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Optimal production and marketing planning with geometric programming approach

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Abstract

One of the primary assumptions in most optimal pricing methods is that the production cost is a non-increasing function of lot-size. This assumption does not hold for many real-world applications since the cost of unit production may have non-increasing trend up to a certain level and then it starts to increase for many reasons such as an increase in wages, depreciation, etc. Moreover, the production cost will eventually have a declining trend. This trend curve can be demonstrated in terms of cubic function and the resulted optimal pricing model can be modeled in Geometric Programming (GP). In this paper, we present a new optimal pricing model where the cost of production has different trends depending on the production size. The resulted problem is formulated as a parametric GP with five degrees of difficulty and it is solved using the recent advances of optimization techniques. The paper is supported with various numerical examples and the results are analyzed under different scenarios.

Keywords: Geometric Programming; Nonlinear Model; Production and Operations Management; Optimal pricing; Marketing planning.

1-Introduction

During the past two decades there have been great efforts devoted to achieve profitability through strategic pricing. Indeed, companies may increase their market shares and maximize customer satisfaction through systematic pricing strategies. The primary concern of most studies is to select one or more appropriate pricing strategies to increase profitability. Pricing strategy also determines how customers view and respond to different products or services. Appropriate pricing strategy will depend on how we wish to position products or services. It should consider the market and competitors before developing pricing strategies. A key point in a competitive environment is associated with marketing and production decisions. Generally, three main strategies have been introduced for production and marketing sectors: Separate strategy, Jointstrategy and coordinated strategy. The recent advances on pricing and marketing strategies lead to increase motivation for having more realistic optimal lot-sizing strategies. Lee and Kim (1993) considered full and partial models of a joint production and marketing problems in order to maximize the profit of a firm facing constant, but price and marketing dependent demand over

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a planning horizon to determine price, marketing cost and lot-size, simultaneously. They used geometric programming method to solve the resulted model and discuss them through managerial discussions. Kim and Lee (1988) investigated the fixed and variable capacity problems of jointly determining an item's price and lot-size for a profit maximizing firm when demand was a function price and marketing expenditures. Chen (2000) introduced a profit-maximizing inventory model for joint optimization of quality level, selling quantity, and purchasing price of a product for intermediate firms by assuming that the selling price, supply rate of the product, and the fixed selling costs were power functions of decision variables. Kochenberger (1971) tackled the nonlinear economic order quantity (EOQ) problem through geometric programming (GP) method (Duffin et al., 1967). Cheng (1989) implemented GP for solving lot sizing problems where the cost per unit was a function of demand. Lee (1993) developed GP formulations for an EOQ problem where lot-size and selling price were decision variables, and cost was considered to be a function of the lot-size. Geometric programming is an optimization technique developed for solving a class of nonlinear optimization problems with some useful theoretical and computational properties. Abuo-El-Ata et al. (2003) introduced a multi-item production lot-size inventory model with a varying order cost under a restriction and solved the resulted model by geometric programming technique. Mandal et al. (2005) solved multi-objective fuzzy inventory model with three constraints by geometric programming approach. Jung and Klein (2005) considered the difference between the optimal order quantities via GP technique by comparing the cost minimization model to the profit-maximization model. Sadjadi et al. (2005) suggested a profit maximizing GP model for optimal production and marketing planning. In their model, demand depends on price and marketing expenditures, and production cost is inversely related to the lot size. They also described a flexible production rate and derived a closed-form solution for the problem. Parlar and Weng (2006) described a coordinating pricing and production decisions in attention to price competition. They presented a method based on GP to find the optimal solution. Liu (2007) proposed a discount model in his profit-maximization problem and suggested a method based on GP to find the optimal solution. Fathian et al. (2009) explained a pricing model for electronic products. They presented a mathematical model assuming that demand depends on product's price, marketing and service expenditures, and unit production cost is a function of demand. For finding the global solution for the problem they used the GP dual method. Sadjadi et al. (2010) addressed a production-marketing problem in the context of the unreliable production process. They used nonlinear posynomial geometric programming and obtained a closed form solution for jointly optimized the lot-size, marketing expenditures, set-up cost, and reliability of the production. Ghosh and Roy (2013) reformulated a goal programming problem by using GP technique. In this paper, we present a new optimal pricing model where the cost of production has different trends depending on the production size. The resulted problem is formulated as a parametric GP with five degrees of difficulty and it is solved using the recent advances of optimization techniques. The paper is supported with various numerical examples and the results are analyzed under various scenarios.

2-Notations and Assumptions

The following notations are adopted for developing the mathematical model:

2-1-Parameters

D Demand
C Unit production cost
i Cost of having one dollar of the item tied up in inventory for a unit time interval
a Set-up cost of the production.
α Price elasticity to demand
δ Marketing expenditure elasticity to demand
Total revenue (PD)
Production cost (CD)
Marketing cost (MD)
Inventory holding cost ($iC.\ Q / 2$)
Set-up cost (aD/ Q)

2-2-Decision Variable

P Selling price per unit
M Marketing expenditure per unit
Q Production lot size (units)

Π (*P*, *Q*, *M*) the profit function
Here (*) represents optimality for *P*, *Q*, *M* and *Π*.
The following assumptions are used in this paper:

$$D = kP^{-\alpha}M^{\delta} \quad (1)$$

Where (*D*) represents demand, which is a function of unit price (*P*) with $\alpha > 1$, $0 < \delta < 1$ and (*k*) is a scaling constant, (α) is price elasticity to demand ($\alpha > 1$) (Lilien et al., 1992). Parameter (*C*) is defined as production cost per unit. In this study, (*C*) is a unit cost and can be discounted with (β). Therefore, we have:

$$C = r_1Q^3 + r_2Q^2 + r_3Q \quad (2)$$

Where (*Q*) is production lot-size (units), (*r*) is the scaling constant for unit production cost.
We assume that $r_1 < 0$ and $r_3 < 0$. Note that the cubic form of the cost function is more realistic terms compared with previously used non-increasing exponential form.

3-The Proposed Model

Regarding mentioned assumptions, the production lot-sizing and marketing model (*Π*) can be written as follows:
In this model, we have determined the price, demand or production volume, and lot-size of the planning horizon, simultaneously.

$$\text{Max} \quad \Pi(P, Q, M) = PD - CD - MD - iC\left(\frac{Q}{2}\right) - \frac{aD}{Q} \quad (3)$$

$$\begin{aligned}\text{Max} \quad \Pi(P, Q, M) &= \text{Total revenue} - \text{Production cost} - \text{Marketing cost} \\ &- \text{Inventory holding cost} - \text{Set up cost}\end{aligned}$$

This particular form of relationship is signomial function, which can be converted into posynomial parametric GP problem.
The optimal solution of such a problem can be determined by GP method which is explained in details in the following section.

4-Optimal Solution of the Model

The problem (3) is a signomial GP problem with 5 degrees of difficulties. We need to make some necessary modifications in order to change the problem into a posynomial GP problem, for this purpose, we have:
$min\ z^{-1}\ or\ \ max\ z$
So that:

$$\begin{aligned}Max \quad \Pi(P, Q, M) &= kP^{1-\alpha}M^{\delta} + kr_1P^{-\alpha}M^{\delta}Q^3 - kr_2P^{-\alpha}M^{\delta}Q^2 \\ &+ kr_3P^{-\alpha}M^{\delta}Q - kP^{-\alpha}M^{\delta+1} + 0.5ir_1Q^4 - 0.5ir_2Q^3 \\ &+ 0.5ir_3Q^2 - kaP^{-\alpha}M^{\delta}Q^{-1}\end{aligned}$$

$$\begin{aligned}Max \quad \Pi(P, Q, M) &= \{kP^{1-\alpha}M^{\delta} + kr_1P^{-\alpha}M^{\delta}Q^3 + kr_3P^{-\alpha}M^{\delta}Q + 0.5ir_1Q^4 \\ &+ 0.5ir_3Q^2\} - \{kr_2P^{-\alpha}M^{\delta}Q^2 + kP^{-\alpha}M^{\delta+1} + 0.5ir_2Q^3 \\ &+ kaP^{-\alpha}M^{\delta}Q^{-1}\}\end{aligned}$$

Let (*n*) and (*m*) be the number of terms and variables in the primal problem (4), respectively. Then the degree of difficulty (*d*) for the resulting GP problem is $d = n - (m + 1) = 1$.
We have, n=9 &m=3
So there, d=9-(3+1) =5

5-Method of Problem Solving

We consider two parameters R_1 and R_2.

$$Max \quad \Pi(P, Q, M) = R_1 - R_2 \quad (6)$$

$$\begin{aligned}\{kP^{1-\alpha}M^{\delta} + kr_1P^{-\alpha}M^{\delta}Q^3 + kr_3P^{-\alpha}M^{\delta}Q + 0.5ir_1Q^4 \\ + 0.5ir_3Q^2\} = R_1\end{aligned} \quad (7)$$

$$\{kr_2P^{-\alpha}M^{\delta}Q^2 + kP^{-\alpha}M^{\delta+1} + 0.5ir_2Q^3 + kaP^{-\alpha}M^{\delta}Q^{-1}\} = R_2 \tag{8}$$

We propose a variable z as follows:

$$Max \quad \Pi(P, Q, M) = Z \tag{9}$$

$$Z \le R_1 - R_2 \to Z + R_2 \le R_1 \to Z + R_2 \le Y \le R_1 \tag{10}$$

$$Z + R_2 \le Y \to ZY^{-1} + R_2Y^{-1} \le 1 \tag{11}$$

$$Y \le R_1 \to R_1Y^{-1} \ge 1 \tag{12}$$

Also, we have:

$$\prod_{i=1}^{N}\left(\frac{u_i}{a_i}\right)^{a_i} \le \sum_{i=1}^{N} u_i \tag{13}$$

$$\sum_{i=1}^{N} a_i = 1 \tag{14}$$

$$\left(\sum_{i=1}^{N} u_i\right)^{-1} \le \prod_{i=1}^{N}\left(\frac{u_i}{a_i}\right)^{-a_i} \tag{15}$$

For the first restriction, we have:

$$R_1 = R_{11} + R_{12} + R_{13} + R_{14} + R_{15}$$

$$R_1Y^{-1} \ge 1 \to R_{11}Y^{-1} + R_{12}Y^{-1} + R_{13}Y^{-1} + R_{14}Y^{-1} + R_{15}Y^{-1} \ge 1$$

$$(R_{11}Y^{-1} + R_{12}Y^{-1} + R_{13}Y^{-1} + R_{14}Y^{-1} + R_{15}Y^{-1})^{-1} \le 1$$

$$(R_{11}Y^{-1} + R_{12}Y^{-1} + R_{13}Y^{-1} + R_{14}Y^{-1} + R_{15}Y^{-1})^{-1} \le \left(\frac{R_{11}Y^{-1}}{T_1}\right)^{-T_1}\left(\frac{R_{12}Y^{-1}}{T_2}\right)^{-T_2}\left(\frac{R_{13}Y^{-1}}{T_3}\right)^{-T_3}\left(\frac{R_{14}Y^{-1}}{T_4}\right)^{-T_4}\left(\frac{R_{15}Y^{-1}}{T_5}\right)^{-T_5}$$

$$T_i = \frac{R_{1i}}{\sum_{i=1}^{5} R_{1i}}$$

We have:

$$R_{11} = kP^{1-\alpha}M^{\delta} \tag{21}$$
$$R_{12} = kr_1P^{-\alpha}M^{\delta}Q^3 \tag{22}$$
$$R_{13} = kr_3P^{-\alpha}M^{\delta}Q \tag{23}$$
$$R_{14} = 0.5ir_1Q^4 \tag{24}$$
$$R_{15} = 0.5ir_3Q^2 \tag{25}$$

And also:

$$W = \sum\nolimits_{i=1}^{5} R_{1i} \tag{26}$$

$$T_1 = \frac{(kP^{1-\alpha}M^{\delta})}{W} \tag{27}$$

$$T_2 = \frac{(kr_1 P^{-\alpha} M^{\delta} Q^3)}{W} \tag{28}$$

$$T_3 = \frac{(kr_3 P^{-\alpha} M^{\delta} Q)}{W} \tag{29}$$

$$T_4 = \frac{(0.5ir_1 Q^4)}{W} \tag{30}$$

$$T_5 = \frac{(0.5ir_3 Q^2)}{W} \tag{31}$$

So we have:

$$\left(\frac{R_{11}Y^{-1}}{T_1}\right)^{-T_1}\left(\frac{R_{12}Y^{-1}}{T_2}\right)^{-T_2}\left(\frac{R_{13}Y^{-1}}{T_3}\right)^{-T_3}\left(\frac{R_{14}Y^{-1}}{T_4}\right)^{-T_4}\left(\frac{R_{15}Y^{-1}}{T_5}\right)^{-T_5} \leq 1$$

For the second restriction, we have:

$$Z + R_2 \leq Y \rightarrow ZY^{-1} + R_2 Y^{-1} \leq 1 \tag{33}$$

$$ZY^{-1} + R_{21}Y^{-1} + R_{22}Y^{-1} + R_{23}Y^{-1} + R_{24}Y^{-1} \leq 1 \tag{34}$$

Finally, we have:

$$min\, Z^{-1}$$

$$\left(\frac{R_{11}Y^{-1}}{T_1}\right)^{-T_1}\left(\frac{R_{12}Y^{-1}}{T_2}\right)^{-T_2}\left(\frac{R_{13}Y^{-1}}{T_3}\right)^{-T_3}\left(\frac{R_{14}Y^{-1}}{T_4}\right)^{-T_4}\left(\frac{R_{15}Y^{-1}}{T_5}\right)^{-T_5} \leq 1$$

$$ZY^{-1} + R_{21}Y^{-1} + R_{22}Y^{-1} + R_{23}Y^{-1} + R_{24}Y^{-1} \leq 1$$

$$min\, Z^{-1} \tag{36}$$

$$\left(\frac{kP^{1-\alpha}M^{\delta}Y^{-1}}{T_1}\right)^{-T_1}\left(\frac{kr_1P^{-\alpha}M^{\delta}Q^3Y^{-1}}{T_2}\right)^{-T_2}\left(\frac{kr_3P^{-\alpha}M^{\delta}QY^{-1}}{T_3}\right)^{-T_3}\left(\frac{0.5ir_1Q^4Y^{-1}}{T_4}\right)^{-T_4} * \left(\frac{0.5ir_3Q^2Y^{-1}}{T_5}\right)^{-T_5} \leq 1$$

$$ZY^{-1} + kr_2P^{-\alpha}M^{\delta}Q^2Y^{-1} + kP^{-\alpha}M^{\delta+1}Y^{-1} + 0.5ir_2Q^3Y^{-1} + kaP^{-\alpha}M^{\delta}Q^{-1}Y^{-1} \leq 1$$

We simplify the model as follows:

$$min\, Z^{-1}$$

$$\left(kP^{1-\alpha}M^{\delta}Y^{-1}\right)^{-T_1}\left(kr_1P^{-\alpha}M^{\delta}Q^3Y^{-1}\right)^{-T_2}\left(kr_3P^{-\alpha}M^{\delta}QY^{-1}\right)^{-T_3}(0.5ir_1Q^4Y^{-1})^{-T_4} * (0.5ir_3Q^2Y^{-1})^{-T_5}{T_1}^{T_1}{T_2}^{T_2}{T_3}^{T_3}{T_4}^{T_4}{T_5}^{T_5} \leq 1 \tag{37}$$

$$ZY^{-1} + kr_2P^{-\alpha}M^{\delta}Q^2Y^{-1} + kP^{-\alpha}M^{\delta+1}Y^{-1} + 0.5ir_2Q^3Y^{-1} + kaP^{-\alpha}M^{\delta}Q^{-1}Y^{-1} \leq 1$$

The final model obtains as follows:

$$min\, Z^{-1}$$

$$0.5^{(-T_4-T_5)}k^{(-T_1-T_2-T_3)}P^{((-T_1(1-\alpha))+((T_2+T_3)\alpha))}M^{(\delta(-T_1-T_2-T_3))}{r_1}^{(-T_2-T_4)}{r_3}^{(-T_3-T_5)} * Q^{(-3T_2-T_3-4T_4-2T_5)}i^{(-T_4-T_5)}Y^{(T_1+T_2+T_3+T_4+T_5)}{T_1}^{T_1}{T_2}^{T_2}{T_3}^{T_3}{T_4}^{T_4}{T_5}^{T_5} \leq 1 \tag{38}$$

$$ZY^{-1} + kr_2P^{-\alpha}M^{\delta}Q^2Y^{-1} + kP^{-\alpha}M^{\delta+1}Y^{-1} + 0.5ir_2Q^3Y^{-1} + kaP^{-\alpha}M^{\delta}Q^{-1}Y^{-1} \leq 1$$

$$P > 0, C > 0, M > 0, i > 0, Q > 0, a > 0, k > 0, r > 0$$

6-Solution Approach and Numerical Experiment

6-1-Solution Approach

Therefore, the model (38) above is a posynomial GP with five degrees of difficulty (Duffin et al., 1967). Moreover, to find the optimal solution of the problem the CVX modeling system (Grant & Boyd (2014)) is used.

6-2-A numerical example

Consider a particular manufacturer who wants to determine the production lot-size, Total revenue, Production cost, Marketing cost, Inventory holding cost, Set-up cost. For this example, assume that the parameters are as follows:

k=10; α=1.25; δ=0.003; a=3; Moreover, let that production cost is calculated as follows. r_1=1.0058; r_2=8.2637; r_3=8.3788; i=1; the implementation of CVX on the proposed GP model of this paper yields Π*=5.3301; Q*=0.842; P*=8.9414; M*=0.0215.

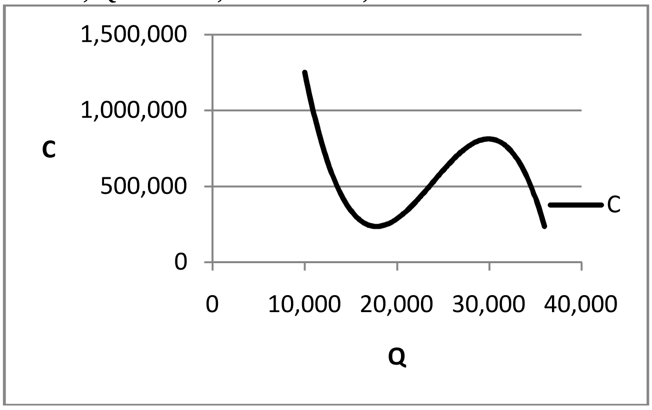

Figure1. The cost of production

From figure 1 it can be seen that as lot-size (Q) value increases, production cost decreases except for a specified period (Range 18,000 to 30,000). The reason of this issue goes to difference between cost of production during the day and night. It should be noted that some expenditures at night are higher than daytime, e.g. wage rates, costs of lighting and heating work spaces, transportation costs and etc.

6-3-Sensitivity Analyses

In order to have a better understanding of the behavior of the algorithm used for the model presented in this paper, we need to carefully analyze the behavior of the optimal solution with regard to changes in each parameter of the GP model. In the following sub-sections we separately consider the effects of changes in different parameters of the proposed power functions, on the optimal values of decision variables. However, due to high degrees of nonlinearities of the model, and interdependencies of some decision variables, it is not always possible to expect a certain behavior of the decision variables.

6-3-1-Changes in Parameters of the Demand Function

At first we considered changes in the input parameters of the demand function on the optimal solution. In the following Figs., the horizontal axis represents different number of instances (n) and the vertical axis shows the optimal selling price per unit (P*), optimal marketing expenditure per unit (M*), optimal production lot-size (Q*) and optimal profit (Π*).

6-3-1-1-Effects of Changes in (α) on Optimal Solution

First, the impacts of changes in values of (α) on the optimal solution are considered. We investigate different values of (α) as follows: $\alpha=\alpha_0+0.25$, where α_0=1andwe observe from Fig. 2 that as (α) increases, the optimal selling price (P) decreases. This is because as price elasticity to demand increases, the company has to decrease its product price in order to maintain the desired market share. As shown in Fig. 3, with the increase in (α), the optimal marketing expenditures decrease. The implications of this situation are that higher values of a cause the total cost to increase, and in order to minimize the total cost, which is the objective function of the problem, marketing expenditures have to decrease. Fig. 4 illustrates the behavior of the lot-size (Q) with consideration of changes in the demand elasticity of price (α) from this figure, it can be seen that when the (α) value increases, the value of lot-size (Q) decreases. As shown in Fig. 5, usually when customers get more sensitive to the price there is a risk for the company of losing some of its customers and therefore reducing its total profit. Though through a reduction in the product's price the market share can be maintained, but sales revenue will fall. Therefore, companies can reduce its total production and inventory holding costs and hence alleviate huge losses in his profit by reducing the lot-size.

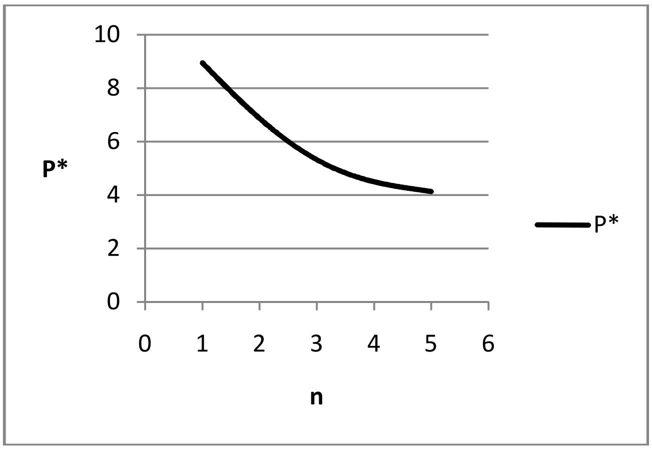

Figure2. Effects of changes in ($\boldsymbol{\alpha}$) on price (P)

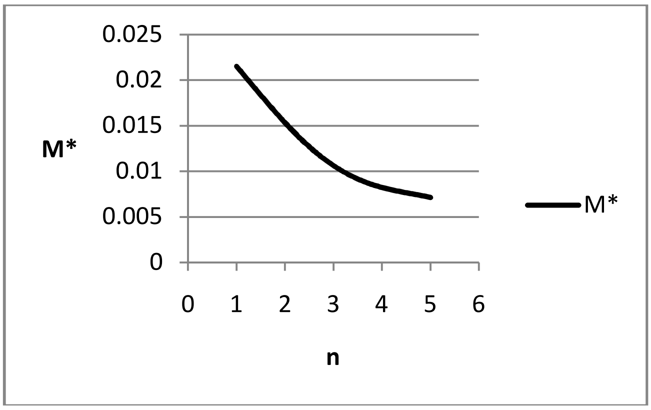

Figure3. Effects of changes in ($\boldsymbol{\alpha}$) on marketing expenditure (M)

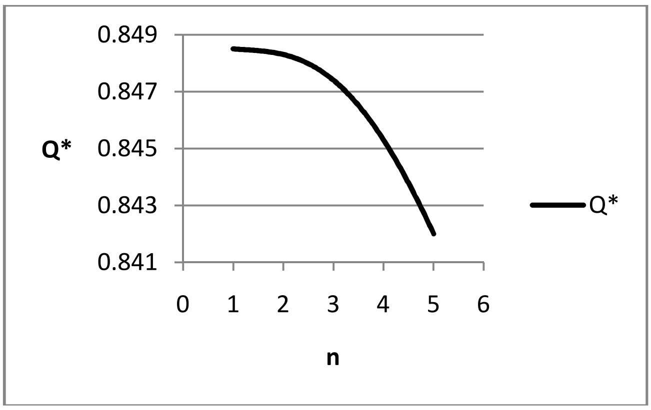

Figure4. Effects of changes in ($\boldsymbol{\alpha}$) on the lot-size (Q)

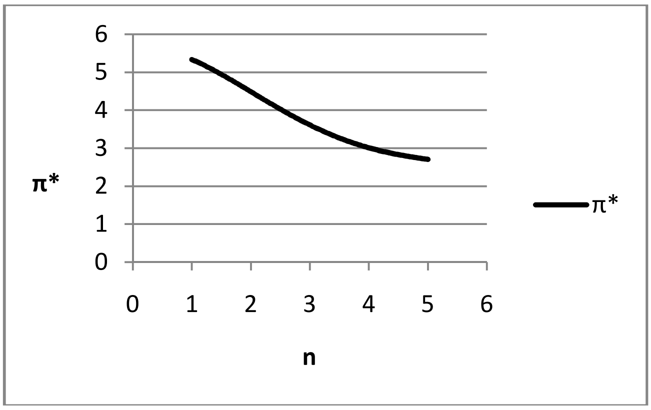

Figure5. Effects of changes in (α) on the profit (Π)

6-3-1-2-Effects of Changes in (δ) on Optimal Solution

In this section, we examine different values of (δ) as follows: $\delta = \delta_0$+0.003, where δ_0 = 0. As can be seen from Fig.6, when marketing expenditure elasticity to demand (δ) increase we see an increase in marketing expenditure (*M*). In order to offset the increase in costs, the selling price per unit (*P*) will increase (Fig.7). Increasing in prices causes a decrease in demand and production lot-size (*Q*) Therefore, the profit will be reduced (Figs.8, 9).

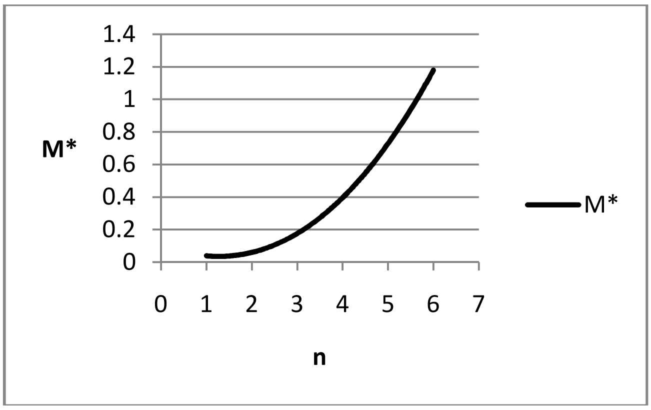

Figure6. Effects of changes in (δ) on marketing expenditures (*M*)

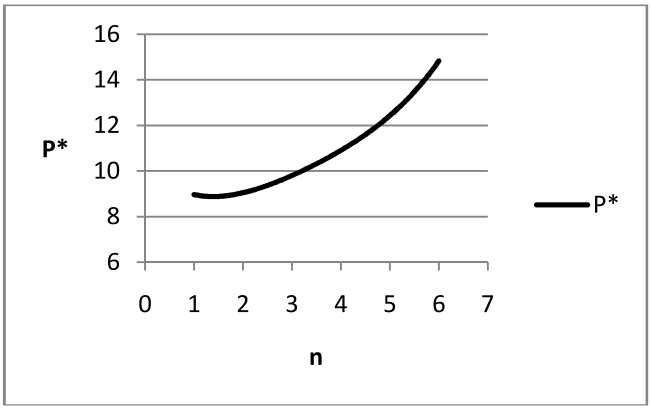

Figure7. Effects of changes in (δ) on price (*P*)

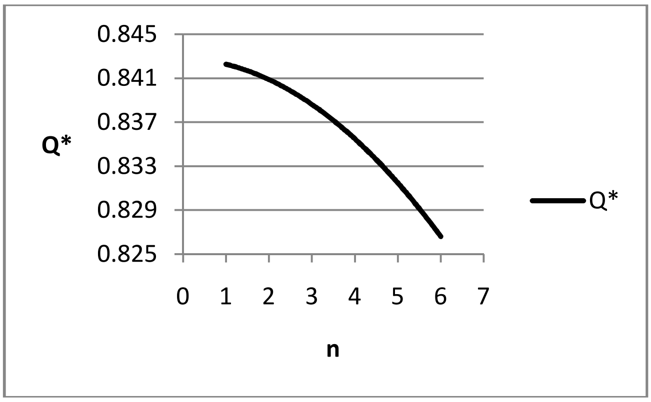

Figure8. Effects of changes in (***δ***) on the lot-size (*Q*)

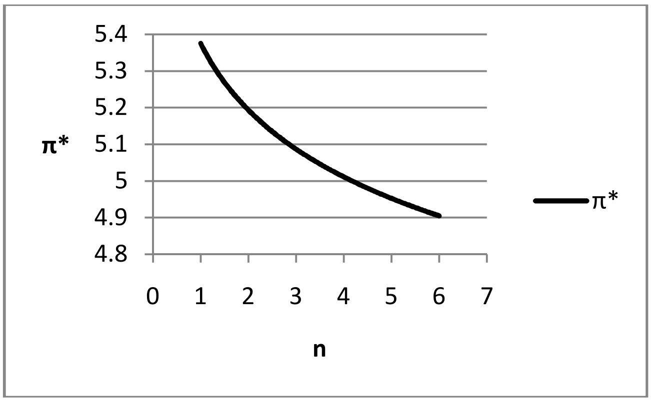

Figure9. Effects of changes in (***δ***) on the profit (*Π*)

6-3-2-Effects of Changes in (*a*) on Optimal Solution

We have investigated different values of (*a*), according to the following assumptions: $a = a_0+1$, where $a_0=2$. Fig.10, represents when set-up costs of the production (*a*) increased, expenditure is also increasing but profit decreased.

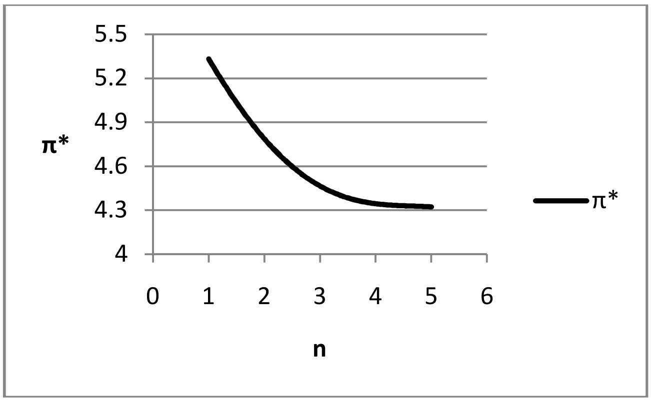

Figure10. Effects of changes in (***a***) on the profit (*Π*)

6-3-3-Effects of Changes in (*k*) on Optimal Solution

We consider different values of (*k*) based on $k=k_0+5$, where $k_0=5$. From Fig. 11, with an increase in (*k*) the demand has increasing trend. Increase in demand results in an increase in production lot-size (*Q*). It is seen from Fig.12 that as (*k*) increases the marketing expenditure decreases. According to Figs. 13-14, in this situation the company has to decrease its selling price per unit in order to maintain the desired market share and increasing its profit.

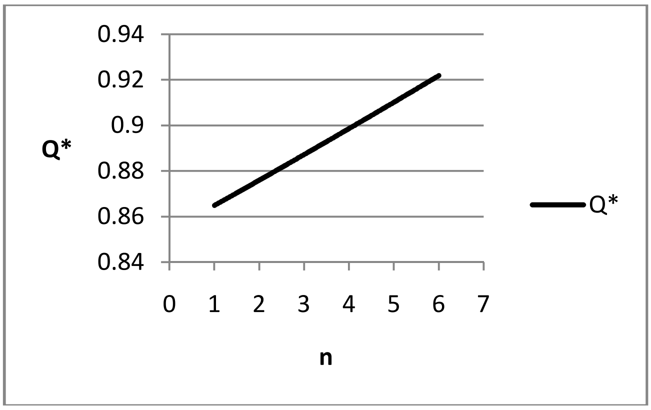

Figure11. Effects of changes in (***k***) on the lot-size (*Q*)

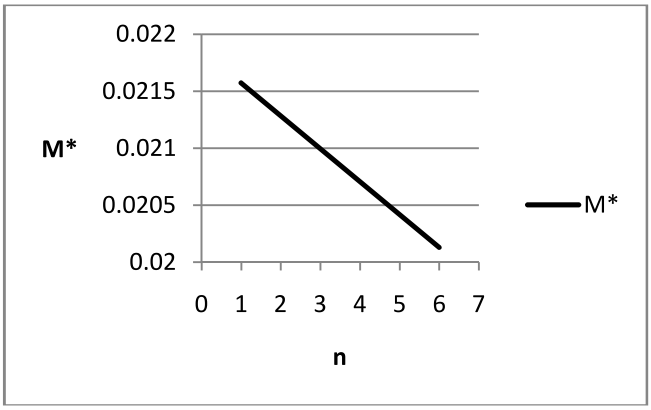

Figure12. Effects of changes in (***k***) on marketing expenditures (*M*)

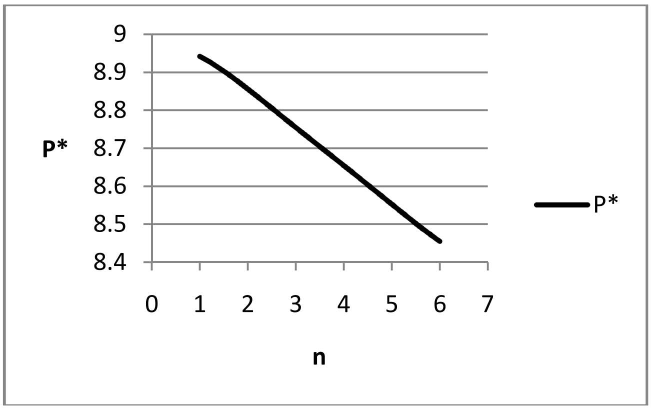

Figure13. Effects of changes in (***k***) on price (*P*)

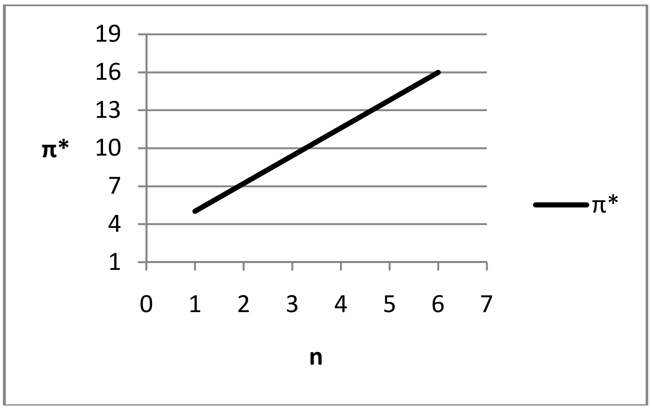

Figure14. Effects of changes in (***k***) on the profit (*Π*)

7-Conclusions

In this paper, we have presented an algorithm to determine the optimal solution of a production lot sizing. We have considered production rate to be a linear function of demand. The model presented in this paper also has considered demand to be a function of price and marketing. The resulting problem formulation is a signomial problem with five degrees of difficulty. We have used Geometric Programming (GP) to determine the optimal solution of the proposed model. Therefore, the CVX optimization toolbox, run in the MATLAB environment, is applied in order to solve this nonlinear optimization problem. Numerical examples have been used to present the implementations of our algorithm. One of the extensions of our model is to consider the proposed model for a multi-product case with some linear constraints. Such a problem cannot be easily converted into a transformed posynomial problem and would be an interesting area for future research.

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