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Cryptocurrency Portfolio Optimization using Conditional Drawdown at Risk Measure with a Novel Approach for Asset Pre-selection

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Abstract

The cryptocurrency market presents enticing yet high-risk investment opportunities, marked by rapid growth, extreme volatility, and substantial uncertainty. Traditional risk management models, which often rely on probabilistic assumptions and historical data, struggle to effectively capture the unique dynamics and unpredictability of this market. In light of these challenges, this paper examines the application of Conditional Drawdown at Risk (CDaR), a prominent downside risk measure, for optimizing cryptocurrency portfolios. Recognizing that portfolio optimization alone may not yield optimal results, this study emphasizes the importance of a rigorous asset pre-selection process to identify assets with strong fundamentals, growth potential, and resilience to market fluctuations. To address this need, we introduce a novel asset pre-selection approach using Multi-Attribute Decision Making (MADM) methods, enabling a systematic evaluation and selection of high-potential cryptocurrency assets prior to applying the CDaR optimization model. By addressing both asset quality and risk-adjusted allocation, this approach bridges a critical gap in current cryptocurrency portfolio management practices, providing a comprehensive tool that aligns with the unique demands of digital asset investing. Tested within the cryptocurrency market, this framework demonstrates promising results, underscoring its effectiveness as a tailored approach for navigating the complexities of digital asset investments.

Keywords: Portfolio optimization, Conditional drawdown at risk, Pre-selection, Cryptocurrency market, Multi attribute decision-making methods

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1. Introduction

Investment is recognized as a key tool for wealth accumulation and securing individuals' and institutions' financial futures (Keller and Siegrist, 2006; Shane, 2012; Greenwood et al., 2000). In recent years, the cryptocurrency market, particularly with the emergence of Bitcoin and altcoins, has become one of the most attractive and rapidly growing investment sectors. This market not only allows investors to capitalize on price fluctuations but also provides new opportunities for portfolio diversification. One of the primary advantages of investing in cryptocurrencies is the high potential for returns, which can significantly exceed those of traditional markets such as stocks and bonds. Additionally, the decentralized nature of the cryptocurrency market offers investors the ability to trade digital currencies without geographical and temporal restrictions. Furthermore, the underlying blockchain technology that forms the basis of cryptocurrencies ensures a high level of transparency and security in transactions, which enhances investor confidence in this market. Consequently, investing in the cryptocurrency market has emerged not only as an appealing financial opportunity but also as a strategic tool for risk management and portfolio diversification (Babaioff et al, 2014; Bouri et al, 2019; Elbahrawy et al., 2017; Wątorek et al, 2021). Given the advantages of investing in cryptocurrencies, such as the potential for high returns and the ability to diversify one's portfolio, it is essential to approach this volatile market with an appropriate strategy. Cryptocurrencies are known for their high volatility, with prices often fluctuating rapidly and unpredictably. This volatility can present both opportunities and risks for investors. On the one hand, the potential for significant gains can be enticing, but on the other hand, the high risk of sudden and substantial losses must be carefully considered. To navigate this volatile market effectively, it is crucial to adopt a well-informed and prudent approach (Almeida et al., 2022; Canh et al., 2019; Movahed et al., 2024).

There are several investment strategies available for navigating the cryptocurrency market, with portfolio optimization being one of the most common and effective approaches. Portfolio optimization involves strategically selecting and allocating assets to maximize returns while minimizing risk (Mansini et al., 2014; Kolm et al., 2014; Khosravi et al., 2024; Nozari et al., 2025). This method allows investors to diversify their holdings across various cryptocurrencies, thereby reducing the impact of volatility in any single asset. By employing optimization techniques, investors can enhance their chances of achieving a favorable risk-return profile, making it a valuable strategy in the ever-changing landscape of digital currencies. Portfolio optimization is recognized as a scientific method for asset allocation and risk management in the financial world. This concept was introduced in the 1950s by Harry Markowitz, an economist and Nobel Prize winner. He demonstrated to investors how they could reduce the overall risk of their portfolio while achieving desirable returns by combining different assets. Markowitz's model is based on the principle that the price fluctuations of assets do not operate independently of one another, and by considering the correlations between assets, one can create a portfolio that carries less risk. This theory quickly gained attention and became a foundation for the development of modern investment management techniques and portfolio optimization. After Markowitz's seminal work on mean-variance portfolio optimization, many researchers have sought to develop new models to address the limitations and challenges of the traditional approach (Ghanbari et al., 2023). One such model that has gained traction in the context of investments is the Conditional Drawdown at Risk (CDaR) model. The CDaR is a risk measure that focuses on drawdown, which is a more suitable risk measure for the highly volatile cryptocurrency market. Drawdown refers to the decline in the value of a portfolio from its peak to its trough, and the CDaR quantifies the expected maximum drawdown given a certain confidence level. This risk measure is particularly relevant for cryptocurrency investors, as the market is characterized by significant price fluctuations and sudden downturns, which can lead to substantial losses if not managed appropriately.

Therefore, this paper intends to explore the application of the CDaR model in the optimization of cryptocurrency portfolios. However, approaches that rely exclusively on portfolio optimization models may not be suitable for achieving optimal investment outcomes. While these models provide valuable frameworks for asset allocation, they often overlook the critical importance of selecting high-quality assets. It is essential to conduct a thorough pre-selection process to identify assets that possess strong fundamentals, growth potential, and resilience to market fluctuations. By focusing on quality assets before applying portfolio optimization models, investors can enhance the effectiveness of their strategies. This pre-selection ensures that the assets included in the portfolio are not only well-positioned for growth but also capable of mitigating risks. Consequently, integrating a robust asset selection process with portfolio optimization techniques can lead to more informed investment decisions and improved overall portfolio performance. In the context of cryptocurrency, the pre-selection process is particularly crucial, as many assets may not hold significant investment value.

This paper aims to explore a novel approach to cryptocurrency portfolio optimization by combining the use of the CDaR risk measure with a pre-selection process for identifying high-quality cryptocurrency assets. While portfolio optimization models provide valuable frameworks for asset allocation, the authors recognize the importance of conducting a thorough pre-selection of assets to ensure that the portfolio is composed of investments with strong fundamentals, growth potential, and resilience to market fluctuations. To this end, this paper introduces a novel pre-selection approach based on Multi-Attribute Decision Making (MADM) methods, which allows for the systematic evaluation and selection of the most promising cryptocurrency assets before applying the CDaR portfolio optimization model.

The remainder of this paper is structured as follows: Section 2 is devoted to a comprehensive literature review, providing an overview of the relevant research on portfolio optimization models, risk measures, and asset selection approaches in the context of cryptocurrency investments. Section 3 presents the preliminaries, outlining the key concepts and methodologies that underpin the proposed framework, including the CDaR risk measure and the MADM-based pre-selection process. In Section 4, the authors present the experimental results, detailing the performance of the integrated approach and comparing it to alternative optimization methods. Section 5 offers a detailed discussion of the managerial insights and practical implications derived from the findings. Finally, Section 6 concludes the paper and outlines potential future directions for research in this rapidly evolving field of cryptocurrency portfolio management.

2. Literature survey

This section presents a thorough literature survey, organized into three essential subsections to provide a comprehensive overview of existing academic research while identifying areas where further investigation is needed. The first subsection explores the role and impact of pre-selection techniques in portfolio optimization, offering an in-depth examination of various strategies used to filter or select assets prior to the application of optimization models. The second subsection provides a detailed review of portfolio optimization models, charting their development from foundational approaches, such as the Markowitz Mean-Variance model, to more advanced contemporary methods. In the final subsection, the findings from the previous sections are synthesized to pinpoint areas where research is either limited or inconsistent, highlighting specific gaps that this study intends to address.

2.1. Literature review on the application of pre-selection in portfolio optimization problems

Pre-selection is a crucial step in portfolio management, focusing on identifying and selecting assets before optimizing the investment portfolio. This phase is especially important in volatile markets like cryptocurrencies, where asset selection can greatly impact portfolio performance. Pre-selection encompasses a range of methodologies designed to filter and identify high-potential assets. This literature review delves into the role of pre-selection in portfolio optimization, highlighting significant findings and advancements that drive this fast-evolving field.

(Lozza et al., 2011) conducted an ex-post comparison of asset pre-selection strategies, utilizing the joint Markovian behavior of returns in relation to market stochastic bounds. Their study, which included approximately 10,000 stocks from 14 different markets, demonstrated that Markovian strategies outperformed classical approaches based on maximizing the Sharpe ratio. This finding suggests that incorporating market dynamics into the pre-selection process can lead to portfolios with more robust performance, especially in large-scale portfolio selection problems. In a similar vein, (Huang, 2012) developed a stock selection model using support-vector regression (SVR) and genetic algorithms (GA). SVR was used to predict the future performance of stocks, while GA optimized the model's parameters and input features. The highest-ranked stocks were then equally weighted to form the portfolio, resulting in superior investment performance compared to traditional benchmarks. This study highlights the potential of machine learning techniques in enhancing the pre-selection process. (Nguyen, 2014) focused on risk measurement for large-scale datasets, incorporating a pre-selection process that removed low-diversification stocks before applying optimization metrics like the Sharpe ratio, Stutzer performance index, and Omega measure. The results showed that pre-selection improved both the performance and diversification of the portfolio, underscoring the importance of filtering assets before optimization. (Rather et al., 2015) proposed a robust hybrid model for stock return prediction, combining linear models like autoregressive moving averages and exponential smoothing with non-linear models like recurrent neural networks (RNNs). This approach significantly improved prediction accuracy, with the optimized model generating ideal portfolio weights using GA. The integration of diverse predictive models during the pre-selection phase was key to enhancing overall portfolio performance. (Le Caillec et al., 2017) introduced a behavioral uncertainty framework combined with probabilistic and possibilistic methods to pre-select stocks using multiple technical indicators. The experimental results indicated that this approach improved portfolio performance by addressing common biases in human decision-making during asset selection. (Fischer and Krauss, 2018) implemented a Long Short-Term Memory (LSTM) neural network to predict stock movements within the S&P 500. Their study found that portfolios based on LSTM predictions outperformed those constructed using other machine learning models, further emphasizing the utility of advanced machine learning techniques in the pre-selection process. In the context of fuzzy environments, (Georgescu and Fono, 2019) presented a possibilistic portfolio choice problem where the return of a risky asset is modeled as a fuzzy number. This approach incorporated uncertainty into the pre-selection process, helping investors select assets that align better with their risk preferences. Huang (2008) and Li et al. (2015) explored fuzzy portfolio selection models, with Huang proposing fuzzy mean-semivariance models and Li et al. developing a model that included background risk based on possibilistic return and risk. These studies illustrate how fuzzy set theory can be effectively applied during the pre-selection phase to manage uncertainty in financial markets. (Parra et al., 2001) and (Li et al., 2010) extended fuzzy set theory applications to portfolio optimization, with Parra et al. introducing a fuzzy goal programming approach and (Li et al., 2010) applying a mean-variance-skewness model considering fuzzy returns. These models further validate the role of fuzzy logic in refining asset selection before optimization. (Jana et al., 2009) and (Qin et al., 2009) contributed to this area by presenting fuzzy cross-entropy models and mean-entropy models for portfolio selection. (Galankashi et al., 2020) also developed a fuzzy analytic network process to assess and select portfolios, emphasizing the importance of multiple criteria in the pre-selection process. Uncertainty theory has also been a focus in pre-selection research. Mehralizade et al. (2020) explored uncertain random portfolio selection problems, introducing a new risk criterion, while Li et al. (2019) proposed an uncertain risk measure for modeling investment risk. These studies underscore the necessity of considering uncertainty in the asset pre-selection process, particularly in volatile markets. Machine learning has been increasingly applied in pre-selection methodologies. Liu and Yeh (2017) used neural networks to build decision support systems for stock selection, while Min et al. (2021) developed hybrid robust portfolio models using LSTM and XGBoost to evaluate market movements. These approaches highlight the potential of machine learning in improving the pre-selection process. Huang (2012) and Chang et al. (2009) further demonstrated the effectiveness of machine learning in pre-selection, with Huang employing SVR and GA for stock selection and Chang et al. concluding that portfolios with fewer assets but higher quality outperform those with a larger number of assets. Wang et al. (2020) utilized deep LSTM methods for asset pre-selection, and Paiva et al. (2019) applied SVM to classify and select the best-performing assets. Hai and Min. (2021) designed a machine-learning-based pre-selection method using random forests and SVM, showcasing the growing relevance of machine learning in refining the pre-selection process. Lozza et al. (2013) and Qu et al. (2017) focused on dimensionality reduction in large-scale portfolio problems using pre-selection techniques. Chen et al. (2021) also developed a portfolio optimization model using extreme gradient boosting for pre-selection, highlighting the utility of machine learning in enhancing the pre-selection phase. Finally, Yang and Feng (2017) and Marasović et al. (2021) applied the expected utility and entropy (EU-E) model to stock selection, demonstrating that efficient portfolios constructed from smaller subsets of pre-selected stocks maintain nearly the same efficient frontier as those constructed from larger sets. This underscores the importance of pre-selection in maintaining portfolio efficiency while reducing complexity.

These studies collectively highlight the essential role of pre-selection in enhancing portfolio optimization, especially in volatile markets like cryptocurrencies. Pre-selection acts as a strategic filter, allowing investors to narrow down a diverse set of assets to those with high potential returns, manageable risks, and favorable growth indicators.

2.2. Literature review on portfolio optimization models

Portfolio optimization has been a central focus in financial research, aiming to identify the most efficient allocation of assets to maximize returns while minimizing risk. The foundational work in this field was laid by Harry Markowitz in the 1950s, and since then, various models have been developed to address the complexities of financial markets. This section reviews the evolution of portfolio optimization models, beginning with Markowitz's seminal Mean-Variance model and progressing to models based on different risk measures. The cornerstone of modern portfolio theory was introduced by Harry Markowitz in 1952 with his Mean-Variance (MV) model, which seeks to optimize the trade-off between expected return and risk, where risk is measured by the variance (or standard deviation) of portfolio returns (Markowitz, 1952). The key contribution of this model is the concept of the efficient frontier, a set of portfolios that offers the highest expected return for a given level of risk or the lowest risk for a given level of return. Markowitz's model assumes that investors are risk-averse and prefer portfolios that minimize risk for a given return. Despite its mathematical elegance and widespread adoption, the MV model has limitations, particularly its reliance on historical data for predicting future returns and variances, and the assumption that asset returns are normally distributed. These limitations have prompted the development of alternative models that incorporate more realistic assumptions about market behavior. In response to the limitations of variance as a risk measure, the Mean Absolute Deviation (MAD) model was proposed as an alternative. Instead of measuring risk through variance, MAD focuses on the average of the absolute deviations of returns from the mean. This model, developed by Konno and Yamazaki (1991), provides a more robust measure of risk, particularly in non-normal return distributions. Unlike variance, which squares the deviations and emphasizes outliers, MAD gives a more intuitive measure of variability by treating all deviations equally. This makes the model less sensitive to extreme values and more appropriate for portfolios that may not follow normal distribution patterns. Markowitz himself recognized the limitations of using variance as the sole measure of risk, as it penalizes both upside and downside fluctuations equally. To address this, the concept of semivariance was introduced Markowitz (1959). Semivariance only considers negative deviations from the mean or a target return, making it a more accurate reflection of the risk that most investors are concerned with—downside risk. As portfolio optimization evolved, new risk measures were introduced to better capture the risks associated with extreme market events. One such measure is Value at Risk (VaR), introduced by J.P. Morgan (1996). VaR measures the maximum potential loss of a portfolio over a specific time period with a given confidence level (Jorion, 1996). While VaR is widely used for its simplicity, it fails to capture tail risk—the risk of extreme losses beyond the VaR threshold. To overcome this limitation, Conditional Value at Risk (CVaR) was introduced by Rockafellar and Uryasev (2000). CVaR focuses on the tail of the loss distribution and calculates the expected loss, given that the portfolio's losses exceed the VaR level. This makes CVaR particularly useful for portfolios exposed to significant downside risk, as it provides a more comprehensive view of extreme market conditions. To better manage the risk of substantial losses, Chekhlov et al. (2005) introduced CDaR. CDaR quantifies the expected maximum drawdown over a specified time period at a given confidence level Unlike traditional models, CDaR focuses on the largest possible loss within the portfolio's performance and provides a more comprehensive measure of the potential risks faced during severe market downturns. This makes CDaR particularly relevant in managing the downside risk in volatile markets such as cryptocurrencies, where large drawdowns are common and can significantly impact the overall performance of a portfolio.

2.3. Research Gap

The rapidly evolving field of cryptocurrency portfolio optimization has garnered significant interest, driven by the unique challenges posed by the volatile and dynamic nature of digital assets. Despite advancements in this area, several critical research gaps remain, particularly in the integration of advanced risk measures and innovative techniques for asset pre-selection. One notable gap is the limited application of the CDaR measure within cryptocurrency markets. Although CDaR has been recognized as an effective downside risk measure in traditional financial markets, its potential for managing the specific risks associated with cryptocurrency portfolios has not been thoroughly investigated. Current studies tend to focus on conventional risk measures which, while useful, may fall short in capturing the extreme volatility and sudden price fluctuations characteristic of digital assets. This underlines the need for a deeper exploration of CDaR applicability in cryptocurrency portfolio management, especially in constructing portfolios resilient to significant drawdowns. Additionally, the field lacks advanced techniques for asset pre-selection, which is a critical step in the optimization process, particularly in high-volatility environments like cryptocurrency markets. While effective asset pre-selection can substantially enhance portfolio performance, many studies either omit this step or employ simplistic methods such as heuristic filters or basic machine learning models. The absence of sophisticated pre-selection frameworks leaves a significant gap in the literature, underscoring the need for a novel, robust approach.

In response to these gaps, this paper aims to introduce a novel framework for asset pre-selection based on MADM methods. Following this, CDaR will be employed as a downside risk measure to effectively allocate weights within the optimized portfolio. This approach not only advances the methodology for cryptocurrency portfolio optimization but also contributes a tailored framework that addresses the specific risk and volatility profiles of digital assets.

3. Methodology

The methodology of this study is structured into two distinct stages. The first stage emphasizes the pre-selection of assets through an innovative approach, while the second stage focuses on the allocation of weights to these assets using the CDaR model. In the initial stage, we implement a novel framework grounded in MADM methods to identify the most promising investment alternatives based on key financial indicators. Recognizing that different MADM methods may yield varying sorting results, we address the challenge of selecting the most effective method, which can complicate the decision-making process for investors. To enhance the robustness of our findings, we apply multiple MADM methods and integrate their results using the BORDA approach, leading to improved overall outcomes. After completing the ranking process, we select the top ten cryptocurrencies, adding Bitcoin to establish a portfolio of eleven assets for optimization. Research suggests that a portfolio comprising ten assets strikes an optimal balance between diversification and risk management. Following the pre-selection phase, we apply the CDaR model with practical constraints to allocate appropriate weights for each asset. All data processing and modeling activities are conducted using Microsoft Excel and Python software. The subsequent sections will offer a detailed exploration of the methodology, including: (3.1) an overview of MADM approaches; and (3.2) the proposed CDaR method.

3.1. Key Concepts and Definitions of MADM Approaches

This section presents the MADM methodologies applied for asset pre-selection in the context of cryptocurrency portfolio optimization. These methods enable a systematic evaluation of multiple criteria, facilitating the selection of assets that meet portfolio objectives, particularly in highly volatile markets such as cryptocurrencies. The following subsections detail the key MADM approaches employed in this study, alongside their mathematical formulations and theoretical underpinnings.

3.1.1. MARCOS (Measurement Alternatives and Ranking according to the Compromise Solution)

The MARCOS method, introduced by Stević et al. (2020), evaluates alternatives by measuring their distance from both ideal and anti-ideal solutions. The process involves calculating utility degrees and multiple utility functions to determine the weight and ranking of each alternative. The steps and associated formulas of the MARCOS method are outlined below.

The extended decision matrix is constructed by including the alternatives and their corresponding criteria, along with the ideal (AI) and anti-ideal (AAI) solutions:

$$X = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \\ AI_1 & AI_2 & \cdots & AI_n \\ AAI_1 & AAI_2 & \cdots & AAI_n \end{pmatrix} \tag{1}$$

where x_{ij} represents the value of the *i-th* alternative against the *j-th* criterion, and AI and AAI denote the ideal and anti-ideal solutions respectively.

The normalization process ensures that all criteria are on the same scale. For beneficial criteria, normalization is performed as:

$$n_{ij} = \frac{x_{ij}}{x_{AI,j}} \tag{2}$$

For cost criteria, the formula is inverted:

$$n_{ij} = \frac{x_{AAI,j}}{x_{ij}} \tag{3}$$

Normalization is applied to the entire extended matrix, including both AI and AAI. The weighted normalized matrix is calculated by multiplying the normalized matrix values by their corresponding criterion weights w_j:

$$v_{ij} = n_{ij} \times w_j \tag{4}$$

This step adjusts the normalized values to account for the relative importance of each criterion.

The utility degree for each alternative is determined by comparing each alternative's weighted value to the ideal and anti-ideal solutions:

$$K_i^+ = \frac{S_i}{S_{AI}} (for\ the\ ideal\ solution) \tag{5}$$

$$K_i^- = \frac{S_i}{S_{AAI}}\ (for\ the\ anti\ ideal\ solution) \tag{6}$$

where S_i represents the sum of the weighted values for each alternative:

$$S_i = \sum_{j=1}^{n} v_{ij} \tag{7}$$

utility functions are calculated for each alternative based on the utility degrees:

$$f(K_i^+) = K_i^+ + \frac{K_i^-}{K_i^+ + K_i^-} \tag{8}$$

$$f(K_i^-) = K_i^- + \frac{K_i^+}{K_i^+ + K_i^-} \tag{9}$$

$$f(K_i) = \frac{K_i^- + K_i^+}{1 + \frac{1 - f(K_i^+)}{f(K_i^+)} + \frac{1 - f(K_i^-)}{f(K_i^-)}} \tag{10}$$

these functions reflect the balance between ideal and anti-ideal solutions. The final ranking of the alternatives is based on the values obtained from the utility functions. The alternative with the highest utility function value is ranked as the best.

3.1.2. CODAS (Combinative Distance-based Assessment)

The CODAS method is a relatively recent Multi-Criteria Decision Making (MCDM) approach that was introduced by Ghorabaee et al. (2016). CODAS evaluates alternatives by measuring their distances from the negative ideal solution (NIS) and uses both Euclidean and Taxicab (Manhattan) distances to establish rankings. This method emphasizes alternatives that are farther away from the NIS, making it particularly useful for decision problems in which minimizing undesired attributes is important.

The decision matrix X is constructed as follows:

$$X = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{pmatrix} \tag{11}$$

where x_{ij} represents the performance value of the i-th alternative on the j-th criterion, n is the number of alternatives, and mmm is the number of criteria. The decision matrix is normalized using linear normalization as follows:

$$For\ beneficial\ criteria: n_{ij} = \frac{x_{ij}}{max\, x_{ij}} \tag{12}$$

$$For\ cost\ criteria : n_{ij} = \frac{min\, x_{ij}}{x_{ij}} \tag{13}$$

weighted normalized decision matrix is calculated as follows:

$$r_{ij} = w_j \times n_{ij} \tag{14}$$

where w_j is the weight of the j-th criterion and $\sum_{j=1}^{m} w_j = 1$.

The negative-ideal solution is determined as:

$$ns = [ns_j] = [min\, r_{ij}] \quad \forall j \tag{15}$$

The Euclidean distance E_i of each alternative from the negative-ideal solution is calculated as:

$$Euclidean\ distance : E_i = \sqrt{\sum_{j=1}^{m} \left(r_{ij} - ns_j\right)^2} \tag{16}$$

The Taxicab distance T_i is calculated as:

$$Taxicab\ distance : T_i = \sum_{j=1}^{m} \left|r_{ij} - ns_j\right| \tag{17}$$

The relative assessment matrix R_a is constructed as:

$$h_{ik} = (E_i - E_k) + \psi(E_i - E_k) \times (T_i - T_k) \tag{18}$$

where k ∈ {1, 2, …, n} and ψ is a threshold function defined as:

$$\psi_{(x)} = \begin{cases} 1\ if\ |x|\ \geq\ \tau \\ 0\ if\ |x|\ <\ \tau \end{cases} \tag{19}$$

The assessment score η_i for each alternative is calculated as:

$$\eta_i = \sum_{k=1}^{n} h_{ik} \tag{20}$$

the alternatives are ranked based on their final scores η_i.

3.1.3. CoCoSo (Combined Compromise Solution)

The CoCoSo method was introduced by Yazdani et al. (2018) as a MCDM technique. This method combines additive and multiplicative aggregation strategies to generate a more comprehensive ranking of alternatives by assessing their performance based on multiple criteria. CoCoSo evaluates alternatives by applying three aggregation approaches: simple additive weighting (SAW), weighted geometric averaging (WGA), and combined compromise. The method allows decision-makers to utilize both types of preference modeling, providing more accurate and balanced results.

The decision matrix is determined where x_{ij} is the value of the *i-th* alternative under the *j-th* criterion. The matrix is represented as:

$$X = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{pmatrix} \tag{21}$$

The criteria values are normalized using the compromise normalization equation:

$$\text{For beneficial criteria: } r_{ij} = \frac{x_{ij} - \min_i x_{ij}}{\max_i x_{ij} - \min_i x_{ij}} \tag{22}$$

$$\text{For cost criteria: } r_{ij} = \frac{\max_i x_{ij} - x_{ij}}{\max_i x_{ij} - \min_i x_{ij}} \tag{23}$$

The total comparability sequence is calculated using two models:

Weighted Sum Model (WSM):

$$S_i = \sum_{j=1}^{n} w_j r_{ij} \tag{24}$$

Weighted Product Model (WPM)**:** The P_i value is calculated using the grey relational generation approach, represented by the following equation:

$$P_i = \sum_{j=1}^{n} (r_{ij})^{w_j} \tag{25}$$

Three aggregation strategies are used to compute the relative weights of the alternatives:
First Strategy (Arithmetic Mean):

$$k_{ia} = \frac{P_i + S_i}{\sum_{i=1}^{m}(P_i + S_i)} \tag{26}$$

Second Strategy (Ratio of WSM and WPM to the Best Values)**:**

$$k_{ib} = \frac{S_i}{min_i\, S_i} + \frac{P_i}{min_i\, P_i} \tag{27}$$

Third Strategy (Balanced Compromise)**:**

$$k_{ic} = \frac{\lambda S_i + (1-\lambda)P_i}{\lambda \max_i S_i + (1-\lambda) \max_i P_i}, \quad 0 \leq \lambda \leq 1 \tag{28}$$

where λ is a decision-maker's preference parameter, typically λ=0.5. The final score k_i for each alternative is calculated as the geometric mean of the three aggregation strategies:

$$k_i = (k_{ia} \cdot k_{ib} \cdot k_{ic})^{\frac{1}{3}} + \frac{1}{3}(k_{ia} + k_{ib} + k_{ic}) \tag{29}$$

the alternatives are ranked based on their final scores k_i, with the highest score representing the most optimal alternative.

3.1.4. WASPAS (Weighted Aggregated Sum Product Assessment)

The WASPAS method is a hybrid Multi-Criteria Decision Making (MCDM) approach that combines the Weighted Sum Model (WSM) and the Weighted Product Model (WPM). Introduced by Zavadskas et al. (2012), WASPAS integrates both additive and multiplicative forms of decision-making to provide a more robust ranking of alternatives.

The decision matrix is determined where x_{ij} is the value of the i-th alternative under the j-th criterion. The matrix is represented as:

$$X = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{pmatrix} \tag{30}$$

The decision matrix is normalized using linear normalization as follows:

For beneficial criteria: $r_{ij} = \frac{x_{ij}}{\max_i x_{ij}}$ (31)

For cost criteria: $r_{ij} = \frac{\min_i x_{ij}}{x_{ij}}$ (32)

The WSM score for each alternative is calculated by summing the weighted normalized values across all criteria:

$$WSM_i = \sum_{j=1}^{n} w_j r_{ij} \tag{33}$$

The WPM score for each alternative is calculated by taking the product of the normalized values, raised to the power of their respective weights:

$$WPM_i = \prod_{j=1}^{n} (r_{ij})^{w_j} \tag{34}$$

The final WASPAS score for each alternative is a weighted combination of the WSM and WPM scores:

$$Q_i = \lambda \cdot WSM_i + (1 - \lambda) \cdot WPM_i \tag{35}$$

where λ is usually set to 0.5, giving equal weight to both methods. The alternatives are ranked based on their combined WASPAS scores Q_i.

3.1.5. TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution)

The TOPSIS method is a well-known Multi-Criteria Decision Making (MCDM) technique introduced by Hwang and Yoon (1981). TOPSIS is based on the concept that the best alternative should have the shortest distance from the positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS).

The decision matrix is determined where x_{ij} is the value of the i-th alternative under the j-th criterion. The matrix is represented as:

$$X = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{pmatrix} \tag{36}$$

To eliminate units and make the criteria values comparable, the decision matrix is normalized using the following formula for each element r_{ij}:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \tag{37}$$

The normalized values are then multiplied by the respective weights of the criteria to obtain the weighted normalized decision matrix v_{ij}:

$$v_{ij} = w_j \cdot r_{ij} \tag{38}$$

The ideal solution A^+ (best-case scenario) and the anti-ideal solution A^- (worst-case scenario) are determined as follows:

$$A^+ = \{ max(v_{ij}) \mid j \in J_1 \}, \{ min(v_{ij}) \mid j \in J_2 \} \tag{39}$$

$$A^- = \{ min(v_{ij}) \mid j \in J_1 \}, \{ max(v_{ij}) \mid j \in J_2 \} \tag{40}$$

Where J_1 represents beneficial criteria, and J_2 represents cost criteria. The Euclidean distance of each alternative from the ideal and anti-ideal solutions is calculated as follows:

Distance to Ideal Solution $\boldsymbol{D_i^+}$**:**

$$D_i^+ = \sqrt{\sum_{j=1}^{n} (v_{ij} - A_j^+)^2} \tag{41}$$

Distance to anti-Ideal Solution $\boldsymbol{D_i^-}$**:**

$$D_i^- = \sqrt{\sum_{j=1}^{n} (v_{ij} - A_j^-)^2} \tag{42}$$

The relative closeness of each alternative to the ideal solution is calculated using the following formula:

$$C_i = \frac{D_i^-}{D_i^+ + D_i^-} \tag{43}$$

Finally, the alternatives are ranked based on their relative closeness values C_i The higher the C_i the closer the alternative is to the ideal solution, and hence the better the alternative.

3.1.6. Borda method

The Borda count method was developed by the French mathematician and political scientist Jean-Charles de Borda in 1770. Originally intended for voting systems, Borda introduced this method to address the limitations of majority voting by taking into account the relative rankings of candidates, not just the top choice of each voter. Over time, the Borda count has been applied beyond elections, particularly in decision-making processes, where it is used to aggregate the rankings of alternatives generated by various decision-making methods, such as those in multi-criteria decision-making (MCDM).

In the Borda count method, a pairwise comparison matrix is generated between the alternatives, based on the results of multiple decision-making methods. The primary goal is to count how many times one alternative outranks another. This is done by comparing alternatives across the different methods used in the decision-making process. At first a pairwise comparison matrix is created between alternatives based on the results of various decision-making methods. Each element in the matrix reflects how often one alternative outranks another.

- If alternative A_i outranks A_j in one method, a value of 1 is assigned to the comparison.
- If alternative A_j outranks A_i a value of 0 is assigned.
- If both alternatives are ranked equally in a method, a value of 0.5 is assigned to each.

This comparison is conducted between all pairs of alternatives, resulting in a matrix of size $m \times m$ times, where m is the number of alternatives. After constructing the pairwise comparison matrix, the row sum for each alternative is calculated. The row sum represents how often each alternative outranks the others across all the decision-making methods. The formula for calculating the total score for each alternative is:

$$S_i = \sum_{j=1, j \neq i}^{m} M_{ij} \tag{44}$$

where:

- S_i is the total score for alternative A_i
- M_{ij} is the value of the comparison between alternative A_i and A_j in the pairwise comparison matrix,
- m is the total number of alternatives.

If an alternative consistently outranks others, it will accumulate a higher total score. In cases where alternatives are ranked equally, the row sum will reflect that by summing intermediate values (e.g., 0.5). After calculating the row sums for each alternative, the alternatives are ranked according to their total scores. The alternative with the highest total score is considered the best, followed by the next highest, and so on. The ranking is based on how often an alternative outranked other alternative in the decision-making process. The number of pairwise comparisons is m(m-1)/2.

3.2. The proposed CDaR Model

Drawdown is defined as the reduction in the value of a portfolio from its previous peak. This measure is particularly important to investors as it highlights how much the portfolio value has decreased from its highest point, making it a useful tool for evaluating the performance of an investment portfolio. Investors often aim to construct portfolios that prevent losses beyond a specific percentage of the portfolio's maximum value. To address these concerns, Chekhlov et al. (2005) introduced the CDaR, a measure that evaluates the risk of a portfolio by focusing on the largest drawdowns during the investment period. CDaR is defined as the average of the worst-case drawdowns. It shares similarities with CVaR but is applied specifically to drawdowns rather than general losses. The CDaR risk measure has key properties such as convexity, positive homogeneity, and non-negativity, which are beneficial in risk management. CDaR allows portfolio managers to assess and control the worst drawdowns within a specific confidence level, thus helping in the construction of portfolios that are better able to manage downside risk.

Let *w(x,t)* represent the uncompounded portfolio value at time t, and let x=(x1,x2,…,xn) be the weights of the assets in the portfolio. The drawdown function at time t is then defined as the difference between the portfolio's peak value up to time t and the portfolio value at time t:

$$f(x,t) = \max_{0\leq\tau\leq t}\{w(x,\tau)\} - w(x,t) \tag{45}$$

Suppose r_{it} is the rate of return of the *i*-th asset during the *t*-th trading period. The uncompounded portfolio value at time t is calculated as:

$$w(x,j) = \sum_{i=1}^{n}\left(1 + \sum_{t=1}^{j} r_{it}\right)x_i \tag{46}$$

The drawdown function at time *j* is defined as the maximum difference between the portfolio's peak value up to time *j* and the portfolio value at time *j*. It is expressed as:

$$f(x,j) = \max_{1\leq k\leq j}\left(\sum_{i=1}^{n}\left(\sum_{t=1}^{k} r_{it}\right)x_i\right) - \sum_{i=1}^{n}\left(\sum_{t=1}^{j} r_{it}\right)x_i \tag{47}$$

The CDaR is defined as the average of the worst-case drawdowns observed in the sample path. The mathematical expression for CDaR is:

$$CDaR_{\alpha}(x,\eta) = \eta_{\alpha} + \frac{1}{(1-\alpha)}\sum_{j=1}^{J} max\{0, f(x,j) - \eta_{\alpha}\} \tag{48}$$

Where:

- η is the threshold drawdown level,
- $\alpha \in [0,1]$ is the confidence level, and
- *J* is the total number of time intervals.

The CDaR model is represented using a threshold η, which is the drawdown level exceeded by *(1−α)J* drawdowns, where $\alpha\in[0,1]$ represents the confidence level. The CDaR model can be expressed as follows:

$$CDaR_{\alpha}(x,\eta) = \eta + \frac{1}{(1-\alpha)J}\sum_{j=1}^{J}\max\left(0, \max_{1\leq k\leq j}\left(\sum_{i=1}^{n}\sum_{t=1}^{k} r_{it}x_i\right) - \sum_{i=1}^{n}\sum_{t=1}^{j} r_{it}x_i - \eta\right) \tag{49}$$

If *(1−α)J* is not an integer, the CDaR function is obtained as the solution to the following minimization problem:

$$CDaR_{\alpha}(x,\eta) = \min_{\eta}\left(\eta + \tfrac{1}{(1-\alpha)J}\textstyle\sum_{j=1}^{J}\max\left(0, \max\limits_{1\leq k\leq j}(\sum_{i=1}^{n}\sum_{t=1}^{k} r_{it}x_i) - \sum_{i=1}^{n}\sum_{t=1}^{j} r_{it}x_i - \eta\right)\right) \tag{50}$$

The linear specification of the portfolio optimization model can be expressed through the following equations:

$$\min_{\eta}\left(\eta + \frac{1}{(1-\alpha)J}\sum_{j=1}^{J} y_j\right) \tag{51}$$

S.t.

$$\sum_{i=1}^{n} \mu_i x_i = \mu_p \tag{52}$$

$$y_j \geq \left(\sum_{i=1}^{n}\left(\sum_{t=1}^{k} r_{it}\right)x_i\right) - \left(\sum_{i=1}^{n}\left(\sum_{t=1}^{j} r_{it}\right)x_i\right) + \eta \tag{53}$$

$$y_j \geq 0 \tag{54}$$

$$\sum_{i=1}^{n} x_i = 1 \tag{55}$$

$$x_i \geq 0, \quad i = 1,2,\dots,n \tag{56}$$

3.2.1. Constraints

In real-world portfolio optimization, additional constraints are typically incorporated to enhance the practical application of the model. This section explores some of the most common practical constraints applied in portfolio optimization.

Cardinality Constraint

The cardinality constraint limits the number of assets in a portfolio, which helps control the number of positions in the optimal allocation. This, in turn, can reduce operational costs. The selection status of an asset under this constraint is represented by the binary variable *Zi* . Therefore, the cardinality constraint can be formulated as:

$$\sum_{i=1}^{N} Z_i = K \tag{57}$$

Where:

- $Z_i \in \{0,1\}, \quad i = 1,2,\dots,n$ represents whether asset *i* is included in the portfolio,
- *K* is the desired number of assets to be held in the portfolio.

Threshold Constraints

Threshold constraints, also known as floor and ceiling constraints, specify the minimum and maximum limits for investment in each asset within a portfolio. These constraints ensure that an asset's investment level remains within certain bounds. The threshold constraints are expressed as follows:

$$l_i Z_i \leq x_i \leq u_i Z_i, \quad i = 1,2,\dots,n \tag{58}$$

Where:

- l_i and u_i represent the lower and upper bounds for the investment in asset iii,
- x_i is the weight of asset *i* in the portfolio.

Additionally, the thresholds are further constrained by the following inequality:

$$0 \leq l_i \leq u_i \leq 1 \tag{59}$$

The proposed portfolio optimization model incorporates both cardinality and threshold constraints. It can be formulated as the following optimization problem:

$$min\, \eta + \frac{1}{(1-\alpha)J} \sum_{j=1}^{J} y_j \tag{60}$$

S.t.

$$\sum_{i=1}^{n} \mu_i x_i = \mu_p \tag{61}$$

$$y_j \geq \left(\sum_{i=1}^{n} \left(\sum_{t=1}^{k} r_{it} \right) x_i \right) - \left(\sum_{i=1}^{n} \left(\sum_{t=1}^{j} r_{it} \right) x_i \right) \tag{62}$$

$$y_j \geq 0 \tag{63}$$

$$\sum_{i=1}^{N} Z_i = K \tag{64}$$

$$l_i Z_i \leq x_i \leq u_i Z_i, \quad i = 1,2,\dots,n \tag{65}$$

$$Z_i \in \{0,1\}, \quad i = 1,2,\dots,n \tag{66}$$

$$\sum_{i=1}^{n} x_i = 1 \tag{67}$$

$$x_i \geq 0, \quad i = 1,2,\dots,n \tag{68}$$

4. Experimental Results

In this section, the experimental results obtained from analyzing the dataset are presented. The main focus is on a comprehensive evaluation of the selected 25 altcoins using the 12 identified metrics through multi-criteria decision-making (MCDM) methods. Initially, the 25 altcoins are ranked based on these metrics to determine their relative performance and investment potential. This ranking phase provides an insight into the strengths and weaknesses of each altcoin. After ranking the altcoins, the top 10 altcoins are selected along with Bitcoin, forming a group of 11 cryptocurrencies that are subsequently used for portfolio optimization. The optimization model aims to construct an optimal investment portfolio using these cryptocurrencies, considering Conditional Drawdown at Risk (CDaR) as the risk measure. This comprehensive approach ensures that both the ranking and optimization aspects are effectively addressed, providing a robust analysis for informed investment decisions.

The results are structured as follows: first, the dataset and criteria used are introduced, followed by the analysis of individual rankings from each MCDM method. Finally, the portfolio optimization process and the results obtained from the selected cryptocurrencies are presented to summarize the findings.

4.1. Case Study (Dataset and Criteria)

In this subsection, the dataset used for the analysis is introduced in detail. The dataset comprises 25 altcoins, each evaluated using 12 different metrics. These altcoins are: Ethereum (ETH), Binance Coin (BNB), Cardano (ADA), Solana (SOL), Polkadot (DOT), Avalanche (AVAX), Chainlink (LINK), Stellar (XLM), VeChain (VET), Terra (LUNA), Algorand (ALGO), Tezos (XTZ), Filecoin (FIL), Theta (THETA), Cosmos (ATOM), Hedera (HBAR), Elrond (EGLD), Aave (AAVE), Maker (MKR), Compound (COMP), SushiSwap (SUSHI), Uniswap (UNI), PancakeSwap (CAKE), Fantom (FTM), Kusama (KSM).

The dataset includes daily historical data and a range of quantitative and qualitative metrics that provide a comprehensive evaluation of the altcoins' performance, covering market, technical, and adoption-related factors. These metrics are critical for applying multi-criteria decision-making (MCDM) methods to determine the relative ranking of the altcoins. The weights for these metrics were assigned based on expert judgment and analysis of expert judgment regarding the relative importance of each criterion in assessing the performance and potential of the altcoins.

The decision matrix, which will be introduced later in this section, includes these 25 altcoins and the 12 metrics listed below:

1. **Market Capitalization**: Total market value of each altcoin.
2. **Trading Volume**: Average daily trading volume.
3. **Transactions per Day**: Number of transactions conducted daily.
4. **Number of Nodes**: Number of nodes in the blockchain network.
5. **Transaction Speed**: Time taken to process a transaction.
6. **Transaction Fee**: Average cost per transaction.
7. **Stable coin Support**: Indicates if the altcoin supports stable coins.
8. **Decentralization Level**: A qualitative measure of how decentralized the network is.
9. **Adoption and Acceptance**: A qualitative metric assessing the level of adoption.
10. **Network Security**: A qualitative evaluation of the security level.
11. **Scalability**: Ability of the network to handle an increasing amount of work.
12. **Historical Incidents**: Record of significant incidents or vulnerabilities in the past.

4.2. Decision Matrix

The decision matrix provides a structured representation of the 25 altcoins evaluated against the 12 metrics mentioned above. Each row in the decision matrix corresponds to an altcoin, and each column represents a specific metric. This matrix serves as the basis for applying the MCDM methods, allowing for a systematic comparison of the altcoins across multiple dimensions. The values in the decision matrix were derived from historical data and expert analysis, ensuring a comprehensive and reliable evaluation. The decision matrix is presented below, providing a clear overview of the evaluation data.

Table 1. *The decision matrix*

Altcoin	Market Cap (Billion $)	Trading Volume (Billion $)	Transactions per Day	Number of Nodes	Transaction Speed (Seconds)	Transaction Fee ($)	Stable coin Support	Decentralization Level	Adoption and Acceptance	Network Security	Scalability	Historical Incidents
(ETH)	220	10	1200000	9000	13	10	Yes	High	High	High	Medium	Low
(BNB)	40	1.5	1700000	21	3	0.1	Yes	Medium	High	High	Medium	Low
(ADA)	12	0.8	60000	2700	20	0.2	Yes	High	Medium	High	Medium	Low
(SOL)	10	0.7	400000	1800	0.4	0.00025	Yes	Medium	Medium	High	High	Medium
(DOT)	8	0.5	30000	300	6	0.5	Yes	High	Medium	High	High	Low
(AVAX)	6	0.4	500000	1100	2	0.005	Yes	High	Medium	High	High	Low
(LINK)	4	0.3	30000	1000	15	0.1	No	High	Medium	High	Medium	Low
(XLM)	3	0.2	1500000	75	5	0.00001	No	Medium	Medium	High	Medium	Medium
(VET)	3	0.2	300000	101	10	0.003	Yes	High	Medium	Medium	Medium	Medium
(LUNA)	2.5	0.1	50000	130	6	0.1	Yes	High	Medium	High	High	Medium
(ALGO)	2	0.1	700000	1300	4	0.001	Yes	High	Low	High	High	Low
(XTZ)	1.8	0.09	40000	450	30	0.01	No	High	Medium	High	High	Low
(FIL)	1.5	0.08	40000	200	30	0.02	Yes	Medium	Low	High	Medium	Medium
(THETA)	1.4	0.07	50000	100	10	0.01	No	Medium	Medium	High	High	Medium
(ATOM)	1.3	0.06	30000	200	7	0.01	Yes	High	Low	High	High	Medium
(HBAR)	1.2	0.05	20000	20	5	0.0001	Yes	Medium	Low	High	High	Medium
(EGLD)	1.1	0.04	15000	50	2	0.0001	Yes	High	Low	High	High	Medium
(AAVE)	0.9	0.03	10000	10	15	0.01	Yes	High	Low	High	High	Low
(MKR)	0.8	0.02	5000	10	30	0.02	Yes	High	Low	High	High	Low

Altcoin	Market Cap (Billion $)	Trading Volume (Billion $)	Transactions per Day	Number of Nodes	Transaction Speed (Seconds)	Transaction Fee ($)	Stable coin Support	Decentralization Level	Adoption and Acceptance	Network Security	Scalability	Historical Incidents
(COMP)	0.7	0.02	5000	10	15	0.02	Yes	High	Low	High	High	Low
(SUSHI)	0.6	0.02	10000	10	15	0.02	Yes	High	Medium	High	High	Low
(UNI)	0.5	0.5	100000	80	13	0.1	Yes	High	High	High	High	Low
(CAKE)	0.5	0.3	80000	60	3	0.2	Yes	High	High	High	High	Low
(FTM)	0.4	0.2	90000	50	1	0.001	Yes	High	Low	High	High	Low
(KSM)	0.4	0.1	20000	50	6	0.02	Yes	High	Medium	High	High	Low
WEIGHT	0.1	0.1	0.1	0.1	0.1	0.1	0.05	0.1	0.1	0.05	0.05	0.05

4.3. Result of Asset Preselection

The results obtained from applying the various MCDM methods are summarized in the following table, which shows the ranking scores of each altcoin as determined by five different MCDM methods: MARCOS, CODAS, COCOSO, WASPAS, and TOPSIS. Each method ranks the altcoins based on their performance across the 12 metrics, providing a comprehensive comparison of the altcoins' strengths and weaknesses. The combined ranking will later be used to select the top altcoins for portfolio optimization.

Table 2. *Result of selected MADM methods*

Altcoins	MARCOS	CODAS	COCOSO	WASPAS	TOPSIS
Ethereum (ETH)	0.733911629	3.123411653	2.793069766	0.3395082	0.629402877
Binance Coin (BNB)	0.481608331	1.112940964	2.30860599	0.227325582	0.462674725
Cardano (ADA)	0.386820581	-0.20789101	2.231311166	0.177984986	0.396946967
Solana (SOL)	0.453153172	0.650954148	2.079962392	0.325813282	0.416252128
Polkadot (DOT)	0.375264873	-0.48708052	2.451171853	0.157271552	0.375365318
Avalanche (AVAX)	0.421954334	0.498347376	2.593528722	0.239630516	0.406582749
Chainlink (LINK)	0.310599992	-2.26123628	1.947237142	0.093351407	0.375215998
Stellar (XLM)	0.428615786	0.957769573	1.623238057	0.128821274	0.420172191
VeChain (VET)	0.324588858	-1.62084181	1.760401512	0.158633399	0.38014701
Terra (LUNA)	0.344042626	-1.17964722	2.187290824	0.140429278	0.377043969
Algorand (ALGO)	0.389607301	-0.13085107	2.359293688	0.211719514	0.405202264
Tezos (XTZ)	0.317193049	-2.05419138	1.914785743	0.095332962	0.360256536
Filecoin (FIL)	0.257668076	-3.73385083	1.126691642	0.108284386	0.355823948
Theta (THETA)	0.259653943	-3.97708244	1.595243737	0.078039477	0.371329301
Cosmos (ATOM)	0.30958666	-2.14758157	1.946934906	0.130861116	0.374916729
Hedera (HBAR)	0.286433684	-2.89747281	1.602146242	0.130311964	0.375091634
Elrond (EGLD)	0.330281441	-1.55559083	1.948667741	0.151372893	0.379329582
Aave (AAVE)	0.327215096	-1.5990928	1.993291694	0.120008161	0.3655307
Maker (MKR)	0.325447786	-1.64318567	1.628743496	0.114512778	0.35506941
Compound (COMP)	0.326696387	-1.61195819	1.84725538	0.115849184	0.365101189
SushiSwap (SUSHI)	0.359254198	-0.79571127	2.138664148	0.127950192	0.36748816
Uniswap (UNI)	0.402426151	-0.05535392	2.477152872	0.158673537	0.377177493
PancakeSwap (CAKE)	0.409069874	-0.0029539	2.528478268	0.159744582	0.383752102
Fantom (FTM)	0.370703452	-0.58726993	2.156051125	0.169736306	0.383429581
Kusama (KSM)	0.364820856	-0.66481238	2.307877063	0.140734909	0.377563373

The Borda count method was applied to aggregate the rankings obtained from the different MCDM methods. The Borda count method assigns points to each altcoin based on its rank in each MCDM method, with the altcoin receiving the highest overall score considered the best. This approach helps to combine the results from multiple methods into a unified ranking, providing a more robust and consensus-based evaluation. The final result of the Borda count method is presented below, summarizing the aggregate ranking of the altcoins based on their performance in the various MCDM methods. The table below shows the Borda scores assigned to each altcoin, with the altcoins receiving the highest scores being ranked the best.

Table 3. *Integrating results using the BORDA approach*

Altcoins	Score
Ethereum (ETH)	24
Binance Coin (BNB)	23
Solana (SOL)	23
Avalanche (AVAX)	23
PancakeSwap (CAKE)	22
Stellar (XLM)	22
Algorand (ALGO)	22
Uniswap (UNI)	21
Cardano (ADA)	20
Polkadot (DOT)	20
Fantom (FTM)	19
Kusama (KSM)	19
Terra (LUNA)	16
Elrond (EGLD)	16
VeChain (VET)	16
SushiSwap (SUSHI)	14
Cosmos (ATOM)	11
Aave (AAVE)	11
Chainlink (LINK)	11
Hedera (HBAR)	10
Compound (COMP)	9
Tezos (XTZ)	9
Maker (MKR)	8
Theta (THETA)	6
Filecoin (FIL)	4

4.4. Results of Portfolio Optimization Using CDaR Model

This section presents the analysis and results of the optimal cryptocurrency portfolio constructed under cardinality, ceiling, and floor constraints. Descriptive statistics of the selected cryptocurrencies are provided in Table 4:

Table 4. *Descriptive statistics of the selected cryptocurrencies*

Asset	Mean	Variance	Standard Deviation	Min Return	Max Return
ETH	0.052571	0.023353	0.152817	-0.19144	0.381255
BNB	0.074807	0.028013	0.167371	-0.10726	0.417467
SOL	0.143298	0.113099	0.336302	-0.46869	0.588001
AVAX	0.062354	0.118927	0.344858	-0.50346	0.635817
CAKE	0.081527	0.095472	0.308986	-0.38566	0.828322
XLM	-0.03969	0.019596	0.139987	-0.27011	0.146076
ALGO	0.019449	0.077389	0.278189	-0.40904	0.511351
UNI	0.035658	0.114124	0.337822	-0.61102	0.618593
ADA	0.01757	0.054955	0.234424	-0.38901	0.457915
DOT	0.008215	0.051579	0.22711	-0.40643	0.404891
BTC	0.066321	0.021878	0.147912	-0.16247	0.362675

After ranking the top 25 altcoins using the Borda count method, the top 10 were selected, and Bitcoin was added to form a group of 11 cryptocurrencies for optimization. The model aimed to minimize portfolio risk by focusing on reducing drawdowns while the optimization process included a confidence level of 95%, a cardinality constraint of 10, ceiling and floor constraints of 35% and 10%, respectively, for each asset, and a constraint ensuring that the weight of Bitcoin was at least 20%. The computational result of the optimization is as follows:

BNB (35%), BTC (20%), ETH (35%), and CAKE (10%)

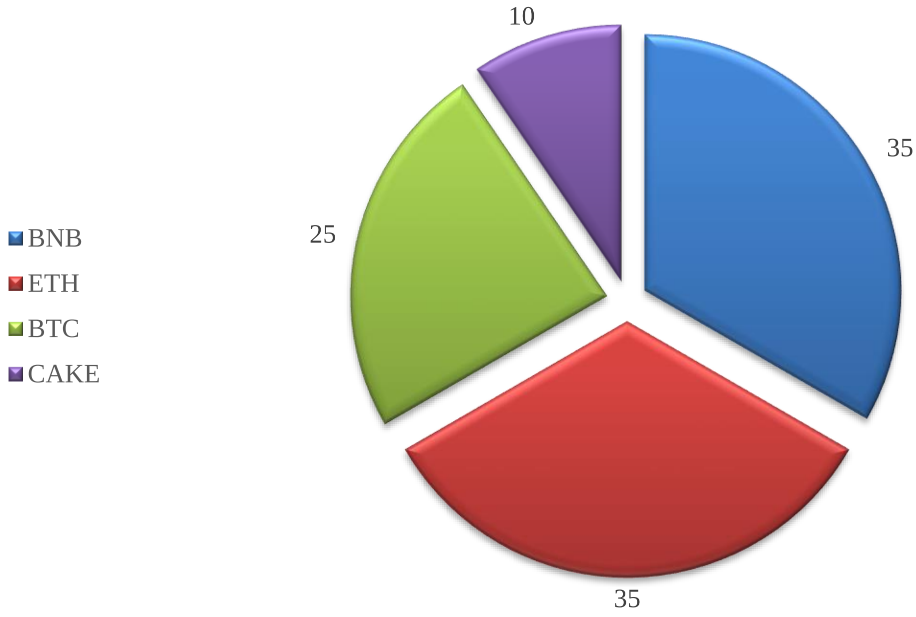

Figure 1. *Results of weight allocation using CDaR model*

BNB had the highest allocation due to its strong risk-adjusted return, while BTC was assigned 20%, influenced by the minimum allocation requirement and its role as the market leader. ETH and CAKE were selected to enhance diversification. The use of CDaR successfully minimized downside risk, leading to a well-balanced portfolio designed to withstand market volatility. The resulting allocations provided a strategic balance between stability (BTC) and growth potential (BNB, ETH, and CAKE), offering a robust approach for managing risks in the cryptocurrency market.

5. Conclusion

In this study, a novel approach to cryptocurrency portfolio optimization was presented by integrating a pre-selection process using Multi-Attribute Decision Making (MADM) methods before applying the CDaR model. The pre-selection ensured that only high-quality assets with strong fundamentals, growth potential, and resilience to market fluctuations were included in the optimization process. This integration significantly enhanced the robustness and overall effectiveness of the portfolio. The CDaR-based optimization was then employed to minimize downside risk, resulting in a resilient cryptocurrency portfolio that demonstrated a strategic balance between stability and growth. BTC was included as a stabilizing asset due to its role as the market leader, while BNB, ETH, and CAKE contributed to the portfolio's growth and diversification. Compared to the Nave approach, the CDaR-based model showed superior performance in managing downside risk, especially in highly volatile market conditions. The Nave approach, which involves equal weight allocation, was effective in providing a simple diversification strategy but did not address extreme drawdowns as comprehensively as CDaR. By focusing on minimizing drawdowns, the CDaR-based optimization produced a more stable portfolio capable of withstanding adverse market movements. It should be noted that the proposed approach serves as a framework for constructing a cryptocurrency portfolio and is not intended as a specific investment recommendation. Investors should consider their own risk tolerance and conduct thorough due diligence before making any investment decisions. The findings of this research demonstrate the potential effectiveness of CDaR as a tool for portfolio construction, but practical application requires careful consideration of individual circumstances and market conditions.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Declaration of generative AI and AI-assisted technologies in the writing process

During the preparation of this work, the authors used OpenAI's tool Chat GPT in order to edit and write some parts of the paper. After using this service, the authors reviewed and edited the content as needed and took full responsibility for the content of the publication.

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