

Development of an Inventory Control Model for Reusable Items: Economic Quantities of Order and Recovery

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Abstract

Today, access to natural resources is much more complex than before. In addition, governments encourage companies to recover and reuse with their modern policies. Recovering and reusing materials is necessary for some companies to compete in the market and improve their performance by reducing costs. Examples of this model can be seen in the system of machine tools, medical equipment, the automobile industry, and furniture, which have been able to implement improvements in their work process with the policies of this model. This paper focuses on developing a mathematical model to manage an inventory of items that can be reused. The model for inventory control that can recover items for reuse will be presented in this article. The purpose of this inventory model is to minimize the costs when there is a possibility of multiple consumptions for each item through the recovery process. This problem aims to simultaneously determine the economic quantities of order and recovery by utilizing an inventory control model. This article works on designing an efficient and effective model to reduce and control costs and increase the system's efficiency. By recovering and reusing consumer goods, the waste will reach its minimum, and the costs will also be minimized.

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1- Introduction

Inventory management is "the continuous process of planning, organizing, and controlling inventory to minimize investment in inventory while balancing supply and demand" (Sings & Verma, 2017). Inventory control is a flow that guarantees that items and inventory of organizations, companies, and stores are provided to customers on time, considering time, place, quantity, quality, and cost. Inventory management is essential in supply chain management because it enhances supply chain performance. The main task of inventory management includes meeting the demands of customers as well as reducing the level of inventory maintenance and related costs. To complete this task, it is necessary to develop appropriate inventory control models by answering two critical questions: When and how many new items should be ordered (Penn et al., 2014). Also, in the new era, this kind of inventory model is used in brilliant factories (Tavakkoli-Moghaddam et al., 2024). Inventory control is the activity that allows the organization to adjust and place the necessary inventory at an acceptable level. The primary objective of inventory control is to maximize profit while minimizing investment in storing inventory in a way that does not impact customer satisfaction (Nozari et al., 2023). Also, inventory control is defined as where the inventory of various items is kept in the warehouse and how much it is at any moment. EOQ, EPQ, EOQ considering shortage, EPQ considering shortage, and inventory control models for perishable items are all types of inventory control models. FOS and FOI are the two primary inventory models that deal with ordering at specific intervals (such as weekly, twice a month, etc.). This model can significantly impact sales businesses, such as pharmacies and supermarkets. This model is utilized because the supplier's policy may entice customers to place orders in fixed intervals.

Reducing shipping costs can be achieved by ordering products from a single supplier. There are times when periodic control is sufficient, as there is no need to monitor the inventory level of the warehouse continuously. A fixed order quantity defines the FOS model. In the FOS ordering model, the warehouse inventory is continuously controlled, and as soon as the inventory level reaches the reorder point, a fixed amount is ordered. In today's world, where raw materials are becoming more expensive daily, it is possible to reduce costs by recovering and reusing goods. The model addresses the inflation rate and minimizes expenses by recovering and reusing. This article defines the problem as providing an inventory control model for reusable items that can improve some current inventory systems. This article's innovation assumes that half of the items are only used once and cannot be recovered.

The supply chain is a dynamic system that includes all activities related to product production, from raw materials to customer delivery. The supply chain is a system of organizations, people, activities, information, and resources that supply a product or service to the consumer. Supply chain activities include transforming natural resources, raw materials, and components into a final product delivered to the final customer. Used products can be recycled at any point in

complex supply chain systems where the remaining amount can be reused. Reuse is using an item for its original purpose (regular reuse) or to perform a different function (creative reuse or reuse). Reusing should be distinguished from recycling, the breakdown of used items to produce raw materials to create new products. Reusability saves time, money, energy, and resources by capturing but not reprocessing previously used items. In the broader economic context, it can make quality products available to people and organizations with limited resources while creating jobs and business activities contributing to the economic cycle. From an environmental point of view, the recovery of items is much more attractive than its destruction (Choi et al., 2007).

Reusing products and raw materials has been discussed, and opinions have been related. In addition, iron scrap, wastepaper, and sedimentation systems for beverage bottles are examples of reusing products and raw materials (Koh et al., 2002). The types of recovery and reuse are distinct, and in some cases, they may overlap. Recovery is divided into two categories. The first category is recovered with repair, but the second is recovered without repair. The second category is also called reuse. Product recovery (repair, renovation, remanufacturing) is attracting more and more attention. In the past, participation in recovery activities was often driven by legislation or an environmentally friendly image. However, today, economics is the main reason companies get involved in product recovery (Tenter, 2004).

The article's structure is that the recovery system is responsible for recovering the defective items and decides to fix possible defects. If the recovery system cannot resolve the defects, the system is forced to destroy the product. If the system can recover the desired fault, the recovery system will start to fix the fault. After the product's defects are fixed, the product is returned to the company for use. This cycle continues until the company uses the products many times. A brief introduction is given at the start of this article, followed by a literature review. In the next part, the modeling process is discussed. Also, in this part, purchase, recovery, ordering, setup, and holding costs are considered to form the objective function. With the sum of the costs, the model reaches the total cost or the objective function. After that, a numerical example will be discussed, in which the model identifies the optimal variables by assigning values to the problem parameters. Sensitivity analysis is also proposed in this article. The model examines the optimal variables by changing the value of the parameters. At the end of the article, the conclusions are the objective function's depreciation and the model outputs' findings.

2- Literature Review

Inventory is a part of any product or resource used in an organization. Inventory management has always been a new issue because any company uses inventory management to reduce costs. Inventory management determines when and how much to order (Pilliner & Borisov, 2015). Inventory consists of raw materials, products in production, and finished goods that are considered part of the assets of a business that are either ready for sale or will be prepared for sale. Inventory is one of the most critical assets of a business (Singes & Verma, 2017). The act

of ordering, handling, storing, and utilizing a company's non-capital assets, including its inventory, is known as inventory management. Raw materials and components may be part of inventory in some businesses, while others may only deal with inventory items ready for sale (Jangali et al., 2021).

Either way, inventory management is all about balance, which means having the right amount of inventory in the right place at the right time. Inventory management is a part of supply chain management that plans, implements, and controls the efficient, effective, forward and reverse flow and storage of goods, services, and relevant information between the point of origin and the point of consumption to meet the needs of customers (Sings and Verma, 2017). The dynamics of supply chains are increasing in response to changing business and technology environments (Yu et al., 2019). The supply chain is one of the supporting concepts. It includes a set of different and diverse activities used to manage the flow of materials and goods during the processes in the industry and organization so that the flow is smooth and appropriate. Make materials and goods possible. The main goal in the activities related to the supply chain is the effective and efficient use of the participating elements in the supply chain, including suppliers, manufacturers, distributors, and customers, to provide value to the organization's final customers. Therefore, according to the case and volume of operations, different institutions and organizations have wide working dimensions in the field of supply chain (Aghaei et al., 2014).

The topic of the supply chain was first proposed by Hahn in 1985. It includes the management of all processes and various activities that create value for the final customer and help create a compelling and improved link between upstream and downstream business. Another essential goal of supply chain management is to coordinate the various activities or constraints of the chain so that the goods can be provided to the customer at the required time when the inventory is low and the costs are low (Mirfallahdemochali et al., 2024). The supply chain includes all the parts directly or indirectly related to each other to meet the customer's needs (Manavizadeh et al., 2022). These sectors include manufacturers, suppliers, carriers, warehouses, retailers, and customers. These departments include activities such as new and updated product development, marketing, executive operations, distribution and delivery, financial services, customer service, and more.

The supply chain includes a dynamic set of information, product, and capital flows among its different levels, in which the customer is only an involved and internal component in the supply chain. So, one of the first goals in the supply chain is to create satisfaction for the customer's needs in the supply process and to generate profit for the supply chains. The supply chain refers to the processes of physical, informational, financial, and knowledge flows that meet the needs of end consumers through products and services related to suppliers (Yarian & Shamsaldini, 2013). Also, the increase in global competition led to placing the supply chain at the center of attention instead of the individual views of companies (Cooper, 1997).

The supply chain is a system of organizations, people, activities, information, and resources that transfer a product or service from the supplier to the customer (Singes & Verma, 2017). The supply chain includes all parties directly or indirectly fulfilling the customer's request. The supply chain in any organization, such as a manufacturer, includes all functions related to receiving and completing customer requests. The supply chain works dynamically and includes a constant flow of information, products, and funds in different stages. The goal of the supply chain is to increase the value of the chain, and the value of the chain is the value that the product has for the final customer, which is independent of the efforts that the chain makes to create the product (Chopra & Mindel, 2016). In a holistic view, the supply chain is a network of organizations that participate and cooperate in various activities, from raw materials to the production, distribution, and delivery of the final product to the target customers (Cooper, 1997). The supply chain includes all the activities related to the preparation of the product, from raw materials to the distribution of the final product, which also functions effectively in the flow of information and financial resources (Biehaki et al., 2008). Supply chains include business activities and organizations that need to design, manufacture, deliver, and use a product or service. Companies that learn to build and participate in supply chains will have a significant market competitive advantage. A supply chain shows its importance and necessity in regulating and aligning companies that offer products or services in the market (Higus, 2018). According to Chopra and Mindel's division in 2007, all supply chain processes are divided into three major processes: customer relationship management, internal supply chain management, and supplier relationship management. A supply chain consists of two essential and integrated processes: production and warehouse control and distribution and logistics. These processes provide a general framework for converting and transferring raw materials to finished products. Carrying out these processes in cooperation produces an integrated supply chain (Beamon, 1998).

The outputs of the supply chain can appear in the form of products and services, appropriate knowledge and information, everyday standards, and a specialized and skilled workforce (Shojaei & Ghajavand, 2012). Companies face intense competition today, and to maintain this competition, they must discover new market opportunities and take advantage of the existing efficiency in their operations (Wu et al., 2017). To properly face global competition, organizations need proper supply chain management (Fang, 2012). Today's supply chains operate in an environment that constantly seeks to reduce costs and find solutions in this direction (Aghaei et al., 2014). The supply chain is an integrated process in which several business entities work together to obtain raw materials, transform these materials into a specified final product, and deliver this product to retailers (Beamon, 1998). Supply chain management seeks to integrate organizational units along the chain and coordinate the flow of raw materials, information, and financial flows to meet customer demand and improve the supply chain's competitiveness (Segsted et al., 2010). In the supply chain, each of the business partners mutually affects the performance of the other partners and creates a kind of mutual dependence between the parties (Nozari et al., 2022). Supply chain management is the efforts of organizations to properly manage this mutual dependence between chain members to

achieve a competitive advantage and realize each company's unique goals and, ultimately, the whole group's goals (Shojaei & Ghajavand, 2013). Supply chain management is a set of methods used to effectively integrate suppliers, manufacturers, warehouses, and stores so that the cost of the entire system is minimized and service level requirements are met (Simchi et al., 2008).

The management of this chain can be considered as cross-sectional integration and coordination between companies and coordination of material, information, and financial flows to use resources as the most logical way along the way of organizations (Ivano et al., 2017). Supply chain management is a unique deliverable activity. It also includes a specific starting and ending point that focuses on the integration and coordination within and between companies and the coordination of material, informational, and financial flows or eliminating costs from the supply chain (Meyer & Torres, 2019). Supply chain management is the result of the evolution of the warehouse management process. In the 1960s, experts reduced their inventory by studying the internal relationship between storage and transportation and their integration, called distribution management studies. During evolution, with the addition of construction management, procurement, and orders, logistics emerged. Supply chain management includes integration in product development, appropriate demand management, planning and forecasting, inventory management, increasing the speed of distribution of goods and services, and improving information integration (Sharbiani et al., 2012). An integrated supply chain indicates effective communication, including cooperation, trust, and participation between different and diverse parts of the supply chain (Sabbaghi & Vaideniten, 2008). Supply chain management is the design, maintenance, and operations related to supply chain processes to meet the final consumer's needs (Yarian & Shamsaldini, 2013). Supply chain management is a set of methods to manage and coordinate all parts of the chain, from suppliers to customers (Chan, 2000). Chain management can be seen as planning, organizing, and controlling all activities in the supply chain (Varma & Deshmaq, 2008). The purpose of supply chain management is to improve the activities of various components and levels of a chain to improve the overall condition of the supply chain system. It is possible to see contradictions and contradictions between the goals of different components and levels of the chain to achieve the overall goals, and these contradictions lead to a decrease in the supply chain's competitiveness over time. Among these conflicts, the costs are related to marketing, advertising, pricing, and inventory (Amozad et al., 2012). Recent research has shown that the performance of supply chains is not significantly better than in the past, and the management and effective performance of the supply chain is increasingly considered a vital factor in achieving companies' competitive advantages (Moradi, 2019). This matter can also result in new research using modern technologies (Nozari et al., 2021). Supply chain networks are all the people and organizations cooperating to produce and supply a product (Motaghi, 2016).

Additionally, for stating the research gap, the following table is presented. After reviewing the literature, it is found that no research can evaluate reusable items after the disposal of half of the items.

Table 1. Background and gap of research

<i>Number</i>	<i>References</i>	<i>Source Title</i>	<i>Findings</i>
1	Fallahi et al., 2022	A constrained multi-item EOQ inventory model for reusable items: Reinforcement learning-based differential evolution and particle swarm optimization	This research aims to present a new variation of hybrid differential evolution (DE) and particle swarm optimization (PSO) algorithms in which a machine learning algorithm manages the value of metaheuristic parameters.
2	Rabata., 2020	An Economic Order Quantity inventory model for a product with a circular economy indicator	Formulation of an inventory model for a product in the circular economy.
3	Ullah et al., 2021	Ramification of remanufacturing in a sustainable three-echelon closed-loop supply chain management for returnable products	Based on the findings, a hybrid policy is the best choice for single- and multi-retailer cases. However, the remanufacturing rate for multiple retailers increases.
4	Nasari et al., 2021	Pricing and inventory control decisions in the stochastic hybrid production systems with multiple recovery options	The results indicate that the purchase price of returned products will increase the number of returned products.
5	Liu et al., 2020	Inventory sharing strategy and optimization for reusable transport items	The developed decision support framework employs A two-stage solution process to optimize distribution flows and dispatch vehicle routes.
6	Current Research	Development of an Inventory Control Model for Reusable Items: Economic Quantities of Order and Recovery	The research gap concerns items that can be used again after being used for the first time. Also, half of them are eliminated for the first time.

3- Problem Description

The proposed model is controlled by Mojo by the company buying a currently used product. If the product fails, it is sent to the recovery system for recovery. At this point of the model, the recovery system examines the product. By answering the question, "Is the product's life over?" The recovery system has two options ahead. If the answer is "yes," then the product is removed from the cycle for "disposal," and if the answer is "no," then the product is stored in the "inventory of recoverable products." Next, the recovery system deals with the products that need recovery and sends them to the "Inventory of Usable Products" after recovery. The products are stored in the usable products warehouse when recovered and ready for use. Finally, the recovered products are sent to the original company. This cycle continues for a product until it can no longer be recovered. Figure 1 shows a view of this inventory cycle.

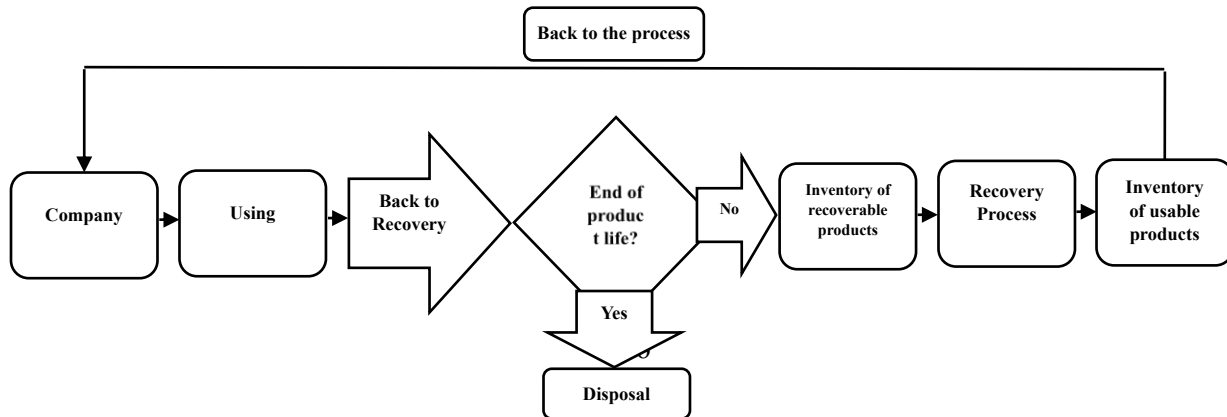


Fig 1. Suggested Qualitative model

3-1 Assumptions

- Only 50% of items are reusable.
- Demand is specific and fixed, and consumption is gradual.
- Products that need to be recovered are sent in batches of q and returned to the company in batches of q .

3-2 Variables

T_1 = The time it takes for 50% of the items to be used for the first time.

t_1 = Inventory cycle to recover and reuse q until reaching the end of its useful life.

T_R = Inventory cycle to recover and reuse category q .

T_C = Cycle length

Q = Order quantity for new items

q = The series of items that are sent for recovery.

p = The result of dividing Q by q

S_u = The result of dividing Q by q

S_R = Area under the graph of recoverable items

TC = Total system cost

n = The result of dividing half of Q by q

3-3 Parameters

m = The number of times that one type of material can be recovered.

D = Total annual demand

C = Purchase price per product unit

A_r = Fixed recovery installation cost

A_u = Fixed ordering cost

$H = h_u + h_R$ = The sum of all maintenance costs

h_u = Maintenance cost for a unit of usable items per unit of time

$h_R =$ Holding cost for a unit of items used per unit of time

$r =$ Recovery cost for each item

The purchase, ordering, installation, recovery, and holding costs are written first to start the modeling. We show purchase costs with PC, ordering costs with OC, installation costs with SC, recovery costs with RC, and holding costs with HC.

3-4 Costs of model

Purchase cost:

$$pc = \frac{CD}{m+1} \quad (1)$$

Ordering cost:

$$Oc = \frac{A_u D}{(m+1)pq} \quad (2)$$

Setup cost:

$$Sc = \frac{A_r D m}{q(m+1)} \quad (3)$$

Recovery cost:

$$Rc = r \left(\frac{Dm}{m+1} - \frac{T_1}{t_q} \right) \Rightarrow Rc = r \left(\frac{Dm}{m+1} - \frac{Q}{2q} \right) \quad (4)$$

Holding cost:

Calculating the holding cost is essential in this study. The research team reached a constant number and calculated H using the following formula to obtain this goal.

$$H = h_u + h_r \quad (5)$$

The area under the diagram of usable items, the area under the diagram of recoverable items, and the total area of usable items and recoverable items are calculated as follows to calculate the average inventory and holding costs.

$$S_u = \frac{3Q^2}{8 * D} + \frac{2pq^2(m+1) - Qq}{4 * D} + \frac{(m+1) \left(\frac{Q^2}{2} + Qq \right)}{4D} \quad (6)$$

$$S_R = \frac{pq^2(m+1)}{2D} \quad (7)$$

$$I = S_u + S_R \Rightarrow I = \left[\frac{3Q^2}{8 * D} + \frac{2pq^2(m+1) - Qq}{4 * D} + \frac{(m+1) \left(\frac{Q^2}{2} + Qq \right)}{4D} \right] + \left[\frac{pq^2(m+1)}{2 * D} \right] \quad (8)$$

To calculate \bar{I} we must divide the total area under the graphs of usable and recoverable items by the total cycle time.

$$\bar{I} = \frac{I}{T_c} \Rightarrow \bar{I} = \frac{\left[\frac{3Q^2}{8 * D} + \frac{2pq^2(m+1) - Qq}{4 * D} + \frac{(m+1) \left(\frac{Q^2}{2} + Qq \right)}{4D} \right] + \left[\frac{pq^2(m+1)}{2 * D} \right]}{\frac{pq}{D} * (m+1)} \quad (9)$$

$$\bar{I} = \frac{(m+1)(8pq^2 + Q^2 + 2Qq) + 3Q^2 - 2Qq}{8pq * (m+1)} \quad (10)$$

Moreover, holding costs are $H\bar{I}$, which requires H and \bar{I} to calculate it.

3-5 Time variables of the model

The required times for the model are T_1, T_c, T_R, t_q The formula for the required times will be as follows:

$$t_q = \frac{q}{D} \quad (11)$$

$$T_R = (m+1) \frac{q}{D} \quad (12)$$

$$T_c = p \times T_R = \frac{pq}{D} (m+1) \quad (13)$$

$$T_1 = \frac{Q}{2D} \quad (14)$$

The following two have been defined before. This is stated here to be remembered and utilized next.

$$Q = pq \quad (15)$$

$$n = \frac{Q}{q} = \frac{Q}{2q} \quad (16)$$

In conclusion, the sum of all the stated costs leads to total costs, which form the objective function. It is written as follows.

$$\begin{aligned}
TC &= pc + Oc + Sc + Rc + Hc \\
\Rightarrow TC &= \frac{CD}{(m+1)} + \frac{A_u D}{(mpq + pq)} + \frac{A_r D m}{(mq + q)} + r \left(\frac{Dm}{m+1} - \frac{Q}{2 * q} \right) \\
&+ H \left(\frac{(m+1)(8pq^2 + Q^2 + 2Qq) + 3Q^2 - 2Qq}{8pq * (m+1)} \right)
\end{aligned} \tag{17}$$

$$Q = pq \tag{18}$$

So, after calculating and solving the equation, this is the new objective function in this research study. The total cost must be minimized.

$$\begin{aligned}
TC(p, q) &= \frac{C * D}{(m+1)} + \frac{A_u * D}{(mpq + pq)} + \frac{A_r * D * m}{(mq + q)} + r \left(\frac{D * m}{m+1} - \frac{p}{2} \right) \\
&+ H \left(\frac{q * (10m + pm + 8 + 4p)}{8 * (m+1)} \right)
\end{aligned} \tag{19}$$

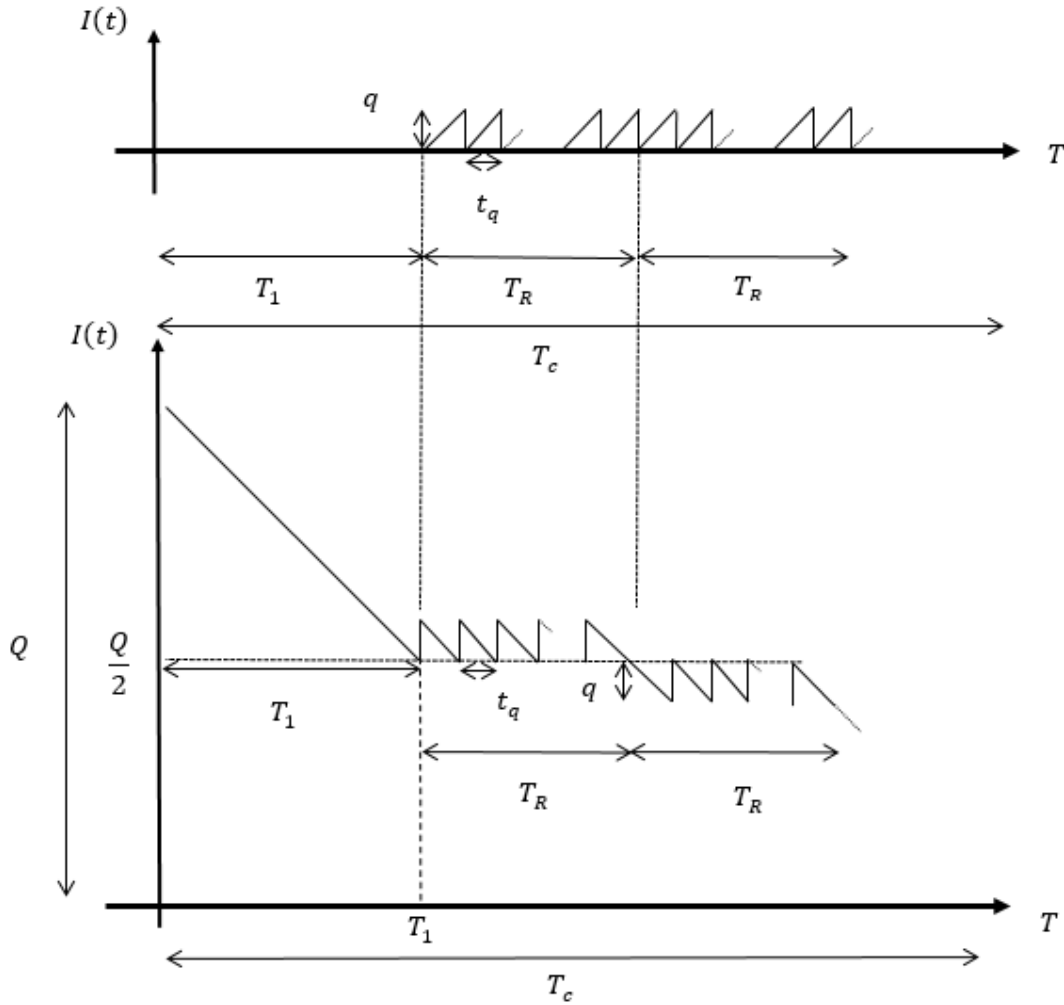


Fig 2. Recoverable and reusable items

Figure 2 shows the recoverable and reusable item graphs in this section. This diagram was drawn in Word software, and several articles are examined at the end of the article. To obtain the values of p and q , the first derivatives taken concerning p and q must be equal to zero. Finally, we arrive at the following two expressions:

$$q = \sqrt{\frac{8D * (A_r m p + A_u)}{pH(10m + pm + 4p + 8)}} \quad (20)$$

$$p = \sqrt{\frac{8A_u D}{Hq^2(m + 4) - 4rq(m + 1)}} \quad (21)$$

As a result, p is equal to the following equation.

$$p = \sqrt{\frac{8A_u D}{H \left(\frac{8D(A_r m p + A_u)}{(m + 4)pH(p + 10m + 8)} \right) (m + 4) - 4r \sqrt{\left(\frac{8D(A_r m p + A_u)}{(m + 4)pH(p + 10m + 8)} \right) (m + 1)}}} \quad (22)$$

Moreover, q is equal to the following equation.

$$q = \sqrt{\frac{8D \left(A_r m \left(\sqrt{\frac{8A_u D}{Hq^2(m + 4) - 4rq(m + 1)}} \right) + A_u \right)}{(m + 4) \left(\sqrt{\frac{8A_u D}{Hq^2(m + 4) - 4rq(m + 1)}} \right) H \left(\left(\sqrt{\frac{8A_u D}{Hq^2(m + 4) - 4rq(m + 1)}} \right) + 10m + 8 \right)}} \quad (23)$$

4- Results and Examples

We will provide examples to check the presented model. For this purpose, values are determined for the parameters.

4-1 Example one

As the first example, for the time when the annual demand is 3000, the number of times of reuse is 2, the purchase cost per item is 50, the cost required for recovery per item is 2, the fixed cost of the order is 1000, the cost of installing recovery is the constant is 200, and the sum of all holding costs is 8. The values of p^* , q^* , Q^* , $TC(p^*, q^*)$ are calculated and are as follows.

$$D = 3000, m = 2, r = 2, A_u = 1000, A_r = 200, H = 8, C = 50$$

$$q^* = 206.83, p^* = 3.42, Q^* = 707.35, TC(p^*, q^*) = 60689.37, t_q = 0.068, T_R = 0.206, T_C = 0.707, T_1 = 0.117$$

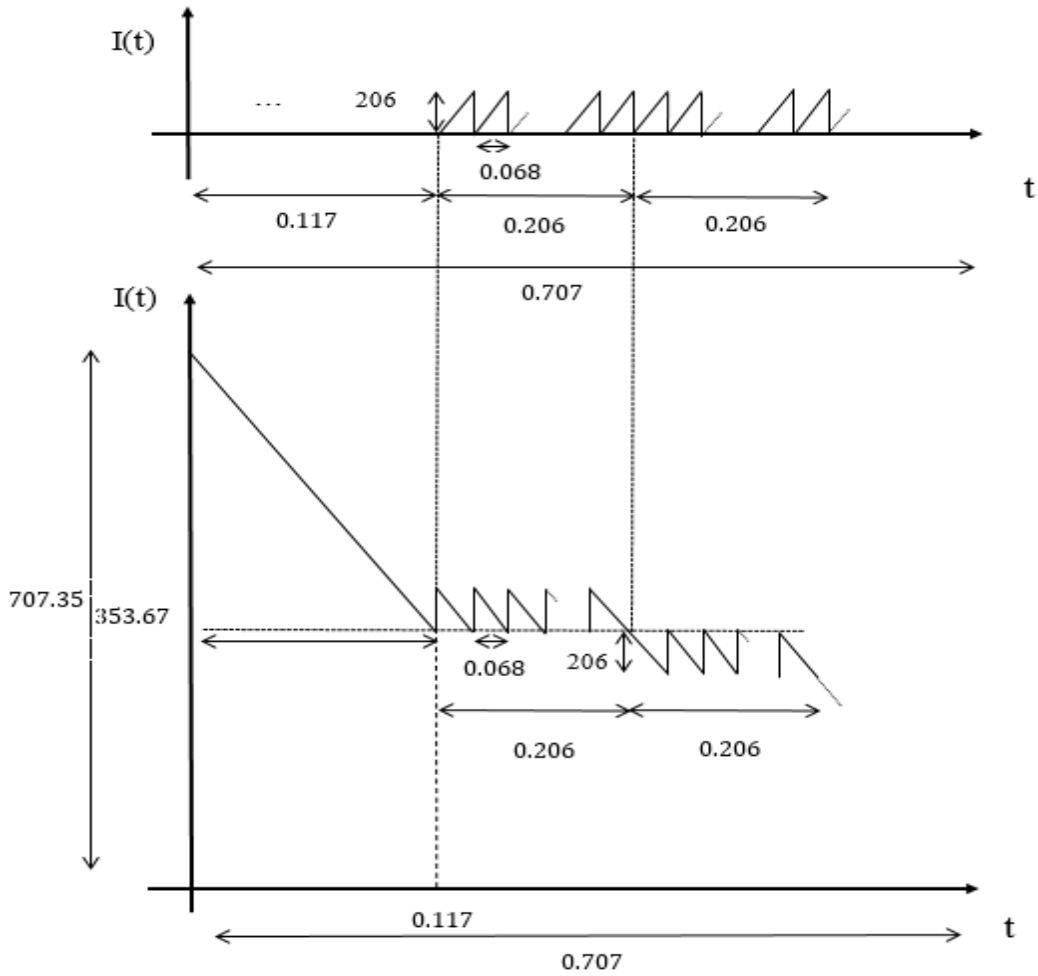


Fig 3. Solution diagram of example one

4-2 Example two

As the following example, for the time when the annual demand is 4000, the number of times of reuse is 2, the purchase cost per item is 50, the cost required for recovery per item is 2, the fixed ordering cost is 1500, the fixed recovery installation cost is 150, and the sum of all holding costs is 10. Obtain the values of p^* , q^* , Q^* , $TC(p^*, q^*)$.

$$D = 4000, m = 2, r = 2, A_u = 1500, A_r = 150, H = 10, c = 50$$

$$p^* = 4.83, q^* = 184.95, Q^* = 893.30, TC(p^*, q^*) = 79189.57, t_q = 0.046, T_R = 0.138, T_C = 0.666, T_1 = 0.111$$

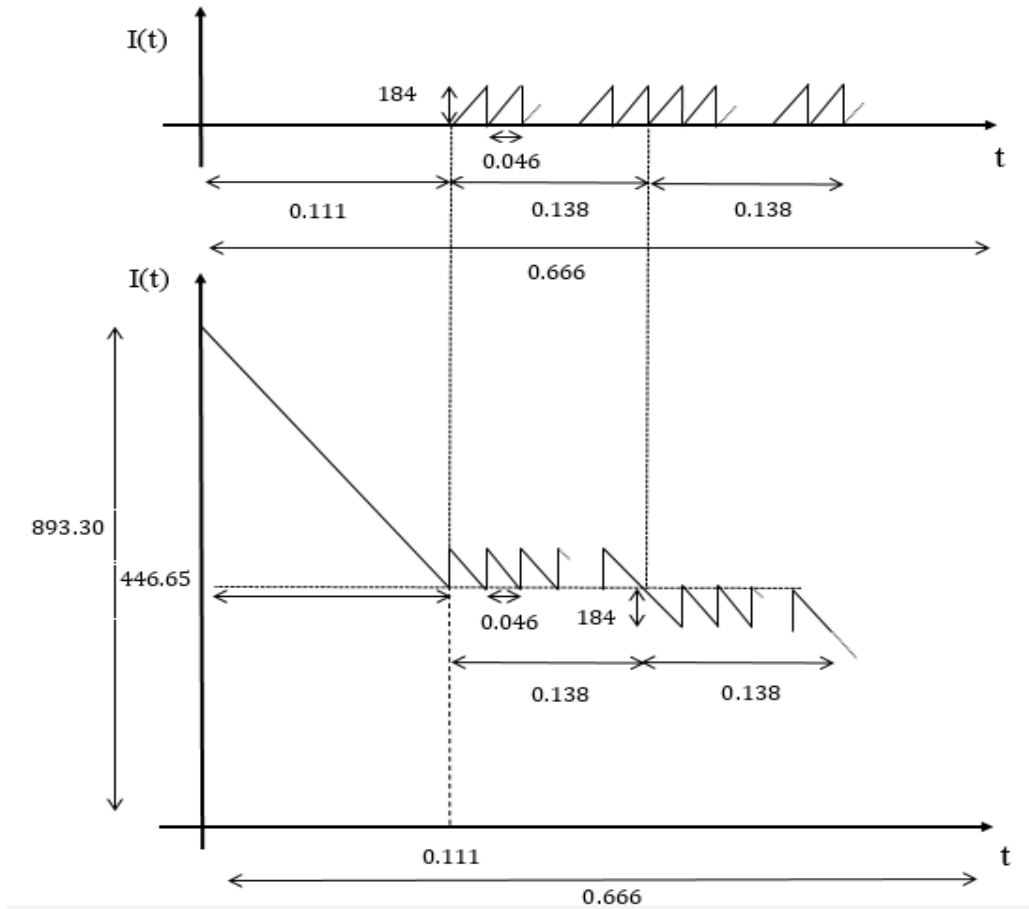


Fig 4. Solution diagram of example two

4-3 Example three

Also, in the third example, for the time when the annual demand is 8000, the number of times of reuse is 4, the purchase cost per item is 100, the cost required for recovery per item is 40, the fixed cost of the order is 2500, and the fixed recovery installation cost is 200. and the sum of all holding costs is 12. Obtain the values of p^* , q^* , Q^* , $TC(p^*, q^*)$.

$$D = 8000, m = 4, c = 100, r = 40, A_u = 2500, A_r = 200, H = 12$$

$$p^* = 4.43, q^* = 295.07, Q^* = 1309.61, TC(p^*, q^*) = 346325.05, t_q = 0.036, T_R = 0.184, T_C = 0.815, T_1 = 0.081$$

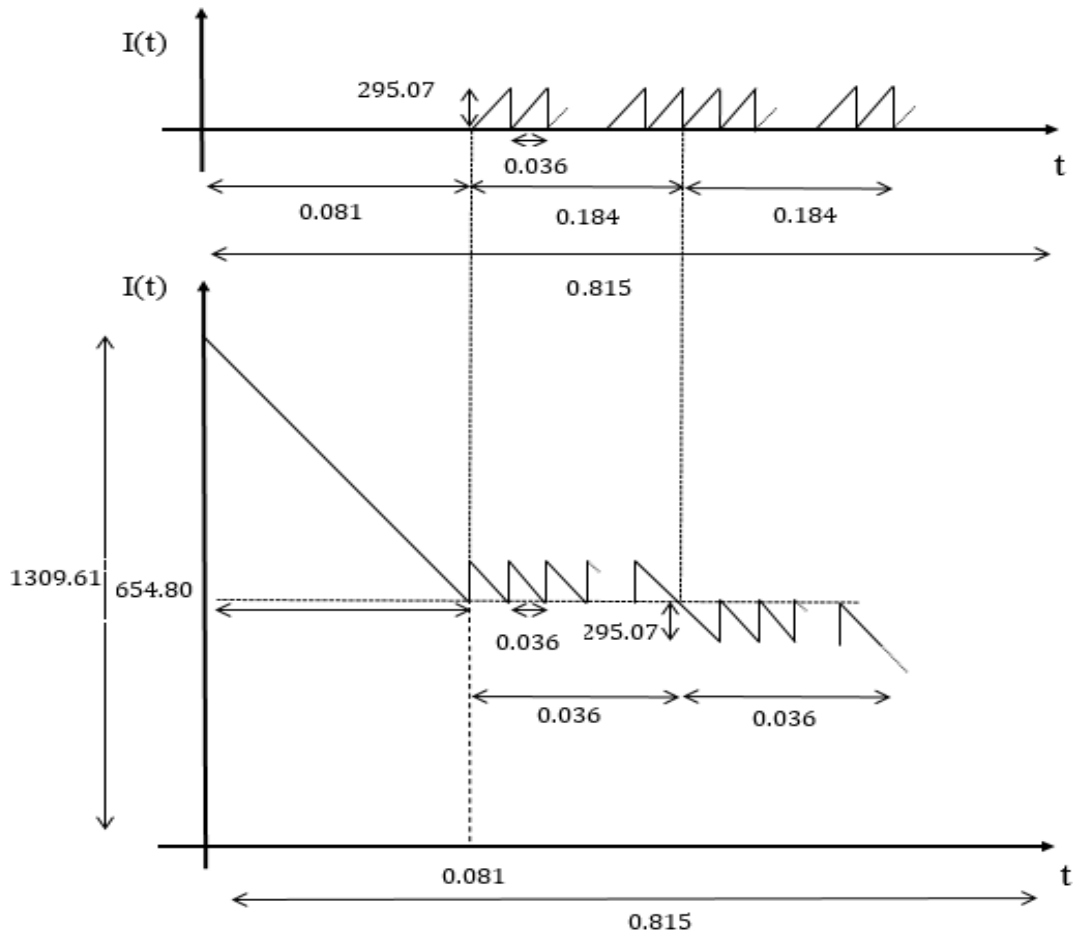


Fig 5. Solution diagram of example three

4-4 Sensitivity analysis

In the real world, changes in the parameters are inevitable. The parameters D, H, A_u, A_r Undergo changes, but the parameters r and c do not change because the cost per unit of goods is considered fixed. In the following tables, we apply a 50% increase and decrease in the amount of demand (D), holding costs (H), and fixed installation costs (A_r) To find the amount of change in the variables p, q, Q, TC .

4-4-1 Change in demand

Table 2 and Figure 6 provide a table and graph for changes in demand.

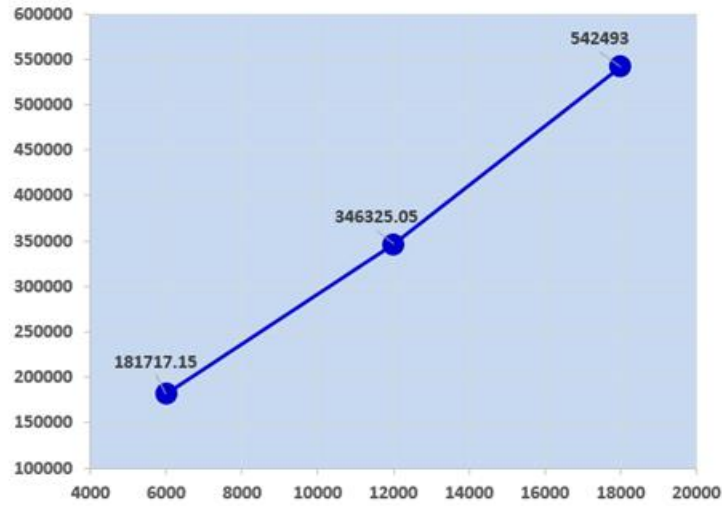


Fig 6. Total cost and demand graph

Table 2. Change in demand

TC	Q	q	p	D	Change in Demand (%)
181717.15	191.23	11.95	16	6000	-50
346325.05	1539.20	102.61	15	12000	0
542493	538.61	41.43	13	18000	+50

4-4-2 Change in holding costs

Table 3 and Figure 7 provide a table and graph for changes in holding costs.

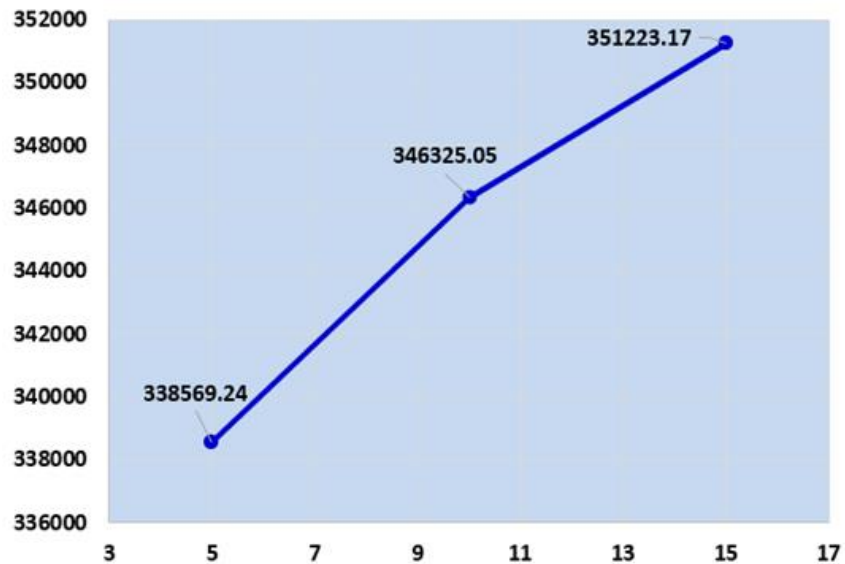


Fig 7. Chart of total cost and holding cost

Table 3. Change in holding costs

TC	Q	q	p	H	<i>Change in Holding Cost (%)</i>
338569.24	1621.2	162.12	10	5	-50
346325.05	1539.20	102.61	15	10	0
351223.17	1338.89	74.38	18	15	+50

4-4-3 Change in setup costs

Table 4 and Figure 8 provide a table and graph for ordering costs.

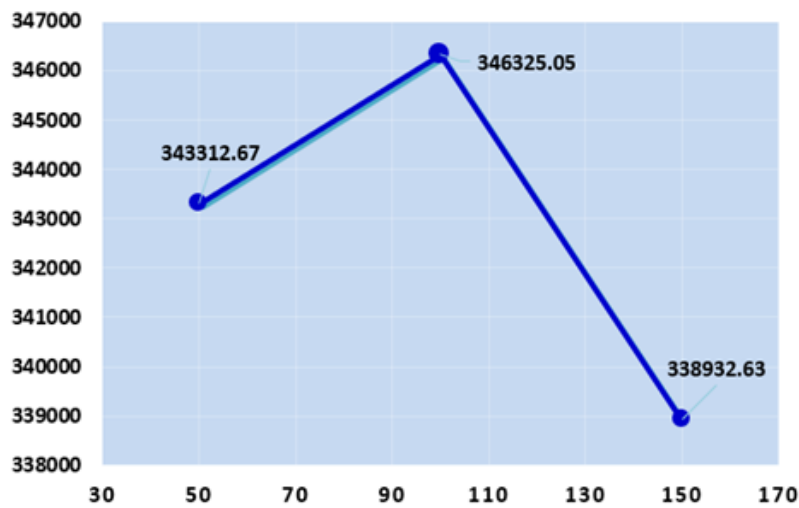


Fig 8. Chart of total cost and installation cost

Table 4. Change in setup costs

TC	Q	q	p	A_r	<i>Change in Setup Cost (%)</i>
343312.67	1460	73	20	50	-50
346325.05	1539.20	102.61	15	100	0
338932.63	1512	126	12	150	+50

5- Conclusions

The conclusion of the current research is divided into three sections. The limitations of the research and the innovations will be explained briefly. Also, the summary of the research is written for the writers who are interested in this article.

5-1 Research innovations

The current study's primary invention is a new inventory control system specifically made for reusable items. This approach is different from traditional inventory models, which usually assume that items are only used once. By focusing on reusable items, the authors have created a model more applicable to real-world scenarios where companies must manage the lifecycle of products that can be used multiple times.

A key innovation is that reusable items can be identified by their optimal economic order quantity (EOQ) and economic recovery quantity (ERQ) simultaneously. Prior investigation has frequently centered on the ordering or the recovery aspect, but not both simultaneously. The model offers a more comprehensive and realistic way of managing reusable item inventories by jointly considering these two decisions.

5-2 Research Limitations

Although the proposed inventory control model is an essential advancement in managing reusable item inventories, it has some drawbacks. Firstly, the model assumes that the demand for reusable items is deterministic and known in advance. This may not always be the case in real-world scenarios where demand can be uncertain and fluctuating. Furthermore, the model does not consider the possibility of product degradation or obsolescence over time, which could impact the most suitable ordering and recovery decisions.

The model assumes no loss or quality degradation during the recovery process, which may not be true of the practical challenges of reusing and refurbishing items. The computationally intensive and significant processing power required by advanced optimization techniques may limit the model's applicability in some business environments. Future research could examine ways to tackle these shortcomings and improve the practical utility of the inventory control model for reusable items.

5-3 Summary of the study

This article presents an inventory control model for recoverable and reusable items. When new orders are received, they are consumed gradually. At first, half of the items are used and cannot be recovered, but the other half enters the recovery process. When an order of new items is received, they are gradually collected, and the used items are simultaneously stored in special conditions until their quantity reaches a predefined number. Then, the recovery process is performed for the current batch of used items so that an item reaches "new" quality. At this point, all recovered items are returned to the parent process for reuse. The recovery (and reuse) process is continuously repeated a predetermined number of times for each case. Reusable items are discarded at the end of their useful life, and another batch of new items enters the process. The use, recovery, reuse, and discard cycle is constantly repeated. In addition, mathematical expressions are developed to determine the economic order quantity for new items and the economic batch quantity for the recovery and reuse process. The goal is to minimize the total cost of the inventory system, including the cost of purchasing new items, ordering new items, the cost of holding new and used items, the cost of setting up the recovery

process, and operating costs. A numerical example is presented, and a sensitivity analysis is performed to identify and depict the most significant influence of the input parameters.

From this article, it can be concluded that in the current situation, due to the increase in the price of raw materials and the increase in environmental pollution, models like ours, which pay special attention to the recovery and reuse of products, can be a good solution for factories and companies. Therefore, in the form of a proposal, this model can be presented to reduce costs, improve performance, and increase customer satisfaction. Among other suggestions in this article, we can mention the creation of warehouses for the items that will be recovered. The initial cost of building such warehouses may seem complicated, but it will be a long-term solution. In this article, the derivation of the objective function and finally reaching the optimal values is mentioned, and due to the lack of games or more powerful software, this article can be considered different from other articles. Other researchers can use various software by considering other influential factors, such as increasing variables and restrictions.

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