.99775

The effect of changing discount factor, d, on the profit of each member is examined. As shown in Fig. 2, by increasing discount factor up to the d_{max} the portion of achieved profit by the manufacturer increases while the retailer's portion decreases. At the value of d, the portion of each member is equal to the other one's portion. This point is d=0.9954 and the portion of each member is about 0.005% of their profitability in the decentralized model. When the percent of increase in profitability in comparison with the decentralized model is the criterion of members' participation, applying Fig. 2 is useful.

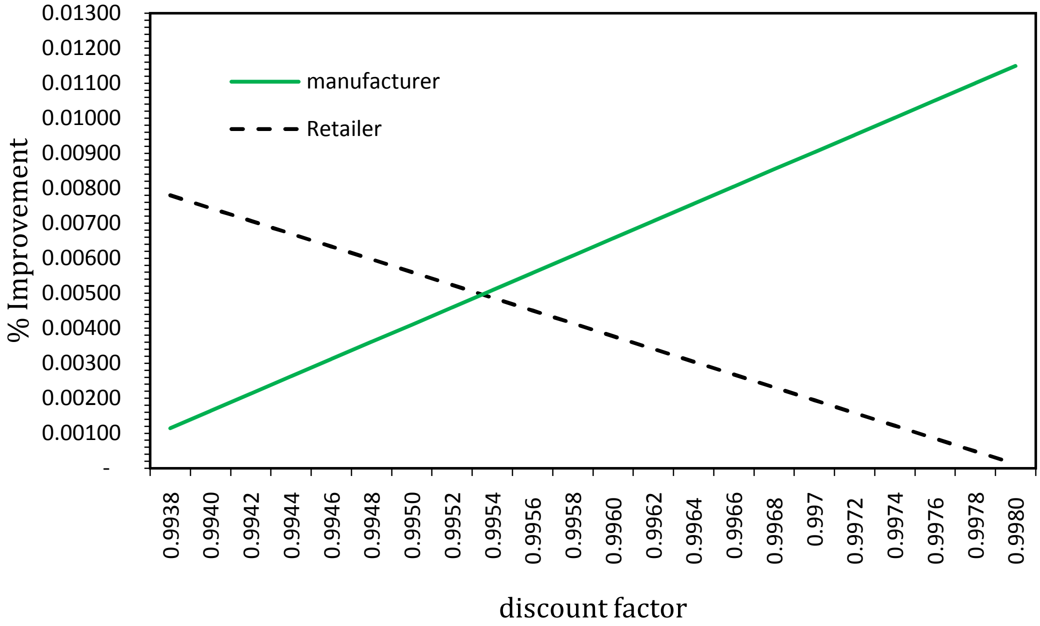

Fig. 2. Members' increased profit (as a percentage of their decentralized profitability) for different d.

Figure 3, shows trend ofd_{min}, d_{max}, and d by increasing uncertainty. As shown in Fig.3, the distance between $d_{min}$and d_{max}decreases by increasing uncertainty under coordination model. In addition, in the high value of uncertainty, the distance between $d_{min}$and $d_{min}$is small and also d is so small which is more applicable in practice. Moreover, the designed d can motivate the downstream to accept coordination policy. Thus, the proposed model is suitable when SC faces a high level of uncertainty. Moreover, interval $[\mathrm{d_{min}},\mathrm{d_{max}}]$ remains a non-empty interval for wide values of σ.

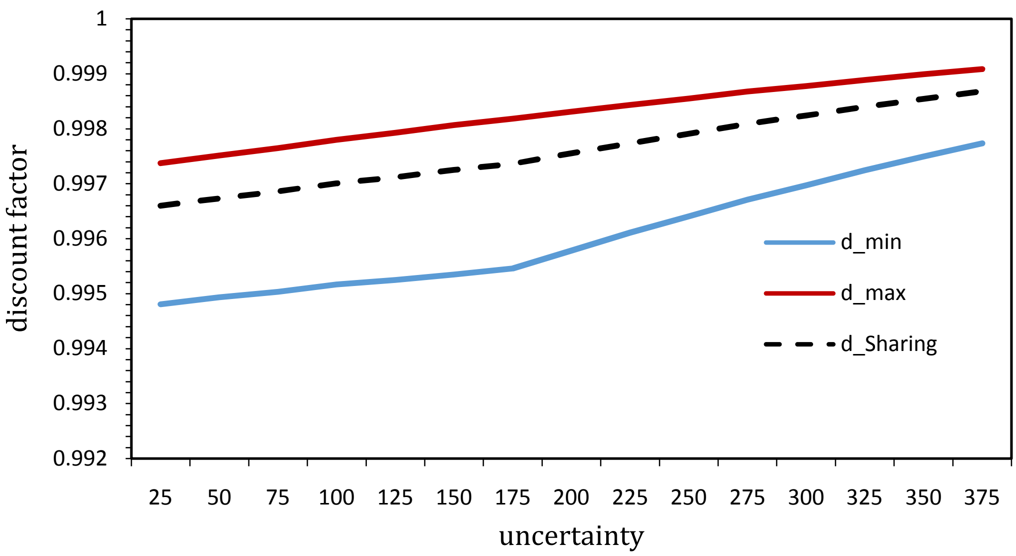

Fig. 3. Interval $[\mathrm{d_{min}},\mathrm{d_{max}}]$ based on changing uncertainty (σ)

Figure 4, indicates the changes of SC profitability in both coordinated and decentralized models by increasingh_r. As shown in Fig. 4, by increasingh_r, SC profitability decreases in both decentralized and coordinated models, which is expected. However, SC profitability in coordinated model indicates less decrease than decentralized model when h_r increases.

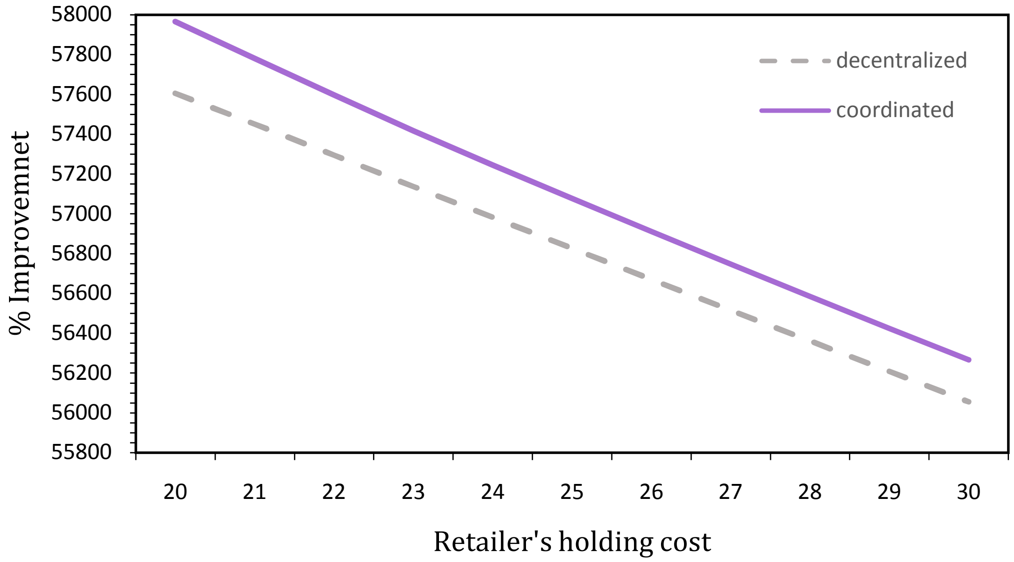

Fig. 4. Changing on SC profit improvement, by increasing h_r

According to the Figure 5, when uncertainty is equal to 150, the percent of each member's improvement in comparison with decentralized model is maximized. As shown in Fig. 5, by increasing σ, the profitability of the model for the both members beside whole SC will decrease. Moreover, in the high value of uncertainty the distance between the two SC members' profit decreases under coordination model. Therefore, using coordination decision making model under uncertainty could be considerable and advantageous.

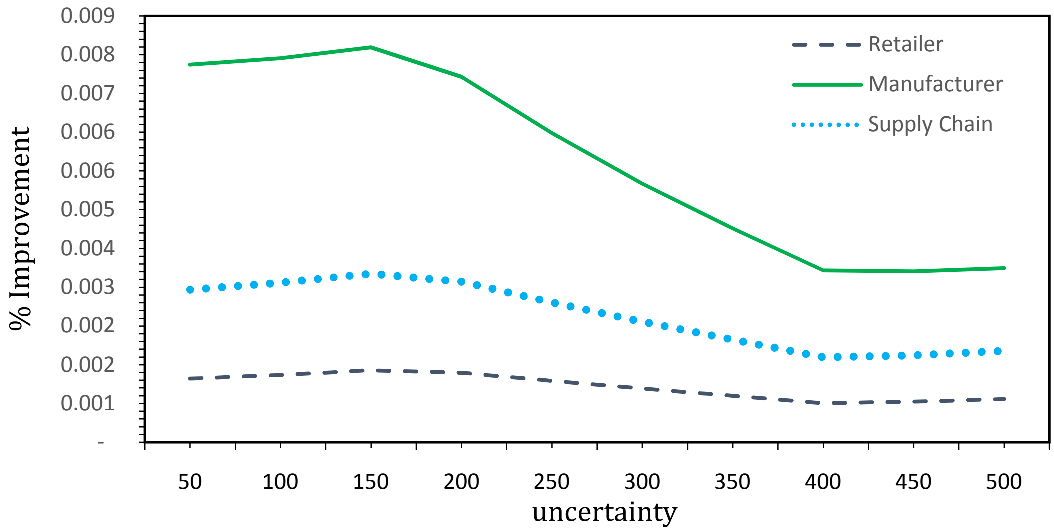

Fig. 5. Comparing improvement by coordination plan under uncertainty (σ)

The managerial implications from our study can be summarized as follows:

(1) In a two-echelon periodic review inventory system, joint decision making on review period, order-up-to-level, and number of shipments from manufacturer to retailer per production run makes more profits for the whole SC. However, this joint decisions may decrease the profitability of the retailer compared to the decentralized decision making structure. Therefore, applying a coordination mechanism is of high importance in order to convince the retailer for participation in the joint decision making plan.
(2) Results of sensitivity analysis show that under periodic review inventory system, applying quantity discount contract as a coordination mechanism can increase the whole SC profits by changing the retailer's replenishment decisions. In addition, the proposed contract will increase both members' profitability compared to the traditional structure by sharing additional benefit between two members fairly. Therefore, proposed coordination mechanism is capable of achieving channel coordination under periodic review inventory system in practice.
(3) The proposed quantity discount contract in the investigated problem results small discount factors for coordinating SC, which is more applicable in practice. Moreover, in the high value of uncertainty, the distance between minimum and maximum amounts of discount is small which makes the model more applicable. Thus, the proposed model is more suitable when SC faces a high level of uncertainty (see Figure 3).

4- Conclusion

In this paper, a two-echelon supply chain consisting of one retailer and one manufacturer under periodic review inventory policy for the downstream member is developed. In this article, simultaneous decision making on review period and order-up-to-level in a two-echelon SC is proposed by modeling SC decision structure under three various models: (1) decentralized decision making,(2) centralized decision making, and coordinated decision making. Centralized decision making conducts a remarkable improvement in the entire SC profit by globally optimizing decisions. Nevertheless, centralized decision making often decreases down stream's profit. To ensure more profitability for all the members, an incentive mechanism is developed which could persuade downstream member to optimize his/her replenishment decisions from the entire SC perspective. A model according to quantity discount contract is proposed to coordinate replenishment decisions throughout the SC. Situations under which both SC members have sufficient incentive to take part in the proposed model are extracted, i.e. maximum discount value from the retailer perspective and minimum discount value from the manufacturer point of view. Numerical examples indicate the effectiveness of the integrated decision making in the proposed SC. Moreover, according to the sensitivity analysis, one can conclude that the developed incentive mechanism is capable of resulting channel coordination. In addition, coordination model increases both SC members' profit alongside SC profit. Moreover, proposed model conducts the small discount interval for SC coordination, which is more applicable in practice and also, can motivate the retailer to accept coordination policy in all tests. Indeed small discounts imply on the applicability of the advanced model. To extend the current model, considering elasticity of customer demand to retailer price or product's quality or rival price are suggested. In addition, it can be interesting to consider other parties in the SC, in our work, one retailer and one manufacturer is considered. Also, coordination scheme would be applied for complicated SC such as multi echelon inventory policy. Moreover, joint contracts to coordinate SC members can be challengeable.

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