Journal of Industrial and Systems Engineering

Vol. 17, No. 2, pp. 166-196

Spring (April) 2025

Information Sharing Decisions in a Manufacturer-Stackelberg Supply Chain Under Different Contracts

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Abstract

Choosing an efficient and flexible contract is crucial in various circumstances. However, most existing studies focus only on defining and comparing various contracts and often overlook the efficiency of information in contract execution. This paper aims to examine the impact of information sharing on supply chain members' behavior and profit under a linear price contract and contract menus. To this end, we use a supply chain model featuring a manufacturer and a retailer to analyze the decision-making process regarding information sharing, considering associated costs. Through a manufacturer-Stackelberg game, we compare the impact of different contract types on member performance and profitability. Our findings indicate that the choice of contract type significantly influences profit distribution within the supply chain and the decision on information sharing. Specifically, the retailer's profit is higher under a linear price contract, while the manufacturer achieves higher profitability with contract menus.

Keywords: Information sharing; Linear Price Contract; Contract Menus; Game Theory; Stackelberg Game

1. Introduction

Researchers and organizations have consistently prioritized two key areas: enhancing productivity and implementing effective supply chain management practices. According to Scarf's study, it was observed that while information and information sharing have become integral to supply chain management in the past few decades, the significance of having up-to-date and efficient information has particularly grown since the 1950s (Scarf, 1959). In the contemporary era, information not only plays a pivotal role in expanding the horizons of supply chain managers for making intelligent decisions but also significantly impacts the profitability of diverse industries,

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ISSN: 1735-8272,

propelling businesses forward. An illustrative example is the sharing of information within recycling chains, which enhances societal awareness and responsibility while simultaneously boosting companies' profitability (Zheng, 2022). In a study examining profitability, Costantino et al (2014) demonstrated that the utilization of relevant information can effectively prevent detrimental occurrences like the Bullwhip effect. According to the Alzoubi (2022) study, sharing valuable information yields a positive impact in multiple dimensions. It directly affects the specific goals of the information provider while also indirectly influencing the long-term objectives of the information recipient. Recently, there has been a substantial increase in the variety of methods available to streamline information. One notable example is the utilization of blockchain technology, which not only facilitates the secure and timely sharing of updated data but also enhances the performance of supply chain financing (Zheng, 2022).

Given the aforementioned factors, supply chain management poses numerous challenges encompassing coordination, information flow, and the procurement of critical data. As a result, owners have grown increasingly concerned and are compelled to prioritize the resolution of these issues.

In addition to the importance of acquiring essential and effective information, two other critical topics that have been discussed are overcoming obstacles related to data utilization and achieving organizational goals. Consequently, the utilization of suitable tools to streamline and enhance supply chain management has captured the keen interest of managers. Current contracts between supply chain members serve as vital tools for coordinating and fostering commitment among the supply chain members, ultimately enhancing the overall performance of the supply chain (Wang, Wang et al. 2013). However, according to El Ouardighi (2014), information sharing and utilization of contracts among supply chain members, which are typical of contemporary supply chains, can yield both advantageous and detrimental outcomes.

Supply chain contracting has become a pivotal driver of companies' performance, particularly in the domain of coordination, amid the contemporary business landscape. Notable contract models encompass revenue-sharing contracts, wholesale-price contracts, flexible contracts, and more (Cachon, 2003). While accepted contract commitments bring forth a significant responsibility for transparency and reliability, the condition factor also holds considerable importance. Members often employ different strategies and pursue diverse goals when utilizing contracts. For instance, contract menus become the focal point when the supply chain encounters issues of asymmetrical information (Shen, Choi, et al, 2019; Abdi, 2023). Additionally, revenue sharing contracts are widely embraced as they effectively foster a sense of trust and accomplishment among members of the chain (Wang, Chen et al, 2022).

This article aims to explore the issue of information sharing in the supply chain and its impact on the behavior and profitability of chain members. We are comparing two types of contracts based on their flexibility difference: linear price contracts and contract menus. Contract menus offer a fixed price for a menu of quantity, commonly used in supply chains involving retailers and manufacturers as a form of non-linear pricing (Corbett, 2004; Movaheh et al., 2024). On the other hand, linear price contracts, often used among supply chain members, are less flexible due to factors such as the Bullwhip effect and the double marginalization effect (Ha and Tong, 2008).

This comparison provides an intriguing exploration of information sharing and different contract types.

The primary objective of this study is to analyze and select the most suitable contract under various scenarios where demand information is limited. It specifically investigates the effectiveness of two linear price contracts and contract menus in addressing this issue. The problem is modeled and examined using a game theory approach. The primary contribution of this study lies in the comparison of two contracts and the identification of their efficacy in dealing with uncertainty regarding market demand information.

The rest of the article is structured as follows: The second section provides a brief review of the relevant literature pertaining to the problem at hand. In Section 3, the problem description is provided. Section 4 analyzes the case of the contract menu. Section 5 presents the case of the linear price contract. In Section 6, a comparison between the contracts is provided. The effect of information cost and the manufacturer's decision is investigated in Section 7. Section 8 provides a detailed numerical example, followed by conclusion and managerial insights presented in Section 9.

2. Literature review

The coordination and cooperation among supply chain members to enhance chain performance have emerged as significant challenges for researchers in recent years. Investigations into supply chain fulfilment encompass various key areas, including the identification of effective factors impacting supply chain performance and the measurement of improvement rates influenced by various ratios. Therefore, this paper addresses previous research that can be categorised into three interconnected aspects. These aspects include contracting within the supply chain, information sharing, uncertainty in demand, and an examination of the behaviour of chain members based on game theory. This article considers these factors collectively due to their profound impact on the behaviour of supply chain members, the profitability of companies, and their influence on other factors.

2-1 Contracting in the supply chain

Implementing intelligent supply chain management practices and mitigating detrimental factors, such as the double marginalization effect, play pivotal roles in enhancing supply chain performance (He, 2023). To overcome marginalization and tackle other issues like moral hazards, there exist several approaches, one of which involves implementing effective contracting strategies (Cai, Choi et al, 2021; Nozari, 2025). Achieving supply chain coordination consistently yields favorable outcomes by aligning members towards attaining desired objectives. In their study, Yan, Liu et al, (2023) demonstrate how a contract combining features of revenue sharing and transfer payment can effectively coordinate the supply chain.

A study conducted by (Zhang and Chen 2013) reveals that revenue sharing contracts, compared to linear price contracts, effectively coordinate supply chains during times of asymmetrical information. While linear price contracts are simple and commonly used within the chain, they also introduce an increasing level of uncertainty and risk. Ha and Tong (2008) acknowledge that linear price contracts are not suitable for addressing asymmetrical information, but propose that contract menus offer greater flexibility.

Furthermore, there have been successful studies that recognise the failure of contracts in coordinating supply chain members in competitive markets with incomplete information, such as the study by Pavlov, Katok et al, (2022). However, supply chain contracts possess specific features preferred by each chain member. For example, manufacturers tend to generate higher profits through long-term contracts, while customers prefer behaviour-based pricing (Wang, Fan et al, 2021). Reducing waste is a critical strategy for enhancing supply chain efficiency. Mohammadi Dolat-Abadi's study illustrates that a two-part tariff (TPT) can be used as a method of price discrimination. He found that implementing TPT within a coordinated strategy results in lower levels of wasted products and retail prices compared to a decentralized strategy. Therefore, decision-makers are recommended to adopt the two-part tariff contract within a coordinated strategy to minimize wasted products without compromising their profits (Dolat-Abadi 2021). In another study, the impact of centralized policies and contract types on profits also discussed (Mohammadi, Ghazanfari et al, 2018). This study research indicates that the contract design effectively streamlines the fresh-product supply chain across various sharing rates. Moreover, the centralized mode demonstrates reduced profit losses compared to the decentralized one, ultimately yielding higher overall channel profits.

A successful supply chain should not overlook the competitive landscape, which includes considering both quantitative discount contracts and linear price contracts. Li, Zhou, et al, (2013) developed a competitive game model for two chains that analyzed the selection of these two contract types. Numerous beneficial studies have investigated the features of revenue-sharing contracts and linear price contracts. Additionally, Ji, Xu et al, (2020) compared these contract types under cap-and-trade regulation and found that revenue sharing contracts significantly contribute to higher social welfare compared to wholesale price contracts.

2-2 Information sharing and uncertainty in demand

In this era of technological revolution, information plays an essential role in effectively coordinating supply chain members and achieving success in a competitive market. To gain trust and facilitate efficient information exchange, Sarfaraz suggests exploring new means such as blockchain technology (BCT) (Sarfaraz, Chakrabortty et al, 2023). Shang et al. (2016) demonstrate that while retailers are content with contractual agreements, manufacturers consistently strive to acquire information to enhance their service provision. Information sharing, encompassing knowledge sharing and other interpretations, has undeniable positive effects (Jen, Hu et al. 2020). Sharing information becomes necessary when faced with uncertain demand, to which manufacturers readily agree. While retailers benefit from information sharing under specific conditions (Zhang, Dan et al. 2019; Nozari & Samadi, 2025), improper management of information sharing in competitive conditions can lead to challenges like the prisoner's dilemma (Guan, Zhang et al. 2020). Manufacturers, who are eager to share information, receive input from various sources but, as revealed in Zhang et al.'s study (2020), prefer utilizing retailer information.

(Sezen 2008) highlights information sharing as a well-established approach to enhance supply chain performance, emphasising the paramount importance of a well-designed chain to achieve this goal. However, contrary to expectations, possessing information does not always lead to improved outcomes. For instance Chen, Pun et al, (2023)demonstrates that the more market demand information manufacturers possess, the less profit retailers earn, intensifying competition among them.

2-3 Supply chain structure and game theory approaches

In modern supply chains, members are increasingly cautious about seeking knowledge and awareness. Game theory provides a valuable way for analyzing the behavior of supply chain actors. Recent research has focused significantly on the competition among members to achieve favorable targets, such as gaining a larger market share and maximizing profits and trust. The structure of the supply chain has a direct impact on how its members operate. For instance, retailers who purchase products from manufacturers compete with each other (Shang, Ha, et al. 2016).

Competition exists not only among downstream members but also between upstream organs, Shamir's study include two manufacturers whose retailers also compete with each other (Shamir 2012). However, under market conditions, these members cooperate, often through information sharing between retailers and suppliers. (Luo et al. 2019). The study of Stackelberg's game with either manufacturer or retailer leadership is a fundamental aspect of game theory research. Stein claims that his study on the Stackelberg game approach can assist individuals in managing their path to achieving desired goals (Stein, Salvioli et al, 2023). (Tang, Wang et al. 2020) examined the manufacturer-led Stackelberg game to demonstrate the manufacturer's power over retailers and various methods such as remanufacturing and returns to control costs and increase demand. However, retailers also have a significant impact on supply chain profit by adhering to nondisclosure agreements rather than disclosure agreements during information sharing and promotional pricing strategies (Tai, Duc et al, 2022).

In Table 1, we have summarized various studies conducted on contracts in the supply chain, information sharing, member performance comparison using specific contracts, structured supply chains, and game theory principles. The review of previous studies reveals a lack of research on structured supply chains, despite the importance of demand uncertainty and information value in maintaining optimal conditions. Although there are studies on contracting in the supply chain to establish trust among chain members, there is currently no effective method for deciding on the appropriate contract during uncertain market times.

Therefore, there is a gap in terms of investigating supply chain member strategies during times of information shortage and selecting suitable contracts in the supply chain. The main contribution of this paper is to attempt an expansion of a supply chain consisting of a single manufacturer and a retailer who utilize two different contracts. Subsequently, it investigates the effects of information sharing on the performance of supply chain members when using linear price contracts and contract menus. Moreover, previous research has not included a comparison of contract efficiency in terms of information availability or shortage. Our paper presents detailed insights into the behavior of leaders and followers in a Stackelberg-manufacturer game. We will discuss the importance of information sharing and non-sharing under specific conditions. The performance of the members is also monitored, taking into account other factors that influence information sharing, such as payments from the manufacturer to the retailer.

Table 1: Summary of relevant studies

Reference	SSCC*	IS**	CMP***	SSC****	GT approach****
(Koo 2022)	√	√		√	
(Roemer, Müller et al. 2023)	√	√		√	√
(Johnsen, Sadrieh et al. 2021)	√	√			√
(Sundram, Chhetri et al. 2020)		√	√	√	
(Huo, Haq et al. 2021)		√		√	
(Guggenberger, Schweizer et al. 2020))		√		√	
(Agrawal, Angelis et al. 2023)	√	√			
(Singh and Kumar 2022)	√	√			
(Qiu, Yu et al. 2022)	√			√	√
(Jia and Wang 2022)	√			√	
(Wang, Peng et al. 2022)		√		√	√
(Ha and Tong 2008)	√	√		√	√
This paper	√	√	√	√	√

*A structured supply chain with at least two members **Information Sharing
*** Compare member performance by different contract **** Study of supply chain contract
***** Game Theory Approach

3- Model Description

We consider a supply chain consisting of a single manufacturer and a retailer. The manufacturer supplies a single product to the retailer, who sells it in the market.

We are evaluating two contract types based on their flexibility: linear price contracts and contract menus. Contract menus, which provide a fixed price for a range of quantities, are frequently used in supply chains with retailers and manufacturers as a non-linear pricing approach (Corbett, 2004). In contrast, linear price contracts, commonly employed among supply chain members, are less flexible due to issues like the Bullwhip effect and double marginalization (Ha and Tong 2008). This comparison offers a valuable examination of information sharing and the impact of different contract types. The demand for the product is uncertain and can be categorized into two possibilities. In fact, the potential market size has two states: high market demand (A_H) with probability θ and low market demand (A_L) with probability $1-\theta$. Therefore, the market demand (represented by q) is defined as follows:

$$q_d = A_d - \beta p_d \qquad d = H,\ L \tag{1}$$

Where d indicates the state of demand, and p_d and β are the retail price and price sensitivity coefficient, respectively.

To simplify matters, we assume that the retailer has a reservation profit of zero. While the retailer is aware of the demand state, the manufacturer remains unaware of the demand situation. We investigate two scenarios: one where the demand information is shared by the retailer and the other where it is not. In the case of information sharing, the manufacturer compensates the retailer for providing demand information. Additionally, this paper considers two types of contracts: contract menus and linear price contracts. Contract menus involve the manufacturer offering a range of quantity and price options to the retailer, who then selects one option from the menu. In the linear price contract, the manufacturer sets a wholesale price, and the retailer pays accordingly for each unit of the product. Moreover, we examine a market where the manufacturer possesses more power than the retailer. Thus, we employ a Stackelberg model to address the competition between the manufacturer (acting as the leader) and the retailer (acting as the follower).

Throughout the paper, the following notations are used:

Notations	Description
O	The fixed cost of the manufacturer's investment to obtain information
S, N	Two uppercase letters indicate the information sharing and no information sharing scenarios
C,M,R	The indexes represent the supply chain, manufacturer, and retailer, respectively.
W_d	The wholesale price of the manufacturer at demand state $d = H, L$.
p_d	The retail price of retailer in demand state $d = H, L$.
q_d	The demand for retailer in demand state $d = H, L$.
F_d	The retailer’s revenue in demand state $d = H, L$.

Notations	Description
R_d	The manufacturer's revenue in demand state $d = H, L$.
U_C^E	The supply chain's profit function under the contract menu at equilibrium E.
U_M^E	The manufacturer's profit function under the contract menu at equilibrium E.
U_R^E	The retailer's profit function under the contract menu at equilibrium E.
$\overline{U}_C^E$	The supply chain's profit function under the linear price contract at equilibrium E.
$\overline{U}_M^E$	The manufacturer's profit function under the linear price contract at equilibrium E.
$\overline{U}_R^E$	The retailer's profit function under the linear price contract at equilibrium E.

4- The case of contract menu

Within the context of the contract menu, the manufacturer presents a range of options to the retailer, who then selects a specific option. Upon acceptance of an offer (Q, R), the retailer receives Q units of the product from the manufacturer and makes a payment equal to R. As the manufacturer assumes the role of the leader, he anticipates the retailer's response and offers the contract menu accordingly. Our study examines the behavior of the supply chain members in two distinct scenarios when utilizing the contract menu. The first scenario (referred to as S) involves information sharing between the retailer and the manufacturer, while the second scenario (referred to as N) does not involve information sharing.

4-1 Contract menu with information sharing

In this scenario, the retailer shares information regarding the state of demand with the manufacturer. As a result, both supply chain members are aware of the demand state ($d = H \; or \; L$). The manufacturer, designs a bid (Q_d, R_d) and presents it to the retailer in the form of a contract menu, taking into account the anticipated best response from the retailer. To determine the retailer's revenue and profit functions, the following expressions can be formulated:

$$F_d(Q_d, p_d) = min\{(A_d - \beta p_d), Q_d\} \times p_d \tag{2}$$

$$U_R = F_d(Q_d, p_d) - R_d \tag{3}$$

Considering the retailer's objective of maximizing profit, the best response function $\hat{p}_d(Q_d)$, can be obtained as follows:

$$p_d = \arg max \; U_R \quad where \; U_R = min\{(A_d - \beta p_d), Q_d\} \times p_d - R_d \tag{4}$$

In relation (4), if $Q_d < \; A_d - \beta p_d$, then U_R will be an increasing function of p_d and $\hat{p}_d(Q_d)$ is the upper bound of p_d, where

$$\hat{p}_d(Q_d) = \frac{A_d - Q_d}{\beta} \quad (5)$$

To determine $\hat{p}_d(Q_d)$, by considering the value of Q_d, when $Q_d < A_d - \beta p_d$ then $p_d < \frac{A_d - Q_d}{\beta}$ and $\hat{p}_d(Q_d)$ can be obtained from relation (5).

For $Q_d > A_d - \beta p_d$, we have $U_R = (A_d - \beta p_d) \times p_d - R_d$ which is a concave function of p_d, and is obtained from the first order condition. That is, $p_d(Q_d) = \frac{A_d}{2\beta}$ for which $q_d = \frac{A_d}{2}$. Therefore, the best response function of the retailer can be calculated as follows:

$$p_d(Q_d) = \begin{cases} \frac{A_d - Q_d}{\beta} & if \;\; Q_d < \frac{A_d}{2} \\ \frac{A_d}{2\beta} & otherwise \end{cases} \quad (6)$$

The retailer's revenue can be obtained by multiplying the quantity by the retail price as below:

$$F_d = \begin{cases} Q_d \times \frac{A_d - Q_d}{\beta} & if Q_d \leq \frac{A_d}{2} \\ \frac{(A_d)^2}{4\beta} & otherwise \end{cases} \quad (7)$$

The manufacturer, aiming to maximize her profit, anticipates the retailer's behavior. Consequently, when designing the proposed contract, the manufacturer sets the second term of the contract equal to the retailer's revenue. In simpler terms, the manufacturer offers $R_d = F_d(Q_d, p_d)$. Furthermore, the manufacturer sets the other term of the contract (Q_d) to maximize her revenue. If $Q_d \leq \frac{A_d}{2}$, then U_M will be a concave function of Q_d, and the following relations are achieved:

$\hat{Q}_d(p_d) = argmax \;\; U_M$ where $U_M = F_d = Q_d \times \frac{A_d - Q_d}{\beta}$, then we have: $\hat{Q}_d(p_d) = \frac{A_d}{2}$.

Therefore, the manufacturer offers (Q_d, R_d) as follows:

$$Q_d(p_d) = \frac{A_d}{2} \quad (8)$$

$$\hat{R}_d = F_d(Q_d) = \frac{(A_d)^2}{4\beta} \quad (9)$$

4-2 Contract menu without information sharing

In this scenario, the manufacturer formulates a contract menu without prior knowledge of the demand state. Her objective is to determine the menu list, $\{(Q_H, R_H), (Q_L, R_L)\}$, in order to maximize her expected profits while taking into account the anticipated behavior of the retailer. It is important to note that the retailer possesses information regarding the demand state, and her best response function aligns with the one observed in the first scenario, as described in Equation (6). Consequently, the manufacturer engages in problem l to determine her decision variables.

$$(problem\ l) \quad \underset{Q_H, R_H, Q_L, R_L}{\text{Max}}\ \theta R_H + (1-\theta) R_L \tag{10}$$

Subject to

$$F_H(Q_H, p_H) - R_H \geq F_H(Q_L, p_H) - R_L \tag{11}$$

$$F_L(Q_L, p_L) - R_L \geq F_L(Q_H, p_L) - R_H \tag{12}$$

$$F_H(Q_H, p_H) - R_H \geq 0 \tag{13}$$

$$F_L(Q_L, p_L) - R_L \geq 0 \tag{14}$$

$$Q_H \geq 0, Q_L \geq 0 \tag{15}$$

The problem encompasses the manufacturer's objective function and logical constraints in the design of the contract menu. Relation (10) which represents the manufacturer's expected profit, takes into account the various demand possibilities. Relations (11) and (12) govern the retailer's truth-telling behavior, ensuring that the contract menu aligns with the anticipated demand state. Relations (13) and (14) guarantee the retailer's motivation to participate in the business, with the requirement that the retailer's profit should meet or exceed its reservation profit. Lastly, the final relation establishes the non-negativity constraint on the order quantities. Let$\{(Q_H^N, R_H^N), (Q_L^N, R_L^N)\}$ denote the optimal solution of the problem l in which the superscript N points to the case of non-sharing information. The solution to problem l is provided in Proposition 1, which indicates the manufacturer's best response functions.

Proposition 1: In the case of a contract menu without information sharing, the best response functions of the manufacturer will be as follows:

$$Q_H^N = \frac{A_H}{2} \tag{16}$$

$$Q_L^N = max\left[\frac{A_L}{2} - \frac{\theta(A_H - A_L)}{2(1-\theta)}, 0\right] \tag{17}$$

$$R_H^N(p_H, p_L) = (Q_H \times p_H) - (Q_L \times p_H) + (Q_L \times p_L) \tag{18}$$

$$R_L^N(p_H, p_L) = Q_L \times p_L \tag{19}$$

Proof: See appendix.

The best response functions depict a player's behavior in response to the strategies employed by other players. Thus, when the best response functions of both players intersect, it signifies the equilibrium point of the game. Proposition 2 offers the equilibrium solution for the present scenario.

Proposition 2: In the case of a contract menu without information sharing, two possibilities arise:

a) If $\theta \leq \frac{A_L}{A_H}$ there is a Bayesian equilibria expressed as follows:

$$p_H^{N\mathrm{I}} = \frac{A_H}{2\beta} \qquad p_L^{N\mathrm{I}} = \frac{A_L}{2\beta} + \frac{\theta(A_H - A_L)}{2\beta(1-\theta)} \qquad Q_L^{N\mathrm{I}} = \frac{A_L}{2} - \frac{\theta(A_H - A_L)}{2(1-\theta)} \qquad Q_H^{N\mathrm{I}} = \frac{A_H}{2}$$

b) If $\theta \geq \frac{A_L}{A_H}$ there is a Bayesian equilibria expressed as follows:

$$p_H^{N\mathrm{II}} = \frac{A_H}{2\beta} \qquad p_L^{N\mathrm{II}} = \frac{A_L}{\beta} \qquad Q_L^{N\mathrm{II}} = 0 \qquad Q_H^{N\mathrm{II}} = \frac{A_H}{2}$$

Proof: See appendix.

4-3 - Comparison of members' profit under contract menu with and without information sharing

To further examine the impact of information sharing on player profitability within the contract menu framework, it is necessary to compare the profits at equilibrium points. Accordingly, let U_M^E, U_R^E and $U_C^E = U_M^E + U_R^E$, respectively, represent the retailer's profit, the manufacturer's profit, and the supply chain's profit at the equilibrium point E, where $E \in \{S, N\mathrm{I}, N\mathrm{II}\}$. The results are presented as the following proposition:

Proposition 3: The following relationships are held for the case of contract menu.

1. $U_C^{N\mathrm{I}} \leq U_C^S$, $U_C^{N\mathrm{II}} \leq U_C^S$

2. $U_M^{N\mathrm{II}} \leq U_M^S,\ U_M^{N\mathrm{I}} \leq U_M^S$

3. $U_R^{N\mathrm{II}} = U_R^S = 0$; $U_R^S \leq U_R^{N\mathrm{I}}$ if $\theta \leq \frac{1}{2}$, otherwise $U_R^S \geq U_R^{N\mathrm{I}}$.

5. The case of linear price contract

Under the linear price contract, the manufacturer sets the wholesale price, and the retailer places an order based on this price. This section focuses on analyzing the behavior of the supply chain members in two scenarios: one with information sharing between the retailer and the manufacturer, and the other without such information sharing.

5-1 Linear price contract with information sharing

As the leader of the supply chain, the manufacturer anticipates the retailer's behavior and incorporates it into her decision-making process by considering her best response function. To analyze the dynamics of the supply chain, we employ the backward induction method, starting with an examination of the retailer's decision and subsequently addressing the manufacturer's concerns. Let $\bar{U}_R$ and $\bar{U}_M$ represent the retailer's, and the manufacturer's profits, respectively. We have

$$\bar{U}_R = (A_d - \beta p_d)(p_d - W_d)$$

$$\bar{U}_M = W_d(A_d - \beta p_d)$$

Given that $\frac{\partial^2 \bar{U}_R}{\partial p_d^2} = -2\beta < 0$, then the retailers' best response could be obtained from the first order condition. Therefore, we have:

$$\frac{\partial \bar{U}_R}{\partial p_d} = 0 \Rightarrow p_d = \frac{A_d + \beta W_d}{2\beta}$$

We define $\bar{p}_d^S$ and $\bar{W}_d^S$ as the retail price and the wholesale price at equilibrium point, respectively. The manufacturer determines W_d regarding the retailer's best response function. Therefore, the wholesale price in equilibrium points could be obtained from the first order condition as bellow:

$$\bar{U}_M = W_d \times \left(A_d - \beta \frac{A_d + \beta W_d}{2\beta}\right) \Rightarrow \bar{W}_d^S = \frac{A_d}{2\beta}$$

In addition, we have $\bar{p}_d^S = \frac{3A_d}{4\beta}$ and $\bar{q}_d^S = \frac{A_d}{4}$.

5-2 Linear price contract without information sharing

In the absence of information sharing between the manufacturer and the retailer, the manufacturer, unaware of the demand state d, sets the wholesale price W that remains unaffected by the market state. However, as the retailer possesses knowledge of the market situation, her profit remains similar to that in the information sharing model. Consequently, the retailer's price can be defined by substituting W_d by W yielding the following expression:

$p_d = \frac{A_d + \beta W}{2\beta}$ for $d = H, L$

The manufacturer anticipates high market demand (A_H) with probability θ and low market demand (A_L) with probability$(1-\theta)$. Therefore, the manufacturer's expectation profit is obtained as bellow:

$$\bar{U}_M = W \times q = W[\theta q_H + (1-\theta) q_L]$$

Considering the best response of the retailer, manufacturer profit is measured as follows:

$$U_M = W \times q_d = W\left[\theta \frac{A_H - \beta W}{2} + (1-\theta) \frac{A_L - \beta W}{2}\right]$$

Since the manufacturer's profit function is a concave function of the wholesale price, the calculation of the manufacturer's best response can be derived from the first-order condition:

$$\frac{\partial^2 U_M}{\partial W^2} = -\beta < 0, \qquad \frac{\partial U_M}{\partial W} = 0 \Rightarrow \bar{W}^N = \frac{\theta A_H + (1-\theta) A_L}{2\beta}$$

where $\bar{W}^N$ represents the equilibrium wholesale price. Further, let $\bar{p}_d^N$ denote the equilibrium retailer price, where $d = H,\ L$. After some algebraic substitution, we have

$$\bar{p}_d^N = \frac{A_d}{2\beta} + \frac{\theta A_H + (1-\theta) A_L}{4\beta}$$

Moreover,

$$\bar{q}_H^N = \frac{A_H}{2} - \frac{\theta A_H + (1-\theta) A_L}{4}$$

$$\bar{q}_L^N = max\left(\frac{A_L}{2} - \frac{\theta A_H + (1-\theta) A_L}{4}, 0\right)$$

5-3 Comparison of members' profit under linear price contract with and without information sharing

Let $\bar{U}_C^E$, $\bar{U}_M^E$, and $\bar{U}_R^E$ be the profit of the supply chain, the manufacturer and the retailer, respectively, at the equilibrium point in linear price contract, where $\bar{U}_C^E = \bar{U}_M^E + \bar{U}_R^E$ and $E = S, N$. Comparing the profits of supply chain members can be illustrated via the following proposition:

Proposition 4: The following relationships are held for the case of linear price contract.

1. $\bar{U}_M^N > \bar{U}_M^S$

2. $\overline{U}_R^N > \overline{U}_R^S$

Proof: See Appendix.

Proposition (4) demonstrates that employing a linear price contract renders information sharing among supply chain members unfavorable. In simpler terms, the manufacturer shows reluctance towards receiving information, while the retailer stands to gain greater profits by withholding information. Therefore, the contract's rigidity may impede its efficacy in addressing uncertain conditions.

6. Comparison of linear price contract and contract menus

In this section, we compare the performance of chain members in different models of information sharing and no information sharing. The earned profits of members at equilibrium points of different contracts are compared using the following proposition:

Proposition 5: The following relationships are held comparing contract menus and linear price contracts:

a) For the case of information sharing

$U_M^S > \overline{U}_M^S, \quad \overline{U}_R^S > U_R^S, \quad U_C^S > \overline{U}_C^S$

b) For the case of no information sharing

$U_M^{N\text{I}} < \overline{U}_M^N\ , \ \ U_M^{N\text{II}} \leq \overline{U}_M^N\ , \ \ U_R^{N\text{I}} \leq \overline{U}_R^N$

$U_R^{N\text{II}} \leq \overline{U}_R^N, \ \ U_C^{N\text{I}} \leq \overline{U}_C^N, \ \ U_C^{N\text{II}} \leq \overline{U}_C^N$

Proof: See Appendix.

In Proposition 5, Part it is revealed that the manufacturer benefits from contract menus when members share information, whereas the retailer prefers the linear price contract. However, information sharing exerts a positive influence on the overall performance of the entire supply chain within contract menus. This indicates that contract menus can effectively coordinate supply chain members towards a common goal, such as improving performance and maximizing mutual benefit, if both retailer and manufacturer tend to share information to have better relations and make more profit in contract menus. Additionally, in Part b of Proposition 5, when information is limited, both the manufacturer and the retailer tend to lean towards employing the linear price contract. This inclination arises from manufacturer’s approach to pricing individual product units.

7. Analyses of the decision to share or not to share information

Upon thorough evaluation of the current circumstances and a comprehensive recognition of the pivotal role played by information, the members of the supply chain arrive at a crucial decision regarding the sharing of information. In light of this decision, we introduce the concept of development profit, which serves as a means to quantify the disparity between the overall profitability of the supply chain when both the manufacturer and retailer engage in information sharing, and the profitability achieved by the supply chain in situations where information sharing is absent. The investment cost, denoted as (O), represents the expense incurred by the manufacturer for acquiring information from the retailer. The profit, denoted as J_P Here, $P = M, C$ (M= manufacturer, C= Supply chain) signifies the difference between the profits derived from information sharing and the absence of information sharing. In the upcoming analysis, we will quantify the value of information sharing by calculating the comparison. Specifically, we will focus on J_P which represents the incremental profit resulting from the disparity between the supply chain's profit without information sharing and the profit achieved by chain members when they actively engage in information sharing. It is pertinent to emphasize that the investment cost is evaluated separately for the entire supply chain and the manufacturer. When the profit exceeds the investment cost, in accordance with the findings of Proposition 3, the manufacturer will willingly proceed with making the required payment to gain access to the valuable information.

The information sharing decisions among supply chains under contract menus and linear price contracts can be described in the following cases.

1. $if\ J_M \geq O\ and\ J_C \geq O$ Sharing information can enhance the profitability of manufacturer and the entire supply chain.
2. $if\ J_M \leq O\ and\ J_C \leq O$ Manufacturer and the entire supply chain do not typically prefer information sharing.
3. $if\ J_M \geq O\ and\ J_C \leq O$ Although manufacturer stand to gain benefit from information sharing, the preference within the entire supply chain is for its members to abstain from sharing information amongst themselves.

Based on the previous propositions and the following formulation, we can describe the above cases in more detail.

$$J_M = U_M^S - U_M^N \tag{20}$$

$$J_C = U_C^S - U_C^N = (U_M^S + U_R^S) - (U_M^N + U_R^N) \tag{21}$$

We can divide the manufacturer's decision into two scenarios:

Based on proposition 3, we have $U_M^S - U_M^N \geq 0 \Rightarrow J_M \geq 0\ and\ U_C^S - U_C^N \geq 0 \Rightarrow J_C \geq 0$. If the supply chain's profitability surpasses the amount paid to the retailer for receiving information, the manufacturer chooses to acquire that information. In simpler terms, if $O \leq J_C$ information sharing becomes the prevailing strategy for the supply chain, Additionally, to examine supply chain coordination, we calculate J_C which represents the positive impact of information sharing on all supply chain members, assuming the information cost is fairly borne by the manufacturer. In the context of a linear price contract, proposition 4, allows us to compare the profits of all chain members under conditions of information sharing or non-sharing. Let define

$$\bar{J}_M = \bar{U}_M^S - \bar{U}_M^N \tag{22}$$

$$\bar{J}_C = \bar{U}_C^S - \bar{U}_C^N = (\bar{U}_M^S + \bar{U}_R^S) - (\bar{U}_M^N + \bar{U}_R^N) \tag{23}$$

Considering Proposition 4, the predominant strategy within the supply chain dictates that the manufacturer refrains from receiving information from the retailer. This characteristic is inherent in the linear price contract, which proves inadequate in effectively aligning the supply chain members toward a shared objective. Consequently, the manufacturer's optimal strategy may involve abstaining from receiving information from the retailer.

8. Numerical example

The purpose of this study is to investigate how information sharing impacts members' profitability and how different contract types within the supply chain framework affect members' actions. Retail pricing sensitivity and market potential are important considerations. The profitability of supply chain members is assessed and clarified through numerical examples. MATLAB software is used to program equilibrium solutions, and tables and graphs display the findings. Comparing the profits associated with each strategy facilitates the determination of the most effective approach. The primary parameters used in the numerical analysis are listed in Table 2, which are essential for describing interactions within the supply chain and determining equilibrium points. The following subsections analyze different contract types.

Table 2- the basic data for numerical example

Parameters	Descriptions	Base values
A_H	Potential market demand (high state)	1000
A_L	Potential market demand (low state)	500
β	Price Sensitivity for Retailer	2
θ	The probability of high state demand	0.5

To assess the willingness of supply chain members to share or withhold information, the difference $\boldsymbol{U^S - U^N}$ is calculated under different contracts and presented in Table 3. The results show that while retailers are generally indifferent to sharing information, manufacturers are more inclined to do so, particularly in the context of contract menus. Moreover, under a linear price contract, the manufacturer is more likely to share information, whereas the retailer tends to avoid it.

Table 3: Comparative analysis of members' profits in various equilibrium scenarios under contract menus

Members	$U^S - U^N$ **(In Contract Menu)**	$U^S - U^N$ **(In Linear Price Contract)**
Manufacturer	15625	2.1484×10^4
Retailer	0	-2.3437×10^4

Furthermore, figure 2 illustrates the effects of different β values on the profits of supply chain members across various scenarios under the contract menu contract. It indicates that as price sensitivity increases in scenarios S and N, the manufacturer's profit decreases linearly. This reduction occurs due to a decline in market demand as β rises. However, for the retailer in scenario N, this represents a clear negative impact, while in scenario S, the retailer's profit remains unchanged.

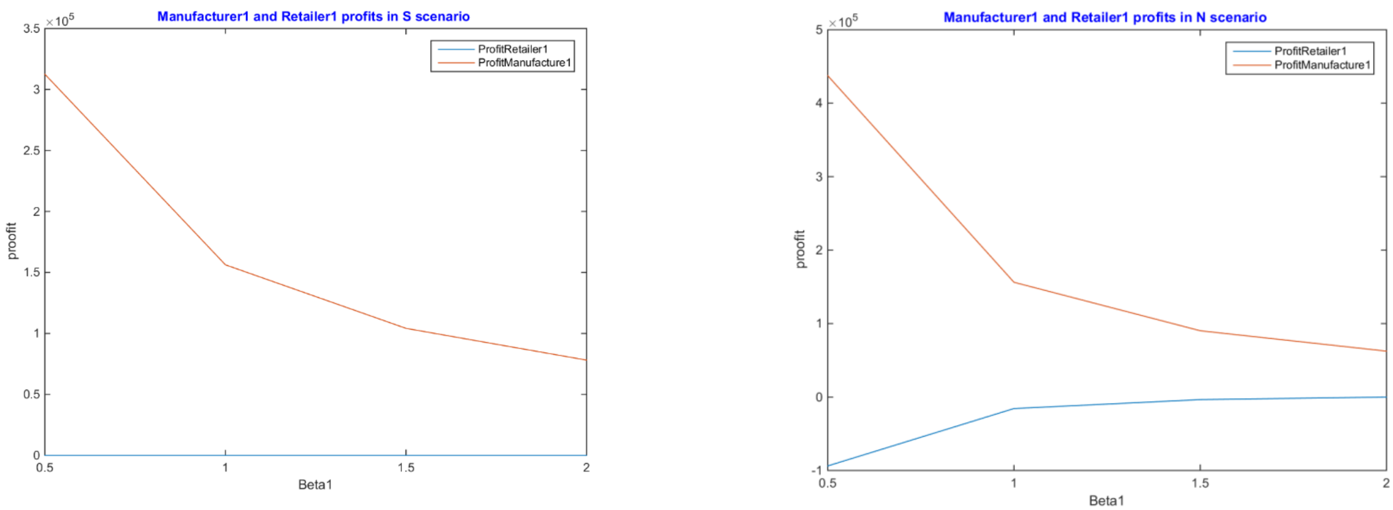

Figure 2. The impact of changes in β on supply chain member profits across different scenarios under contract menus.

he profits of both the retailer and manufacturer decrease as price sensitivity (β) increases. Notably, in scenario S for the manufacturer and in scenario N for the retailer, the rate of profit decline is more pronounced.

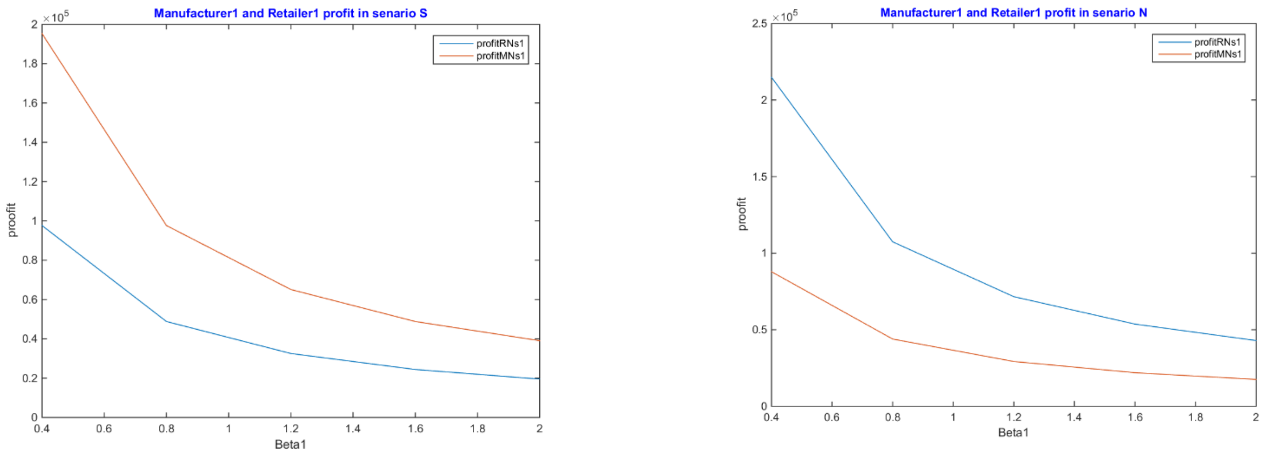

Figure 3: The impact of changes in β on supply chain member profits across different scenarios under a linear price contract.

9. Conclusion and Managerial insight

In this study, we looked at how contracting and information sharing work in a supply chain with one manufacturer and one retailer. We focused on the case where the retailer knows about market demand. We first explored the best decisions for both contract menus and linear price contracts, considering both information-sharing and non-information-sharing scenarios. We then compared the performance of these two contract types, taking into account whether the manufacturer knows about the market demand information or not. To analyze this, we used a Stackelberg game, where we found the solution by determining the equilibrium point.

Based on our mathematical analysis, several important managerial insights emerge:

Contract Menus: Information sharing consistently improves the performance of the entire supply chain, making it an effective strategy. Specifically, information sharing benefits the manufacturer. For the retailer, the preference for sharing or not sharing information depends on the value of θ. If $\theta \geq \frac{A_L}{A_H}$, the retailer remains neutral regarding information sharing. However, if $\theta \leq \frac{A_L}{A_H}$ the retailer's preference varies with θ: if $\theta \leq \frac{1}{2}$, the retailer prefers not to share information; otherwise, it opts for sharing.

Linear Price Contracts: Information sharing among supply chain members is generally unfavorable. The manufacturer is reluctant to receive information, while the retailer can achieve higher profits by withholding information. This rigidity in the contract may limit its effectiveness in managing uncertain conditions.

Additionally, Table 4 compares the contract preferences of supply chain members regarding information sharing, highlighting the differences in member profits under shared and non-shared information conditions for two contract types.

Table 4: Supply Chain Members' Contract Preferences in Different Conditions

members	Information sharing	Non-Information sharing
manufacturer	Contract menus	Linear price contract
retailer	Linear price contract	Linear price contract
Supply chain	Contract menus	Linear price contract

Our model assumes the retailer is reliable and does not consider dishonesty. Future research could look into scenarios where the retailer might be dishonest using game theory. Additionally, the problem could be framed as a cooperative game to better reflect real-world situations.

Disclosure: Declaration of Generative AI

In the course of preparing this work, the authors employed Chat-GPT 3.5 to assist in proofreading and improving the paper's readability. Subsequently, the authors carefully reviewed and edited the content as required, assuming full responsibility for the final publication's content.

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Appendix

Proof of Proposition 1

To prove this proposition, we firstly ignore constraints (12) and (13) and obtain the solution, and then show that our solution satisfies these constraints, and therefore, they are the solution of problem *l*. Disregarding constraints (12) and (13), constraints (11) and (14) must be binding, otherwise the manufacturer could increase her profit by increasing R_H and R_L. Thus we have

$$R_L = F_L(Q_L, p_L)$$

$$R_H = F_H(Q_H, p_H) - F_H(Q_L, p_H) + R_L$$

Next, we substitute the derived relationships into the objective function

$$U(Q_H, Q_L) = \theta R_H + (1-\theta)R_L$$

$$= \theta\big(F_H(Q_H, p_H) - F_H(Q_L, p_H) + F_L(Q_L, p_L)\big) + (1-\theta)F_L(Q_L, p_L)$$

$$U(Q_H, Q_L) = \theta\big(F_H(Q_L, p_H)\big) + F_L(Q_L, p_L) - \theta(F_H(Q_L, p_H))$$

According to Equation (7), $F_d(Q_d, p_d)$ is increasing in Q_d when $Q_d \leq \frac{A_d}{2}$ whereas it remains constant when$Q_d \geq \frac{A_d}{2}$. Therefore, the optimal value Q_d denoted as Q_d^* must adhere to the constraint$Q_d^* \leq \frac{A_d}{2}$. Hence, we can use Equation (7) and rewrite the above expression as

$$U(Q_H, Q_L) = \theta(F_H(Q_H)) + F_L(Q_L) - \theta(F_H(Q_L))$$

$$= \theta\left(Q_H \times \frac{A_H - Q_H}{\beta}\right) + \left(Q_L \times \frac{A_L - Q_L}{\beta}\right) - \theta(Q_L \times \frac{A_H - Q_L}{\beta})$$

The function $U(Q_H, Q_L)$ is separable with respect to Q_H andQ_L. Since $\frac{\partial^2 U}{Q_L^2} = \frac{-2(1-\theta)}{\beta} < 0$ and$\frac{\partial^2 U}{Q_H^2} = \frac{-2}{\beta} < 0$, the best values of Q_L and Q_H could be obtained from the first-order conditions. Hence,

$$\frac{\partial U}{\partial Q_H} = 0 \Rightarrow Q_H = \frac{A_H}{2}, \text{ and } \frac{\partial U}{\partial Q_L} = 0 \Rightarrow Q_L = \frac{A_L}{2} - \frac{\theta(A_H - A_L)}{2(1-\theta)}$$

This is a feasible solution when $\frac{A_L}{A_H} > \theta$ for boundary point we consider $Q_L = 0, Q_H = \frac{A_L}{2}$ which is a feasible solution when $\frac{A_L}{A_H} \leq \theta$.

Now, it is necessary to demonstrate the validity of the answers obtained for constraints 12 and 13. In constraint 12, we have the variable $F_L(Q_L, p_L) - R_L \geq F_L(Q_H, p_L) - R_H$. By considering the relationships derived from constraints 11 and 14, we establish the following relations.

$R_H = F_H(Q_H, p_H) - F_H(Q_L, p_H) + R_L$

and

$R_L = F_L(Q_L, p_L)$

Substituting these relations into constraint 12 yields the desired outcome.

$F_L(Q_L, p_L) \geq F_L(Q_H, p_L) - F_H(Q_H, p_H) + F_H(Q_L, p_H)$

Given that و $Q_L \leq \frac{A_L}{2} \leq \frac{A_H}{2}$, $Q_H = \frac{A_H}{2}$ and considering relation (7), the following relation can be derived.

$$Q_L \times \frac{A_L - Q_L}{\beta} \geq Q_H \times \frac{A_L - Q_H}{\beta} - Q_H \times \frac{A_H - Q_H}{\beta} + Q_L \times \frac{A_H - Q_L}{\beta} \Leftrightarrow Q_H \geq Q_L$$

The above relationship is established. Now, considering the relations derived from constraints 11 and 14, we can analyze constraint 13.

$$F_H(Q_H, p_H) - R_H \geq 0 \quad \Leftrightarrow F_H(Q_L, p_H) - F_L(Q_L, p_L) \geq 0 \Leftrightarrow$$

$$\left(Q_L \times \frac{A_H - Q_L}{\beta}\right) - \left(Q_L \times \frac{A_L - Q_L}{\beta}\right) \geq 0 \Leftrightarrow$$

$$A_H - A_L \geq 0$$

To demonstrate the validity of $Q_H = \frac{A_H}{2}, Q_L = 0$ for constraints 12 and 13, we first analyze constraint 12. $F_L(Q_L, p_L) - R_L \geq F_L(Q_H, p_L) - R_H$

By utilizing the relations obtained from constraints 11 and 14, specifically the relations $R_H = F_H(Q_H, p_H) - F_H(Q_L, p_H) + R_L$ and $R_L = F_L(Q_L, p_L)$

we establish that

$F_L(Q_L, p_L) \geq F_L(Q_H, p_L) - F_H(Q_H, p_H) + F_H(Q_L, p_H)$ Placing these relations in the limit12.

Considering that $Q_L \leq \frac{A_L}{2} \leq \frac{A_H}{2}$ and $Q_H = \frac{A_H}{2}$are dependent on relation (7), we derive the following relation:

$$Q_L \times \frac{A_L - Q_L}{\beta} \geq Q_H \times \frac{A_L - Q_H}{\beta} - Q_H \times \frac{A_H - Q_H}{\beta} + Q_L \times \frac{A_H - Q_L}{\beta} \Leftrightarrow Q_H \geq Q_L \Rightarrow Q_H \geq 0$$

confirming the aforementioned relationship. Moving on to constraint 13, by considering the relationships derived from constraints 11 and 14, we find that

$F_H(Q_H, p_H) - R_H \geq 0 \quad \Leftrightarrow F_H(Q_L, p_H) - F_L(Q_L, p_L) \geq 0 \Leftrightarrow$

$$\left(Q_L \times \frac{A_H - Q_L}{\beta}\right) - \left(Q_L \times \frac{A_L - Q_L}{\beta}\right) \geq 0 \Leftrightarrow$$

$A_H - A_L \geq 0$

satisfying this constraint. In conclusion, the existence of relations

$R_H = F_H(Q_H, p_H) - F_H(Q_L, p_H) + R_L$ and $R_L = F_L(Q_L, p_L)$ enables us to easily comprehend relations (18) and (19)

Proof of Proposition 2:

After determining the best response functions through their intersection, we identify the equilibrium points for the manufacturer and retailer in the Stackelberg model, considering both high and low market demand scenarios. To begin, we establish the proof for part (a) of Proposition 2.

$$p_d = \begin{cases} \frac{A_d - Q_d}{\beta} & if\ Q_d < \frac{A_d}{2} \\ \frac{A_d}{2\beta} & otherwise \end{cases}$$

And from Proposition 1 we have:

$$Q_H^* = \frac{A_H}{2}$$

$$Q_L^* = \frac{A_L}{2} - \frac{\theta(A_H - A_L)}{2(1-\theta)}$$

It is clear that $Q_L^* \leq \frac{A_L}{2}$ and $Q_H^* \geq \frac{A_H}{2}$ so according to relation (7)

$$p_H = \frac{A_H}{2\beta}$$

$$p_L = \frac{A_L}{2\beta} + \frac{\theta(A_H - A_L)}{2\beta(1-\theta)}$$

Similarly for $Q_L = 0$ and $Q_H = \frac{A_H}{2}$ since $Q_L^* \leq \frac{A_L}{2}$ and $Q_H^* \geq \frac{A_H}{2}$ so according to relation (7)

$p_H = \frac{A_H}{2\beta}, p_L = \frac{A_L}{\beta}$ ■

Proof of Proposition 3:

We start by substituting the equilibrium points obtained from Proposition 2 into the given equations to compute the profits of the chain members. This allows us to derive the results presented below.

The equations representing the profits are as follows:

$$R_H^N(P_H, P_L) = (Q_H \times p_H) - (Q_L \times p_H) + (Q_L \times p_L)$$

$$R_L^N(P_H, P_L) = (Q_L \times p_L)$$

And the expected utility functions for the respective agents are defined as:

$$U_M^S = \theta\left(\frac{{A_H}^2}{4\beta}\right) + (1-\theta)(\frac{{A_L}^2}{4\beta})$$

$$U_R^S = 0$$

$$U_C^S = \theta\left(\frac{{A_H}^2}{4\beta}\right) + (1-\theta)(\frac{{A_L}^2}{4\beta})$$

$$U_M^{NI} = \frac{\theta A_H(A_H - A_L)}{4\beta(1-\theta)} + \frac{{A_L}^2}{4\beta} - \frac{\theta^2(A_H - A_L)^2}{4\beta(1-\theta)^2}$$

$$U_C^{N\mathrm{I}} = \theta\left(\frac{{A_H}^2}{4\beta}\right) + (1-\theta)(\frac{{A_L}^2}{4\beta} - \frac{\theta^2(A_H - A_L)^2}{4\beta(1-\theta)^2})$$

$$U_R^{N\mathrm{I}} = \frac{\theta((1-\theta)A_L - \theta A_H)(A_H - A_L)}{4\beta(1-\theta)} + \frac{\theta^3(A_H - A_L)^2}{4\beta(1-\theta)^2}$$

$$U_R^{N\mathrm{II}} = 0$$

$$U_M^{N\mathrm{II}} = \theta(\frac{{A_H}^2}{4\beta})$$

$$U_C^{N\mathrm{II}} = \theta(\frac{{A_H}^2}{4\beta})$$

For the manufacturer, comparing under conditions of information sharing U_M^S and non-sharing $U_M^{N\mathrm{I}}, U_M^{N\mathrm{II}}$, the comparison yields:

$$U_M^S - U_M^{N\mathrm{I}} = \frac{\theta(1-2\theta)(1-\theta)(A_H - A_L)A_L}{4\beta(1-\theta)^2} + \frac{\theta^3(A_H - A_L)^2}{4\beta(1-\theta)^2} = \frac{\theta(A_H - A_L)\left((\theta^2 - 3\theta + 1)A_L + \theta^2 A_H\right)}{4\beta(1-\theta)^2}$$

To prove $U_M^S - U_M^{NI} \geq 0$, it is sufficient to show $(\theta^2 - 3\theta + 1)A_L + \theta^2 A_H \geq 0$. Since $\theta \leq \frac{A_L}{A_H}$ in equilibrium *NI*, we have

$$(\theta^2 - 3\theta + 1)A_L + \theta^2 A_H \geq (\theta^2 - 3\theta + 1)\theta A_H + \theta^2 A_H = \theta A_H(\theta^2 - 2\theta + 1) = \theta A_H(\theta - 1)^2 \geq 0$$

In comparing $U_M^S and\ U_M^{NII}$ we have

$$U_M^S - U_M^{NII} = (1-\theta)\frac{{A_L}^2}{4\beta} \geq 0.$$

For U_R^S, U_R^{NI}, and U_R^{NII}, the comparisons and their implications follow in a similar manner.

$$U_R^{NI} - U_R^S = \left(\frac{\theta\big((1-\theta)A_L - \theta A_H\big)(A_H - A_L)}{4\beta(1-\theta)} + \frac{\theta^3(A_H - A_L)^2}{4\beta(1-\theta)^2}\right)$$

$$= \frac{\theta(A_H - A_L)(A_L - \theta A_H)(1 - 2\theta)}{4\beta(1-\theta)^2}$$

To prove $U_R^{NI} - U_R^S \geq 0$, we should show $\theta(A_H - A_L)(A_L - \theta A_H)(1 - 2\theta) \geq 0$.

Since $\theta \leq \frac{A_L}{A_H}$ in equilibrium *NI*, the expression $A_L - \theta A_H \geq 0$ is valid. Therefore, if $\theta \leq \frac{1}{2}$ $U_R^{NI} - U_R^S$ will be positive and $U_R^{NI} \geq U_R^S$ otherwise $U_R^S \leq U_R^{NI}$.

Finally, the comparison of the supply chain profits under conditions of information sharing and non-sharing is as follows:

$$U_C^S - U_C^{NI} = \left(\frac{\theta(A_H - A_L)^2}{4\beta(1-\theta)}\right) \geq 0$$

$$U_C^S - U_C^{NII} = (1-\theta)\frac{{A_L}^2}{4\beta} \geq 0$$

■

Proof of proposition 4: Analysis of Linear Price Contract in Supply Chain

The profit functions in equilibrium are depicted as follows:

$$\overline{U}_M^S = \theta\left(\frac{{A_H}^2}{8\beta}\right) + (1-\theta)\left(\frac{{A_L}^2}{8\beta}\right)$$

$$\overline{U}_R^S = \theta\left(\frac{{A_H}^2}{16\beta}\right) + (1-\theta)\left(\frac{{A_L}^2}{16\beta}\right)$$

$\overline{U}_C^S = \theta\left(\frac{3A_H{}^2}{16\beta}\right) + (1-\theta)\left(\frac{3A_L{}^2}{16\beta}\right)$

$\overline{U}_M^N = \left(\frac{(\theta A_H+(1-\theta)A_L)(A_H+A_L)}{4\beta}\right) - \left(\frac{(\theta A_H+(1-\theta)A_L)^2}{4\beta}\right)$

$\overline{U}_C^N = \left(\frac{A_H{}^2}{4\beta}\right) + \left(\frac{A_L{}^2}{4\beta}\right) - \left(\frac{(\theta A_H+(1-\theta)A_L)^2}{8\beta}\right)$

$\overline{U}_R^N = \left(\frac{A_H{}^2}{4\beta}\right) + \left(\frac{A_L{}^2}{4\beta}\right) - \left(\frac{(\theta A_H+(1-\theta)A_L)(A_H+A_L)}{4\beta}\right) + \left(\frac{(\theta A_H+(1-\theta)A_L)^2}{8\beta}\right)$

For the retailer,

$\overline{U}_R^N - \overline{U}_R^S = \left(\frac{(A_L-\theta(A_H-A_L)(A_L)}{4\beta}\right) + 3(1-\theta)\left(\frac{A_H(A_H-A_L)}{16\beta}\right) + (1-\theta)^2\left(\frac{A_H(A_H-A_L)}{16\beta}\right) + (1-\theta)^2\left(\frac{A_L{}^2}{16\beta}\right) + \theta(1-\theta)\left(\frac{A_L(A_H-A_L)}{16\beta}\right) + 3\theta(1-\theta)\frac{A_HA_L}{16\beta} + \theta^2\left(\frac{A_H{}^2}{16\beta}\right)$

To compare $\overline{U}_R^{N\text{I}}$ and $\overline{U}_R^S$, we need to examine $\overline{q}_L^N = max\left(\frac{A_L}{2} - \frac{\theta A_H+(1-\theta)A_L}{4}, 0\right)$ so $if\ \overline{q}_L^N > 0$, implies $\theta < \frac{A_L}{A_H-A_L}$. Consequently, the term $\left(\frac{((A_L-\theta(A_H-A_L))}{4\beta}\right)$ remains positive, and the other components$3(1-\theta)\left(\frac{A_H(A_H-A_L)}{16\beta}\right) + (1-\theta)^2\left(\frac{A_H(A_H-A_L)}{16\beta}\right) + (1-\theta)^2\left(\frac{A_L{}^2}{16\beta}\right) + \theta(1-\theta)\left(\frac{A_L(A_H-A_L)}{16\beta}\right) + 3\theta(1-\theta)\frac{A_HA_L}{16\beta} + \theta^2\left(\frac{A_H{}^2}{16\beta}\right)$ are evidently positive as well. Therefore,

$\overline{U}_R^N - \overline{U}_R^S \geq 0$

For the manufacturer,

$$\overline{U}_M^N - \overline{U}_M^S = \theta(1-\theta)\left(\frac{(A_H-A_L)^2}{4\beta}\right) + \left(\frac{(A_H(A_L-\theta(A_H-A_L))}{8\beta}\right) + \left(\frac{(1-\theta)A_L(A_H-A_L)}{8\beta}\right)$$

We know $\overline{q}_L^N = max\left(\frac{A_L}{2} - \frac{\theta A_H+(1-\theta)A_L}{4}, 0\right)$. Thus, $\overline{q}_L^N > 0$ indicates that $\theta < \frac{A_L}{A_H-A_L}$. As a result, the expression $\left(\frac{(A_H(A_L-\theta(A_H-A_L))}{8\beta}\right)$ becomes positive. Additionally, the expressions $\theta(1-\theta)\left(\frac{(A_H-A_L)^2}{4\beta}\right)$ and $\left(\frac{(1-\theta)A_L(A_H-A_L)}{8\beta}\right)$ are always positive. Consequently, the inequality $\overline{U}_M^N - \overline{U}_M^S \geq 0$ holds true. ■

Proof of Proposition 5:

To compare the performance of members in scenarios with or without information sharing, we examine profits under two linear price contracts and contract menus

a) **Information Sharing Between Manufacturer and Retailer (S):**

We have

$$U_M^S - \overline{U}_M^S = \theta\left(\frac{{A_H}^2}{8\beta}\right) + (1-\theta)\left(\frac{{A_L}^2}{8\beta}\right) \geq 0$$

$$\overline{U}_R^S - U_R^S = \theta\left(\frac{{A_H}^2}{16\beta}\right) + (1-\theta)\left(\frac{{A_L}^2}{16\beta}\right) \geq 0$$

$$U_C^S - \overline{U}_C^S = \theta\left(\frac{{A_H}^2}{16\beta}\right) + (1-\theta)\left(\frac{{A_L}^2}{16\beta}\right) \geq 0$$

b) **Lack of Information Sharing:**

We have

$$\overline{U}_M^N - U_M^{NI} = \frac{\theta^3(A_H - A_L)(A_H - 2A_L)}{4\beta(1-\theta)^2} + \frac{\theta^3(1-\theta)(A_H - A_L)^2}{4\beta(1-\theta)^2} + \frac{(1-2\theta)^2 A_L(A_H - A_L)}{4\beta(1-\theta)^2} + \frac{\theta(1-\theta)(A_L - \theta A_H)(A_H - A_L)}{4\beta(1-\theta)^2}$$

To simplify the expression, we can factor out the positive term $\frac{(A_H - A_L)}{4\beta(1-\theta)^2}$. Thus, $\overline{U}_M^N - U_M^{NI} = \theta^3(A_H - 2A_L) + \theta^3(1-\theta)(A_H - A_L) + (1-2\theta)^2 A_L + \theta(1-\theta)(A_L - \theta A_H)$. The term $\theta^3(1-\theta)(A_H - A_L)$ is positive, and for $\theta(1-\theta)(A_L - \theta A_H)$ according to Proposition 2, the range for θ is $\theta \leq \frac{A_L}{A_H}$. Thus, the expression $A_L - \theta A_H \geq 0$ holds true. Then for the expressions $\theta^3(A_H - 2A_L)$ it is positive If $A_H \geq 2A_L$; otherwise, it can be verified that

$\theta^3(A_H - 2A_L) + \theta^3(1-\theta)(A_H - A_L) + (1-2\theta)^2 A_L + \theta(1-\theta)(A_L - \theta A_H)$ is positive in range $\theta \leq \frac{A_L}{A_H}$.

Furthermore,

$$\overline{U}_M^N - U_M^{N\mathrm{II}} = \frac{(1-2\theta)(A_L A_H)}{4\beta} + \theta^2\left(\frac{A_H(2A_L - A_H)}{4\beta}\right) + \theta(1-\theta)\frac{{A_L}^2}{4\beta}$$

The expression $\theta(1-\theta)\frac{{A_L}^2}{4\beta}$ is consistently positive. Additionally, $\frac{(1-2\theta)(A_L A_H)}{4\beta}$ exhibits a decreasing trend concerning the value of θ and remains positive in the interval where $\theta \leq \frac{1}{2}$. The expression $\theta^2\left(\frac{A_H(2A_L-A_H)}{4\beta}\right)$ demonstrates an upward trajectory with the increase of θ. If $2A_L \geq A_H$, which aligns with the conditions specified in the proposition 2, and considering $\theta \geq \frac{A_L}{A_H}$, we can deduce that $\theta \geq \frac{1}{2}$, ensuring the perpetually positive nature of this expression. Conversely, if $2A_L \leq A_H$, which constitutes the limit of the theorem, and under the condition $\theta \geq \frac{A_L}{A_H}$, we encounter the expression $\frac{(A_H-2A_L)(A_L)(A_H-A_L)}{4\beta}$, which holds true. Consequently, the expression $\overline{U}_M^N - U_M^{N\text{II}} \geq 0$ is consistently valid.

$$\overline{U}_R^N - U_R^{N\text{I}} = (1-\theta)\left(\frac{(A_H-A_L)^2}{4\beta}\right) + \frac{(1-\theta)A_L(A_H-A_L)}{4\beta} + \left(\frac{\theta^2(A_H-A_L)^2}{8\beta}\right) + \frac{\theta^2(A_H-A_L)(A_L-\theta A_H)}{4\beta(1-\theta)^2} + \frac{\theta(A_H-A_L)(\theta A_H)}{8\beta(1-\theta)} + \frac{(A_L-\theta A_H)(\theta A_L)}{8\beta(1-\theta)} + \frac{(A_L-\theta A_H)^2}{8\beta(1-\theta)}$$

The expression $(1-\theta)\left(\frac{(A_H-A_L)^2}{4\beta}\right) + \frac{(1-\theta)A_L(A_H-A_L)}{4\beta} + \left(\frac{\theta^2(A_H-A_L)^2}{8\beta}\right) + \frac{\theta(A_H-A_L)(\theta A_H)}{8\beta(1-\theta)}$ are positive. According to Proposition 2, the range for θ is $\theta \leq \frac{A_L}{A_H}$. Consequently, the expression $A_L - \theta A_H \geq 0$ holds true. In this manner, the positive nature of $\frac{\theta^2(A_H-A_L)(A_L-\theta A_H)}{4\beta(1-\theta)^2} + \frac{(A_L-\theta A_H)(\theta A_L)}{8\beta(1-\theta)} + \frac{(A_L-\theta A_H)^2}{8\beta(1-\theta)}$ is also established. Hence, $\overline{U}_R^N - U_R^{N\text{I}} \geq 0$ holds true.

$$\overline{U}_C^N - U_C^{N\text{I}} = (1-\theta)\left(\left(\frac{{A_H}^2}{8\beta}\right) - \left(\frac{{A_L}^2}{8\beta}\right)\right) + \left(\frac{A_L(A_L-\theta A_H)}{8\beta}\right) + \frac{\theta^3(A_H-A_L)^2}{8\beta(1-\theta)} + \theta\left(\frac{(A_H-2A_L)^2}{8\beta}\right) + (1-\theta)\frac{(A_H-A_L)A_H}{8\beta} + \frac{(A_H-A_L)A_L}{8\beta}$$

To simplify, we factor out the positive term $\frac{1}{8\beta(1-\theta)}$, resulting in:

$$(1-\theta)^2\left({A_H}^2 - {A_L}^2\right) + ((1-\theta)A_L(A_L-\theta A_H)) + \theta^3(A_H-A_L)^2 + \theta(1-\theta)(A_H-2A_L)^2 + \theta(1-\theta)(A_H-A_L)A_H + (1-\theta)(A_H-A_L)A_L$$

based on the Proposition 2 and the condition $\theta \leq \frac{A_L}{A_H}$ the expression $\frac{A_L(A_L-\theta A_H)}{8\beta}$ is positive. The other expression $(1-\theta)^2\left({A_H}^2 - {A_L}^2\right) + \theta^3(A_H - A_L)^2 + \theta(1-\theta)(A_H - 2A_L)^2 + \theta(1-\theta)(A_H - A_L)A_H + (1-\theta)(A_H - A_L)A_L$ are all positive. Therefore, $\overline{U}_C^N - U_C^{N\mathrm{I}} \geq 0$ holds true.

$$\overline{U}_C^N - U_C^{N\mathrm{II}} = (1-\theta)(\theta)\left(\frac{(A_H - A_L)^2}{8\beta}\right) + (2-3\theta)\left(\frac{{A_H}^2}{8\beta}\right) + (1+\theta)\left(\frac{{A_L}^2}{8\beta}\right)$$

The expression $(1-\theta)(\theta)\left(\frac{(A_H-A_L)^2}{8\beta}\right) + (1+\theta)\left(\frac{{A_L}^2}{8\beta}\right)$ yields a positive value. Additionally, $(2-3\theta)\left(\frac{{A_H}^2}{8\beta}\right)$ is positive as θ ranges between 0 and 2/3, For values of θ greater than 2/3, it is noteworthy that the expression remains positive until $\theta\left(\frac{{A_H}^2}{8\beta}\right) \leq (1-\theta)(\theta)\left(\frac{(A_H-A_L)^2}{8\beta}\right) + 2(1-\theta)\left(\frac{{A_H}^2}{8\beta}\right) + (1+\theta)\left(\frac{{A_L}^2}{8\beta}\right)$ Furthermore, considering that $(2-3\theta)\left(\frac{{A_H}^2}{8\beta}\right)$ reaches its minimum at $\theta = 1$, and $\theta \geq \frac{A_L}{A_H}$ based on Proposition 2 $\overline{U}_C^N - U_C^{N\mathrm{II}} \geq 0$ holds true.■