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Chance-constrained programming for Cryptocurrency portfolio optimization using Conditional Drawdown at Risk

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Abstract

Portfolio optimization is a widely studied problem in financial engineering literature. Its objective is to effectively distribute capital among different assets to maximize returns and minimize the risk of losing capital. Although portfolio optimization has been extensively investigated, there has been limited focus on optimizing portfolios consisting of cryptocurrencies, which are rapidly growing and emerging markets. The cryptocurrency market has demonstrated significant growth over the past two decades, offering potential profits but also presenting heightened risks compared to traditional financial markets. This situation creates challenges in constructing portfolios, necessitating the development of new and improved risk management models for cryptocurrency funds. This paper utilizes a new risk measurement approach called Conditional Drawdown at Risk (CDaR) in constructing portfolios within high-risk financial markets. Traditionally, portfolio optimization has been approached under certain conditions, considering risk and profit as decision criteria. However, recent approaches have addressed uncertainty in the decision-making process. The results obtained from this model can provide valuable guidance in making investment decisions in high-risk financial markets.

keywords: Portfolio selection, Conditional drawdown at risk, Stochastic programming, Chance constrained programming, Cryptocurrency.

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1. Introduction

Investment plays a crucial role in the economic development of nations and is often seen as a sign of progress. It provides various benefits such as economic stability, societal well-being, and the opportunity for investors to earn profits while protecting their capital from inflationary losses (Bagheriyan et al., 2023; Fooeik et al., 2022; Ghanbari et al., 2022). This has resulted in a significant increase in global investors participating in financial markets. However, it is important to recognize that financial markets inherently carry the risk of capital loss, prompting investors to adopt strategies that minimize investment risks (Fong et al., 2005; Mousavi Loleti et al., 2024). Portfolio diversification, introduced by Harry Markowitz in 1952, is a popular investment strategy that is widely recognized as a novel model for investment (Bielstein, 2023; Eskorouchi et al., 2023; Ghanbari, et al., 2023a; Koumou, 2020). Although extensive research has been conducted on portfolio optimization, only a few studies have focused on optimizing portfolios containing cryptocurrencies, which are emerging and growing markets characterized by uncertainty. Uncertainty presents challenges in problem-solving, as it represents incomplete knowledge of future events that can only be partially mitigated through information gathering (Abdollahi Moghadam et al., 2019; Hauser et al., 2018; Khalili-Fard et al., 2024; Larni-Fooeik, Sadjadi, et al., 2024; Mohammadi et al., 2018; Nozari et al., 2022). Investors cannot accurately predict the exact returns of cryptocurrencies, and the presence of uncertainty introduces risks into the decision-making process (Kurosaki & Kim, 2022). Therefore, this study aims to construct a portfolio of cryptocurrencies using a probabilistic-constrained approach under uncertain conditions .

Cryptocurrencies have emerged as a revolutionary and rapidly expanding phenomenon within the realm of financial markets, exerting profound effects on industrial productivity, global financial systems, energy consumption, and international trade. This transformative trend has captured the attention of both developed and developing countries, who perceive it as a unique opportunity and have implemented policies to facilitate investor engagement in this dynamic market (Hrytsiuk et al., 2019; Ma et al., 2020; Rahmaty & Nozari, 2023). The genesis of the cryptocurrency market dates back to 2009 when Bitcoin, the pioneering and widely accepted cryptocurrency, was introduced. Since its inception, this market has experienced extraordinary growth, boasting a total market capitalization that surpasses 2 trillion dollars and encompassing a vast array of over 10,000 distinct cryptocurrencies. While this market holds immense potential for unparalleled growth and lucrative profit opportunities, it is crucial to acknowledge its inherent volatility and the substantial investment risks it carries (Aljinović et al., 2021; Gaskin et al., 2023; Ma et al., 2020). The impact of cryptocurrencies extends far beyond their financial implications. These digital assets possess the potential to revolutionize traditional financial systems, enhance transparency, and foster financial inclusion. Moreover, the underlying technology behind cryptocurrencies, known as blockchain, holds promise for various industries, including supply chain management, healthcare, and cybersecurity. However, it is imperative for investors and market participants to approach the cryptocurrency market with caution and conduct thorough research before engaging in transactions. Understanding the intricacies of the market, staying informed about regulatory developments, and employing effective risk management strategies are vital in navigating this ever-evolving landscape. Therefore, the primary objective of this research is to construct an optimal portfolio in the cryptocurrency market by employing the Conditional Drawdown at Risk (CDaR) measure, specifically under uncertain market conditions. The CDaR measure is a downside risk measure that takes into account the probabilistic distribution of asset returns, enabling a comprehensive assessment of risk and the optimization of the portfolio (Ghanbari, , et al., 2023b). In this study, the data utilized is collected from the comprehensive Investing database, spanning a period of 10 months. The inclusion of this dataset ensures a robust foundation for the analysis and enhances the reliability of the findings .

The focus of this paper is on portfolio construction, with a specific emphasis on employing the CDaR measure in the context of the cryptocurrency market, an area that has remained relatively unexplored in previous research. Recognizing the inherent uncertainties in the cryptocurrency market, the study aims to address these uncertainties by implementing a chance-constrained programming (CCP) model (Li et al., 2010). By doing so, the proposed model is transformed into a deterministic multi-objective model, allowing for a more comprehensive analysis and decision-making process (Chen et al., 2020; Li et al., 2010). The transformed CCP model is then applied to construct an optimal portfolio from a carefully selected pool of six cryptocurrencies. Through diligent consideration of the CCP-based approach, the study seeks to provide valuable insights into portfolio construction and risk management under uncertain market conditions in the cryptocurrency domain. The findings and analyses derived from the CCP-based approach are compared with those of a deterministic scenario, enabling a comprehensive evaluation of the effectiveness and robustness of the proposed methodology. By leveraging the power of the CCP model and incorporating the CDaR measure, this research aims to contribute to the existing body of knowledge regarding investment strategies in the cryptocurrency market. The insights gained from this study have the potential to assist investors in making informed decisions, effectively managing risk, and optimizing their portfolios in the face of market uncertainty. Furthermore, the findings may have implications for the broader field of finance and risk management, offering valuable lessons that can be applied beyond the realm of cryptocurrencies.

The subsequent sections of this paper are organized as follows: Section 2 presents a comprehensive review of relevant literature, providing the theoretical foundation for the study. Section 3 introduces the proposed mathematical model and details the data utilized in the research. Section 4 examines and analyzes the findings, offering a thorough interpretation of the results. Section 5 is dedicated to discussion and managerial insights, highlighting practical implications and strategies for risk management. Lastly, Section 6 summarizes the conclusions and offers recommendations for potential future research directions.

2. Literature Survey

The literature review of this study is organized into three comprehensive sections to provide a thorough understanding of the existing research and to identify the gaps that this study aims to address. Section 2.1 discusses various portfolio optimization models, exploring traditional and contemporary approaches to constructing optimal investment portfolios. Section 2.2 delves into the application of chance-constrained programming for portfolio optimization, highlighting how probabilistic constraints can be used to manage risk in uncertain markets. Finally, Section 2.3 identifies the research gap, emphasizing the limited exploration of portfolio optimization in the context of cryptocurrencies and the need for innovative strategies to address the unique challenges posed by this emerging asset class.

2.1. Portfolio optimization models

The development of the modern investment theory by Markowitz (1952) introduced the mean-variance model as a systematic approach to constructing portfolios that aim to maximize returns given a specific level of risk or minimize risk given a certain level of returns (Larni-Fooeik et al., 2024). The appeal of portfolio optimization has prompted researchers to expand and enhance the fundamental mean-variance model by incorporating various market and investor factors. Constraints such as liquidity constraints (Tobin, 1958), transaction cost constraints (Konno & Wijayanayake, 2001), capital preservation constraints (Crama & Schyns, 2003), cardinality constraints (Chang, 2005), and more have been incorporated into the mean-variance model to account for additional market aspects and investor preferences. The importance of investment in today's society has generated significant interest from researchers in optimizing portfolios, leading to the development of diverse models based on different risk measures. Key risk measures that have been introduced include variance (Markowitz, 1991), semi-variance (Markowitz, 1959), standard deviation (Konno & Yamazaki, 1991), semi-standard deviation (Gotoh & Konno, 2000), conditional value at risk (Rockafellar & Uryasev, 2000), and CDaR (Chekhlov et al., 2000).

2.2. Chance-constrained programming for portfolio optimization

Traditional portfolio theory considers portfolio construction under certainty, and mathematical approaches have been proposed to solve the extensive Markowitz mean-variance model (Zarezade et al., 2023). However, the existence of uncertainty in the real world has recently motivated the adoption of stochastic portfolio models, which treat asset returns as random variables and utilize stochastic programming approaches to strike a balance between returns and risk in uncertain environments (Nouri et al., 2023). One approach to address the challenge of uncertainty is CCP, a class of models introduced by Charnes and Cooper (1959). CCP involves interpreting criteria under random constraints that occur at least α times, where α represents the desired safety margin of the decision maker. CCP has been applied in portfolio analysis by treating parameters and returns as random variables (Jiang & Guan, 2016). In certain decision-making contexts, decision-makers can accurately determine the values of certain parameters, but these values become random in the portfolio selection problem (Peykani et al., 2021). Therefore, stochastic programming, particularly multi-objective stochastic programming models, is utilized to address this issue (Mohammadi, 2018). Various methods, such as two-stage stochastic programming and CCP, have been widely employed in practical applications of multi-objective stochastic programming for stock selection (Ogryczak, 2000). The Markowitz model has been extended to a multi-objective linear programming framework in a model proposed by Shing and Nagasawa (1999), where the means and variances of asset returns are characterized by known probabilities.

2.3. Research Gap

The research gap addressed in this study lies in the limited exploration of portfolio construction using the Conditional Drawdown at Risk (CDaR) measure under uncertain market conditions in the cryptocurrency market. While cryptocurrencies have gained significant attention in recent years, there is a lack of comprehensive research that specifically focuses on optimizing portfolios and managing risk in this volatile and uncertain market environment. Existing studies often concentrate on the performance analysis of individual cryptocurrencies or general risk measures without considering the unique characteristics and challenges of the cryptocurrency market. Furthermore, the incorporation of downside risk measures, such as CDaR, in portfolio optimization under uncertain market conditions remains relatively unexplored. By filling this research gap, this study aims to contribute to the existing literature by providing insights and methodologies for constructing optimal portfolios in the cryptocurrency market. The utilization of the CDaR measure and the implementation of a chance-constrained programming (CCP) model address the need for more sophisticated risk management techniques that consider the probabilistic distribution of asset returns. The research also contributes to the broader field of finance by offering valuable insights into portfolio construction and risk management in highly volatile and uncertain markets. The findings and analyses derived from this study may have implications beyond the realm of cryptocurrencies, providing valuable lessons that can be applied to other asset classes and investment domains.

3. Preliminaries

This section comprises two subsections that elucidate the mathematical models and data employed in the research. The explanations provided are comprehensive, aiming to enhance the reader's understanding and facilitate their engagement with the subject matter.

3.1. Mathematical Models

This section explains the application of CDaR in portfolio risk optimization. A model is introduced for optimizing portfolios, considering certain parameters as random variables following a normal distribution. This is achieved through the utilization of a CCP model, employing a stochastic programming approach. Finally, the proposed model is presented. The calculation of the return rate for each cryptocurrency is performed using equation (1).

$$r_t = Ln(\frac{p_t}{p_{t-1}}) \tag{1}$$

Definition 1. Conditional Drawdown at Risk (CDaR)

The measure of CDaR was initially introduced by Chekhlov et al. (2004) and is considered to be a more recent risk measure with undesirable characteristics. Its performance closely resembles that of the value-at-risk conditional risk measure, which was first studied by Rockafellar and Uryasev (2000). Krokhmal et al. (2005) conducted a comparative analysis of these two risk measures, along with the capital-at-risk conditional risk and value-at-risk conditional risk measures, in the context of risk hedging funds. The findings of their study indicated that the CDaR measure is more conservative, while the value-at-risk conditional risk measure offers greater flexibility. The CDaR measure is defined as the expected value of the worst 1-α percent capital losses (average worst level of capital losses) over different periods (Ghanbari et al., 2023b), as represented by Equation (2).

$$CDaR_\alpha(x.\eta) = \min_{\eta} \left\{ \eta + \frac{1}{(1-\alpha)} \frac{1}{J} \sum_{j=1}^{J} max \left\{ 0. \max_{1 \le k \le j} \left[\sum_{i=1}^{n} \left(\sum_{t=1}^{k} r_{it} \right) x_i \right] - \sum_{i=1}^{n} \left(\sum_{t=1}^{j} r_{it} \right) x_i - \eta \right\} \right\} \quad (2)$$

Equations (3) to (8) demonstrate the existence of a linear correlation between the portfolio optimization model and the value at risk measure.

$$min\, \eta + \frac{1}{(1-\alpha)} \frac{1}{J} \sum_{j=1}^{J} (y_j) \quad (3)$$

Subjected to

$$\sum_{i=1}^{n} \mu_i x_i = \mu_p \quad (4)$$

$$y_j \ge \left\{ \sum_{i=1}^{n} \left(\sum_{t=1}^{k} r_{it} \right) x_i \right\} - \left\{ \sum_{i=1}^{n} \left(\sum_{t=1}^{j} r_{it} \right) x_i \right\} - \eta \quad (5)$$

$$y_j \ge 0 \quad (6)$$

$$\sum_{i=1}^{n} x_i = 1 \quad (7)$$

$$x_j \geq 0 \quad i = 1.2. \dots n \tag{8}$$

Within this mathematical framework, equation (3) characterizes the objective function, quantifying the value at risk associated with the portfolio. Equation (4) corresponds to the relationship that equates the portfolio's return with the investor's expected return. Equation (5) pertains to the computation of the average worst-case capital loss over a specific period, while equation (6) mandates that the average worst-case capital loss be positive. Equation (7) sets forth the budget constraint, ensuring that the total investment ratios sum to 1. Lastly, constraint (8) stipulates that short selling is prohibited, disallowing the investment ratios to assume negative values for each asset.

Definition 2. Cardinality Constraints

When constructing a practical portfolio model, it is crucial to consider factors beyond just risk and returns. This enables the mathematical model to closely align with the real world and generates more realistic outcomes. Consequently, in this study, the proposed model incorporates cardinality constraints, which significantly influence the formation of an optimal portfolio. The constraint of cardinality determines the maximum number of assets that can be included in a portfolio. In this constraint, a binary variable Z_i represents the selection status of each asset. The expression for the cardinality constraint is as follows:

$$\sum_{i=1}^{N} Z_i = K \tag{9}$$

$$Z_i \in \{0.1\} \qquad i = 1.2. \dots . n \tag{10}$$

Definition 3. Floor and Ceiling Constraints

The minimum and maximum investment amounts for each asset within a portfolio can be represented through ceiling and floor Constraints. These Constraints define the upper and lower bounds for the allocation of funds to each asset. One way to express the ceiling and floor Constraints is as follows:

$$l_i Z_i \leq x_i \leq u_i Z_i. \quad i = 1.2. \dots . n \tag{11}$$

$$0 \leq l_i \leq u_i \leq 1 \tag{12}$$

Definition 4. The proposed CDaR with Practical Constraints

The proposed portfolio optimization model with cardinality and floor and ceiling constraints can be formulated as follows:

$$min\, \eta + \frac{1}{(1-\alpha)} \frac{1}{J} \sum_{j=1}^{J} (y_j) \tag{13}$$

Subjected to

$$\sum_{i=1}^{n} \mu_i x_i = \mu_p \tag{14}$$

$$y_j \geq \left\{ \sum_{i=1}^{n} \left(\sum_{t=1}^{k} r_{it} \right) x_i \right\} - \left\{ \sum_{i=1}^{n} \left(\sum_{t=1}^{j} r_{it} \right) x_i \right\} - \eta \tag{15}$$

$$y_j \geq 0 \tag{16}$$

$$\sum_{i=1}^{N} Z_i = K \tag{17}$$

$$l_i Z_i \leq x_i \leq u_i Z_i. \quad i = 1.2. \dots . n \tag{18}$$

$$Z_i \in \{0.1\} \qquad i = 1.2. \dots . n \tag{19}$$

$$\sum_{i=1}^{n} x_i = 1 \tag{20}$$

$$x_j \geq 0 \quad i = 1.2. \dots . n \tag{21}$$

Definition 5. Stochastic Programming

Stochastic programming has found applications in diverse areas including production planning, energy investment, water management, and financial engineering. In the context of single-objective problems, two prevalent approaches are employed: the resource approach and the CCP approach. The CCP approach seeks to maximize the expected value of functions while adhering to a prescribed level of uncertainty for stochastic constraints (Sahinidis, 2004). When pursuing the maximization of these functions, a formulation of multi-objective stochastic programming can be represented by equations (22) and (23):

$$max \sum_{J=1}^{n} \tilde{R}_{ij} x_j \quad \forall_i = 1.2. \dots . m \tag{22}$$

Subjected to

$$\sum_{i=1}^{n} \tilde{r}_i x_i = \tilde{b}_k \quad \forall_i = 1.2. \dots . k \tag{23}$$

$$x \in X$$

Definition 6. Chance Constrained Programming (CCP)

The chance-constrained approach is a prominent technique utilized to address optimization problems in the presence of uncertainties. This strategy formulates the problem in a manner that guarantees the probability of satisfying a specific constraint surpasses a predefined threshold. In essence, it confines the feasible solution space to achieve a desired level of confidence in the obtained solutions. While the CCP approach is considered robust, its practical implementation often poses challenges (Ackooij et al., 2011; Ghazanfari et al.,2019; Dolat-Abadi, 2021). In the field of financial engineering, CCP plays a crucial role due to the prevalence of uncertainties in factors such as prices, demand, supply, exchange rates, and recovery rates. Applications of the CCP approach span across various domains, including water reservoir management and financial risk management (Geletu & Ilmenau, 2012). In the aforementioned research, the stochastic constraint approach is applied in the context of cryptocurrencies under uncertain conditions. It takes into account the expected returns for investors and the probabilistic returns associated with each stock. This implies that both sides of the constraint equation (24) are considered in a probabilistic manner:

$$\sum_{\forall i} \tilde{a}_i x_i \geq \tilde{b} \tag{24}$$

To cope with these uncertainties, the notion of a probabilistic constraint is proposed, as depicted in equations (25), (26), and (27).

$$P\left(\sum_{\forall i} \tilde{a}_i x_i \geq \tilde{b}\right) \geq 1 - \alpha \tag{25}$$

$$P\left(\sum_{\forall i} \tilde{a}_i x_i - \tilde{b} \geq 0\right) \geq 1 - \alpha \Rightarrow \qquad \tilde{s} = \tilde{a}_i x_i - \tilde{b} \qquad P(\tilde{s} \geq 0) \geq 1 - \alpha \tag{26}$$

$$1 - F_S(0) \geq 1 - \alpha \;\Rightarrow\; F_S(0) \leq \alpha \tag{27}$$

The symbol α denotes the degree of confidence. Let us examine the constraint (28).

$$\tilde{a}_1 x_1 + \tilde{a}_2 x_2 + \cdots + \tilde{a}_n x_n \geq \tilde{b} \tag{28}$$

If the technical coefficients exhibit a non-standard normal distribution, it is feasible to standardize them without difficulty.

$$\tilde{a} \sim N(\mu_a . \sigma_a^2) \tag{29}$$

$$\tilde{b} \sim N(\mu_b . \sigma_b^2) \tag{30}$$

$$P\left(\tilde{a}_1 x_1 + \tilde{a}_2 x_2 + \cdots + \tilde{a}_n x_n - \tilde{b} \geq 0\right) \geq 1 - \alpha \implies \qquad F_S(0) \leq \alpha \tag{31}$$

$$\tilde{s} = \tilde{a}_1 x_1 + \tilde{a}_2 x_2 + \cdots + \tilde{a}_n x_n - \tilde{b} \tag{32}$$

The precise average is calculated by employing equations (33) and (34).

$$\mu_{\bar{s}} = E(\bar{s}) = E\left(\tilde{a}_1 x_1 + \tilde{a}_2 x_2 + \cdots + \tilde{a}_n x_n - \tilde{b}\right) \tag{33}$$

$$\mu_{\bar{s}} = x_1 E(\tilde{a}_1) + x_2 E(\tilde{a}_2) + \cdots + x_n E(\tilde{a}_n) - E(\tilde{b}) \tag{34}$$

The exact variance is calculated using equations (35) and (36).

$$\sigma_{\bar{s}}^2 = Var(\tilde{s}) = Var\left(\tilde{a}_1 x_1 + \tilde{a}_2 x_2 + \cdots + \tilde{a}_n x_n - \tilde{b}\right) \tag{35}$$

$$\sigma_{\bar{s}}^2 = x_1^2\, Var(\tilde{a}_1) + x_2{}^2\, Var(\tilde{a}_2) + \ldots + x_n{}^2\, Var(\tilde{a}_n) + Var\left(\bar{b}\right) \tag{36}$$

The calculated mean and variance are plugged into functions (37) and (38).

$$Z = \emptyset\left(\frac{0 - \mu_{\bar{s}}}{\sigma_{\bar{s}}}\right) = F_S(0) \tag{37}$$

$$F_S(0) \leq \alpha \tag{38}$$

In this study, the selected confidence level is denoted by α = 0.95. The mean and variance obtained from equations (34) and (36) are subsequently substituted into equation (37) to decisively determine the constraint, which is then integrated into the objective function (3) constraints. Following the required calculations on the provided data, including mean and variance determination and their inclusion in the cumulative distribution function as depicted in equation (37), solving the equation leads to a quadratic equation as described in equation (39).

$$\sum_{i=1}^{n} r_i x_i{}^2 - \sum_{t=1}^{k} b_i\ x_i \geq d \tag{39}$$

Now, it is crucial to incorporate this equation as a constraint in the main problem. Ultimately, the mathematical representation of the problem is structured as follows:

$$Z_1 = max \sum_{j=1}^{6} \tilde{r}_j x_j \tag{40}$$

$$Z_2 = min\,\eta + \frac{1}{(1-\alpha)}\frac{1}{J}\sum_{j=1}^{J}(y_j) \tag{41}$$

Subjected to:

$$y_j \geq \left\{\sum_{i=1}^{n}\left(\sum_{t=1}^{k} r_{it}\right)x_i\right\} - \left\{\sum_{i=1}^{n}\left(\sum_{t=1}^{j} r_{it}\right)x_i\right\} - \eta \tag{42}$$

$$\sum_{i=1}^{n} r_i x_i^{\,2} - \sum_{t=1}^{k} b_i\ x_i \geq d \tag{43}$$

$$\sum_{i=1}^{n} x_i = 1 \tag{44}$$

$$\sum_{i=1}^{N} Z_i = K \tag{45}$$

$$l_i Z_i \leq x_i \leq u_i Z_i. \quad i = 1.2. \dots .\, n \tag{46}$$

$$Z_i \in \{0.1\} \qquad i = 1.2. \dots .\, n \tag{47}$$

$$y_j \geq 0 \tag{48}$$

3.2 Data Description

To conduct this study, we gathered comprehensive data from the Investing.com database, covering a span of 10 months. This extensive dataset provided us with the necessary information to analyze the performance of various cryptocurrencies over a significant period. By selecting a 10-month duration, we aimed to capture a range of market conditions, including periods of high volatility and relative stability. This approach ensures a robust analysis of the cryptocurrencies in different market scenarios. The dataset included detailed information on six carefully selected cryptocurrencies. These cryptocurrencies were chosen based on their market relevance, trading volume, and overall impact on the digital asset market. The selected assets are Algorand (ALGO), Chainlink (LINK), Ethereum (ETH), Ethereum Classic (ETC), FTX Token (FTT), and Tezos (XTZ). Each cryptocurrency represents a different facet of the market, providing a diversified sample for our analysis. The inclusion of both well-established and emerging cryptocurrencies allows us to explore a broad spectrum of investment opportunities and risks. For a more granular analysis, we divided the data into 25 sub-intervals and calculated the return rates for each cryptocurrency over 10-day periods. This division into 10-day intervals enabled us to observe the short-term performance and volatility of each cryptocurrency, providing insights into their dynamic behavior. Table 1 below presents the selected assets, their abbreviations, and their market values, offering a snapshot of the data used in our study. This structured approach to data collection and analysis lays the groundwork for constructing an optimal cryptocurrency portfolio using advanced optimization techniques.

Table 1. Selected asset data

Row	Property	Abbreviation	Market value
A_1	Algorand	ALGO	11.15 B
A_2	Chainlink	LINK	16.34 B
A_3	Ethereum	ETH	17.39 B
A_4	Ethereum Classic	ETC	7.78 B
A_5	FTX Token	FTT	6.72 B
A_6	Tezos	XTZ	4.28 B

4. Empirical Results and Discussion

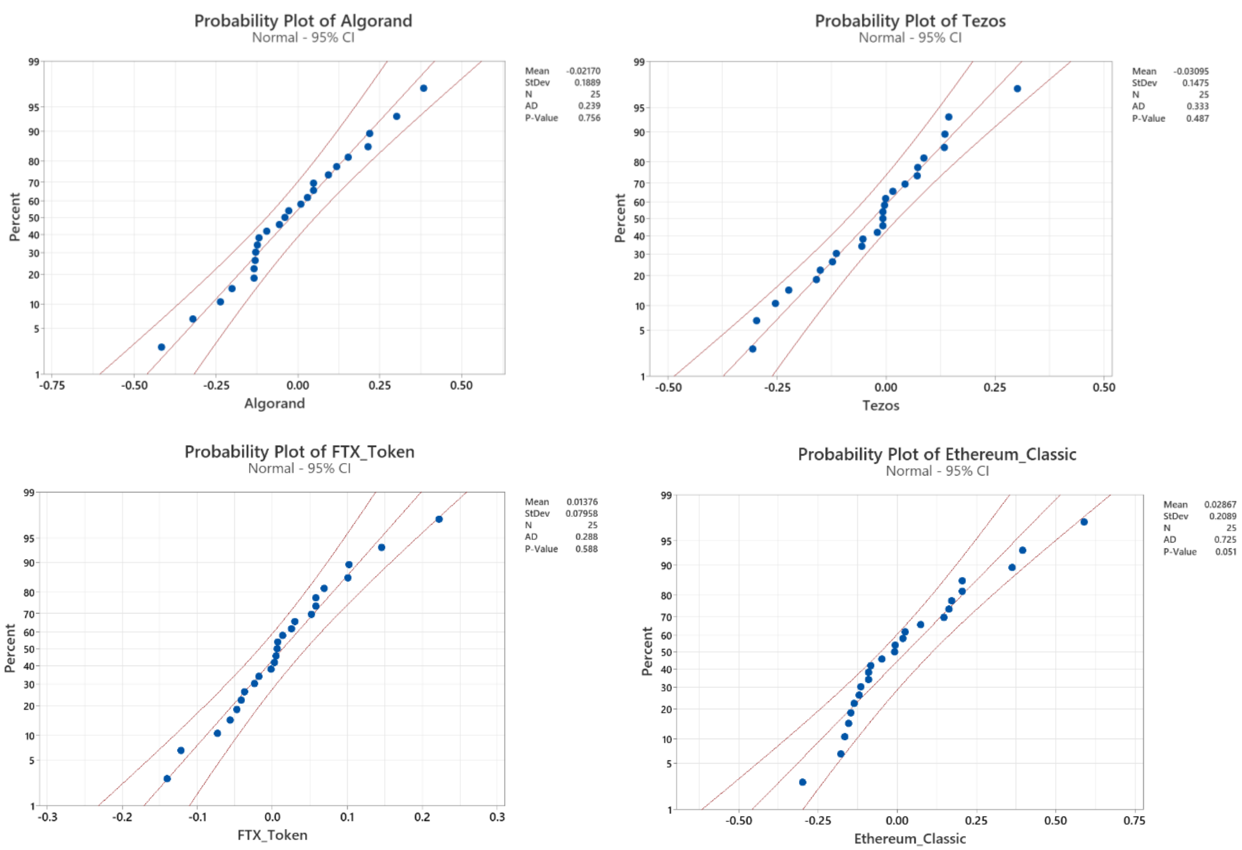
This section focuses on the presentation and analysis of the outcomes derived from constructing an optimal portfolio consisting of cryptocurrencies, taking into account both uncertain and certain problem parameters. The study expands on the previous research by developing and evaluating the model under two distinct circumstances: one characterized by uncertainty and the other by certainty. This approach allows for a comprehensive validation of the research findings. To facilitate the validation process, the outcomes obtained from both scenarios are documented and compared. The research findings are presented in Tables 3 and 5, providing a clear juxtaposition of the results between the uncertain and certain scenarios. These tables serve as a valuable reference for understanding the impact of uncertainty on the optimal portfolio construction process. Furthermore, Table 2 presents descriptive statistics for the selected cryptocurrencies in this study. These statistics provide a comprehensive overview of the characteristics and performance of the chosen cryptocurrencies. The inclusion of these tables and descriptive statistics enhances the transparency and clarity of the research outcomes. It allows readers to assess and evaluate the effectiveness of the model in both uncertain and certain scenarios, as well as gain a deeper understanding of the characteristics of the selected cryptocurrencies.

Table 2. Descriptive statistics of the selected cryptocurrencies

Asset	Mean	Variance	Standard Deviation	Minimum Return	Maximum Return
ALGO	-0.0217	0.0343	0.1851	0.3834	-0.4161
LINK	0.0207	0.0447	0.2113	0.3810	-0.3333
ETH	0.0190	0.0240	0.1551	0.4324	-0.2106
ETC	0.0287	0.0419	0.2047	0.5893	-0.2989

FTT	0.0138	0.0061	0.0780	0.2226	-0.1401
XTZ	-0.0310	0.0209	0.1445	0.3020	-0.3053

In this section, we delve into the analysis of return distributions for the selected cryptocurrencies to assess their normality. The data is collected and processed, and Minitab software is employed for determining the distribution type. Figures 1 present the observed P-Values for each cryptocurrency: Algorand (0.756), Tezos (0.487), FTX_Token (0.588), Ethereum_Classic (0.051), Ethereum (0.613), and Chainlink (0.319). The majority of the examined cryptocurrencies exhibit P-values greater than 0.05, indicating that their returns adhere to a normal distribution at a 95% confidence level. This finding suggests a level of normality in the data, providing insights into the statistical characteristics of the returns for each cryptocurrency. To further supplement the analysis, Table 2 provides the mean and standard deviation of the return data. These descriptive statistics offer a summary of the central tendency and dispersion of returns for the selected cryptocurrencies. They enable a better understanding of the average performance and variability associated with each cryptocurrency.



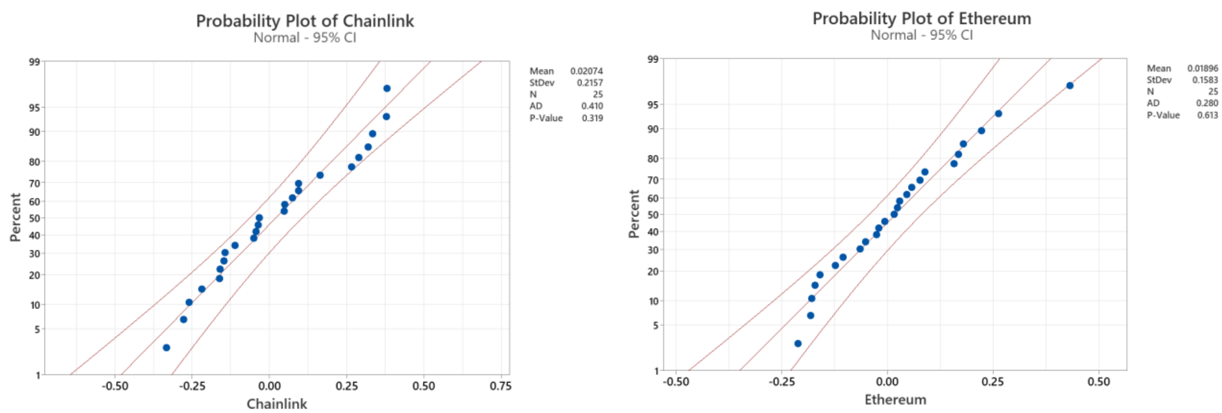


Figure 1. Probability plot of cryptocurrencies

In this study, the proposed model incorporates a confidence level of 95% ($\alpha = 0.95$) to ensure a robust and reliable analysis of the portfolio construction process. Furthermore, a cardinality constraint is specified, limiting the portfolio to a predetermined number of assets, in this case, k=3, implying a portfolio consisting of three cryptocurrencies. This constraint aids in diversification and risk management. To further refine the model, the research establishes lower and upper constraints for each individual cryptocurrency within the portfolio. The lower constraint, $l_i = 0.1$, ensures that the allocation to each cryptocurrency is at least 10% of the total portfolio value, promoting a minimum level of exposure to each asset. On the other hand, the upper constraint, $u_i = 0.50$, sets a maximum limit of 50% allocation to any single cryptocurrency in the portfolio, preventing overexposure and promoting diversification. With these defined parameters and constraints, the model is solved, producing the following results:

Table 3. The outcomes derived from the process of resolving a problem Under conditions of uncertainty

the optimal portfolio value	ALGO	LINK	ETH	ETC	FTT	XTZ
0.36	0.45	0	0	0.45	0	0.1

The analysis conducted, as shown in Table 3, highlights those assets such as ALGO, ETC, and XTZ are identified as constituents of the optimal portfolio. These specific cryptocurrencies contribute to the overall portfolio value, which amounts to 0.36. Figure 2 visually represents the allocation ratios of these assets within the optimal portfolio, providing a clear illustration of their respective weights. It is noteworthy that the optimal portfolio, as determined by the model, does not include other cryptocurrencies such as LINK, ETH, and FTT. This suggests that the model's calculations and constraints have led to the exclusion of these particular assets from the portfolio composition. The model's selection process prioritizes ALGO, ETC, and XTZ, considering them as the most suitable choices based on the defined parameters and constraints. To further elucidate the distribution of individual cryptocurrencies within the portfolio under uncertain circumstances, Table 4 presents the percentages of each cryptocurrency's allocation. These percentages offer insights into the relative weightings of the chosen cryptocurrencies within the portfolio composition. They provide a comprehensive understanding of how each asset contributes to the overall risk and return profile of the portfolio when faced with uncertain conditions.

Table 4. The outcomes derived from the process of resolving a problem Under conditions of uncertainty

Property	Abbreviation	Market value	percentage
Algorand	ALGO	11.15 B	0.45
Chainlink	LINK	16.34 B	0.00
Ethereum	ETH	17.39 B	0.00
Ethereum Classic	ETC	7.78 B	0.45
FTX Token	FTT	6.72 B	0.00
Tezos	XTZ	4.28 B	0.1

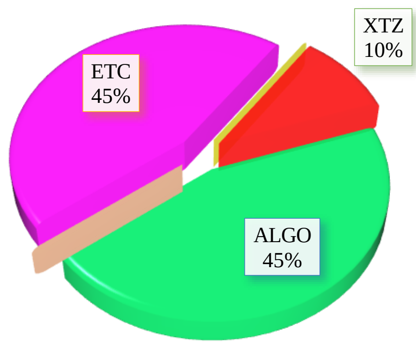


Figure 2. The proportional distribution of cryptocurrencies within the optimal portfolio Under conditions of uncertainty

In this particular section, the primary objective is to assess the reliability and robustness of the research model in circumstances where modeling certainty is present. The allocation table presented offers valuable insights into the composition of the portfolio under these conditions. Each cryptocurrency included in the portfolio is assigned a specific allocation, and the optimal portfolio value is calculated as a result. The allocation table provides a comprehensive overview of the distribution of investments among the selected cryptocurrencies. It highlights the specific weights assigned to each asset, offering a clear understanding of the relative importance of each cryptocurrency within the portfolio. By examining this table, readers can discern the proportion of the portfolio's value allocated to each individual cryptocurrency, allowing for a detailed analysis of the asset composition. Furthermore, the optimal portfolio value holds considerable significance. It represents the total value of the portfolio generated by the model under circumstances of modeling certainty. This value serves as an important metric for evaluating the effectiveness and performance of the research model. By comparing the optimal portfolio value across different scenarios or benchmarking it against alternative portfolio constructions, investors and researchers can gain valuable insights into the efficiency and potential returns of the proposed model. In summary, the allocation table and the optimal portfolio value presented in this section offer crucial information regarding the composition and performance of the portfolio under circumstances of modeling certainty. This analysis enhances the reliability and robustness of the research model, facilitating informed decision-making in portfolio construction and allocation.

Table 5. The outcomes derived from the process of resolving a problem Under conditions of certainty

the optimal portfolio value	ALGO	LINK	ETH	ETC	FTT	XTZ
-0.055	0.00	0.20	0.35	0.00	0.45	0.00

According to the findings presented in Table 5, the analysis reveals that assets such as LINK, ETH, and FTT are identified as constituents of the optimal portfolio under certain circumstances. These particular cryptocurrencies have been deemed suitable for inclusion based on the defined parameters and constraints of the research model. The optimal portfolio value for the examined cryptocurrencies is calculated to be -0.055, which indicates the expected performance of the portfolio under certain conditions. Figure 3 visually represents the allocation ratios of LINK, ETH, and FTT within the optimal portfolio. This graphical representation provides a clear visualization of the relative weights assigned to each cryptocurrency, giving readers a comprehensive understanding of the composition of the portfolio. It is important to note that the optimal portfolio, as determined by the model, does not include other cryptocurrencies apart from LINK, ETH, and FTT. This implies that the model's calculations and constraints have led to the exclusion of these additional assets from the portfolio composition. The research model has identified LINK, ETH, and FTT as the most suitable cryptocurrencies for inclusion, based on the defined parameters and constraints. To further enhance the understanding of the distribution of individual cryptocurrencies within the portfolio under certain circumstances, Table 6 presents the percentages of allocation for each cryptocurrency. These percentages provide insights into the relative weightings of LINK, ETH, and FTT within the portfolio composition. They offer valuable information regarding the contribution of each cryptocurrency to the overall risk and return profile of the portfolio when faced with certain conditions.

Table 6. The outcomes derived from the process of resolving a problem Under conditions of uncertainty

Property	Abbreviation	Market value	percentage
Algorand	ALGO	11.15 B	0.00
Chainlink	LINK	16.34 B	0.20
Ethereum	ETH	17.39 B	0.35
Ethereum Classic	ETC	7.78 B	0.00
FTX Token	FTT	6.72 B	0.45

Tezos	XTZ	4.28 B	0.00

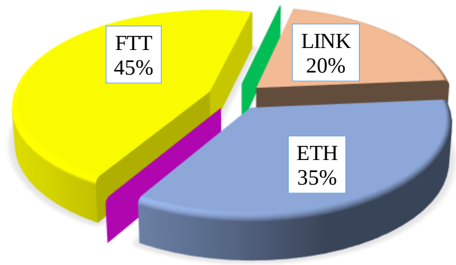


Figure 3. The proportional distribution of cryptocurrencies within the optimal portfolio Under conditions of certainty

By comparing the outcomes of two models, one deterministic and the other non-deterministic, as presented in Tables 5 and 3 respectively, a clear distinction emerges in terms of portfolio composition and performance under different circumstances. When faced with uncertain circumstances, as indicated in Table 3, the optimal portfolio consists of the stocks ALGO, ETH, and XTZ, with respective weights of 0.45, 0.45, and 0.1. The allocation of these assets within the portfolio reflects the model's response to uncertainty, aiming for diversification and risk management. The resulting optimal portfolio value is calculated to be 0.36, indicating the expected performance under uncertain conditions. On the other hand, in a scenario of certainty as reflected in Table 5, the selected stocks for the portfolio consist of LINK, ETH, and FTT, with respective weights of 0.2, 0.35, and 0.45. These allocations represent the deterministic model's optimal portfolio composition, which may prioritize different factors or constraints compared to the uncertain scenario. Surprisingly, the optimal portfolio value in this case is -0.055, significantly lower than the value obtained from the model under uncertain conditions. The stark difference in the optimal portfolio value between the deterministic and non-deterministic models indicates that the portfolio constructed under uncertain circumstances holds greater value. This suggests that the non-deterministic model, which accounts for uncertainty and incorporates a broader range of possible outcomes, is better equipped to adapt to changing market conditions and exploit potential opportunities. The inclusion of ALGO, ETH, and XTZ in the uncertain scenario, with their respective higher weights, may reflect the model's attempt to navigate uncertain conditions and capitalize on potential gains. It is important to note that the optimal portfolio composition and performance can vary significantly depending on the presence or absence of uncertainty. This highlights the importance of considering and accounting for uncertainty in portfolio construction and decision-making processes. By comparing the outcomes of both models, researchers and investors can gain valuable insights into the impact of uncertainty on portfolio outcomes and make more informed investment decisions.

5. Discussion and managerial insights

The findings of this study underscore the importance of employing advanced risk management strategies in the volatile and rapidly evolving cryptocurrency market. By utilizing the CDaR measure and a CCP model, this research provides a robust framework for constructing an optimal cryptocurrency portfolio under uncertain market conditions. This approach not only addresses the inherent risks associated with cryptocurrencies but also offers practical insights into managing these risks effectively.

5.1. Discussion

The application of the CDaR measure in portfolio optimization highlights its effectiveness in capturing downside risks and providing a more accurate risk assessment compared to traditional measures. The probabilistic nature of the CCP model further enhances this framework by accommodating uncertainties and transforming them into a deterministic multi-objective model. This transformation allows for a more precise optimization process, ultimately leading to a portfolio that balances risk and return more efficiently. The comparative analysis between the CCP-based approach and a deterministic scenario reveals that incorporating probabilistic constraints significantly improves the portfolio's performance. This improvement is particularly evident in the context of cryptocurrencies, where market conditions are highly unpredictable and volatile. The ability to adapt to these uncertainties through advanced modeling techniques is a key contribution of this study, offering a pathway for more resilient investment strategies in the digital asset space.

5.2. Managerial Insights

From a managerial perspective, the insights gained from this study are crucial for investors and financial managers who are navigating the complexities of the cryptocurrency market. Here are several key takeaways:

1. **Risk Management**: Managers should prioritize advanced risk management tools like CDaR to better understand and mitigate downside risks. This study demonstrates that traditional risk measures may not be sufficient in capturing the full spectrum of risks inherent in cryptocurrency investments.

2. **Diversification**: While diversification remains a cornerstone of investment strategy, this research highlights the need for sophisticated models that can handle the unique volatility and uncertainty of cryptocurrencies. Financial managers should consider incorporating probabilistic constraints into their diversification strategies to enhance portfolio resilience.

3. **Policy Implications**: As governments and regulatory bodies continue to develop policies to facilitate cryptocurrency investments, financial managers should stay informed about these changes and adapt their strategies accordingly. Understanding the regulatory landscape is essential for optimizing investment outcomes and ensuring compliance.

4. **Future Preparedness**: Given the rapid evolution of the cryptocurrency market, managers should foster a culture of continuous learning and adaptation. Keeping abreast of the latest research and technological advancements will be critical in maintaining a competitive edge and capitalizing on emerging opportunities.

In conclusion, this study provides valuable insights into optimizing cryptocurrency portfolios under uncertainty. By employing advanced risk measures and probabilistic modeling, investors can better navigate the complexities of the digital asset market, ultimately enhancing their investment outcomes. Future research should continue to explore innovative approaches to risk management and portfolio optimization, further contributing to the understanding and development of this burgeoning field.

6. Conclusion and further research directions

In this research, we have explored the optimization of a portfolio comprising cryptocurrencies, which is an emerging and rapidly expanding phenomenon in financial markets. Considering the extreme fluctuations and high risk associated with investing in the cryptocurrency market, we have employed the prudent risk measure of capital loss risk under conditional risk. This risk measure falls into the category of downside risk measure, which assesses and optimizes the portfolio by taking into account the probability distribution of asset returns. Traditionally, portfolio optimization has been approached under certain conditions, considering risk and profit as decision criteria. However, recent approaches have addressed uncertainty in the decision-making process. To contribute to the advancement of scientific knowledge in this field, this paper proposes a new mathematical formulation of CDaR based on a CCP approach for portfolio optimization. To enhance the realism and cater to investors' additional preferences, we incorporated cardinality Constraint, as well as upper and lower bounds, into the modeling process. The analysis of the findings demonstrates the efficiency of the proposed model in constructing an optimal portfolio. These outcomes possess utility for various stakeholders, including researchers, investors, investment fund managers, and other participants in the financial market. In future research, scholars can explore and contrast alternative portfolio optimization models within the cryptocurrency market. Additionally, incorporating uncertainty approaches into mathematical modeling by considering the non-deterministic aspects of the problem can be beneficial. To address investors' preferences and enhance result validity, the modeling process incorporated cardinality Constraints, ceilings, and floors. Analysis of the findings demonstrates the efficacy of the proposed model for constructing an optimal portfolio. This research's results have broad applicability, benefiting researchers, investors, fund managers, and other participants in financial markets. In future studies, scholars can explore and compare alternative portfolio optimization models, as well as consider different uncertainty approaches, not only in the cryptocurrency market but also in other financial markets.

Furthermore, researchers can enhance the realism of the mathematical model and its outcomes by incorporating additional practical constraints in portfolio optimization, such as liquidity

Constraints and transaction costs. This approach would consider the real-world complexities and challenges faced by investors in their decision-making process.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Declaration of Generative AI and AI-assisted technologies in the writing process

During the preparation of this work, the authors used OpenAI's tool Chat GPT to edit and write some parts of the paper. After using this service, the authors reviewed and edited the content as needed and took full responsibility for the content of the publication.

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