

## **Genetic algorithm-based optimization approach for an uncapacitated single allocation P-hub center problem with more realistic cost structure**

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### **Abstract**

A p-hub center network design problem consists of some nodes as hubs and allocation of non-hub nodes to them wherein the maximum travel times between any pair of nodes is minimized. The distinctive feature of this study is proposing a new mathematical formulation for modeling costs in a p-hub center problem. Here, instead of considering costs as a linear function of distance, for the first time, we formulate costs as a summation of different parts: fixed cost, health, safety and environment (HSE) cost, energy cost and personnel cost. Such integrated model results in a hard-to-solve nonlinear formulation. To validate the proposed model, a small scale problem instance of CAB dataset solved by LINGO software. We also prepared a Genetic Algorithm (GA) by MATLAB software to solve complete problems of CAB and AP datasets.

**Keywords:** Hub location problem, un-capacitated single allocation p-hub center problem, stepwise cost function, genetic algorithm.

### **1- Introduction**

Hubs are some facilities that are widely used in transportation, telecommunication and logistic networks such as airline passengers, data transmissions, and express packages to organize the network routs, taking into account cost considerations. Aggregating some allocated non-hub nodes with indirect routs in order to decrease the cost of flows is the function of a hub in such systems. Therefore, determining the location of hub nodes and allocating non-hub nodes to them is a challenging task and plays potentially results in lower network costs.

Many studies in the context of hub location problems have been focused on two significant criteria (Farahani et al., 2013). The first one tries to minimize the maximum distance between each pair of nodes that is called p-hub center problem and originally represented by Campbell (1994) while the other one attempts to minimize the total sum of distances in the network and is frequently referred to as p-hub median problem (Campbell, 1996). The way in which non-hub nodes may be allocated to hub nodes is another approach to classify the previous studies. Accordingly, Hub Location Problems (HLPs) can be

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categorized into two separate groups, including single allocation HLPs when a non-hub node must be allocated to a single hub (Abyazi-Sani and Ghanbari, 2016) and multiple allocation HLPs when multiple allocation of non-hub nodes is permitted (Rabbani and Kazemi, 2015).

Kara and Tansel (2000) developed a new modeling for single-assignment p-hub center problem and investigated the computational performance of the proposed model and various linear versions of Campbell's model. The results of this study indicated that the new model is substantially superior than linearization method of the old model in terms of required CPU times. Formulation of the single p-hub center allocation problem was improved by Ernst et al. (2002). They defined maximum gathering costs between hub  $k$  and its allocated nodes as radius of hub  $k$ . By considering the number of hubs as a decision variable of the problem that must be minimized regarding all trip times between each pair of nodes are within predetermined bounds, Kara and Tansel (2003) proposed an integer programming formulation of the hub covering problem. The better performance in both CPU time as well as core storage requirements was superiority of their model than the linearization of the old model. A real-world application of single allocation HLP was studied by Aversa et al. (2005). They developed a mixed integer programming model to select a transportation hub port in the East Coast of South American among 11 alternatives with the least port and shipping costs. Unlike previous studies that assume direct connections between hub center and its assigned demand centers, Yaman et al. (2007) proposed a new formulation which allows stopovers for the latest arrival hub location problem. In their new modeling for ground p-hub location problem, some overlooked aspects like loading and unloading time were taken into account. Eiselt and Marianov (2009) formulated the hub location problem by considering customers' interests. They assumed the customers choose an airline depending on a mixture function of flying time and fare. This study consists of three phases; at first they tried to study customer's behavior as the flow, then formulated the model and finally solved some instances of the model. A study on un-capacitated hub center problems with either single or multiple allocations was conducted by Ernst et al. (2009). They investigated the problem of finding an optimal single allocation with integer programming formulations for both problems. Hwang and Lee (2012) developed an un-capacitated single allocation p-hub maximal covering problem in a network with the assumption of specific hub number and deadline for traveling time in order to maximize the demand centers covered. O'Kelly (2012) considered environmental factors in hub location problems. In this study, the author assumed the amount of consumed fuel as an indicator for environmental cost. Rabbani et al. (2013) tried to propose a new model on economic aspects by considering the total flow between nodes in a p-hub center single allocation problem. An extension of the classical single allocation HLP considered multiple types of flow was introduced by Correia et al. (2014). Three mixed integer programming formulation were proposed for the purpose of minimizing the total cost, including setup costs for the hubs, setup cost for each product in each hub, and flow routing costs. Investigation of hub maximal covering problem under uncertain shipments was carried out by Zade et al. (2014). A bi-objective mathematical formulation was developed for this problem based upon two objectives of maximizing coverage as well as maximizing safety of the paths. The work was done by Rahimi et al. (2016) is another study that addresses HLP under uncertainty. Rostami et al. (2015) tried to make HLPs more realistic through applying a stepwise cost function rather than a linear function of the transport volume in modeling the problem. In other words, this cost function considers vehicle sizes directly in the model. A recently introduced extension of the well-known HLP, i.e. tree HLP with an incomplete graph was addressed by Sedehzadeh et al. (2015). In addition to environmental aspects, their study considers different transportation modes between hubs as well as a set of capacity levels for each potential hub.

Ernst et al. (2009) claimed that the problem of finding an optimal single allocation with respect to a given set of hubs is a NP-hard problem. They defined an integer programming formulation for both p-hub center and p-hub median problems. They used branch and bound method for solving the problem and showed that their formulation is better than the previous one on the basis of numerical result. Meyer et al. (2009) proposed a two-phase algorithm to solve p-hub center single allocation problem. In the first phase, they computed a group of optimal hub combinations by a method based on branch and bound algorithm. In phase 2, they continued with a degraded size formulation to complete the allocation. They used an Ant Colony Optimization (ACO) approach to achieve a good upper bound. According to the authors' claim, it

is possible to achieve optimal solution for larger problems in shorter times by using the algorithm. Consequently, they could provide the exact solution for problems larger than those studied before.

Topcuoglu et al. (2005) tried to use the pattern of Genetic Algorithm (GA) to solve un-capacitated hub location problems. They compared the results obtained through GA with optimal solutions and they presented a new solution based on a genetic pattern framework. The objective was to exploit the features of GAs in finding the number of hubs, the location of hubs, and the assignment of spokes to the hubs but they did not consider stepwise functions in the algorithm. Kratica and Stanimirović (2006) pointed out that GA is potentially useful metaheuristic for solving other similar hub location problems. They solved un-capacitated multiple allocation p-hub center problem by this method and concentrated on an exacting variant of hub location problem. Their solution approach required a reasonable amount of computational time, even for problem instances of higher dimensions. Computational results have demonstrated that the GA can reach all optimal solutions obtained by commercial optimization solvers for smaller problem dimensions (Stanimirović, 2012, Damgacioglu et al., 2015). Bashiri et al. (2013) applied a GA based heuristic algorithm to handle the capacitated p-hub center problem which considers qualitative parameters such as quality of service and capability for development in the future along with quantitative parameters. The results of this study showed a reasonable average gap between optimal solutions and obtained solutions by the GA heuristic approach. The study was carried out by Alizadeh et al. (2016) is a sample for application of GAs in multi-objective capacitated hub maximal covering problem. Authors also worked on other metaheuristics such as tabu search algorithm (Abyazi-Sani and Ghanbari, 2016) and Particle Swarm Optimization (PSO) algorithm (Ghaderi and Rahmaniani, 2016).

By considering the aforementioned literature on the HLPs, it is obvious that economic aspects play conventionally the most significant role on choosing a network. However, nowadays sole consideration of these aspects leads to immature modeling of the real challenges. This study aims to overcome this shortcoming. The unique feature of this study is developing an integrated model for p-hub center problem considering flow dependent costs that encompass hubs establishment fixed costs, health, safety and environmental (HSE) costs, and energy and personnel costs. These costs are modeled by step functions of flow. Such modeling of costs results in a more balanced and fair design of hub networks.

The rest of this study is structured as follows: Section 2 provides a description of the problem and its formulation. Methodology is presented in Section 3. Section 4 is assigned to the parameters tuning for solution approach and numerical results for problem instances. Finally, Conclusion remarks are provided in Section 5.

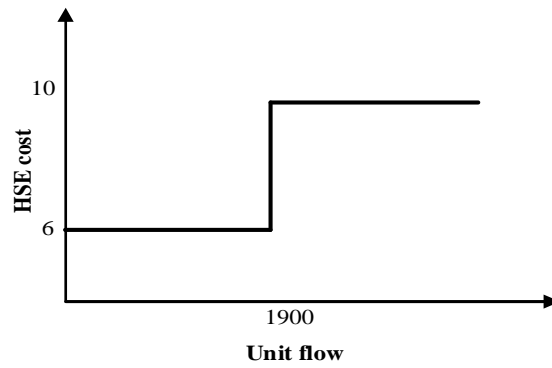
## 2- Problem definition and formulation

While keeping the structure of the classical un-capacitated single allocation HLP, this study tries to provide a new approach for computing the system costs. In other words, we assumed that the costs are step functions of bigness of hub. This assumption is based on this real fact that every time a hub covers more flow, this does not mean this hub should be larger. As the covered flow increases, form a critical flow (the break points in the diagrams) we must establish a larger hub, consequently we must pay more and here is where the diagram jump to next step.

Consider a hub-spoke network that  $p$  hubs must be selected among all possible locations with the following assumptions that are as follows:

- This is a single allocation problem with direct way between all hubs.
- It is assumed that hubs have infinite capacity.
- Each non-hub node must be assigned to only one hub.
- Direct shipment between non-hub nodes is not permitted.
- Sum of total flow which goes from a hub is an indicator for estimating bigness of hub.
- It is assumed that the costs, including personnel costs, energy costs, and HSE costs are step functions of bigness of hub and a fixed cost for its establishing.
- The mentioned stepwise functions are supposed to be determined by experts.
- A sample stepwise function for HSE costs is illustrated in Figure 1.

- All of the parameters assumed to be deterministic.



**Figure 1.** A sample of stepwise function for HSE costs of a given hub

By considering the aforementioned assumptions, a new mixed integer formulation is presented for p-hub center problem in this section based upon following notations. The objective function and constraints of this formulation are modelled in terms of equations (1) to (9).

#### Indices

$i$	index of nodes; $i = 1, \dots, n$
$j$	index of nodes; $j = 1, \dots, n$
$k$	index of hubs; $k = 1, \dots, p$
$m$	index of hubs; $m = 1, \dots, p$

#### Input parameters

$w_{ij}$	flow between node $i$ and $j$
$d_{ij}$	distance between node $i$ and $j$
$\theta_{1k}$	personnel cost for $k$ -th hub
$\theta_{2k}$	energy cost for $k$ -th hub
$\theta_{3k}$	HSE cost for $k$ -th hub
$f_i$	fixed cost of establishing a hub at node $i$
$\alpha$	cost discount factor

### Decision variables

$Z_{ik}$	binary variable ( $Z_{ik} = 1$ if spoke $i$ is servicing by $k$ -th hub; otherwise 0)
$Y_k$	binary variable ( $Y_k = 1$ if node $k$ is hub; otherwise 0)
$C_{ij}$	cost of a unit of flow between node $i$ and $j$
$A_k$	the bigness of $k$ -th hub
$\theta_k$	consists of personnel costs $\theta_{1k}$ , energy costs $\theta_{2k}$ , and HSE costs $\theta_{3k}$

$$\text{Min } X \quad (1)$$

s. t.

$$w_{ij} \times (Z_{ik} \times C_{ik} + Z_{ik} \times Z_{jm} \times \alpha \times C_{km} + Z_{jm} \times C_{jm}) \leq X \quad ; \forall i, j, k, m \quad (2)$$

$$C_{ij} = \sum_k \sum_m (Z_{ik} \times d_{ik} + Z_{ik} \times Z_{jm} \times \left\{ \frac{(\theta_k + f_k)}{A_k} + \frac{(\theta_m + f_m)}{A_m} + d_{km} \right\} + Z_{jm} \times d_{jm}) \quad ; \forall i, j \quad (3)$$

$$\theta_k = \theta_{1k} + \theta_{2k} + \theta_{3k} \quad ; \forall k \quad (4)$$

$$A_k = \sum_i Z_{ik} \times w_{ik} + \varepsilon \quad ; \forall k \quad (5)$$

$$\sum_k Y_k = p \quad (6)$$

$$\sum_k Z_{ik} = 1 \quad ; \forall i \quad (7)$$

$$Z_{ik} \leq Y_k \quad ; \forall i, k \quad (8)$$

$$Z_{ik}, Y_k \in \{0,1\} \quad ; \forall i, k \quad (9)$$

Both of equations (1) and (2) denote that this is a p-hub center problem which tries to minimize the maximum travelling cost between every pair of origin-destination points. Equations (3) state the way of calculating cost of a unit of flow between two nodes ( $C_{ij}$ ) used in equations (2). Actually,  $C_{ij}$  is the amount of cost for traveling from origin node  $i$  to hub  $k$ , then to hub  $m$  and finally arrive to destination node  $j$ . At this point, different types of cost, including fixed cost of establishing a hub center and other cost with respect to bigness of each hub center are considered. According to equations (4), these costs consist of three major parts, including personnel costs, energy costs, and HSE costs as mentioned before. In equations (5), summation of flows which goes through the hub represents an indicator for estimating

bigness of each hub. Constraint (6) guarantees that the number of hubs will be equal to  $p$ . Single allocation characteristic of the problem is ensured through constraints (7). Constraints (8) denote that a non-hub node can only be assigned to a node, only if a hub has been previously established at this node. Finally, constraints (9) define the binary variables in the model.

### 3- Methodology

Non-linear nature of the proposed model as well as the NP-hard nature of the problem proven in previous studies makes us to apply a proper metaheuristic algorithm for designing large scale hub-spoke networks in a reasonable time. The big networks have been solved through GA because of time limitation for network designing, like a network with 15 nodes that have been solved by LINGO software in one hour. Therefore, applicability of GA in handling large scale hub location problem is the main reason of its implementation in this study.

#### 3-1- Solution representation

Genetic procedure begins with some random solutions and tries to find better hub locations and its nodes allocation in order to optimize the better previous solutions. So we need to have a chromosome that it should be able to distinguish hubs and its allocated nodes. Consider a hub-spoke network consisting of 3 hubs must be selected among 10 locations that are shown in Figure 2. In this figure, established hubs are shown with black pentagons (nodes 4, 5, and 9) and white circles (1,2,3,6,7,8, and 10) are related to spokes which are served by hubs.

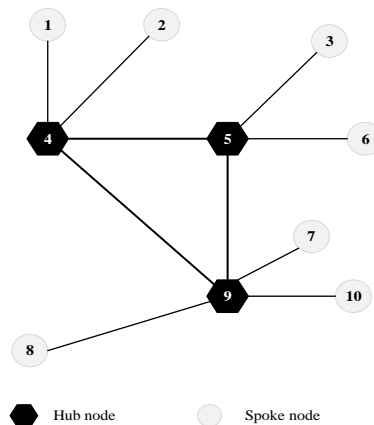


Figure 2. A single allocation hub-spoke network

A sample chromosome for this system has 10 genes whose values are equal to location number if the location is a hub node, otherwise the number of its allocated hub. Figure 3 shows an example of a chromosome. As you can see in the figure, nodes 4, 5 and 9 are selected as hub centers and their values are equal to their numbers. Furthermore, the values of nodes 1 and 2 that assigned to hub 4 are equal to 4. The value 5 is assigned to nodes 3 and 6 and finally the value 9 is assigned to nodes 7, 8, and 10. Consequently, we would have a chromosome with three different values 4, 5 and 9.

1	2	3	4	5	6	7	8	9	10
4	4	5	4	5	5	9	9	9	9

Figure 3. Proposed chromosome structure

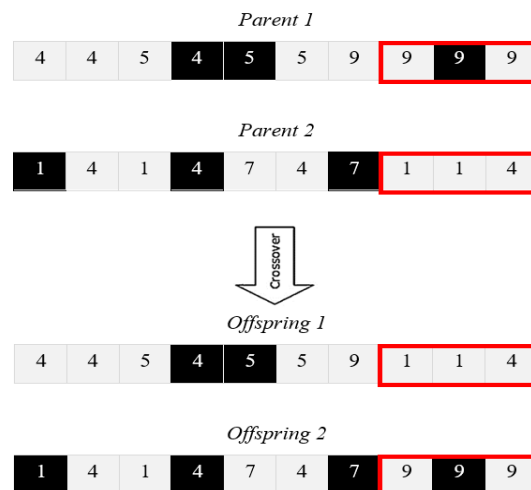
### 3-2- Proposed GA

GA is a well-suited optimization approach for handling large scale optimization problems in various areas that introduced by (Holland, 1975). A genetic search procedure begins with a randomly generated initial population. Therefore, first of all we need to have some chromosomes formed in two steps:

- Step 1: Select  $p$  nodes randomly as hubs and give them their order number.
- Step 2: Randomly choose a hub value and assign it to the first non-hub node. Keep on this procedure with the next non-hub nodes.

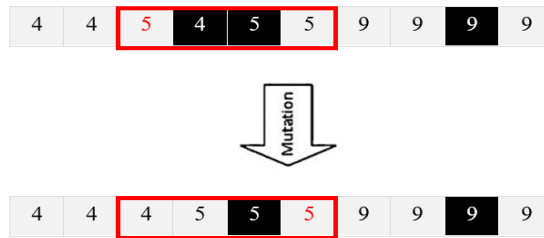
The randomly generated population members are evaluated based upon a fitness function computed by objective function value of related solution. The members are further manipulated by using three main operators to generate the next generation that are as follows:

- Selection: This operator selects individuals in the population that will be survive. Roulette Wheel selection method as a stochastic sampling operator is used for this purpose in this study. According to this method, better-fitted solutions will survive with higher probability.
- Crossover: The operation of crossover in the proposed GA is performed based upon a single-point crossover operator. In this method, an integer value is randomly selected as crossover point. Afterwards, two selected parents swap their gens. For example, consider two selected chromosomes with length 10 and random crossover point 7. The last three genes of each parents (i.e. genes 8, 9, and 10) changed between them and developed couple of new chromosomes as two Offspring. Figure 4 depicts this procedure.



**Figure 4.** Implementation of single-point crossover operator

- Mutation: This operation tries to produce irregular chromosomes to exit for local optimum solutions. New chromosome consists of a selected parent by selection operator with changes between gen associated with the first random integer value and its next gens until another random integer value. For example, Figure 5 shows how to implement this operator for a selected chromosome with length 10 and two random integer values 3 and 6.

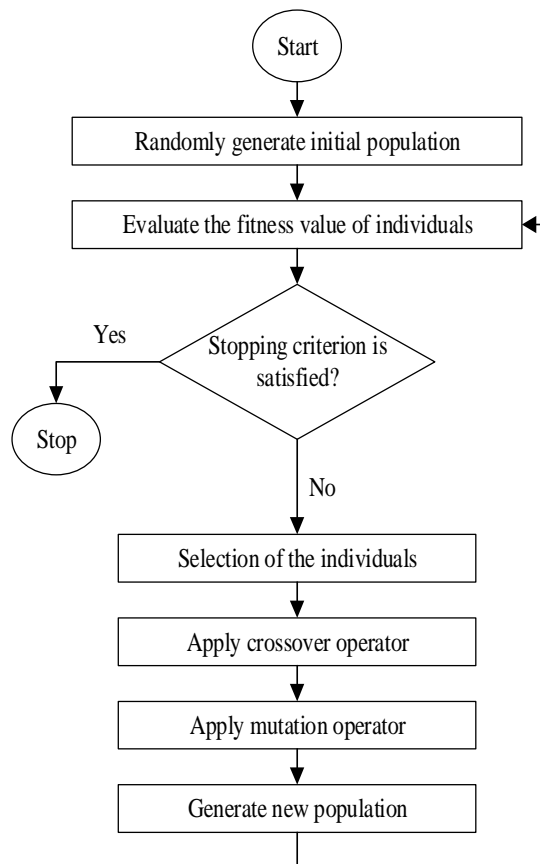


**Figure 5.** Implementation of mutation operator

According to figures 4 and 5, it is obvious that applying this crossover and mutation techniques do not guarantee the feasibility of the newborn chromosomes. In other words, there is a possibility that some newborn chromosomes gain more or less hubs than their parents as well as there exist nodes that are allocated to non-hub nodes. Therefore, overcoming this infeasibility needs to follow these steps are listed below.

- Step 1: Set different values assigned to offspring genes as possible hub nodes.
- Step 2: Choose  $p$  hubs randomly among possible hub nodes as Offspring hubs and assign not selected possible hubs to offspring hubs randomly.
- Step 3: Randomly choose offspring hubs for not correct allocated nodes.
- Step 4: Set correct offspring as new born chromosome.

The steps involved in the proposed GA is illustrated in flowchart of Figure 6.



**Figure 6.** The flowchart of GA



## 4- Numerical results

### 4-1- Model validation

To obtain a profound understanding of the novel formulation, in this section the model is solved for a sample data set taken from CAB dataset. Dataset have accessed through the following website: <http://brunel.ac.uk>.

For this example, we have extracted the needed data from CAB dataset for a simple problem with total number of three nodes where number of hubs ( $p$ ) is set to two. While distance data ( $d_{ij}$ ) and flow data ( $w_{ij}$ ) given in Tables 1 and 2 respectively are taken from CAB, the other required data such as data used for fixed cost, Health, Safety and Environment (HSE) cost, energy cost and personnel cost are randomly generated by the authors. Since these costs, as mentioned above, are stepwise functions of flow routed from a hub node, simple two-step functions with one breaking point are considered for each of them given in Table 3. For example, if the flow passing through a hub node is less than 7000 units, the associated HSE costs will be equal to 6 units, otherwise it takes the other value (here for example 10). This is also the same for the other functions. The data for fixed costs of establishing a hub center at nodes 1, 2 and 3 are randomly generated as 84, 92 and 70, respectively. Also, the parameters  $\alpha$  and  $\varepsilon$  are assumed to be equal to 0.8 and 1, respectively.

**Table 1.** The distance data taken from CAB dataset

To From		Node		
		1	2	3
Node	1	0	576.9631	946.4954
	2	576.9631	0	369.5327
	3	946.4954	369.5327	0

**Table 2.** The flow data taken from CAB dataset

To From		Node		
		1	2	3
Node	1	0	6469	7629
	2	6469	0	12999
	3	7629	12999	0

**Table 3.** The stepwise functions for different costs

Costs	Break point	Low step	High step
HSE cost	7000	6	10
Energy cost	6000	16	18
Personnel cost	8000	8	15

This small scale test problem was coded and solved by LINGO 11.0 on a PC with 2.2 GHz CPU and 2 MB RAM. The following global optimum solution was found at iteration 858 in less than 1 second. Table 4 shows optimum solution costs and flow for the problem, Table 5 shows the hub and its nodes connections. The optimum objective function value for the problem is equal to 4.8829E+06 unit that is

the minimum of maximum travelling cost between every pair of origin-destination points which each pair of connectivity cost shows in Table 6.

**Table 4.** Optimum solution (part 1)

	Node		
	1	2	3
Hubs	✗	✓	✓
HSE cost	6	6	6
Energy cost	16	18	16
Personnel cost	8	8	8
Bigness of hub	1	6470	1

**Table 5.** Hubs and their nodes connection (part 2)

To		Node		
		1	2	3
Node	1	0	0	0
	2	1	1	0
	3	0	0	1

**Table 6.** Travelling cost between different nodes (part 3)

To		Node		
		1	2	3
Node	1	1153.9	577.0	577.0
	2	577.0	2217214.0	375.6
	3	577.0	4894604.0	160.0

Note that the answer obtained through proposed genetic algorithm is equal to the global optimum value reported above.

#### 4-2- Test problems

In this part, CAB as well as Australia Post (AP) datasets and their reduced examples with different number of hubs ( $p$ ) will be solved by the proposed GA is programmed in MATLAB R2012a. Datasets have accessed through the following website: <http://people.brunel.ac.uk>.

Similar to the small scale illustrative example, distance and flow are taken from CAB and AP and the other data are randomly generated. Since cost data, as mentioned in the previous sub-section, are stepwise functions of flow routed form a hub node, simple two-step functions with one breaking point are considered for these costs presented in Table 3 with discount factor 0.8, 0.2, 0.5, and 1.0 and parameter  $\varepsilon$  that is assumed to be equal to 1. Furthermore, since fixed cost and overhead costs are not considered in datasets, we generate them randomly that listed below.

- For AP dataset: 84, 92, 70, 80, 90, 28, 98, 58, 45, 54, 65, 88, 88, 48, 15, 45, 45, 54, 45, 45, 85, 45, 65, 44, 55.
- For CAB dataset: 68, 63, 73, 67, 71, 76, 57, 69, 54, 70, 72, 52, 75, 51, 59, 54, 56, 55, 57, 65, 74, 77, 61, 53, 58, 68, 68, 74, 58, 67, 54, 57, 74, 78, 67, 68, 79, 78, 76, 57, 54, 78, 66, 52, 57, 66, 50, 53, 78, 51, 77, 55, 54, 59, 63, 80, 51, 77, 58, 55, 77, 66, 67, 53, 65, 68, 60, 79, 69, 77, 72, 60, 73, 51, 55, 57, 65, 54, 68, 78, 66, 70, 74, 57, 64, 56, 77, 53, 77, 68, 79, 56, 64, 67, 71, 72, 74, 69, 76, 51, 60, 64, 59, 75, 52, 76, 68, 74, 76, 51, 63, 79, 71, 78, 72, 61, 74, 76, 58, 50, 66, 65, 75, 73, 67, 57, 57, 54, 59, 67, 50, 52, 70, 79, 58, 58, 58, 60, 64, 80, 78, 52, 58, 59, 62, 70, 79, 53, 59, 54, 75, 61, 56, 77, 65, 72, 72, 79, 62, 54, 58, 62, 50, 76, 61, 51, 53, 65, 66, 68, 65, 70, 61, 59, 65, 76, 61, 70, 73, 50, 69, 60, 58, 64, 62, 75, 60, 55, 68, 59, 74, 71, 64, 60, 69, 60, 77, 73, 79, 56.

The computational results of the capacitated p-hub center location and allocation problem which tries to minimize the maximum route cost of network are reported below. Table 7 to Table 10 is assigned to results of problem instances related to CAB dataset. Results of AP dataset instances are shown in Table 11 too.

**Table 7.** Center CAB data set for  $n=25$

Problem no.	$p$	$\alpha$	Best cost	Mean cost	CP time
1	10	1	187912693	187912693	126
2	10	0.8	166951006	166951006	136
3	10	0.5	164933311	164933311	129
4	10	0.2	156391190	156391190	150
5	8	1	263788155	263788155	131
6	8	0.8	207076726	207076726	135
7	8	0.5	200531548	200531548	134
8	8	0.2	156391190	156391190	131
9	5	1	305559694	305559694	138
10	5	0.8	254226613	254226613	130
11	5	0.5	160853784	175636459	140
12	5	0.2	156890646	156890646	138
13	3	1	290742000	290742000	141
14	3	0.8	278750590	317391732	142
15	3	0.5	254286521	254286521	137
16	3	0.2	186045690	186045690	137

**Table 8.** Center CAB data set for  $n=20$ 

Problem no.	$p$	$\alpha$	Best cost	Mean cost	CP time
17	10	1	175329211	175329211	80.25
18	10	0.8	161485849	161485849	80.75
19	10	0.5	145569629	145569629	79.38
20	10	0.2	127427450	127427450	80.39
21	8	1	215985701	224038745	86.64
22	8	0.8	185144638	185144638	87.47
23	8	0.5	135647829	135647829	83.99
24	8	0.2	124565457	124565457	86.00
25	5	1	250114515	250114515	91.91
26	5	0.8	246444939	246444939	85.08
27	5	0.5	166926712	166926712	85.95
28	5	0.2	129423145	129423145	83.48
29	3	1	250114419	250114419	86.91
30	3	0.8	211562201	211562201	85.44
31	3	0.5	173057839	173057839	85.04
32	3	0.2	171965906	175240703	88.24

**Table 9.** Center CAB data set for  $n=18$ 

Problem no.	$p$	$\alpha$	Best cost	Mean cost	CP time
33	10	1	252878545	252878545	105
34	10	0.8	161485849	161485849	104
35	10	0.5	129423295	129423295	73.10
36	10	0.2	89990422	89990422	90.97
37	8	1	198750834	198750834	65.96
38	8	0.8	175329211	175329211	69.23
39	8	0.5	129423453	129423453	72.09
40	8	0.2	105956506	105956506	73.09
41	5	1	252878545	252878545	64.45
42	5	0.8	250537869	250537869	67.20
43	5	0.5	160617354	160617354	68.35
44	5	0.2	129423600	129423600	63.37
45	3	1	227703688	251553552	84.69
46	3	0.8	219894703	219894703	66.03
47	3	0.5	187344189	187344189	68.25
48	3	0.2	186045750	186045750	67.99

**Table 10.** Center CAB data set for  $n=15$ 

Problem no.	$p$	$\alpha$	Best cost	Mean cost	CP time
49	10	1	53561144	53561144	56.33
50	10	0.8	48158465	48158465	47.04
51	10	0.5	47103750	47103750	60.28
52	10	0.2	29184301	29184301	55.79
53	8	1	72430908	72430908	84.68
54	8	0.8	57628265	57628265	45.84
55	8	0.5	60805771	60805771	81.77
56	8	0.2	45524600	45524600	67.66
57	5	1	93755196	93755196	67.60
58	5	0.8	81718253	90870116	55.63
59	5	0.5	50459684	50459684	89.47
60	5	0.2	42921079	42921079	50.37
61	3	1	81600531	101398957	71.38
62	3	0.8	70532584	82462805	51.86
63	3	0.5	55031268	84686130	47.45
64	3	0.2	53020024	53020024	88.91

**Table 11.** Center Australia Post data set

$n$	$p$	$\alpha$	Best cost	Mean cost	CP time
200	100	0.8	489677.3	489677.3	2589
200	100	0.5	443264.9	443264.9	2785
200	50	0.8	467242	467242	2598
200	50	0.5	425666	425666	2325
100	50	0.8	61469.34	61469.34	756
100	50	0.5	60428.88	60428.88	723
100	20	0.8	120385.7	120385.7	598
100	20	0.5	102484.7	102484.7	696

Three important points can be derived from the computational results:

- Discount factor is a very effective parameter for a problem and Figure 7 shows this fact. In this figure, the total cost of the first four problem instances of CAB dataset taken from Table 7 are compared (four problems with  $n = 25$  and  $p = 10$  under different  $\alpha$  values).
- There exists an indirect relation between the bigness of network and the minimum solution for this new formulation. This point is illustrated in Figure 8.
- Number of nodes in a network effects on computational time that is depicted in Figure 9.

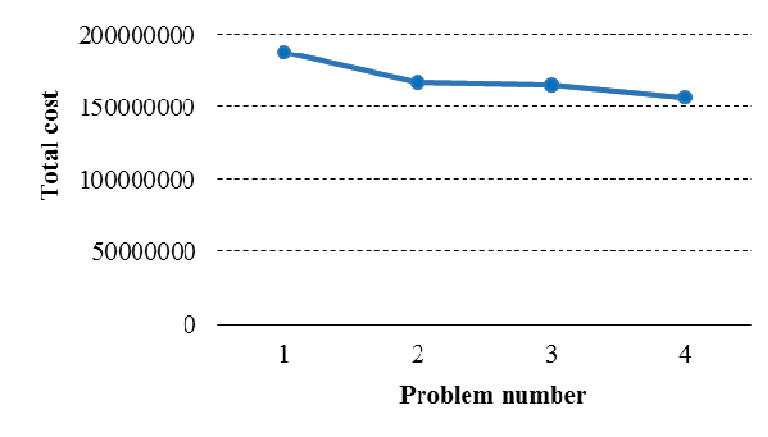


Figure 7. Discount factor evaluation

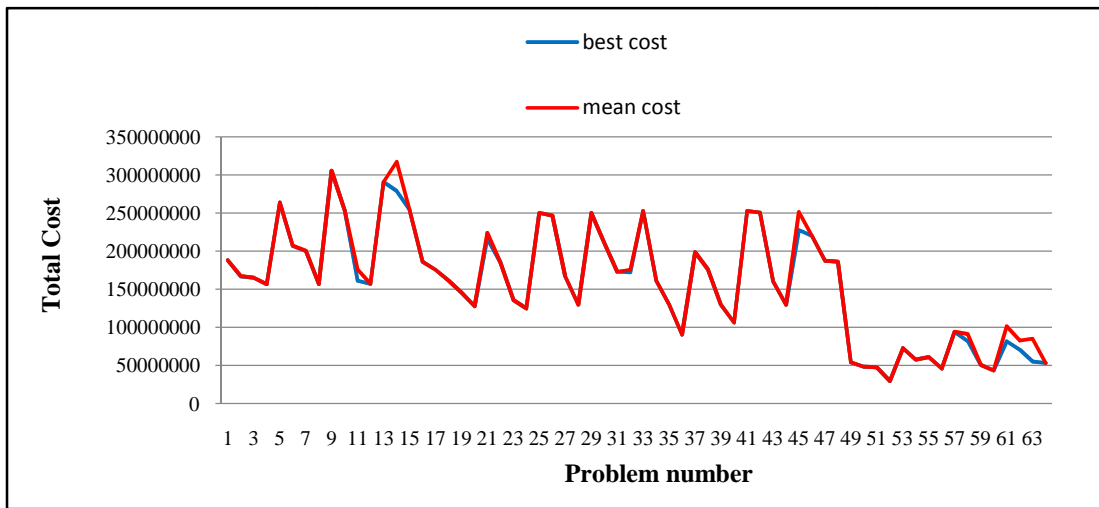


Figure 8. Cost evaluation (taken from Tables 7 to 10)

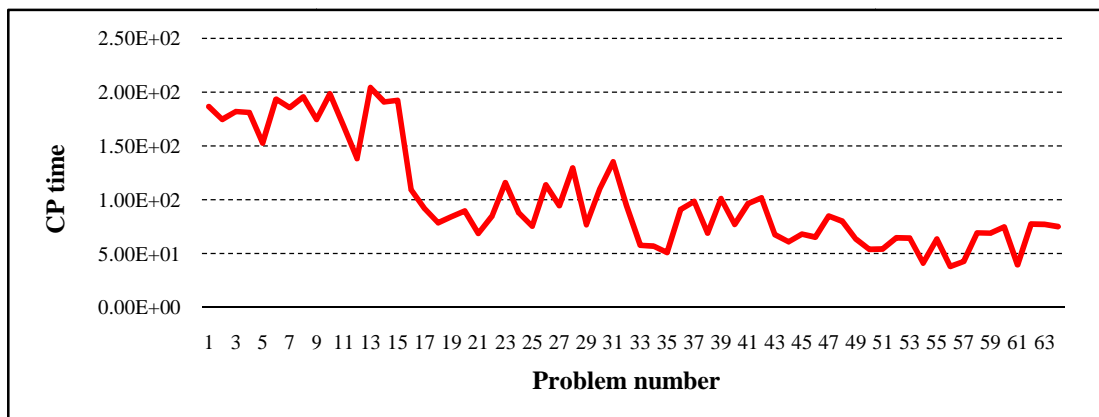


Figure 9. Computational time evaluation (taken from Tables 7 to 10)

## 5- Conclusions

This paper developed a new mathematical formulation for a p-hub center problem in terms of modeling the costs. Actually, instead of modeling costs as a linear function of distance, we can formulate costs as a combination of different parts: fixed cost, HSE cost, energy cost and personnel cost. After presenting the model along with its in-depth explanation, the model was solved for a small scale test problem to show its validity and provide a deep understanding for the respected reader. The only problem with the proposed modeling is its high degree of nonlinearity. Accordingly, we applied a Genetic Algorithm (GA) to solve the problems, especially medium and large scale problems in a reasonable computational time.

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