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An evolutionary game theory approach for financing in a financially-constrained supply chain

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Abstract

This research uses an evolutionary game-theoretic model to investigate the long-term effects of financing in a supply chain (SC) including a one-population of financially-constrained manufacturers and one well-funded e-tailer. The e-tailer and the manufacturers decide on selling price decisions. The manufacturers can either choose a financing scheme from the e-tailer or not. Hence, each manufacturer has two strategies: financing from the e-tailer or non-financing. Considering the two strategies of the manufacturers, this study investigates three strategy profiles in basic and evolutionary game theoretic models. Through the evolutionary game theoretical model, an evolutionary stable strategy solution is determined implying the long-term strategy implemented by most of the manufacturers. Also, numerical analyses are provided to examine the effectiveness of the proposed models. Our result demonstrates that most of the manufacturers (about 71%) will implement the non-financing strategy in the long term suggesting innovative or hybrid financing schemes to financially-constrained manufacturers. Moreover, our result indicates that as the interest rate of the e-tailer increases (about 70%), the manufacturers and the e-tailer offer higher prices to customers. Also, the result shows that when pricing competition is high (about 2.2), all competing SC actors (the manufacturers and the e-tailer) gain more benefits.

Keywords: Evolutionary game theory; financing; financially-constrained manufacturers; supply chain.

1- Introduction

In today's business market, there are many enterprises (e.g., manufacturers) which significantly affect the economic growth. Such manufacturers usually face financial constraints (Yan et al., 2020a). Therefore, they need to finance for continuing their operational and production activities. There exist various financing options including financing from banks and financing from well-funded SC members such as large retailers on e-commerce platforms (Tunca and Zhu, 2018). However, as manufacturers have not enough tangible assets, banks' loans are not an appropriate financing scheme for them (Tang et al., 2018). Accordingly, *financing from e-commerce platforms* is of great importance for financially-constrained manufacturers which is extensively provided by large e-tailers (e.g., JD.com and Amazon) to their suppliers (Tsai and Kuan-Jung, 2017; Yan et al., 2020a).

Therefore, in the business market, manufacturers can gain financing support from e-tailers and hence their financing from e-tailers results in financing competition among them. Thus, investigating *financing competition* among a population of manufacturers is of high significance. In addition to the

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financing, manufacturers face the challenge of how selling their products in the market: direct selling, via e-commerce platforms (i.e., e-tailer), or dual sales channel. Therefore, the manufacturers competitively decide on selling prices in the market. Accordingly, analyzing *pricing competition* among manufacturers and e-tailers is essential. To investigate competitive interactions among actors in the market and supply chains (SCs), the *game theory* is used (Barron, 2013; Abdi et al., 2023).

The prior literature has mainly analyzed the effects of financing schemes in the short term. However, as noted by the study of Johari and Hosseini-Motlagh (2023), it is of great investigation to analyze the long-term effects of financing schemes. Moreover, investigating pricing competition under long-term financing schemes is underexplored. Also, the literature has mainly considered one or two manufacturers when investigating financing schemes. However, in the competitive business market, there are many manufacturers.

Motivated by the observed practical cases and research gaps, this study contributes to the literature by proposing an evolutionary game-theoretic model to analyze the long-term financing scheme of financially-constrained manufacturers in an SC including one population of manufacturers and one e-tailer. The manufacturers can either choose financing from the e-tailer or not. In addition, pricing competition is explored among the manufacturers and the e-tailer. Also, this study investigates basic and evolutionary game theory models. In the basic model, three strategy profiles are analyzed and the optimal pricing decisions of the manufacturers and the e-tailer are obtained. Moreover, in the evolutionary game theory model, the probabilities of financing options are incorporated and the ESS solution in the long term is determined. The result demonstrates that under intense competition on pricing (about 2.2), all competing SC members (the e-tailer and the manufacturers) gain more profits. Also, the result indicates that most of the manufacturers (about 71%) will choose the non-financing scheme in the long term. This finding implies that financially-constrained manufacturers will choose other financing modes, such as innovative or hybrid financing schemes. Moreover, the result indicates that by increasing the e-tailer's interest rate (about 70%), the manufacturers and the e-tailer benefit from offering higher prices to customers.

The research questions of this research are as follows. First, what is the long-term strategy of financially-constrained manufacturers? Second, what are the optimal selling prices of the competitive manufacturers and the e-tailer? Third, what are the effects of initial capital and interest rate on the payoffs of SC actors?

The rest of this study is as follows. In Section 2, we review the relevant literature in the fields of SC finance and the game theory. Section 3 provides the problem under study, notations, model formulations, and optimal solutions. In Section 4, sensitivity analyses are discussed. Section 5 concludes the paper and provides future research paths.

2- Literature review

In this section, relevant and recent studies in the fields of SC finance and the game theory are reviewed to clearly identify research gaps.

Using the game theory, Tang et al. (2018) consider an SC comprising of one financially-constrained supplier, one manufacturer, and one bank. They investigate the effectiveness of two financing schemes, including buyer direct financing provided by the manufacturer and purchase order financing provided by the bank. Li et al. (2018) propose a game-theoretical model to examine the efficiency of an SC with one capital-constrained retailer and one risk-averse supplier under two financing schemes, including trade credit financing and partial credit guarantee. Yang et al. (2019) consider SC financing in an SC with one retailer and two manufacturers in which one of the manufacturers faces capital constraint. This study uses a Stackelberg game to investigate optimal pricing under two financing modes for the capital-constrained manufacturer, including financing from the retailer and financing from the bank. Yan et al. (2020a) investigate pricing competition in a capital-constrained SC using the game theory. They examine the effectiveness of the e-tailer's financing on the performance of one supplier who has budget constraints on its production. Using the game theory, Wang et al. (2020) examine how different financing options affect the service provided by a capital-constrained downstream. In this study, order quantity and pricing decisions are compared under three financing approaches, including bank credit financing, financial leasing, and trade credit financing. Reza-Gharehbagh et al. (2020) analyze how government's regulations affect two competitive financial SCs using the game theory. Qin et al. (2020) apply the game-theoretical approach to examine how advancement payment and bank financing schemes affect carbon emission reduction in an SC including one retailer and one manufacturer under two scenarios: (1) when the manufacturer has capital constraints and (2) when the manufacturer has not capital constraints. Yan et al. (2020b) investigate how investment and loan as two financing modes provided by one retailer affect the optimal production quantity of one capital-constrained supplier using the game-theoretic approach. An et al. (2021) analyze carbon emission reduction for a capital-constrained manufacturer-well-funded supplier SC using a Stackelberg game. In this study, two credit financing options: (1) green credit financing by the bank and (2) trade credit financing by the supplier are compared.

Also, optimal pricing, production quantity, and green investment decisions are determined. Shi et al. (2021) utilize the game theory to examine how two SC financing schemes influence order quantity and pricing decisions of a capital-constrained SC with a demand uncertainty reduction approach. This study considers one supplier and one retailer with capital constraints who can obtain (1) credit financing from the bank and (2) trade credit financing from the supplier. Using the game theory, Dong et al. (2021) investigate two SC financing schemes to analyze the optimal operations strategy with and without blockchain technology. Hosseini-Motlagh et al. (2021) investigate sustainable financing for a grain SC with one population of producers who are financially constrained and one distributor utilizing the evolutionary game theory. Fu et al. (2021) examine how external financing influences the efficiency of technology innovation operations of a third-party logistics firm in a platform SC using the game theory. Wang et al. (2021) utilize the game-theoretical approach to examine the effects of order financing on the green activities of firms with different technologies and the bank's loan decision. Hadi et al. (2021) investigate the effect of government's financial schemes on the efficiency of regular and closed-loop SCs using the game theory. Zhang and Zhang (2022) investigate channel competition in an SC including two capital-constrained retailers and one manufacturer under both external financing and trade credit financing using the game theory. Zhang et al. (2022) model a four-level SC game including one supplier, one retailer, one private lender or one fourth-party logistics (4PL) firm, and one capital-constrained third-party logistic (3PL) firm under two financing schemes: (1) external financing from the private lender and (2) financing from the 4PL. Han et al. (2023) conduct a practical study to address the development of agriculture green and green finance using an integrated approach (coordination, coupling, and game theory). Zhu and Ou (2023) examine the effects of two financing schemes (financing provided by one retailer and financing provided by one bank via loans) to one supplier in a supplier-retailer-bank SC utilizing a game theory method. Li and Qu (2023) investigate how the cooperative interactions of SC members influence their decisions in an SC finance system under blockchain technology. Bai and Lin (2024) use the game theory to investigate green innovation and green finance in terms of practical and theoretical considerations. Nozari et al. (2025) investigate how two financing modes: retailer financing and bank financing affect the performance of a competitive SC under risk aversion. Hong and Qin (2024) use the game theory to investigate green SC financing issues and SC members' decision-making under a digital technology platform.

According to the reviewed literature in the SC finance, the following research gaps are identified. First, prior studies have mainly investigated the short-term effects of financing schemes on optimal decisions and profits of SC members using the game theory. Second, pricing competition among all SC levels under financing schemes in the long term is underexplored. Third, previous studies have considered one or two manufacturers when investigating the impacts of financing schemes. However, in the competitive business market, there are many manufacturers.

To sum up, the long-term impacts of financing modes are of great importance as emphasized in the literature (e.g., Johari and Hosseini-Motlagh, 2023). Therefore, the innovations of this study are: (1) proposing an evolutionary game-theoretic model to investigate the long-term effectiveness of a financing scheme provided by a well-funded e-tailer on the performance of a population of manufacturers, (2) discovering the ESS solution implemented by most of the manufacturers in the long term, and (3) analyzing pricing competition among the manufacturers and the e-tailer.

3- Problem definition

This study considers an SC consisting of one e-tailer and one population of manufacturers. The manufacturers are financially constrained regarding their production. The manufacturers can either choose financing from the e-tailer or not. Thus, according to the evolutionary game theory, the set of strategies of the manufacturers are: Strategy 1 in which they receive credit financing from the e-tailer and (2) Strategy 2 in which they do not receive credit financing from the e-tailer. Also, the manufacturers can sell their products directly or via the e-tailer. Hence, the e-tailer and the manufacturers competitively decide on selling price to customers. To model the interactions among the e-tailer and the manufacturers' population based on the evolutionary game theory, two manufacturers are randomly selected from the population. The two chosen manufacturers compete in the Cournot game and they play the Stackelberg game with the e-tailer. The leader of the market is the e-tailer and the two manufacturers are its followers. In this study, the game among the manufacturers and the e-tailer are modeled in basic and evolutionary game-theoretic models. Three strategy profiles are modeled: (1) Strategy profile 1 where both manufacturers receive credit financing from the e-tailer, (2) Strategy profile 2 in which only Manufacturer 1 receives credit financing from the e-tailer, and (3) Strategy profile 3 in which both manufacturers do not receive credit financing from the e-tailer. Considering the evolutionary game theory, the probability of each strategy is incorporated and it is examined how the manufacturers' financing scheme will be evolved, and finally the ESS solution for the manufacturers is determined. The parameters and decision variables are defined in Table 1.

Table 1. Notations.

Parameters		
	α	Market size (units)
	β	Self-price coefficient of demand
	γ	Cross-price coefficient of demand
	w_{m_i}	Wholesale price of Manufacturer *i* (\$/unit)
	c_{m_i}	Production cost of Manufacturer *i* (\$/unit)
	k	Manufacturers' initial capital (\$)
	i	Interest rate
Decision variables		
	p_{m_i}	Unit price of product i sold by Manufacturer i, i=1, 2 (\$/unit)
	p_{e_i}	Unit price of product i sold by e-tailer (\$/unit)

In the next subsection, the basic model under three strategy profiles is discussed.

3-1- The basic model

Under the basic model, three strategy profiles are investigated. First, we provide the demand functions of the e-tailer and the manufacturers in Eqs. (1-4).

$$D_{m_1}(p_{m_1}, p_{m_2}, p_{e_1}, p_{e_2}) = \frac{\alpha}{2} - \beta p_{m_1} + \gamma(p_{m_2} + p_{e_1} + p_{e_2}) \tag{1}$$

$$D_{m_2}(p_{m_1}, p_{m_2}, p_{e_1}, p_{e_2}) = \frac{\alpha}{2} - \beta p_{m_2} + \gamma(p_{m_1} + p_{e_1} + p_{e_2}) \tag{2}$$

$$D_{e_1}(p_{m_1}, p_{m_2}, p_{e_1}, p_{e_2}) = \frac{1-\alpha}{2} - \beta p_{e_1} + \gamma(p_{m_1} + p_{m_2} + p_{e_2}) \tag{3}$$

$$D_{e_2}(p_{m_1}, p_{m_2}, p_{e_1}, p_{e_2}) = \frac{1-\alpha}{2} - \beta p_{e_2} + \gamma(p_{m_1} + p_{m_2} + p_{e_1}) \tag{4}$$

In the following, the payoffs of SC members under three strategy profiles are modeled.

3-1-1- Strategy profile 1

In this situation, the two manufacturers have insufficient initial production budget and hence they are financially constrained. The two manufacturers receive credit financing from the e-tailer. In the following, the payoff functions of SC actors are modeled in Eqs. (5-7).

$$\pi_{m_1}^{s_1} = (p_{m_1} - c_{m_1}) \times D_{m_1} + (w_{m_1} - c_{m_1}) \times D_{e_1} - (c_{m_1} \times (D_{m_1} + D_{e_1}) - k) \times i \quad (5)$$

$$\pi_{m_2}^{s_1} = (p_{m_2} - c_{m_2}) \times D_{m_2} + (w_{m_2} - c_{m_2}) \times D_{e_2} - (c_{m_2} \times (D_{m_2} + D_{e_2}) - k) \times i \quad (6)$$

$$\pi_{e}^{s_1} = (p_{e_1} - w_{m_1}) \times D_{e_1} + (p_{e_2} - w_{m_2}) \times D_{e_2} + (c_{m_1} \times (D_{m_1} + D_{e_1}) - k) \times i + (c_{m_2} \times (D_{m_2} + D_{e_2}) - k) \times i \quad (7)$$

Eqs. (5) and (6) model the profits of the financially-constrained manufacturers. To be specific, In Eqs. (5) and (6), the first part shows the revenue received through direct sales minus production cost. The second part shows the revenue obtained through wholesaling. The third part indicates the financing loan given by the e-tailer.

The e-tailer's profit is shown in Eq. (7) in which the two first parts indicate the revenue of retail selling of the manufacturers' products, and the two second parts imply the revenue obtained through interest of providing loan to the two manufacturers.

3-1-2- Strategy profile 2

In this case, Manufacturer 1 is financially constrained and receives credit financing from the e-tailer. However, Manufacturer 2 is unconstrained. The payoff functions of SC actors are modeled in Eqs. (8-10).

$$\pi_{m_1}^{s_2} = (p_{m_1} - c_{m_1}) \times D_{m_1} + (w_{m_1} - c_{m_1}) \times D_{e_1} - (c_{m_1} \times (D_{m_1} + D_{e_1}) - k) \times i \quad (8)$$

$$\pi_{m_2}^{s_2} = (p_{m_2} - c_{m_2}) \times D_{m_2} + (w_{m_2} - c_{m_2}) \times D_{e_2} \quad (9)$$

$$\pi_{e}^{s_2} = (p_{e_1} - w_{m_1}) \times D_{e_1} + (p_{e_2} - w_{m_2}) \times D_{e_2} + (c_{m_1} \times (D_{m_1} + D_{e_1}) - k) \times i \quad (10)$$

3-1-3- Strategy profile 3

This strategy profile investigates the condition in which both manufacturers have sufficient initial production budget or both manufacturers don't receive financing from the e-tailer. The payoffs of SC members are modeled in Eqs. (11-13).

$$\pi_{m_1}^{s_3} = (p_{m_1} - c_{m_1}) \times D_{m_1} + (w_{m_1} - c_{m_1}) \times D_{e_1} \quad (11)$$

$$\pi_{m_2}^{s_3} = (p_{m_2} - c_{m_2}) \times D_{m_2} + (w_{m_2} - c_{m_2}) \times D_{e_2} \quad (12)$$

$$\pi_{e}^{s_3} = (p_{e_1} - w_{m_1}) \times D_{e_1} + (p_{e_2} - w_{m_2}) \times D_{e_2} \quad (13)$$

In the next subsection, the optimal decision variables of SC members under each strategy profile are determined.

3-2- Optimal solutions

To determine the optimal values of pricing decisions of all SC actors in each strategy profile, the concavity of the payoff functions should be proved. Therefore, we check the profits' concavity though following theorems.

As the Stackelberg game is played among the SC members, the backward induction method should be used to obtain the optimal solutions. Hence, at first, the manufacturers decide on their pricing decisions. Then, considering the manufacturers' pricing decisions, the e-tailer announces the optimal retail prices. Theorems 1 and 2 examine the payoffs' concavity in Strategy profile 1.

Theorem 1. *The payoff of Manufacturer i is concave w.r.t.* p_{m_i}.

Proof. The second-order derivative of the Manufacturer i's payoff w.r.t. p_{m_i} is $\frac{\partial^2 \pi_{m_i}^{s_1}}{\partial^2 p_{m_i}} = -2\beta$ which is always negative since $\beta > 0$. Thus, $\pi_{m_i}^{s_1}$ is concave w.r.t. p_{m_i}. The optimal p_{m_i} is determined as Eqs. (14-15).

$$p_{m_1} = \frac{1}{2(2\beta^2 - 3\beta\gamma - 3\gamma^2)(4\beta^2 - \gamma^2)}(4\beta^3(\alpha + 2c_1(1+i)\beta) + 4\beta^2(1 - 2\alpha + (c_2 - c_1(5+6i) + 3w_1 + w_2)\beta)\gamma + \beta(-5\alpha + 2(c_2(i-3) + c_1(2+6i) - 8w_1)\beta)\gamma^2 + (-1 + (c_2(9i-2) + c_1(10+21i) - 17w_1 - 11w_2)\beta)\gamma^3 + (c_1(i-2) + c_2(4+7i) - w_1 - 7w_2)\gamma^4) \tag{14}$$

$$p_{m_2} = \frac{1}{2(2\beta^2 - 3\beta\gamma - 3\gamma^2)(4\beta^2 - \gamma^2)}(4\beta^3(\alpha + 2c_2(1+i)\beta) + 4\beta^2(1 - 2\alpha + (c_1 - 5c_2 - 6c_2 i + w_1 + 3w_2)\beta)\gamma + \beta(-5\alpha + 2c_1(i-3)\beta + 4(c_2 + 3c_2 i - 4w_2)\beta)\gamma^2 + (-1 + (c_1(9i-2) + c_2(10+21i) - 11w_1 - 17w_2)\beta)\gamma^3 + (c_2(i-2) + c_1(4+7i) - 7w_1 - w_2)\gamma^4) \tag{15}$$

Theorem 2. *The payoff of the e-tailer is concave w.r.t.* p_{e_1} *and* p_{e_2}.
Proof. The Hessian matrix of the e-tailer's payoff is calculated. If the Hessian matrix is negative definite, then the e-tailer's payoff is concave w.r.t. p_{e_1} and p_{e_2}.

$$H(\pi_e^{s_1}) = \begin{bmatrix} \frac{\partial^2 \pi_e^{s_1}}{\partial^2 p_{e_1}} & \frac{\partial^2 \pi_e^{s_1}}{\partial^2 p_{e_1} p_{e_2}} \\ \frac{\partial^2 \pi_e^{s_1}}{\partial^2 p_{e_2} p_{e_1}} & \frac{\partial^2 \pi_e^{s_1}}{\partial^2 p_{e_2}} \end{bmatrix} \tag{16}$$

$H_{11} = \frac{\partial^2 \pi_e^{s_1}}{\partial^2 p_{e_1}} = -4\gamma + 2\beta(-5 - \frac{8\beta}{-2\beta+\gamma})$ which is negative under the condition $4\beta > \gamma + \sqrt{17}\gamma$.
$H_{22} = \frac{4(\beta+\gamma)(2\beta^2 - 3\beta\gamma - 3\gamma^2)}{2\beta - \gamma}$ which is positive under the condition $4\beta > (3 + \sqrt{33})\gamma$.

Therefore, the optimal p_{e_1} and p_{e_2} are as Eqs. (17-18).

$$p_{e_1} = \frac{4(w_1 - c_1 i)\beta^2 + 2\beta(1 - \alpha + (c_1 + c_2 + 5c_1 i + 2c_2 i - 3w_1)\gamma)}{4(2\beta^2 - 3\beta\gamma - 3\gamma^2)} + \frac{\gamma(3\alpha - 1 - 2(c_1 + c_2 - 2c_1 i + c_2 i + 2w_1 - w_2)\gamma)}{4(2\beta^2 - 3\beta\gamma - 3\gamma^2)} \tag{17}$$

$$p_{e_2} = \frac{4(w_2 - c_2 i)\beta^2 + 2\beta(1 - \alpha + (c_1 + c_2 + 2c_1 i + 5c_2 i - 3w2)\gamma)}{4(2\beta^2 - 3\beta\gamma - 3\gamma^2)} + \frac{\gamma(3\alpha - 1 - 2(c_1 + c_2 + c_1 i - 2c_2 i - w_1 + 2w_2)\gamma)}{4(2\beta^2 - 3\beta\gamma - 3\gamma^2)} \tag{18}$$

Theorems 3 and 4 analyze the payoffs' concavity in Strategy profile 2.

Theorem 3. *The payoff of Manufacturer i is concave w.r.t.* p_{m_i}.

Proof. The second-order derivative of the Manufacturer i's payoff w.r.t. p_{m_i} is $\frac{\partial^2 \pi_{m_i}^{s_2}}{\partial^2 p_{m_i}} = -2\beta$ which is always negative since $\beta > 0$. Thus, $\pi_{m_i}^{s_2}$ is concave w.r.t. p_{m_i}. The optimal p_{m_i} is obtained as Eqs. (19-20).

$$p_{m_1} = \frac{1}{2(2\beta^2 - 3\beta\gamma - 3\gamma^2)(4\beta^2 - \gamma^2)}(4\beta^3(\alpha + 2c_1(1+i)\beta)$$
$$+4\beta^2(1 - 2\alpha + (c_2 - c1(5+6i) + 3w_1 + w_2)\beta)\gamma$$
$$+\beta(-5\alpha + 2(-3c_2 + c_1(2+6i) - 8w_1)\beta)\gamma^2$$
$$+(-1 + c_1(10+21i)\beta - (2c_2 + 17w_1 + 11w_2)\beta)\gamma^3$$
$$+(4c_2 + c_1(i-2) - w_1 - 7w_2)\gamma^4) \quad (19)$$

$$p_{m_2} = \frac{1}{2(2\beta^2 - 3\beta\gamma - 3\gamma^2)(4\beta^2 - \gamma^2)}(4\beta^3(\alpha + 2c_2\beta)$$
$$+4\beta^2(1 - 2\alpha + (c_1 - 5c_2 + w_1 + 3w_2)\beta)\gamma$$
$$\beta(-5\alpha + 2c_1(i-3)\beta + 4(c_2 - 4w_2)\beta)\gamma^2$$
$$+(-1 + (10c_2 + c_1(9i-2) - 11w_1 - 17w_2)\beta)\gamma^3$$
$$-(2c_2 - c_1(4+7i) + 7w_1 + w_2)\gamma^4) \quad (20)$$

Theorem 4. *The payoff of the e-tailer is concave w.r.t.* p_{e_1} *and* p_{e_2}.
Proof. The Hessian matrix of e-tailer's payoff is calculated as follows.

$$H(\pi_e^{s_2}) = \begin{bmatrix} \frac{\partial^2 \pi_e^{s_2}}{\partial^2 p_{e_1}} & \frac{\partial^2 \pi_e^{s_2}}{\partial^2 p_{e_1} p_{e_2}} \\ \frac{\partial^2 \pi_e^{s_2}}{\partial^2 p_{e_2} p_{e_1}} & \frac{\partial^2 \pi_e^{s_2}}{\partial^2 p_{e_2}} \end{bmatrix} \quad (21)$$

$H_{11} = \frac{\partial^2 \pi_e^{s_2}}{\partial^2 p_{e_1}} = -4\gamma + 2\beta(-5 - \frac{8\beta}{-2\beta+\gamma})$ which is negative under the condition $4\beta > \gamma + \sqrt{17}\gamma$.
$H_{22} = \frac{4(\beta+\gamma)(2\beta^2 - 3\beta\gamma - 3\gamma^2)}{2\beta - \gamma}$ which is positive under the condition $4\beta > (3 + \sqrt{33})\gamma$.
Therefore, the optimal p_{e_1} and p_{e_2} are as Eqs. (22-23).

$$p_{e_1} = \frac{4(-c_1 i + w_1)\beta^2 + 2\beta(1 - \alpha + (c_1 + c_2 + 5c_1 i - 3w_1)\gamma)}{4(2\beta^2 - 3\beta\gamma - 3\gamma^2)} + \frac{\gamma(-1 + 3\alpha - 2(c_1 + c_2 - 2c_1 i + 2w_1 - w_2)\gamma)}{4(2\beta^2 - 3\beta\gamma - 3\gamma^2)} \quad (22)$$

$$p_{e_2} = \frac{4w_2\beta^2 + 2\beta(1 - \alpha + (c_1 + c_2 + 2c_1 i - 3w_2)\gamma)}{4(2\beta^2 - 3\beta\gamma - 3\gamma^2)} + \frac{\gamma(-1 + 3\alpha - 2(c_1 + c_2 + c_1 i - w_1 + 2w_2)\gamma)}{4(2\beta^2 - 3\beta\gamma - 3\gamma^2)} \quad (23)$$

The payoffs' concavity under Strategy profile 3 is examined through theorems 5 and 6.

Theorem 5. *The payoff of Manufacturer i is concave w.r.t.* p_{m_i}.

Proof. The second-order derivative of the Manufacturer i's payoff w.r.t. p_{m_i} is $\frac{\partial^2 \pi_{m_i}^{s_3}}{\partial^2 p_{m_i}} = -2\beta$ which is always negative since $\beta > 0$. Thus, $\pi_{m_i}^{s_3}$ is concave w.r.t. p_{m_i}. The optimal p_{m_i} is obtained as Eqs. (24-25).

$$p_{m_1} = \frac{4\beta^3(\alpha + 2c_1\beta) + 4\beta^2(1 - 2\alpha + (-5c_1 + c_2 + 3w_1 + w_2)\beta)\gamma + \beta(-5\alpha + 2(2c_1 - 3c_2 - 8w_1)\beta)\gamma^2}{2(2\beta^2 - 3\beta\gamma - 3\gamma^2)(4\beta^2 - \gamma^2)} \quad (24)$$

$$+\frac{(-1+10c_1\beta-(2c_2+17w_1+11w_2)\beta)\gamma^3-(2c_1-4c_2+w_1+7w_2)\gamma^4}{2(2\beta^2-3\beta\gamma-3\gamma^2)(4\beta^2-\gamma^2)}$$

$$p_{m_2}=-\frac{-4\beta^3(\alpha+2c_2\beta)-4\beta^2(1-2\alpha+(c_1-5c_2+w_1+3w_2)\beta)\gamma+\beta(5\alpha+2(3c_1-2c_2+8w_2)\beta)\gamma^2}{2(2\beta^2-3\beta\gamma-3\gamma^2)(4\beta^2-\gamma^2)} \quad (25)$$

$$+\frac{(1+(2c_1-10c_2+11w_1+17w_2)\beta)\gamma^3+(-4c_1+2c_2+7w_1+w_2)\gamma^4}{2(2\beta^2-3\beta\gamma-3\gamma^2)(4\beta^2-\gamma^2)}$$

Theorem 6. *The payoff of the e-tailer is concave w.r.t.* p_{e_1} *and* p_{e_2}.
Proof. The Hessian matrix of the e-tailer's payoff is calculated as follows.

$$H(\pi_e^{S_3})=\begin{bmatrix}\frac{\partial^2\pi_e^{S_3}}{\partial^2 p_{e_1}} & \frac{\pi_e^{S_3}}{\partial^2 p_{e_1}p_{e_2}}\\ \frac{\partial^2\pi_e^{S_3}}{\partial^2 p_{e_2}p_{e_1}} & \frac{\partial^2\pi_e^{S_3}}{\partial^2 p_{e_2}}\end{bmatrix} \quad (26)$$

$H_{11}=\frac{\partial^2\pi_e^{S_3}}{\partial^2 p_{e_1}}=-4\gamma+2\beta(-5-\frac{8\beta}{-2\beta+\gamma})$ which is negative under the condition $4\beta>\gamma+\sqrt{17}\gamma$.
$H_{22}=\frac{4(\beta+\gamma)(2\beta^2-3\beta\gamma-3\gamma^2)}{2\beta-\gamma}$ which is positive under the condition $4\beta>(3+\sqrt{33})\gamma$.
Therefore, the optimal p_{e_1} and p_{e_2} are as Eqs. (27-28).

$$p_{e_1}=\frac{4w_1\beta^2+2\beta(1-\alpha+(c_1+c_2-3w_1)\gamma)+\gamma(-1+3\alpha-2(c_1+c_2+2w_1-w_2)\gamma)}{4(2\beta^2-3\beta\gamma-3\gamma^2)} \quad (27)$$

$$p_{e_2}=\frac{4w_2\beta^2+2\beta(1-\alpha+(c_1+c_2-3w_2)\gamma)+\gamma(-1+3\alpha-2(c_1+c_2-w_1+2w_2)\gamma)}{4(2\beta^2-3\beta\gamma-3\gamma^2)} \quad (28)$$

3-3- The evolutionary game theory model

In the evolutionary game theory model, we examine the evolution of manufacturers' strategies and determine the ESS solution of the manufacturers which indicates the strategy implemented by most of the manufacturers. In line with the evolutionary game theory literature (e.g., Barron, 2013), the ESS is determined through Definition 1.

Definition 1. *Strategy* S^* *is the ESS w.r.t. strategies* $S_1, S_2, \dots, S_n$ *if and only if either Eq. (29) or Eq. (30) holds:*

$$u(S^*,S^*)>u(S,S^*)\ \forall\ 0\le S\le 1, S\neq S^*, \quad (29)$$

$$u(S^*,S^*)=u(S,S^*)\Rightarrow\ u(S^*,S)>u(S,S)\qquad \forall\ S\neq S^*. \quad (30)$$

In this study, each manufacturer can either choose financing form the e-tailer or not. Hence, the manufacturers' strategies are:

$$S_1: the\ strategy\ in\ which\ manufacturers\ receive\ financing\ from\ the\ e-tailer, \quad (31)$$

$$S_2: the\ strategy\ where\ manufacturers\ don't\ receive\ financing\ from\ the\ e-tailer \quad (32)$$

Eqs. (33-34) show the probabilities of strategies.

$$x: the\ proportion\ of\ manufacturers\ choosing\ the\ financing\ scheme, \quad (33)$$

$$1-x: the\ proportion\ of\ manufacturers\ choosing\ the\ non-financing\ scheme. \quad (34)$$

Integrating the payoffs calculated in the basic model and the probability of choosing strategies in the evolutionary game theory model, we provide the payoff matrix of the game played between the two selected manufacturers in Table 2.

Table 2. Payoff matrix.

		Manufacturer 2	
	Strategy	financing (x)	non-financing $(1-x)$
Manufacturer 1	financing (x)	$(\pi_{m_1}^{s_1}, \pi_{m}^{s}$	$(\pi_{m_1}^{s_2}, \pi_{m_2}^{s_2}$
	non-financing $(1-x)$	$(\pi_{m_2}^{s_2}, \pi_{m}^{s}$	$(\pi_{m_1}^{s_3}, \pi_{m_2}^{s_3}$

In the following, we calculate the expected payoffs of strategies.

$$EP_1 = x \times \pi_{m_1}^{s_1} + (1-x) \times \pi_{m_1}^{s_2}, \tag{35}$$

$$EP_2 = x \times \pi_{m_2}^{s_2} + (1-x) \times \pi_{m_1}^{s_3}. \tag{36}$$

The average payoff is shown in Eq. (37).

$$AP = x \times EP_1 + (1-x) \times EP_2. \tag{37}$$

Based on the concept of the evolutionary game theory, most of the population will choose a strategy which has the expected payoff higher than the average payoff. To determine such a strategy, the replicator dynamics equation is considered as Eq. (38).

$$\frac{dx}{dt} = x(t) \times (EP_1 - AP). \tag{38}$$

Eq. (38) is simplified to Eq. (39) by inserting Eqs. (35) and (37) into Eq. (38).

$$\frac{dx}{dt} = x(t) \times \big(1 - x(t)\big) \times [x(t) \times (\pi_{m_1}^{s_1} + \pi_{m_1}^{s_3} - \pi_{m_1}^{s_2} - \pi_{m_2}^{s_2}) + \pi_{m_1}^{s_2} - \pi_{m_2}^{s_2}]. \tag{39}$$

By setting $\frac{dx}{dt}$ to zero in Eq. (39), the ESS solution which does not alter in the long term is determined as follows:

$$x(t) = 0, \qquad x(t) = 1, \qquad x(t) = \frac{\pi_{m_2}^{s_2} - \pi_{m_1}^{s_2}}{\pi_{m_1}^{s_1} + \pi_{m_1}^{s_3} - \pi_{m_1}^{s_2} - \pi_{m_2}^{s_2}}. \tag{40}$$

Therefore, to know which of the determined solutions is the ESS, some conditions, which are indicated in Proposition 1, should be checked in line with the evolutionary game theory.

Proposition 1. *For the payoff matrix* $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$*, the ESS is determined through checking the following conditions:*

$$x(t) = 1 \qquad , if\,(a_{11} - a_{21}) > 0, (a_{22} - a_{12}) < 0, \tag{41}$$

$$x(t) = \frac{a_{22} - a_{12}}{a_{11} + a_{22} - a_{21} - a_{12}} \qquad , if\,(a_{11} - a_{21}) < 0, (a_{22} - a_{12}) < 0, \tag{42}$$

$$x(t) = 0 \qquad , if\,(a_{11} - a_{21}) < 0, (a_{22} - a_{12}) > 0, \tag{43}$$

$$x(t) = 0 \; and \; 1 \qquad , if\,(a_{11} - a_{21}) > 0, (a_{22} - a_{12}) > 0. \tag{44}$$

4- Sensitivity analysis

To examine the usefulness of the proposed models, in this section, numerical analyses are discussed. The data of parameters are $w_1 = 2, w_2 = 1.5, c_1 = 1, c_2 = 0.5, \alpha = 15, \beta = 5, \gamma = 2, k = 3, i = 0.3$.

Fig. 1 shows the strategy adoption of the manufacturers in the long term. From Fig. 1, most of the manufacturers (about 71%) will implement the non-financing strategy. This result implies that as innovative financing schemes are developed in the market, it is of great importance for financially-constrained manufacturers to choose innovative financing or hybrid financing options.

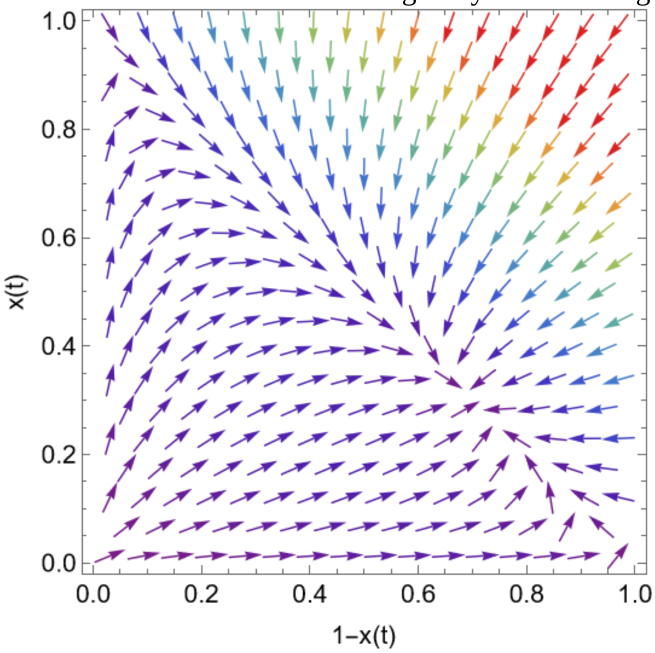

Fig. 1. The long-term behavior of manufacturers.

Fig. 2 indicates how the financing strategy of the manufacturers alters over time. As the ESS solution is about 29% (shown in Fig. 1), the inside paths (solid and dotted-dash lines) converge to the ESS over time. However, the boundary paths stay unchanged over time.

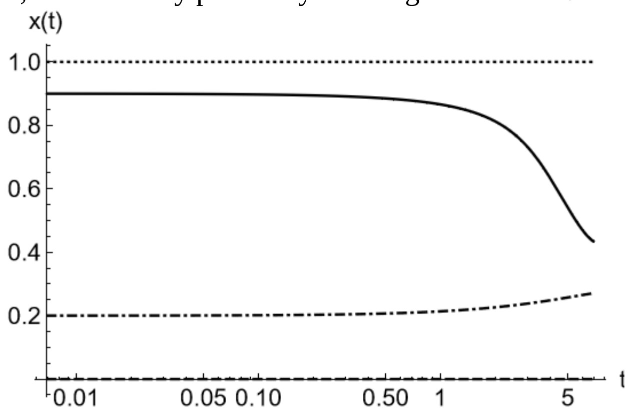

Fig. 2. The convergent strategy.

Fig. 3 shows the trend of SC members' payoffs under cross-price coefficient of demand, γ. According to Fig. 3a, as pricing competition increases, the manufacturers' payoff grows under three strategy profiles. However, when most of the manufacturers are financially constrained (e.g., Strategy profile 1), intense pricing competition (the competition degree is about 2.2) results in more economic benefits for the manufacturers rather than the two other strategy profiles. As shown in Fig. 3b, when pricing competition is weak, the e-tailer's payoff decreases. However, when pricing competition is high (the competition degree is about 2.2), the e-tailer's payoff improves. Therefore, when financially-constrained manufacturers and the e-tailer strongly compete on pricing decisions, all SC members can achieve more benefits.

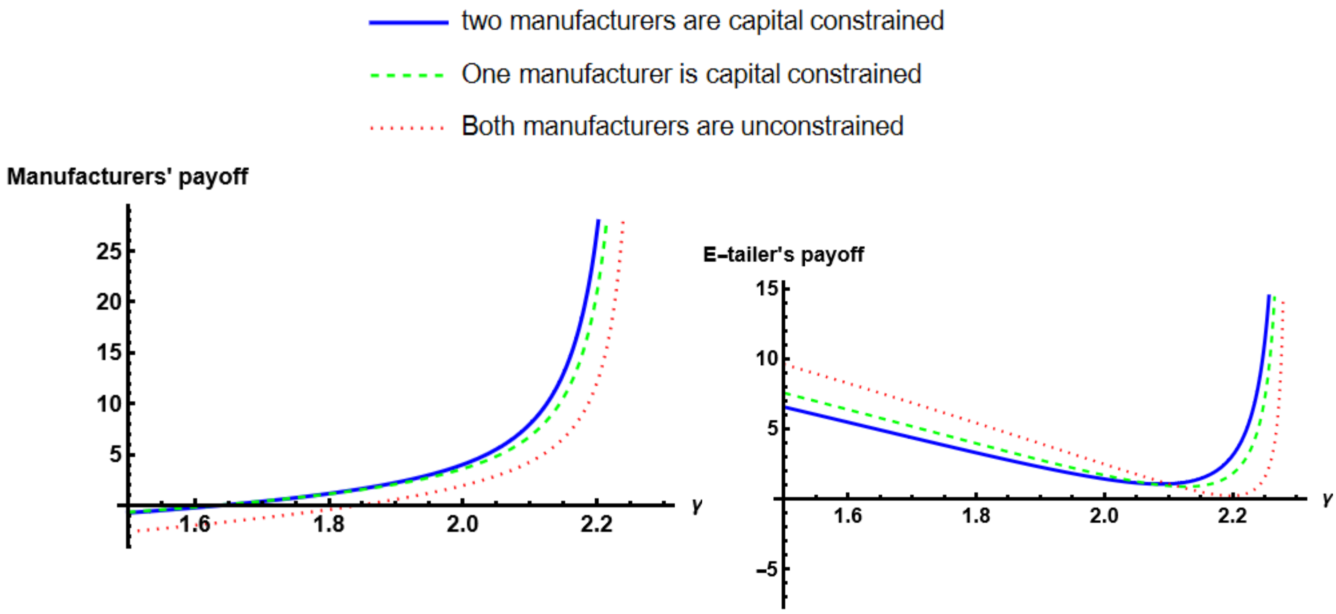

Fig. 3. The SC members' payoff w.r.t. pricing competition. (a) Manufacturers. (b) E-tailer.

Figs. 4 and 5 show how the optimal pricing decisions and SC members' payoffs are affected by the interest rate, i. As shown in Fig. 4a, by growing the interest rate of the e-tailer regarding the finance scheme provided to the manufacturers, the manufacturers offer higher prices to customers. As a result, due to the pricing competition in the market, the e-tailer can also provide higher selling prices to customers as depicted in Fig. 4b. Also, regarding the SC members' payoff changes, as shown in Fig. 5a, by growing the interest rate, the manufacturers' benefit increases under both strategy profiles (two manufacturers are financially constrained and one manufacturer is financially constrained). However, when both manufacturers are financially constrained, they can gain more benefit compared to the case in which one manufacturer is financially constrained. Moreover, Fig. 5b depicts that the e-tailer's payoff decreases by growing the interest rate. As a result, these findings highlight the effectiveness of financing schemes on the optimal decisions and payoffs of SC actors.

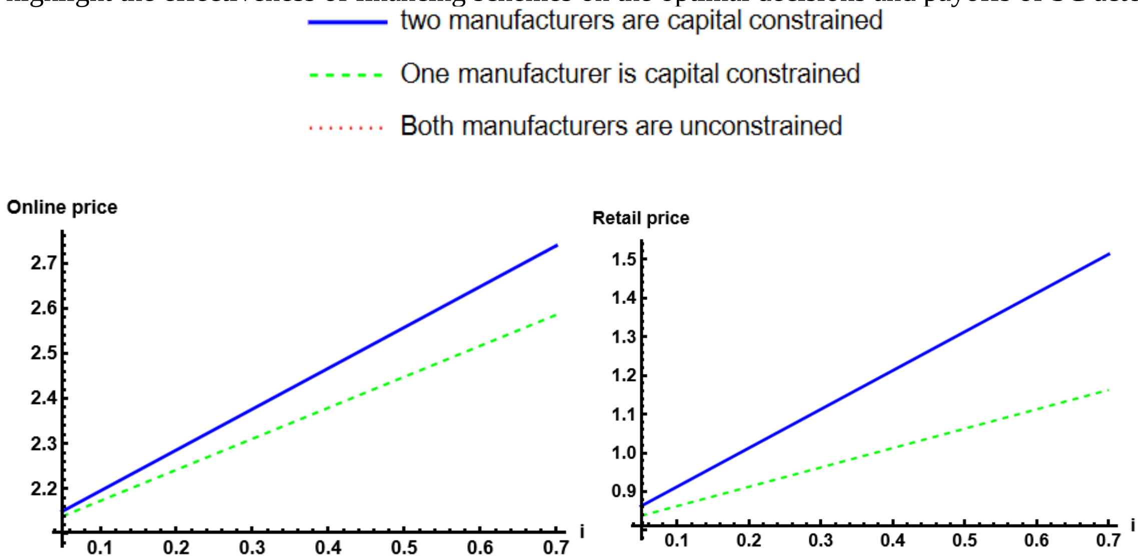

Fig. 4. The selling price under interest rate. (a) Manufacturers. (b) E-tailer.

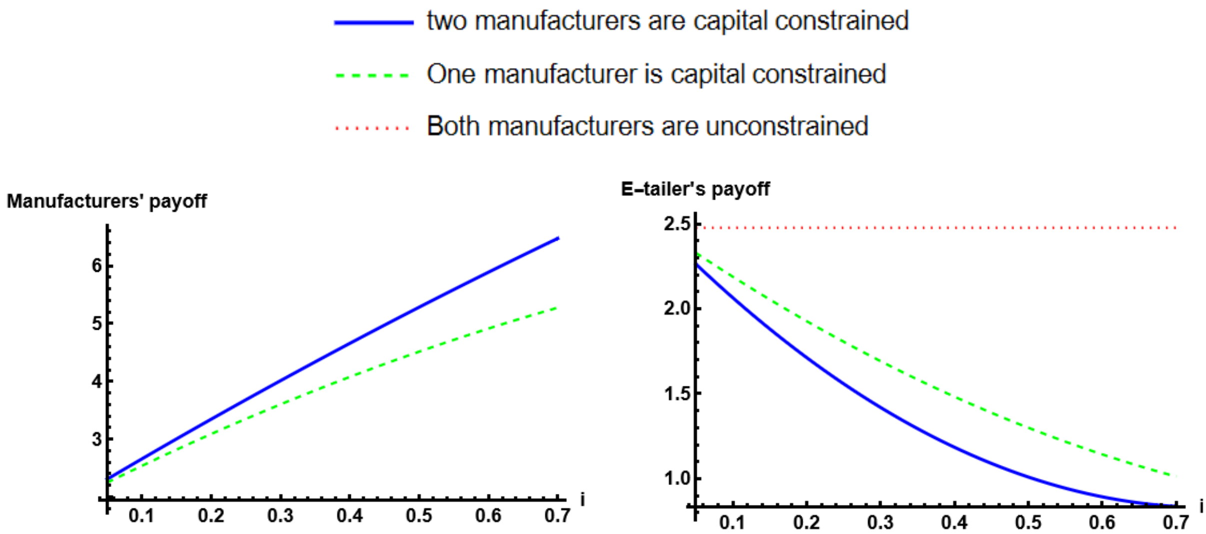

Fig. 5. The SC members' payoff under interest rate. (a) Manufacturers. (b) E-tailer.

4-1- Managerial insights

Our study suggests the following management insights:

- **The long-term financing strategy of financially-constrained manufacturers.** Our finding shows that the long-term stable strategy for the population of financially-constrained manufacturers is to choose other financing schemes, such as innovative or hybrid financing schemes.
- **The effect of the interest rate on the optimal pricing decisions of SC members.** The result indicates that when manufacturers face financial constraints, the higher interest rate provided by the e-tailer leads to the higher selling prices of both manufacturers and the e-tailer.
- **The relationship between pricing competition and SC members' benefits.** Our result demonstrates that the intense competitive situation among the manufacturers and the e-tailer brings more benefits for all SC actors (i.e., the e-tailer and the manufacturers).

5- Conclusions

Motivated by the observed practical issues and research gaps, this study contributes to the literature by developing an evolutionary game theory model to examine the long-term financing scheme of financially-constrained manufacturers in an SC including one e-tailer and one population of manufacturers. The manufacturers can either choose financing from the e-tailer or not. Moreover, pricing competition is investigated among the e-tailer and the manufacturers. This study investigates both basic and evolutionary game theory models. Under the basic model, three strategy profiles are investigated and the optimal pricing decisions of the manufacturers and the e-tailer are determined. Also, under the evolutionary game theory model, the probabilities of implementing financing options are considered and the ESS solution adopted by most of the manufacturers in the long term is obtained. The finding shows that most of the manufacturers (about 71%) will choose the non-financing scheme in the long term. This result implies that financially-constrained manufacturers will adopt other financing modes, such as innovative or hybrid financing schemes. In addition, the result indicates that under strong competition on pricing (about 2.2), all competitive SC members (the e-tailer and the manufacturers) achieve more profits. Also, the finding demonstrates that by rising the e-tailer's interest rate (about 70%), the manufacturers and the e-tailer can benefit from providing higher prices to customers.

This study can be developed in some paths. First, this research explores one population of manufacturers with financial constraints. It will be challenging to investigate a two-population evolutionary game theory model including a population of manufacturers and a population of retailers. Second, this study considers that only manufacturers face financial constraints. This study can be extended by analyzing financial constraints for both SC levels, including manufacturers and retailers.

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