

A New Fuzzy System Approach Based on Z-Number to Solve MADM Problems

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Abstract

Fuzzy rule-based systems have been applied as a powerful approach to deal with uncertainty in multi-attribute decision-making (MADM) problems. However, according to experts' opinions and decision making rules, most studies utilizing fuzzy rule-based systems do not involve reliability. Indeed, considering the reliability of information makes decisions more accurate. To do so, the concept of Z-number as a key role to obtain more precise decisions is applied as an efficient approach in the decision-making process in the description of real-world information. This paper proposes an approach to design a fuzzy rule-based system under Z-number for solving multi-attribute group decision-making (MAGDM) problems. As a case-based problem, a stock selection problem on a research is investigated as a factor in financial portion. Validity and verifying of the proposed approach is demonstrated with importing the data of a numerical example of a research and comparative analysis for the problem at hand.

Keywords: Z-number, Fuzzy systems, MADM, Group decision making, Linguistic variables.

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1. Introduction

The decision making process is one of the most important tasks that has received a lot of attention in many domains such as in engineering, management, mathematics, agriculture and related problems in other fields (Zavadskas et al., 2016; Salih et al., 2019). However, decision making process in the field of engineering often takes place in a complex environment and involves multiple conflicting criteria. Multi criteria decision-making (MCDM) techniques can be roughly divided into two main groups, multiple objective decision-making (MODM) problems and multi-attribute decision-making (MADM) dealing with complex engineering problems (Tavana & Hatami-marbini, 2011).

In the MADM problems, the decision-maker (DM) attempts to choose the best alternatives which are characterized by a set of multiple attributes. In real-world MADM issues, it is hard to consider all the aspects of decision problems for the individual DMs due to the limited practical experience and knowledge. Thus, to ensure the accuracy and rationality of decisions, multiple experts are used about the specific field as DMs or group decision making (GDM) play a key role in this respect (Tavana & Hatami-Marbini, 2011; Ashoka and Keihani, 2020).

In real world situations, experts express their opinions about alternatives by using linguistic variables (Zadeh, 1975; Chen, 2000). Naturally, performed linguistic variables are considered imprecise, vague, uncertain or incomplete data. Fuzzy set theory introduced by Zadeh (zadeh, 1965), provides a powerful tool for dealing with ambiguous data. Also, this theory is used to model uncertainty in decision making models happening owing to lack of perfect information. Over the years, there have been successful applications and implementations of fuzzy set theory in MADM. To do so, a wide range of studies have combined MADM techniques with fuzzy set theory called fuzzy MADM (FMADM) (Mardani et al., 2015). Some of them are fuzzy TOPSIS (Su et al., 2013; Ashtiani et al., 2009; Liao & Kao, 2011), fuzzy AHP (Kahraman & Cebeci, 2003; Chen & Yang, 2011; Chen, 2015), fuzzy VIKOR (Å & Wang, 2009; Shemshadi et al., 2011), fuzzy ELECTREE (Taylor & Sevkli, 2010; Rougendegh & Erkan, 2012), fuzzy ANP (Vinogh et al., 2011; Amy & Yang, 2012; Pang & Bai, 2013) and fuzzy PROMETHEE (Chen et al., 2011; Gupta et al., 2012).

It is logical and acceptable that reliable information will result in reliable and accurate decisions (Sari, 2017). Because of different knowledge, expertise and experience, reliability of experts' opinions and judgments have become an essential property of information. Therefore, in real problem situations it is necessary to considering the reliability of information in decision making process in order to arrive at suitable decisions. Zadeh (Zadeh, 2011) introduced the Z-number concept as a more adequate notion for the description of real-world information. This idea has the potential for important applications in decision making process which is used in many areas especially in the field of engineering and economic problems.

In everyday decision-making, most decisions are in the form of Z-numbers. The Z-numbers have more potential to explain human knowledge compared to the classical fuzzy numbers. Thus, some researchers have investigated the concept of Z-number in decision-making problems. (Kang et al., 2012a) presented a method for multi-criteria decision-making using Z-numbers. Criteria weights and criteria values of alternatives were provided as Z-numbers in the proposed methodology. To simplify and diminish computational complications, they converted Z-numbers to crisp values and then ranked the alternatives (Kang et al., 2012b). (Aliev & Zeinalova, 2014; Nozari et al., 2021) suggested a method for making decisions under uncertainty in which decision-relevant information is characterized by Z-numbers. In the proposed decision analysis, Z-numbers were first transformed to fuzzy numbers by using the methodology given in (Kang, 2012a), then they computed the fuzzy utility function values for alternatives, and the best was chosen based on the fuzzy utility value. (Azadeh *et al.*, 2013) propounded an application of the AHP approach beneath the Z-number. (Zeinalova, 2014) developed an approach founded on the concept of expected utility to solve the MADM problem under Z-information. (Yaakob & Gegov, 2015) introduced Z-TOPSIS, a unique variation of the TOPSIS approach to ease MCDM issues based on the notion of Z-numbers. They used a fuzzy rule basis technique to modify the Z-TOPSIS technique proposed in (Sotoudeh-Anvari & Sadi-Nezhad, 2015). In (Yaakob & Gegov, 2015) the authors feed data into TOPSIS and utilized the results to create rules to improve their capacity to cope with ambiguity and reflect judgments more accurately. The main objective of this rectification was to demonstrate the high ability of the fuzzy rule-based system. Indeed, the proposed technique demonstrated that the experts' empirical

knowledge may be taken into account in the decision-making process by using a fuzzy rule. (Peng et al., 2019) introduced a reliable tool to solve hesitant uncertain linguistic Z-numbers problems which integrated linguistic variables using VIKOR and ERP. As well, (Azadeh & Kokabi, 2016) proposed a Z-number DEA model to transform into possible linear programming by applying an alternative α -cut approach. (Qiao *et al.*, 2020) proposed a simple computational method to rank Z-numbers for solving MADM problems by extending the PROMETHEE technique. In addition, (Shen *et al.*, 2019) combined the classical Multi-Attribute Border Approximation area Comparison (MABAC) approach with a Z-number environment to answer choice issues using the directed distance and regret theory. In (Peng et al., 2019), a simple and efficient technique for modeling Z-number with low computational complexity has been used to solve game problems. In (Sotoudeh-Anvari & Sadi-Nezhad, 2015), strategies from different players were compared systematically. (Nuriyev, 2020) proposed the Z-numbers hybrid method with the aid of TOPSIS, PROMETHEE, and AHP techniques for ranking and selection of energy resources. (Dong et al., 2020) proposed a unique technique for solving evaluation alternatives when attribute values are extreme by integrating the best-worst method (BWM), the maximizing deviation method (MDM), and the reference ideal method (RIM) in a Z-number environment.

On the other hand, one of the most key challenges in decision making process is to estimate utility function of DM. It is usual and convenient for DM to present his/her knowledge, utility and preferences as linguistic terms in form of IF-THEN rules. Fuzzy rule-based system (also known as fuzzy inference system) based on IF-THEN rules has been widely implemented to improve intelligence system for modeling experts' knowledge and utilities. Ease the computing process to solve complex issues in MADM is emphasized by the researchers (Wang, 2015; Sadi-Nezhad & Sotoudeh-Anvari, 2015; Kang et al., 2018). For this reason, fuzzy rules base systems due to simplicity and flexibility (Amindoust et al., 2012; Junior et al., 2013) can better overcome to this problem. Indeed, multiple studies have been devoted to solve MADM problems by means of fuzzy rule-based systems such as (Carrera & Mayorga, 2008; Amindoust et al., 2012; Junior et al., 2013; Kumar et al., 2013; Rezaei & ortt, 2013; Ghadimi et al., 2017; Zanon et al., 2019).

Literature review on both field of fuzzy inference system based MADM methods and decision making under Z-numbers reveals a gap. Indeed, that means level of reliability as a critical property of information has been disregarded in FRBS reviewed papers. With considering experts' opinion by Z-numbers and in form of linguistic can able to increase the level of reliability in make-decisions in order to overcome this problem. In this study, to solve multi-attribute group decision-making (MAGDM) problems a new approach is designed through fuzzy rule-based system under Z-numbers in order to make more precise decisions.

The main reasons motivating to combine Z-number concept and fuzzy rule-based system are: enhancing accuracy in decision making by paying attention to the Z-number concept, modeling utility function and preferences of DM by applying fuzzy rule-based systems using IF-THEN rules and also removing computational complexity utilized in papers related to Z-number and decision making by combining fuzzy rule-based system and Z-number.

As a summary, the main contributions are presented as (1) an approach is proposed to consider Z-rule in MADM problem for ranking with simplifying the computational process. (2) This paper considers experts' opinions in form of Z-number, including two components, performance rating and its reliability. (3) Experts' opinions are collected in form of fuzzy linguistic variables, whereas most studies considered crisp value for experts' opinions (Zanon et al., 2019). (4) A MADM problem under GDM is considered and is solved with the propose approach MAGDM.

The rest of the paper is organized as follows. Preliminary concepts including fuzzy set theory, concept of Z-number and basis of fuzzy rule base system are defined in Section 2. In Section 3, the propose approach is demonstrated to solve MADM problem. In Section 4, the validity and verifying of the proposed method is presented by using data imported from an evaluated research. Finally, the conclusion is given in Section 5.

2. Preliminaries

In this section, the main idea of Z-number, and some basic concepts and definitions related to fuzzy system is reviewed which will be needed for the proposed method of selection process.

2.1 Fuzzy Set Theory

Fuzzy set theory has been studied extensively over the past 45 years. It is pertained to represent uncertainty in human cognitive processes and it is applied to problems in engineering, medical, businesses, and other sciences. Fuzzy set theory is an established and growing research discipline and is useable for modeling and analyzing decision systems and its ability to quantitatively and qualitatively model problems that involve vagueness and imprecision (Carrera & Mayorga, 2008). Zadeh has introduced the fuzzy set theory to cope up with the imprecision and uncertainty which is inherited from the human judgments in decision-making processes through the use of linguistic terms and degrees of membership. A fuzzy set is a class of objects with grades of membership. A normalized membership function is between zero and one (Zadeh, 1965). These grades present the degree of stability with which a special element belongs to a fuzzy set. To express fuzzy sets from the mathematical point of view, consider a set of objects X . The set is explained as follows:

$$X = \{x_1, x_2, \dots, x_n\}; \quad (1)$$

Where x_i is an element in the set X . A membership value (μ) expresses the grade of membership related to each element x_i in a fuzzy set A which shows a combination as below:

$$A = \{\mu_1(x_1), \mu_2(x_2), \mu_3(x_3), \dots, \mu_n(x_n)\} \quad (2)$$

The key reasons why fuzzy set theory is relevant to this area of research including: firstly, imprecision and vagueness are inherent to the decision maker's mental model of the problem under this study. Secondly, in making decisions environments, the information required to model a decision might be vague or not precisely measurable. Thirdly, impression, vagueness, and subjective opinion may further dampen the quality and quantity of available information. Also, the fuzzy set theory is applied to consider the decision makers' preferences to performance indicators to calculate the weights of them and also about the suppliers' performance concerning these indicators.

2.2 Concept of Z-number

In the real life decision making situations, dealing with uncertainty is an unavoidable aspect for decision makers. In most of the real decision making cases, experts are not able to give their knowledge and experience as exactly as numerical values. In such cases, linguistic assessments are alternatively used instead of numerical values to express preference (Adamopoulos & Pappis, 1996; Sets et al., 1997; Wang & chuu, 2004). Fuzzy set theory is one of the well-known frameworks to encounter with uncertainty, vagueness and ambiguity and to model linguistic variables. It is important to mention that reliable information significantly affects the reliability of decision making. In view of this, Zadeh in (Zadeh, 2011) suggested the concept of a Z-number as a more adequate concept for the description of real-world information. A Z-number is an ordered pair of fuzzy numbers denoted as $Z = (\tilde{A}, \tilde{B})$. The first component, \tilde{A} , is a restriction on the values which a real-valued uncertain variable, X is allowed to take. The second component, \tilde{B} , is a measure of reliability (certainty) of the first component. Typically, \tilde{A} and \tilde{B} are described in a natural language and are also fuzzy numbers. Consider a company which wants to assess his/her own potential suppliers. DM is asked to evaluate the suppliers with respect to a given criterion. Assume that DM gives his/her opinion as "very perfect". Then, DM is asked about reliability (certainty) of his/her opinion. Let's consider his answer is "likely". This evaluation can be formalized as a Z-number " X is $Z = (\tilde{A}, \tilde{B})$ ", where X is the evaluation score of criterion, \tilde{A} is *very perfect* as a fuzzy set is used to describe X and \tilde{B} is *likely* as a fuzzy number is used to describe reliability of \tilde{A} . The example mentioned above is a simple form of Z-number and is demonstrated in Figure 1. Other simple examples of Z-number are as follows (Zadeh, 2011):

(Anticipated budget deficit, close to 2 million dollars, very likely)

(Population of Spain, about 45 million, quite sure)

(Price of oil in the near future, significantly over 100 dollars/barrel, very likely)

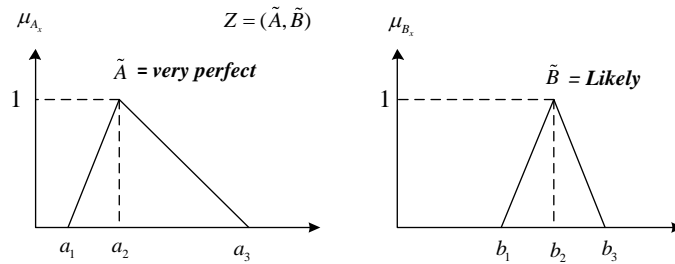


Figure 1. A simple Z-number

Ability to model experts' knowledge in form of fuzzy IF-THEN rules is one of the robustness of fuzzy logic in many applications. Generally, a simple fuzzy rule can be expressed as: *if X is A Then Y is B*, where *A* and *B* are fuzzy numbers. The meaning of such a rule is defined as:

$$\text{if } X \text{ is } A \text{ then } Y \text{ is } B \rightarrow (X, Y) \text{ is } A \times B \quad (3)$$

Where, $A \times B$ are the Cartesian product of *A* and *B*. Zadeh in (Zadeh, 2011) introduced the concept of *Z-rules – IF-THEN rules* in which the antecedents and/or consequents involve Z-numbers. A Z-rule can be shown as:

$$\text{If } (X, A_X, B_X) \text{ then } (Y, A_Y, B_Y) \quad (4)$$

Where, A_X and A_Y are restrictions on *X* and *Y*, respectively. Also, B_X and B_Y are reliabilities on A_X and A_Y , respectively. Let's consider a company wants to evaluate the performance rating of suppliers in terms of the suppliers' relationship. A rule may be denoted as "if (relationship, High, very likely) then (performance rating, moderate, very likely)". A *Z-rule- IF-THEN rule* may be expanded as:

$$\text{If } (X_1, A_{X1}, B_{X1} \text{ and/or } X_2, A_{X2}, B_{X2} \text{ and/or } \dots \text{ and/or } X_n, A_{Xn}, B_{Xn}) \text{ then } (Y, A_Y, B_Y) \quad (5)$$

Where, X_1, X_2, \dots, X_n are input variables and *Y* is output variable.

2.3 Basis of Fuzzy Rule Base Systems

A fuzzy rule-based system (also known as a fuzzy inference system) is a system that can formulate the map of the input variables to an output variable. Generally speaking, the fuzzy rule-based systems are categorized into three types (Rezaei & Ortt, 2013b): the Mamdani fuzzy rule-based system (Mamdani & Assilian, 1975), the Sugeno fuzzy model (Takagi & Sugeno, 1985), and the Tsukamoto fuzzy models (Tsukamoto, 1979). The Sugeno method in interpreting information in form of linguistic has difficulties while, using the Mamdani method to model human expert knowledge has better performance (Jang, 1993) and it is more popular and practical (Lee, 1990). A fuzzy rule-based system is composed of four functional blocks (Jang, 1993), as shown in Figure 2.

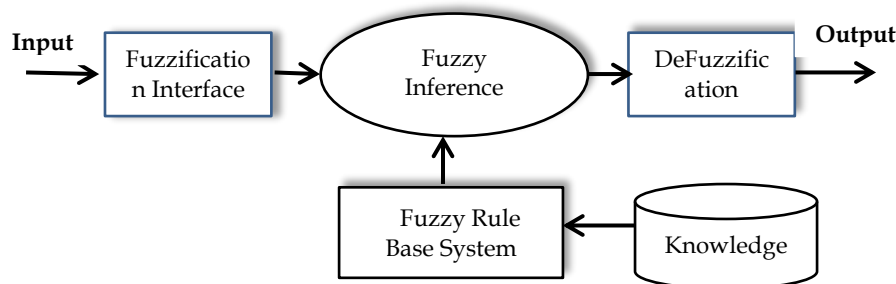


Figure 2: The Mamdani's fuzzy rule base system (Osiro et al., 2014)

The fuzzification interface involves measuring the values of input variables, performing a scale mapping that transfers the range of values of input variables into corresponding universes of discourse, and performing the function of fuzzification that convert input data into suitable linguistic values. The knowledge base refers to the opinion of experts related to an application domain that governs the relationships between inputs and outputs variables. These relationships will be illustrated in form of rules. Containing the number of fuzzy IF-THEN rules and the membership functions of the

fuzzy sets is used in the fuzzy rule base system. The fuzzy rule base is developed by the experience and knowledge of one or more experts. A typical fuzzy rule has the form:

IF antecedent THEN consequent

The number of rules is a function of the number of the input variables and the number of linguistic values that have been defined for inputs. If there are n inputs and k linguistic values for each input, then the total number of rules is k^n . The Fuzzy inference system is as a system controller and the decision-maker performs the inference operations on the rules. This section simulates human decision-making in inferring fuzzy control actions based on the rules of inference in fuzzy logic. The defuzzification interface transforms the fuzzy results of the inference. The output of the fuzzy inference engine is a fuzzy number, but decision-makers need a crisp number. The defuzzification interface, which is the final operation, converts the fuzzy output into a crisp output by using various methods.

3. Proposed the Z-FRBS Method

As described in the introduction, data uncertainty is the degree to which data is inaccurate and untrusted. The inadequacy of information in expert opinion is one of the degrees of uncertainty that leads to inaccuracy in decision making. To increase the level of reliability and complete the information using experts' opinion in form of Z-numbers to make more precise decisions is one of the ways that are effective in the certainty of make decisions.

Therefore, the main objective in this research is that by using FRBS under Z-numbers (is called Z-FRBS method) MADM problems can be solved with more reliable. The Z-FRBS method is proposed a general solution to solve group MADM by using FRBS with considering Z-numbers in it. This method can solve complex issues raised in decision-making with multi-criteria each of which contains several sub-criteria with multi-attribute. So the experts evaluate alternatives concerning the sub-criteria in form of Z-numbers. Thus, the rating of each alternative to sub-criteria is an ordered pair of fuzzy numbers, $Z = (\tilde{A}, \tilde{B})$, where \tilde{A} is the performance rating of alternatives concerning the sub-criteria in the form of linguistic variables and also \tilde{B} is a linguistic variable that exhibits the reliability of experts on their opinion. Since the input values have two components (performance rating and reliability), it is reasonable that the final score of the alternatives contains two components (score of performance and reliability).

Estimating the utility function of DM is an important issue in the decision-making process, it is usual more convenient for a decision-maker to present his/her knowledge as linguistic terms in the form of IF-THEN rules. As a main contribution a new approach is proposed to establish a fuzzy rule-based system under Z- number. A fuzzy IF-THEN rule under Z- number may be exhibited as:

If X_1 is (A_{X1}, B_{X1}) and X_2 is (A_{X2}, B_{X2}) then Y is (A_Y, B_Y)

Where, A_{X1} , A_{X2} , and A_Y are restricted on variables of X_1 , X_2 , and Y , respectively. Also, B_{X1} , B_{X2} , and B_Y are reliabilities on A_{X1} , A_{X2} , and A_Y , respectively. It is rational and acceptable that the measure of reliability in antecedents influences the amount of reliability in consequents. It means that if an expert expresses his opinion with high certainty as an input of a rule, the output of a rule has more strength and reliability. In other words, the reliability of output is dependent on the reliability of the input. Therefore, is concluded that the aforementioned rule can be divided into two rules:

if X_1 is (A_{X1}, B_{X1}) and X_2 is (A_{X2}, B_{X2}) then Y is (A_Y, B_Y)

↓

(a) *if* X_1 is A_{X1} and X_2 is A_{X2} then Y is A_Y

(b) *if* X_1 is B_{X1} and X_2 is B_{X2} then Y is B_Y

Where rule (a) is defined in the domain of variables and rule (b) is defined in the domain of reliabilities. The range of output variable in rule (a) depends on its nature but in rule (b) it is between 0 and 1. Therefore, in our approach two fuzzy rule-based systems are considered to determine the performance score of alternatives and to determine the value of reliability.

The graphical separation scheme for these two parts is shown in Figure 3.

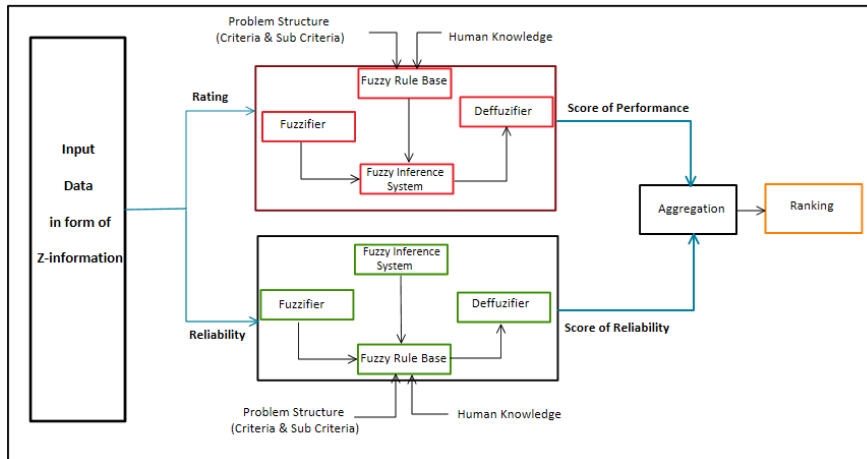


Figure 3: Scheme of separation rating and reliability in fuzzy system

Before describing the computational, the stages of the proposed approach are assumed that DM has determined suitable criteria, potential alternatives, and a committee of experts. And assume that experts have been asked about the performance rating of the alternative concerning sub-criteria and their opinions have been collected in the form of Z-numbers.

A graphical scheme of the proposed approach is depicted in Figure 4. The overall block diagram of it is shown in Figure 4 has been implemented by MATLAB programming software to pass to the method through four stages. The methodology followed during this research is shown in Figure 4.

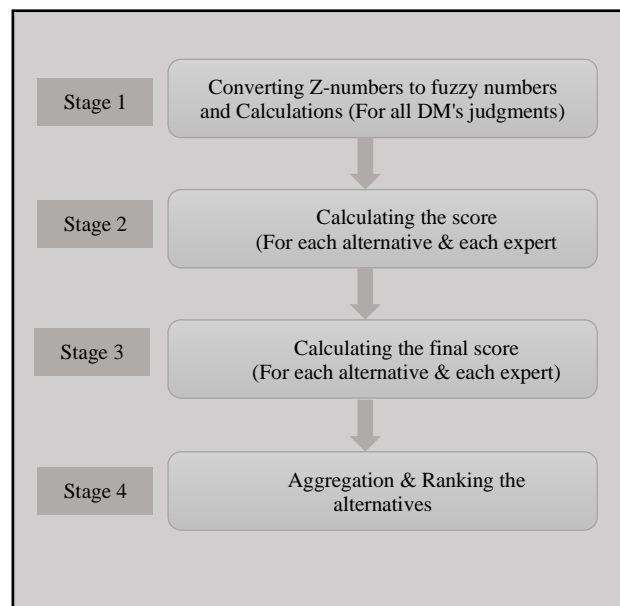


Figure 4: The Methodology of the proposed method

Stage1. In this stage, collected opinions in the form of Z-numbers are converted into appropriate fuzzy numbers. Next, fuzzy numbers of each sub-criterion are multiplied by the weight of the sub-criterion. And then weighted fuzzy numbers are converted into crisp values as inputs of fuzzy rule-based systems for the performance and the reliability separately.

Meanwhile, in this proposed method linguistic variables are defined by triangular fuzzy number (TFN). The TFN membership functions, because it gives a reasonably good performance in terms of theoretical calculations as compared to other shapes. In addition, TFN is widely used in decision-making because of its intuitive membership functions and computational simplicity (Madi et al., 2017). A TFN, denoted as $M = (L, M, U)$, can be defuzzified to a crisp number as follow (Guo & Zhao, 2017; Kang, Hu, Deng, & Zhou, 2016):

$$Crisp = \frac{(L+4 \times M+U)}{6} \quad (6)$$

Stage2. The Score of performance and reliability of each main criterion for each alternative and each expert is calculated using a designed fuzzy rule-based system. The crisp values determined in stage1 are inputs of two fuzzy systems. “Rules” play a vital role in fuzzy expert systems. In our approach, DM defines the fuzzy IF-THEN rules based on his/her experience and utility. After designing two fuzzy rule bases of performance and reliability are utilized the Mamdani method.

Stage3. The final score of performance and its reliability is calculated for each alternative for each expert. To do so, two fuzzy systems are utilized to estimate the score of performance and its reliability. Outputs of the second stage are considered as input values of fuzzy systems in stage 3. Considering the importance of criteria is an important issue in decision-making process. Hence the importance of criteria is specified by DM in the form of linguistic variables. To consider the importance of criteria, the fuzzy weight of criteria is defuzzified to the crisp number and multiplied by its related output value in stage 2.

Stage4. In this stage, all outputs of the fuzzy system collected are aggregated by the ordered weighted averaging (OWA) operator that is a common aggregation operator (Rosanisah et al., 2017). The OWA has been widely used because of its ability to model linguistically expressed aggregation. In final ranking of alternatives is performed using an expected value concept. To do so, for each alternative, an expected value is computed as the follow equation.

$$E_i = \sum_k^{k=1} y_p^{ik} \cdot y_r^{ik} \quad ; \quad i = 1, 2, \dots, s \quad (7)$$

Based on the calculated expected values, the alternatives are ranked in descending order.

4. Validation and Verifying

To show the validity of the Z-FRBS proposed method and its performance, it will be compared and evaluate with a method that has used FRBS systems and Z-number inside of its method. Also, to investigate that the proposed method has high performance and it has strong validity so is needed to find research that, firstly, it has been used conventional method and fuzzy rules to solve MADM problems, and secondly, it has been used a statistical method to measure performance. Unfortunately found only one paper that had these features. Eventually, was founded (Yaakob & Gegov, 2015) paper which is named Yaakob's method. The approach by the name of Z-FRBS TOPSIS was introduced in Yaakob's method, which has been modified through FRBS. They illustrated empirical knowledge of an expert which can be taken into account in the decision-making process through using FRBS.

To show the accuracy and the validation of the ranking method in Yaakob's method which has been used Spearman rho correlation (SRC) statistical method. The reasons for using this method include the simplicity in terms of algebraic structure, intuitively simple interpretation, and less sensitive to bias due to the outlier's effect to reduce the outlier's weight. In general, the coefficient of rho (ρ) measures the strength of association between two ranked variables. The formula used to calculate Spearman's rank is shown below.

$$\rho = 1 - \frac{6 \sum \partial_i^2}{n^3 - n}$$

Where ∂_i represents the difference between the ranks and n shows number of alternatives considered. The Spearman correlation coefficient (ρ) can take values between +1 to -1. If $\rho = 1$ indicates a perfect relationship of ranks if $\rho = 0$ shows no relationship between ranks and $\rho = -1$ indicates a perfect negative association of ranks. The closer ρ is to zero, the weaker the relationship between the ranks.

The analysis of Spearman rho correlation for Yaakob's method is shown in Table 1. The values of this table are exactly taken from the output of Table 7 from (Yaakob & Gegov, 2015) paper. According to this table, the output of the new Yaakob's method is compared with the previous

method. This table indicated that the value of SRC for the previous method (Z-TOPSIS) when FRBS has not been used, is equal to 0.845 and when in their new proposed method (Z-FRBS TOPSIS) which FRBS has been used, it is equal to 0.850. This betterment shows that the use of FRBS has increased the accuracy of the decision-making process. The last step in YAAKOB's method is the ranking order of all alternatives base on the TOPSIS technique is presented in Table 1.

Table 1: TOPSIS Ranking Performance based on (Yaakob & Gegov, 2015) method with its previous method which evaluated with Spearman Rho Correlation

STOCK	Rank	Z-TOPSIS			Z-FRBS TOPSIS		
	Actual	Rank	∂_i	∂_i^2	Rank	∂_i	∂_i^2
S1	2	7	-5	25	5	-3	9
S2	4	9	-5	25	10	-6	36
S3	1	1	0	0	1	0	0
S4	21	20	1	1	20	1	1
S5	19	24	-5	25	24	-5	25
S6	11	8	3	9	9	2	4
S7	17	17	0	0	16	1	1
S8	24	18	6	36	18	6	36
S9	23	25	-2	4	25	-2	4
S10	22	14	8	64	14	8	64
S11	8	13	-5	25	13	-5	25
S12	13	16	-3	9	15	-2	4
S13	25	22	3	9	22	3	9
S14	9	10	-1	1	8	1	1
S15	3	3	0	0	3	0	0
S16	5	2	3	9	2	3	9
S17	18	21	-3	9	21	-3	9
S18	12	19	-7	49	19	-7	49
S19	15	11	4	16	11	4	16
S20	16	15	1	1	17	-1	1
S21	7	4	3	9	4	3	9
S22	20	23	-3	9	23	-3	9
S23	6	5	1	1	7	-1	1
S24	14	6	8	64	6	8	64
S25	10	12	-2	4	12	-2	4
			0	404		0	390
Spearman Rho Coefficient				0.845	0.850		

In order to accomplish the validation and the comparison Z-FRBS method, all data in Yaakob's paper are imported into the method. These definitions of data in the method include the definition of linguistic variables (including a linguistic variable for the important weight of each criterion from Table 1 and linguistic variable for expert reliability from Table 3 in (Yaakob & Gegov, 2015) paper, the definition of the table of criteria importance (including Table 5 from in (Yaakob & Gegov, 2015) paper), rating based table on three decisions makers' opinion (all data in Table 6 (A), 6(B), 6(C)) from (Yaakob & Gegov, 2015) paper).

Thereafter, all rules in the propose method are defined into the fuzzy system into Matlab software. For instance these rules are defined employing fuzzy environment into Matlab. In this example, 7 linguistic variables are defined and 3 input variables are used. So will be exists 7^3 equal to 343 statuses for the rules. Some instances of the definition of rules as follow are defined inside of a fuzzy environment for both rating and reliability separately.

Rule no.	Rule definition (for rating)
1	If (input1 is VL) and (input2 is VL) and (input3 is VL) then (output is VL)
...	...
150	If (input1 is M) and (input2 is VL) and (input3 is ML) then (output is L)
...	...
343	If (input1 is VH) and (input2 is VH) and (input3 is H) then (output is VH)

Rule no.	Rule definition (for reliability)
1	If (input1 is SU) and (input2 is SU) and (input3 is SU) then (output is SU)
...	...
150	If (input1 is N) and (input2 is SU) and (input3 is SWU) then (output is U)
...	...
343	If (input1 is SL) and (input2 is SL) and (input3 is SL) then (output is SL)

One of the capabilities and prominences of the proposed method is its flexibility. Because, the method can operate in two modes including utilizing either 5 linguistic variables or 7 linguistic variables. For the first time the code written was run with 5-Linguistic Variables (5-LV) and for the second time make it run with 7-Linguistic Variables (7-LV). Then, the output generated in each step should be shown in the table. The output obtained in each case is shown in Table 2.

Table 2: Ranking Performance Based on the proposed method and evaluated with Spearman Rho Correlation

STOCK	Actual	Results of the Z-FRBS Method with 5-LV			Results of the Z-FRBS Method with 7-LV		
		Rank	∂_i	∂_i^2	Rank	∂_i	∂_i^2
S1	2	2	0	0	2	0	0
S2	4	3	1	1	5	-1	1
S3	1	1	0	0	1	0	0
S4	21	20	1	1	19	2	4
S5	19	22	-3	9	22	-3	9
S6	11	8	3	9	7	4	16
S7	17	11	6	36	14	3	9
S8	24	13	11	121	20	4	16
S9	23	23	0	0	25	-2	4
S10	22	21	1	1	18	4	16
S11	8	15	-7	49	16	-8	64
S12	13	12	1	1	13	0	0
S13	25	25	0	0	21	4	16
S14	9	9	0	0	11	-2	4
S15	3	7	-4	16	8	-5	25
S16	5	4	1	1	4	1	1
S17	18	24	-6	36	23	-5	25
S18	12	14	-2	4	15	-3	9
S19	15	16	-1	1	10	5	25
S20	16	19	-3	9	12	4	16
S21	7	6	1	1	6	1	1
S22	20	18	2	4	24	-4	16
S23	6	5	1	1	3	3	9
S24	14	10	4	16	9	5	25
S25	10	17	-7	49	17	-7	49
		0	366		0	360	
		Spearman Rho Coefficient		0.859			0.862

By comparing the output of the two tables (Table 1 & Table 2), it was revealed that the amount of SRC in evaluation for the Yaakob's method (Z-FRBS TOPSIS) is equal to 0.850 and this amount in the proposed method (Z-FRBS) with 5-LV that is equal to 0.859 and with 7-LV is equal to 0.862. This implies that the accuracy of decision-making in the proposed method is higher than Yaakob's method. Also, the comparison of SRC between Yaakob's method and the proposed method is illustrated in Figure 5.

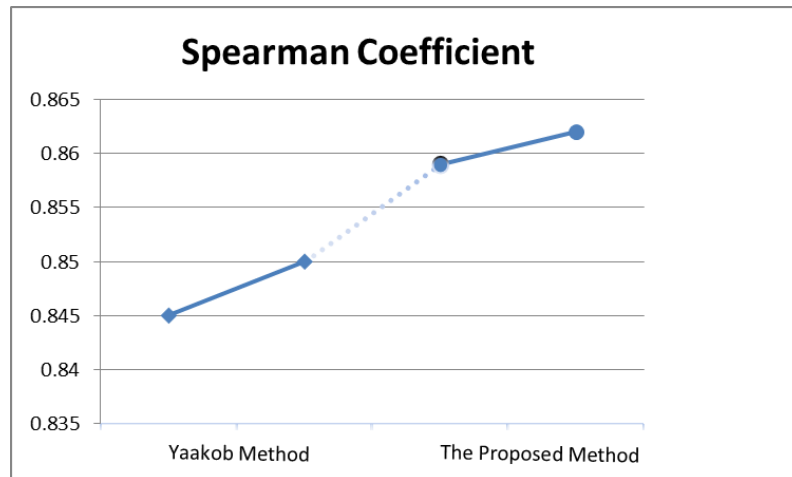


Figure 5: Comparison SRC between Yaakob and Proposed Method

The goal is to compare the performance between two methods in terms of the manner of operational, similarities and differences, strengths and weaknesses that it's summary are shown in Table 3.

Table 3: Comparison of the Methods in terms of performance and properties

Compared Components	Yaakob Method	The proposed Method
FRBS	✓	✓
Z-number	✓	✓
TOPSIS	✓	-
Flexibility	-	✓
Maximum of matches with benchmarks (actual value)	3	5
The strength of association	0.850	0.862
Ranking method	Z-FRBS +TOPSIS	Z-FRBS
Computational Body	Dependent	Independent
Computationally	In Microsoft Excell	Systematically
Fuzzy Membership Function	TrFN	TFN
Unified Z-numbers	In First Step	In Last Step
Aggregation type	By TOPSIS	By equation (7)
Use of Experts' Knowledge	Up to Certain Level	Full

Table 3 shows that both methods has used fuzzy rule base in form of Z-numbers with the difference that they are not same in the computing body. Because in the Yaakob's method, all calculations involved in evaluating the ranking and the weight of each criterion have been conducted in Excell software whereas in the proposed method is performed systematically through the code written in the Matlab software and completely independently. One of the other advantages of the proposed method is that it is flexible. Because the proposed method can works with 5-LV or 7-LV variables defined by linguistics. This is another contribution of the proposed method. As well, the

more number of matches (5 in number) and the numbers of association strength indicate the high accuracy of the proposed method. One of the weaknesses of the Yaakob's method was that they could not able to do their method with the maximum degree of experts influence and the least of computational process. Whereas in the propose method, the processes that had complicated calculations without compromising decision accuracy have been avoided. Indeed, in the proposed method in order to achieve the goal of ease computing process in make-decision all simplification components have been included in the implementation. Even in the use of membership function that in the proposed method has been used Triangular Fuzzy Number (TFN) in the implementation. While the TFN is widely used in decision-making because of its intuitive membership functions and computational simplicity (Madi et al., 2017).

Another difference between the two methods is that the Z-numbers in Yaakob's method have been unified in first step but in the proposed method is done in last step. As mentioned in Yaakob's paper, in the first step the Z-numbers that are type-2 of fuzzy number converted to a type-1 fuzzy number by using Kang's method (Kang et al., 2012) that this conversion is accompanied by data loss. But by referring to Figure 4 (the methodology), it is shown that the Z-numbers till the last step is preserved in form of type-2. These two fuzzy numbers for all experts' opinions are merged in the aggregation process utilizing the equation of (7) in stage 4. Whereas the aggregation process in Yaakob's is performed by TOPSIS and as mentioned is not systematic. As well, one of the advantages of the proposed method is that it is general and it can solve each issue defined in the area of MADM with any criteria and sub-criteria. But the evidence of the Yaakob's method shows that it is not general for solving MADM problems because the calculations in it are done manually and as well its authors have not mentioned this issue. In sum, the result shows that the proposed method has the potential to solve the MADM problems more accurately.

5. Conclusion

Many real-world decision making problems inherently involve uncertainty, vagueness, and impreciseness particularly when human judgments are considered as linguistic variables. Fuzzy logic based on IF-THEN rules has been widely implemented to provide intelligence system for modeling experts' knowledge and utilities. In this paper, a new approach was proposed based on fuzzy system to solve MAGDM problem under Z- number. In the proposed approach, experts' opinions were considered in terms of Z-number, including two components, score of performance and its reliability. Experts' opinions were collected in form of fuzzy linguistic variables, whereas most of the related studies which implemented fuzzy rule-based systems to solve MADM problem considered crisp value for experts' opinions. Furthermore, MADM problem was considered under GDM situation and proposed an approach to solve it. To do so, the problem was solved for each individual expert without aggregating their rating at first; then, at the end of ranking process the concept of the expected value was used to aggregate experts' scores.

In order to validity and verifying of the proposed method a numerical example data of a research imported into program and comparative analysis was accomplished for the problem at hand. Indeed, by measuring the performance of the method by using the statistical analysis demonstrated that the method has strong robustness. As a result, ranking with FRBS by considering Z- number may be more efficient in real world decision making.

The main advantages provided by our proposed approach are (I) enhancing accuracy and flexibility in decision making by considering Z-number concept, (II) modeling utility function and preferences of decision maker by applying fuzzy rule-based system and (III) removing computational complexity by combining fuzzy rule-based system and Z-number.

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