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A new data-driven decision-making method for therapist-patient allocation and scheduling

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Abstract

One of the constant problems that people with mental health conditions are faced with now is that they cannot establish a good relationship with their therapist, or the client's disease type is not in the therapist's specialty. These clients may not receive adequate treatment and stop the therapy before feeling well. Therefore, the classification of mental patients based on their disorder types and allocating a therapist with the same expertise to them could lead to better treatment and improve the quality of the therapy sessions. This paper will compare several machine learning (ML) algorithms to classify patients with mental conditions. Moreover, benefiting from the best ML algorithm, patients will be categorized into different classes based on their disorder types. Finally, a mathematical model will be developed to determine the allocation policy of therapists to each group of patients to maximize the summation of the utilization between therapists and patients. To explore the implementation of the proposed method, we have conducted a real-life case study to assess the validation of the model.

Keywords: Mental Health; Data-Driven Decision-Making; Scheduling; Mathematical Modeling; Machine Learning; Patient Allocation

1. Introduction

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With the development of psychology, mental disorders have become more and more noticed and seen. It has been proved that public mental health plays a significant role in society, especially after witnessing mental disorders increased by 13 percent in the last decade (Hameed et al., 2018); a lot of attention has been drawn to this issue. The public health crisis in 2019 and the COVID-19 quarantine aggravated this issue. So, the reported cases of mental disorders increased significantly during the pandemic time (Vindegaard & Benros, 2020; Rahmaty & Nozari, 2023), which clarifies the importance of knowing about this topic more than ever. By 2022, it is declared that roughly 1 billion of the world's population will be dealing with mental illness, mildly and severely. In some developing countries, the percentage of people with a mental health condition could even exceed 70 percent (Schmalbach et al., 2022; Ghahremani-Nahr et al., 2023; Najafi et al., 2022).

However, the treatment for mental illnesses is still widely unknown, leading to misdiagnoses and wrong medical treatment for the patients. This issue could lead to deep depression and even suicide. Approximately 90 percent of suicides are caused by a mental disorder (Duko & Ayano, 2018; Nozari et al., 2022). Therefore, a practical approach and suitable treatment are necessary for mental health patients because they could drive progress in symptom reduction and improve patients' general mental health condition (Delgadillo & Gonzalez Salas Duhne, 2020).

Baker (2020) has conveyed that one person experiences common mental disorders from every six people in England. It has also been declared that approximately 66 percent of the patients felt improvement in their disease. About 75 percent of the patients who have participated in talk therapy in the United States felt some improvement in their condition. So, it can be concluded that this number could even exceed a higher level with the correct diagnosis and treatment approach. Generally, a more efficient outcome could be achieved by allocating selected therapists to the patients based on their personalities and disorder types.

After noticing the importance of the topic, some valuable research has been done on this subject in recent years, and several similar papers will be reviewed as follows. Soto et al. (2016) have investigated the application of ML in the mental health system. This paper introduces a classification algorithm to recognize potential autism cases at very young ages to control this disorder. To complete this topic, two years later, Shatte et al. (2019), in their review article, comprehensively explained the application of ML algorithms in the mental health system. By reviewing 300 papers in this field, it has been demonstrated that the ML algorithm is used for four aims: detection and diagnosis, prognosis, treatment and support, public health, and research and clinical administration. On the same subject, Delgadillo & Gonzalez Salas Duhne

(2020) have been concerned about the increase in depression cases. With little knowledge about treatment approaches for this mental disorder, there is no best selection for curing depression. So, he compared two treatment approaches based on ML algorithms to find which one was more effective. Schwartz et al., (2021) have also recommended ML algorithms for getting optimal treatment based on their personality; He has reported a big difference in the outcome of the patients with optimal and non-optimal treatment.

In many cases, ML in healthcare systems is used to predict diseases, and extract medical Concepts (Sajdeya et al., 2023). For example, an ensemble-based framework named Meta-Health Stack to predict breast cancer were proposed by Samieinasab et al. (2022). Maleki et al., (2022), the article suggests a method utilizing ML algorithms to enhance the precision of coronary artery disease (CAD) prediction. It utilizes the Cleveland Hospital heart disease dataset, comprising 303 instances with 14 characteristics. Abdali et al., (2024) As per the research outcomes, ML algorithms enhanced CAD diagnosis accuracy. also introduced a policy at vaccination centers to determine the prioritization of applicants when the number of vaccines is insufficient. Dehghan-Bonari et al. (2023) presented a novel data-driven optimization method for managing mental disorders among students.

Noticing the application of ML in the mental healthcare system through the mentioned articles, there are some other exciting contexts in this topic, which will be discussed in the future. Sterling et al. (2001) analyzed the relationship between therapy results and the similarity between the therapist and the patients. This assessment can be helpful in adequately assigning therapists to patients in different regions, cities, and cultures. Tyrer et al. (2015) have announced that personality disorders are rising in every community, and in many cases, mental illness is misdiagnosed, and the wrong treatment method is prescribed for it. The wrong treatment method could lead to the deterioration of the patient's health. Therefore, this provides a classification method to help the treatment system increase its diagnosis's accuracy. The importance of developing apps dedicated to improving the management and monitoring of individuals with bipolar disorder is underscored by (Heydarian et al., 2023) to increase efficiency and minimize the risk of relapse and adverse effects. Clark et al. (2017) have proposed three approaches for understanding and classifying mental illness with particular attention to the four critical factors that lead to the mental disorder. This paper has investigated how these factors are related to the proposed approaches. Werbart et al. (2018) have examined the effect of a patient-therapist match on patients' treatment outcomes. This paper has explained that by initially matching patients and therapists, more remarkable improvement in the treatment of the patients could be achieved compared to single patients and therapists.

To sum up, this article is going to answer the questions below:

- How to classify mental disorders
- How to allocate therapists to the patients to achieve more progress in the treatment process
- How to create the optimal system in which the total time and costs of the mental treatment system will be minimized

To answer the above questions, first, mental disorders should be classified into different groups based on the symptoms and severity with the help of ML algorithms. Then, the therapists will be allocated to the patients to maximize the efficiency of the therapy outcome and minimize the wasted time between the therapist meeting and the total costs of the treatment system.

Many types of research have been done on mental health and therapist-patient pairing. However, to the best of our knowledge, there is no scientific paper for classifying people with mental health conditions based on their symptoms and allocating therapists to each class according to the problem and the therapist's expertise to get the best result from the treatment program. Though some papers introduced some approaches to increase the efficiency of mental treatment, no scientific work classifies patients with ML algorithms, assigns a therapist to them considering their problem and the therapist's profession, and schedules treatment sessions in the form of a patch. Comparing and using ML algorithms alongside the allocation and scheduling model is an innovative approach for this article.

Many research papers on the same topic, such as (J. Morrice et al., 2020), place great emphasis on the therapist scheduling system. However, the contribution of this research lies in firstly establishing a strong match and rapport between a patient and a specific therapist, and then utilizing the scheduling of the mental health clinic. This approach not only maximizes the efficient use of time in the clinic but also enhances the effectiveness of consultation sessions and increases the treatment success rate by ensuring ongoing contact between patients and their therapists.

The rest of the article is organized as follows. The table of notation and model assumption can be seen in the next section, and then, the problem is described. After that, a description of the used dataset is given, and ML algorithms are defined, evaluated, and compared. The mathematical model is developed to optimize each class of patients, and the solution methodology is described in the third section. In the fourth section, the proposed model is studied in a real-life case study to analyze the method's applicability. The result of the case study is discussed in the following section. The fifth section is dedicated to the sensitivity analysis of the model, and the effect of the parameters is investigated and analyzed. The following section gives managerial insight, and the crucial points are identified for decision-makers.

2. Problem description and formulation

2.1.Notations

The set of used notations for developing a mathematical model is shown in Table 1

Table 1. Notation table

Index	
i	Index of the group of patients with the same mental problem of type i = {1, …, I}
j	Index of the number of patients in each group
k	Index of the therapists in the medical center
Parameters	
u_{ijk}	The utility and effectiveness of the treatment process for patient number j in group i, under the supervision of therapist k
L_k	The working hours of the therapist k in a week
N_{ij}	The number of therapy sessions in a week for patient number j with mental illness type i
d_{ij}	The time of the therapy session for patient number j with mental problem type i
Variables	
X_{ijk}	The Binary variable that indicates whether patient number j from group i is assigned to the therapist number k

2.2.Assumption

To enhance the adequacy of the desired model, the following assumptions are incorporated into the mathematical framework.

1) All therapist knows how to treat different types of mental illnesses; however, they are more capable of treating some problem types according to their studies and experiences.
2) The treatment process differs for each patient, including the number and time of therapy sessions.
3) The number and the duration of the therapy session for patients are crisp and do not include uncertainty.

Based on the assumptions above, this research specifically targets a mental healthcare center. However, other mental health centers may also find the proposed approach beneficial and applicable.

2.3.Problem definition

With the development of human knowledge, it has become much more accessible than ever to deal with many life-threatening diseases. However, mental illnesses are still largely unknown to scientists. In addition, mental illnesses are prevalent in every society now. For that reason, noticing and curing mental illnesses has attracted lots of attention in recent years. The most important key to coping with mental illnesses is to diagnose the disorder type correctly and start therapy with a specialist who is fully aware of that type of disease. Therefore, in this research, ML algorithms will be compared to each other to find the most accurate one, and patients with different kinds of mental diseases will be classified with the help of the chosen ML algorithm. Afterward, therapist allocation and appointment scheduling will be done based on a mathematical model. Two objectives will be examined in the following. First, the efficiency of the treatment process, and second, the minimum wasted time of the patients and therapist between therapy sessions.

Figure 1. summarizes the defined problem in this study.

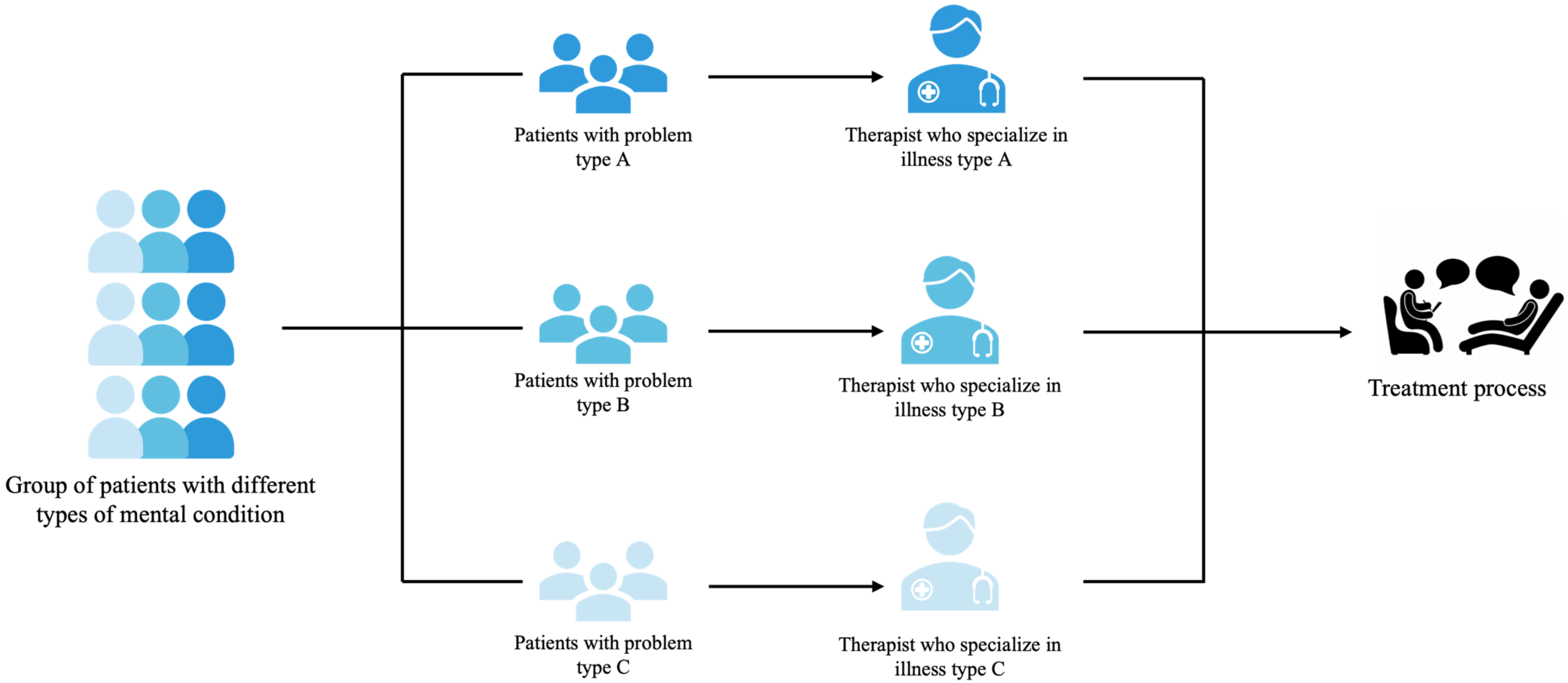

Figure 1. An overview of the problem under study

According to Figure 1, patients with different types of mental health problems arrive at the hospital. Then, with the help of the ML algorithms, these patients will be classified according to their symptoms and behaviors. As a result, patients with the same symptoms and mental problems will be placed into the same class. This classification could help the therapist diagnose the illness type more accurately, leading to a better treatment method and more efficient outcome. According to their studies and research, the selected therapists are experts in a particular area. Thus, they are fully aware of the problem and the treatment methods for the specific illness type. So, after classifying patients, therapists will be assigned to each group of patients according to their skilled profession and the mental kind of problem with the help of the mathematical model. By allocating therapists to the patients when he is genuinely aware of their situation, and it is in their profession, the outcome of the treatment process will be increased. As mentioned in Table 1, the effectiveness and the utility of the therapy session between patients and therapists will be identified by parameter u_{ijk} which can be calculated by Multi-Criteria Decision Making (MCDM) methods, including Bayesian Best Worst (BBWM) and fuzzy Weighted Aggregated Sum Product Assessment (WASPAS). After allocating therapists to each group of patients, a scheduling plan for therapy sessions should be designed to avoid wasting the patients' and therapists' time.

2.4.Mathematical model

In this section, a mathematical model will be developed to ensure that the effectiveness and utility of the therapy sessions will be maximized while considering the limitations of the patients and the therapist.

$$Max\ z = \sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{k=1}^{K} x_{ijk}u_{ijk} \tag{1}$$

S.t.

$$\sum_{i=1}^{I}\sum_{j=1}^{J} x_{ijk}N_{ij}d_{ij} < L_k \qquad \forall k=1,\ldots,K \tag{2}$$

$$\sum_{k=1}^{K} x_{ijk} = 1 \qquad \forall i = 1,\ldots,I \ \ \forall j = 1,\ldots,J \tag{3}$$

$$x_{ijk} = 0,1 \tag{4}$$

As mentioned, the primary purpose of this research is to maximize the effectiveness of the outcome of the treatment process. So, Eq. (1) shows the objective function of the mathematical model. It is tried to maximize the total utility of each patient's treatment session for a better outcome. In Eq. (1), the parameter u_{ijk} indicates the effectiveness level of the therapy session for each patient. This parameter could be identified from Multi-Criteria Decision Making (MCDM) methods, including Bayesian Best Worst (BBWM) and fuzzy Weighted Aggregated Sum Product Assessment (WASPAS), which will be evaluated in continue. Eq. (2) is a constraint to emphasize that the total time of the therapy session of the patient allocated to a therapist should not exceed the working hours of that therapist. Eq. (3) implies that a patient without a therapist is unacceptable, and every patient should be assigned a therapist. Eq. (4) addresses that x_{ijk} a binary variable shows whether patient number j from group i is assigned to therapist k.

3. Methodology

Classifying patients based on their symptoms and assigning skilled therapists according to their professions is expected to observe a higher improvement rate and symptom reduction in the therapy process. Therefore, Figure 2 provides an overview of the problem-solving steps used in this paper. According to Figure 2, the patients will be classified into different classes using ML algorithms to determine their problem types. Then, therapists will be assigned to each patient based on the problem type and the therapist's expertise. After that, the therapy appointment will be optimized to prevent wasting time.

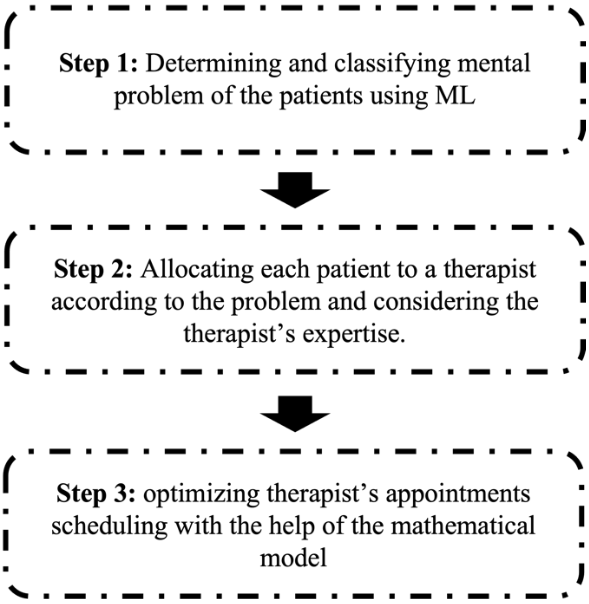

Figure 2. An overview of the problem-solving steps used in this research

3.1.Classifying patients

To spread knowledge about mental disorders, in 2013, the Diagnostic and Statistical Manual of Mental Disorders, Fifth Edition (DSM-5) was published by the American Psychiatric Association, which is used as a tool to understand different disorder types and their symptoms. In DSM-5, seventeen kinds of mental disorders are explained. Hence, based on this standard and experts' opinions, 14 criteria for people with mental conditions are evaluated in this paper to be used for the ML algorithms. Age, cognitive disability, which include speech disorder, movement disorder, communication disorder, drug usage, psychosis, depression symptoms, phobia and anxiety, mental obsession, unpleasant experience, appetite status, sleep status, sexual abnormalities, behavioral abnormalities, identity disorder, and personality disorder are some important factors to diagnose the illness type based on them.

Based on these features and the supervised ML algorithms, the applicants will be classified into different groups, making diagnosing the problem and starting the treatment more accessible.

3.2. Data collecting

The dataset used in this paper is collected from Roozbeh Psychiatric Hospital, Tehran, Iran, including the information on the 1547 patients with a mental condition. After removing the 312 instances from the dataset, 1235 cases remained, used for training and testing the ML algorithms. More than 25 features exist in this dataset; however, after deleting unrelated elements and combining some features for better results, 14 features remain to classify applicants based on them. Table 1 shows the mentioned elements of the applicants collected in the dataset.

Table 2. Data description

Feature	Type
ID	Numeric
Age	Numeric
Sleep status	Boolean
Appetite status	Boolean
Cognitive disability	Boolean
Drug usage	Boolean
Psychosis	Boolean
Depression symptoms	Boolean
Phobia and anxiety	Boolean
Mental obsession	Boolean
Unpleasant experience	Boolean
Sexual abnormalities	Boolean
Behavioral abnormalities	Boolean
Identity disorder	Boolean
Personality disorder	Boolean

The experts label the patients in the dataset based on their symptoms and diagnosis. In this research, several ML algorithms are used for classification because the medical data are imbalanced; ensemble algorithms (combining the prediction of two or more models) bring more accurate results. The bagging approach (bagged decision tree, random forest, and extra tree) and the boosting approach (Stochastic Gradient Boosting and CatBoost) are examples of ensemble algorithms used in this research, which they have already proven their abilities on similar datasets before.

3.3.Identifying the utility index for patients and therapists (u_{ijk})

One of the essential parameters in the proposed mathematical model is the utility index (u_{ijk}) for the therapy session, which indicates the therapy session's effectiveness and usefulness for each patient and therapist. Thus, in this section, two MDCM algorithms will be explained to determine this parameter.

3.3.1. Bayesian Best Worst Method (BBWM)

Mohammadi & Rezaei (2020) have introduced The Bayesian Best-worst method (BBWM), which is a multi-criteria decision-making (MCDM) method that seeks to optimize the result according to the expert opinion. This method is incredibly new and accurate. This paper uses this method to determine each queue's importance (weight) to achieve a better result for the mathematical model. The algorithm definition is as follows

In this method, the joint statistical distribution of the random variable is needed. Therefore, Eq. (A-1) shows Bayesian law. The overall optimal weight is indicated by W^{agg} and the optimal weight of each decision-maker is demonstrated by W^{K}. $A_W^{1:K}$ and $A_B^{1:K}$ are two vectors that indicate the relative importance of each criterion relative to the worst and best criterion, respectively.

$$\mathbf{P}\left(\mathbf{W^{agg}},\mathbf{W^{1:K}}\middle|\mathbf{A_B^{1:K}},\mathbf{A_W^{1:K}}\right) \tag{A-1}$$

After calculating the probability according to the Bayesian law, the probability distribution of each variable can be calculated with the help of Eq. (A-2) in which, x and y are some random variables.

$$\mathbf{P}(\mathbf{X}) = \sum_{\mathbf{y}} \mathbf{P}(\mathbf{x},\mathbf{y}) \tag{A-2}$$

$$\mathbf{P}\left(\mathbf{A_W^K}\middle|\mathbf{W^{agg}},\mathbf{W^K}\right) = \left(\mathbf{A_W^K}\middle|\mathbf{W^K}\right) \tag{A-3}$$

Eq (A-3) asserts that A_W^K is independent of W^{agg}.

$$\begin{aligned}\mathbf{P}\left(\mathbf{W^{agg}},\mathbf{W^{1:K}}\middle|\mathbf{A_B^{1:K}},\mathbf{A_W^{1:K}}\right) &= \mathbf{P}\left(\mathbf{A_B^{1:K}},\mathbf{A_W^{1:K}}\middle|\mathbf{W^{agg}},\mathbf{W^{1:K}}\right) \times \boldsymbol{P}\left(\boldsymbol{w^{agg}},\boldsymbol{w^{1:K}}\right) \\ &= \boldsymbol{P}(\boldsymbol{W^{agg}}) \prod_{\boldsymbol{k=1}}^{\boldsymbol{K}} \boldsymbol{P}\left(\boldsymbol{A_W^k}\ \middle|\boldsymbol{w^k}\right)\ \boldsymbol{P}\left(\boldsymbol{A_B^k}\ \middle|\boldsymbol{w^k}\right)\boldsymbol{P}\left(\boldsymbol{w^k},\boldsymbol{w^{agg}}\right)\end{aligned} \tag{A-4}$$

In Eq. (A-4), by considering all independency between variables and using Bayesian rules in Eq. (A-1) to reach the

$$\begin{aligned}&\mathbf{A_B^k}\ |\mathbf{w^k} \sim \mathbf{multinomial}\left(\frac{\mathbf{1}}{\mathbf{w^k}}\right) \\ &\mathbf{A_B^k}\ |\mathbf{w^k} \sim \mathbf{multinomial}\left(\mathbf{w^k}\right)\end{aligned} \tag{A-5}$$

Eq. (A-5) is used to calculate the distribution of variables.

$$\mathbf{w^k}\ |\mathbf{w^{agg}} \sim \mathbf{Dir}(\boldsymbol{\gamma} \times \mathbf{w^{agg}}) \qquad \forall k=1,\ldots,K \tag{A-6}$$

$$\boldsymbol{\gamma} = \mathbf{gamma}(\mathbf{a},\mathbf{b}) \tag{A-7}$$

$$\mathbf{w^{agg}} \sim \mathbf{Dir}(\boldsymbol{\alpha}) \tag{A-8}$$

The therapist's profession, experience, reputation, sex, and age are the selected factors for evaluating the compatibility between patients and therapists based on the expert's opinion.

3.3.2. Fuzzy Weighted Aggregated Sum Product Assessment (WASPAS)

The fuzzy weighted aggregated sum product assessment method, or WASPAS-F, was introduced by (Turskis et al., 2015). As with the WASPAS method, this method combines two methods: the WSM (total weight model) and the WPM (weight multiplication model) in a fuzzy set. In a decision-making problem, consider m alternatives $A_{1,}, A_{2,}, \dots, A_m$ are assessed based on n criteria $C_{1,}, C_{2,}, \dots, C_n$. IVIFNs are used to evaluate the performance and rating of each alternative i in a criterion j. The group of k decision-makers (D.M.s) experience in the decision operation. In this case, the kth decision-maker makes decisions on his/her inclinations or evaluations of alternatives based on criteria matching the following matrix:

$$\tilde{X}^K = \begin{bmatrix} \tilde{x}_{11}^K & \tilde{x}_{12}^K & \cdots & \tilde{x}_{1n}^K \\ \tilde{x}_{21}^K & \tilde{x}_{22}^K & \dots & \tilde{x}_{2n}^K \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1}^K & \tilde{x}_{m2}^K & \cdots & \tilde{x}_{mn}^K \end{bmatrix}, \tag{B-1}$$

$\tilde{x}_{ij}^K$ = ([μ_{Lij}^K, μ_{Uij}^K], [v_{Lij}^K, v_{Uij}^K]) is the IVIFN of alternative i against a criterion j that could be calculated in different ways (including D.M. directly distinguishing, transferring D.M. preferences to the IVIF by the typical pathways, benefiting intuitionistic fuzzy scale).

The following step is to characterize a set of weights for evaluation criteria. $\widetilde{W}^K$ = ($\widetilde{w}_1^k, \widetilde{w}_2^k, \dots, \widetilde{w}_n^k$) that $\widetilde{w}_j^k$ = ([μ_{Lj}^K, μ_{Uj}^K], [v_{Lj}^K, v_{Uj}^K]) is the kth expert's decision on the significance of the jth criterion and can be confined by diverse methods.

Afterward, the aggregated decision makers' matrices are calculated, and the aggregated weights of the criteria will be defined. Based on IIFWA, the aggregated decision matrix $\tilde{X}$ according to the following:

$$\tilde{X} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \dots & \tilde{x}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{bmatrix}, \tag{B-2}$$

Where $\tilde{x}_{ij}$= ([μ_{Lij}^K, μ_{Uij}^K], [v_{Lij}^K, v_{Uij}^K]). Similarly, the aggregated weight vector of criteria is denoted $\widetilde{W}$ = ($\widetilde{w}_1, \widetilde{w}_2, \dots, \widetilde{w}_n$) and $\widetilde{w}_j$ = ([μ_{Lj}, μ_{Uj}], [v_{Lj}, v_{Uj}]).

Normalizing $\tilde{X}$ in WASPAS-IVIF criteria requires two classifications: a subset of benefit criteria, B and a cost criterion, C. The linear normalization for $\forall j \epsilon B$ is:

$$\tilde{\tilde{x}}_{ij} = \frac{\tilde{x}_{ij}}{max_i \tilde{x}_{ij}}, \tag{B-3}$$

$max_i\tilde{x}_{ij}$ can be written as:

$$max_i\tilde{x}_{ij} = ([max_i\mu_{Lij}, max_i\mu_{Uij}], [max_i v_{Lij}, max_i v_{Uij}]). \quad \text{(B-4)}$$

Next, the linear normalization for $\forall j\epsilon C$ is:

$$\tilde{\tilde{x}}_{ij} = \frac{min_i\tilde{x}_{ij}}{\tilde{x}_{ij}}, \quad \text{(B-5)}$$

Where $min_i\tilde{x}_{ij}$ is extracted in the following equation:

$$max_i\tilde{x}_{ij} = ([min_i\mu_{Lij}, min_i\mu_{Uij}], [min_i v_{Lij}, min_i v_{Uij}]). \quad \text{(B-6)}$$

Obviously, $\tilde{x}_{ij} \leq max_i\tilde{x}_{ij}, \forall i, j: i = 1,2, \ldots, m, j\epsilon C$ and $max_i\tilde{x}_{ij} \leq \tilde{x}_{ij}, \forall i, j: i = 1,2, \ldots, m, j\epsilon C$. The decision matrix $\tilde{X}$ is converted into the normalized matrix $\tilde{\tilde{X}}$:

$$\tilde{\tilde{X}} = \begin{bmatrix} \tilde{\tilde{x}}_{11} & \tilde{\tilde{x}}_{12} & \cdots & \tilde{\tilde{x}}_{1n} \\ \tilde{\tilde{x}}_{21} & \tilde{\tilde{x}}_{22} & \ldots & \tilde{\tilde{x}}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\tilde{x}}_{m1} & \tilde{\tilde{x}}_{m2} & \cdots & \tilde{\tilde{x}}_{mn} \end{bmatrix}. \quad \text{(B-7)}$$

WASPAS method defines $\tilde{Q}_i^{(1)}$ as the sum total relative importance of alternative i, expressed as that is an IVIFN:

$$\tilde{Q}_i^{(1)} = \sum_{j=1}^{n} \tilde{\tilde{x}}_{ij} \,.\, \tilde{w}_j. \quad \text{(B-8)}$$

$\tilde{Q}_i^{(2)}$ represents the relative importance of alternative i as a product-total, which is defined according to the following formula:

$$\tilde{Q}_i^{(2)} = \prod_{j=1}^{n} \left(\tilde{\tilde{x}}_{ij}\right)^{\tilde{w}_j}. \quad \text{(B-9)}$$

With the help of extension principal besides considering $\tilde{x}_{ij} = ([\mu_{Lij}, \mu_{Uij}], [v_{Lij}, v_{Uij}])$ and $\tilde{w}_j = ([\mu_{Lj}, \mu_{Uj}], [v_{Lj}, v_{Uj}])$, the $\left(\tilde{\tilde{x}}_{ij}\right)^{\tilde{w}_j}$ can be obtained as:

$$\left(\tilde{\bar{x}}_{ij}\right)^{\tilde{w}_j} = \left(\left[\min(\mu_{Lij}, \mu_{Lj}), \min\left(\mu_{Uij}, \mu_{Uj}\right)\right]\right), \times \left(\left[\min(v_{Lij}, v_{Lj}), \min\left(v_{Uij}, v_{Uj}\right)\right]\right) \quad \text{(B-10)}$$

After implementing the product operation in Equation (16) on the $\left(\tilde{\bar{x}}_{ij}\right)^{\tilde{w}_j}$, $j = 1,2, \dots, n$, the $\tilde{Q}_i^{(2)}$ is an IVIFN. For ranking alternatives, the weighted aggregated sum product assessment (WASPAS) is calculated as follows:

$$\tilde{Q}_i = 0.5\tilde{Q}_i^{(1)} + 0.5\tilde{Q}_i^{(2)} \quad \text{(B-11)}$$

In the last step, the grade and accurateness of functions are calculated for $\tilde{Q}_i, i = 1,2, \dots, m$ and the final rankings of alternatives are determined based on the descending order of $\tilde{Q}_i, i = 1,2, \dots, m$.

4. Real-Life case study

For a better understanding of the presented model, in this segment, the mathematical model and its applications have been evaluated with a real-life case study from Roozbeh mental clinic for patients with mental conditions in one week.

According to Figure 1, mental patients arrive at the clinic to register their request for therapy. In this case study, people with mental health conditions visit a random therapist after arriving at the clinic without considering the therapist's profession and therapist-patient compatibility. This situation could cause a low efficiency in the therapy session, and most applicant could drop their therapy because they don't feel connected with their therapist. This problem could lead to a life-threatening issue in intense cases.

In this mental clinic, 19 patients registered their request for therapy treatment in one week. Three therapists have worked in this clinic four days a week and 6 hours daily. Based on the ML algorithms, patients are classified into four groups, and their information can be found in Table 3.

Table 3. Classification results in the mental clinic

Class	Disorder definition	number
Class A	Patients with obsessive-compulsive disorder (OCD) and anxiety issue	6
Class B	Patients with schizophrenia and delusion	2
Class C	Patients with different levels of depression issue and bipolar disorder	7
Class D	Patients with an eating disorder	4

Due to the policy of the health clinic, the total number of therapy sessions (N_{ij}) and the therapy session duration (d_{ij}) is the same for every patient in the same class. So, based on this classification, the other parameters of the mathematical model can be found in Table 4.

Table 4. The parameter's value for the mathematical model

parameter	N_{1j}	N_{2j}	N_{3j}	N_{4j}	d_{1j}	d_{2j}	d_{3j}	d_{4j}
Value	1	2	2.5	1	1	1.5	1.25	1

4.1. Experiments

Some ensembled classification algorithms have been introduced in the previous section. It is imperative to establish certain metrics and indicators to evaluate these algorithms and select the most advantageous one for this dataset.

Classification algorithms are evaluated by four important parameters Alipour-Vaezi et al., (2021), including True Positive (T.P.), True Negative (T.N.), False Positive (F.P.), and False Negative (F.N.). With the help of these parameters, some accuracy indicators can be defined to assess each algorithm.

$$\text{Accuracy} = \frac{TP+TN}{TP+FP+FN+TN} \tag{5.1}$$

$$\text{Recall} = \frac{TP}{TP+FN} \tag{5.2}$$

$$\text{Precision} = \frac{TP}{TP+FP} \tag{5.3}$$

$$\text{F1 score} = 2 * \frac{Precision*Recall}{Precision+Recall} \tag{5.4}$$

$$\text{MCC score} = \frac{(TP*TN)-(FP*FN)}{\sqrt{(TP+FP)(TP+FN)(TN+FP)(TN+FN)}} \tag{5.5}$$

Eq. (5.1) represents the accuracy score. Eq. (5.2) shows the ability of algorithms to predict positives correctly out of all actual positives. Eq. (5.3) addresses the ability of the algorithm to predict positives correctly out of all positive predictions that it makes, and Eq. (5.4) and Eq. (5.5) delineate the F1 score and MMC score, respectively, to evaluate the algorithm's accuracy.

The experiment was performed using an Apple MacBook Pro with an M1 chip and 8 GB RAM with the help of Python, an open-source language program that is top-rated among data scientists. Table 5 shows the result of the assessment of the proposed classification algorithms.

Table 5. Table of algorithm results evaluation

Algorithm	**Precession**	**Recall**	**F1 Score**	**Accuracy score**	**MMC score**
random forest	0.938	0.936	0.936	0.936	0.867
extra tree	0.934	0.931	0.932	0.931	0.859
Stochastic Gradient Boosting	0.928	0.926	0.927	0.926	0.849
CatBoost	0.937	0.931	0.931	0.931	0.860

According to the above table, Random Forest has an excellent performance on this data set with the highest MMC score, and the result shows that it could classify patients with very high accuracy. Based on this assessment, the Random Forest algorithm can benefit from this problem. Figure 3 guarantees that overfitting does not happen in the ML algorithm since there are no significant differences between training and testing results in different tests of different algorithms.

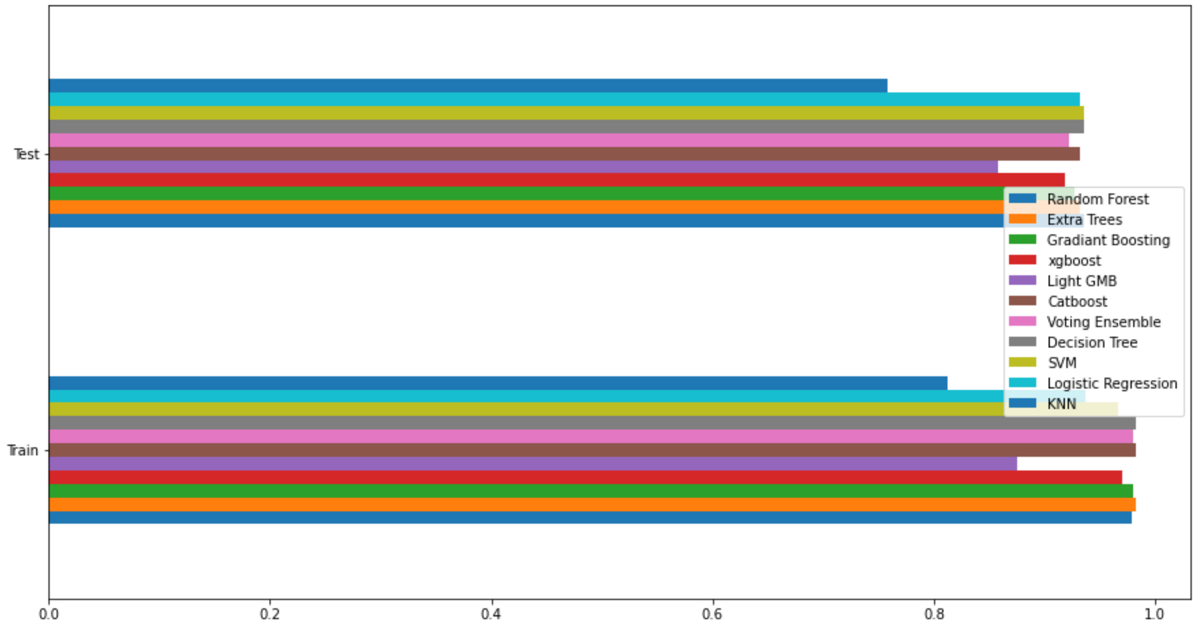

Figure 3. Comparison between Test and Train data in different algorithms

4.2.ML results

In this section, the result of the selected algorithm is reviewed and used in the mentioned case study. As mentioned before in Table 3, patients with mental health conditions are classified into four groups based on their symptoms and behavior. The results of the ML algorithm in this case study also can be found in Table 3.

4.3.MCDM results

One of the main parts of the mathematical model was identifying the utility of the therapy session between patient and therapist. The result of the MCDM methods, including BBWM and fuzzy WASPAS for discovering the value of the utility index between each patient and therapist for this real-life case study, is revealed in this segment.

4.3.1. BBWM results

As mentioned, the BBWM method is used for determining decision-making feature weights. Therefore, the final result of the BBWM method is shown in Table 6.

Table 6. The result of the BBWM methods

Features	Weight
Profession	0.54895914
Experience	0.14186584
Reputation	0.17733231
sex	0.08866615
age	0.04317656

4.3.2. Fuzzy WASPAS results

After calculating each feature's importance, the fuzzy WASPAS method could achieve the utility index. Table 7 shows the utility index for each patient and therapist based on the selected features, which is extracted from the WASPAS solver

Table 7. The fuzzy WASPAS method results

K =1								
i	**j**	u_{ij}	**i**	**j**	u_{ij}	**i**	**j**	u_{ij}
1	1	0.063078	2	2	0.13944	3	7	0.051304
1	2	0.067954	3	1	0.056411	4	1	0.136558
1	3	0.065239	3	2	0.048676	4	2	0.142139
1	4	0.060389	3	3	0.053525	4	3	0.142139
1	5	0.063078	3	4	0.063752	4	4	0.142139
1	6	0.065239	3	5	0.053875			
2	1	0.13944	3	6	0.056176			
K =2								
i	**j**	u_{ij}	**i**	**j**	u_{ij}	**i**	**j**	u_{ij}
1	1	0.174023	2	2	0.095515	3	7	0.119095
1	2	0.171049	3	1	0.127151	4	1	0.127151
1	3	0.159359	3	2	0.127151	4	2	0.12205
1	4	0.171263	3	3	0.12205	4	3	0.12205
1	5	0.174023	3	4	0.12205	4	4	0.12205
1	6	0.159359	3	5	0.127151			
2	1	0.095515	3	6	0.119467			

K =3								
i	**j**	u_{ij}	**i**	**j**	u_{ij}	**i**	**j**	u_{ij}
1	1	0.128254	2	2	0.101658	3	7	0.169606
1	2	0.133389	3	1	0.172249	4	1	0.123211
1	3	0.128316	3	2	0.172249	4	2	0.128316
1	4	0.123211	3	3	0.177874	4	3	0.128316
1	5	0.128254	3	4	0.175222	4	4	0.128316
1	6	0.128316	3	5	0.169606			
2	1	0.101658	3	6	0.175222			

4.3.3. Assigning patients to the therapist results

After calculating the utility and effectiveness of the treatment process for each patient and therapist, the best policy for assigning patients to therapists can be defined by solving mathematical models and finding optimal results. Table 8 addresses the optimal results for the mathematical model obtained with GAMS software.

Table 8.The optimal results for the model's variables

Model variables							
x_{ij}	**K=1**	**K=2**	**K=3**	x_{ij}	**K=1**	**K=2**	**K=3**
x_{11}		1		x_{33}			1
x_{12}		1		x_{34}			1
x_{13}		1		x_{35}			1
x_{14}		1		x_{36}			1
x_{15}		1		x_{37}			1
x_{16}		1		x_{41}	1		
x_{21}	1			x_{42}	1		
x_{22}	1			x_{43}	1		
x_{31}			1	x_{44}	1		
x_{32}			1				

Table 8 shows the optimal policy for assigning patients to therapists; based on this case study, the maximum amount of the utility function (Z) will be 3.063. This number could decrease if the working hours of the therapist would be lower.

4.4.System improving

According to the optimal result of the mathematical model, a significant improvement in the therapy efficiency could be noticed. Before applying this method to this mental clinic, patients were assigned to each therapist randomly without considering the therapist's expertise and abilities. In this case, the total utility index was calculated at 2.184 for 24 patients, which means 0.091 utility for each patient. After applying the proposed approach to the clinic, the summation of the utility index for 19 patients was 3.063, which means 0.161 for each patient. Therefore, this method increased the efficiency of the treatment system by 77 percent. This considerable improvement could ensure an increase in the quality of the treatment for each patient and a better outcome of the therapy for the mental clinic. Table 9 shows the improvement factors before and after applying this method in the clinic

Table 9. The summary of system improvement factors

Factors	**Before**	**After**
Number of patients	25	19
Total amount of utility function	2.184	3.063
An average utility for each patient	0.091	0.161

5. Sensitivity analysis

After computing the mathematical model's optimal results, this section shows the effect of different parameters such as L_k and N_{ij} on the objective function will be investigated to analyze the model behavior.

5.1.The effect of parameter L_k on the utility function (z)

The parameter L_k which is noted for the working hours of each therapist in the week is the first parameter to investigate. According to Figure 4, with an increase in the operating hours of each therapist, the total utility will be increased because the patient could be assigned to the suitable therapist without worrying about the working time. In this case study, the maximum level of utility could be achieved even with the low working hours for therapists 1 and 2. However, for therapist 3, the minimum working hours for achieving the highest utility is 24 hours a week. For most registered applicants, therapist 3 is the most suitable choice.

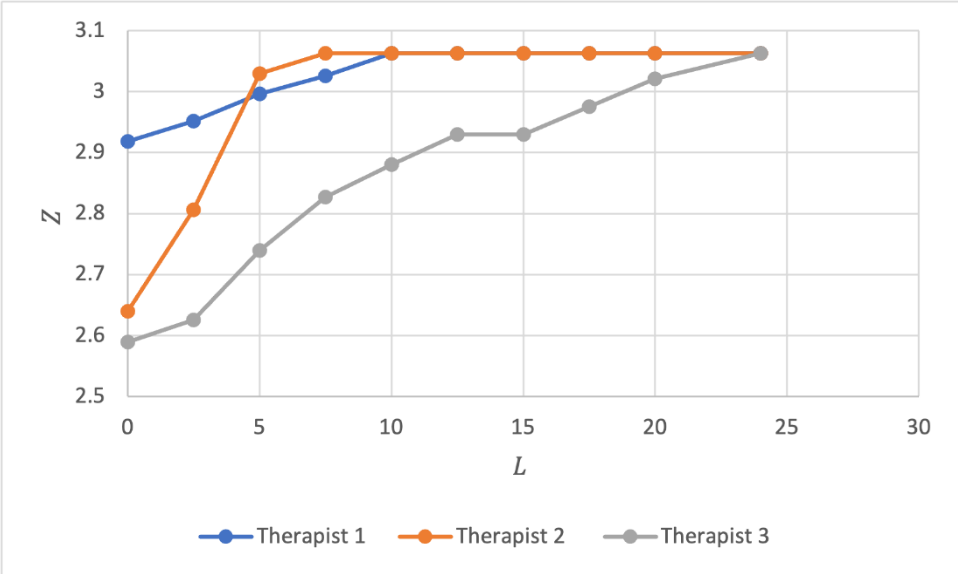

Figure 4. The effect of the parameter L_k on Z

5.2.The effect of parameter N_{ij} on the utility function (z)

As discussed, the working hours of the therapists could prevent the clinic from reaching its maximum efficiency. Another factor that could decrease the amount of the utility function is the needed number of therapy sessions for each group of patients. Figure 5 shows that with the increase in the number of therapy sessions in a week for each group of patients, the total utility will be decreased because of the limited working time of each therapist. If the number of therapy sessions increases, all patients cannot be assigned to a suitable therapist. So, the utility function will be decreased.

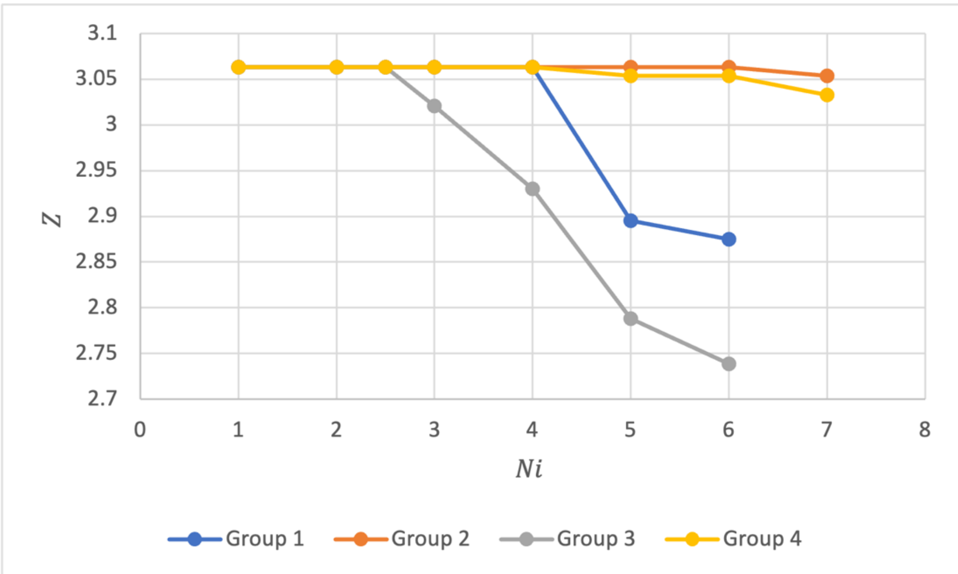

Figure5 . The effect of parameter N_i on z

The mentioned issue could be more noticed with the increase in the therapy session number for patients in groups 1 and 3; because of the higher number of patients in these groups, the decrease in the utility function is more intense.

6. Discussion and managerial insights

To the best of our knowledge, this article significantly improved compared to the previous research on patients with mental conditions. By applying the proposed method, mental health applicants will be categorized into different groups based on their symptoms. With the help of the ML algorithm, therapists can diagnose the illness more accurately. Then, with the MCDM methods, including BBWM and fuzzy WASPAS, the utility index between each patient and therapist will be calculated with particular attention to the therapist's profession. Then, with the help of the mathematical formulation, the patients will be assigned to each therapist in order to maximize the sum of the utility indexes while considering the limits.

In many types of research, the study's main aim is the clinic's total expenses or efficient scheduling. In others that discussed therapist-patient matches, most were just theoretical research, and no mathematical model existed. However, our study considered many factors with different weights to understand the best therapist for each patient. A mathematical model was developed to assign patients to therapists and consider their time limits.

As discussed in the sensitivity analysis section, according to Figure 4, the objective function will increase with the increase in the working hours of the therapist. Notably, the therapist's working hours depend on the number of applicants and the number of patients in a group with a similar mental problem. Therefore, it is recommended for the decision-maker (D.M.) to determine a policy with flexible working hours for each therapist. It is essential, and it could even reduce clinic costs. Based on Figure 5, the D.M. should not accept applicants more than its capacity because, in some cases, if the patients need more therapy sessions, the D.M. should hire an extra therapist or deal with the overtime work of the existing therapist, which increases the clinic's expenses in each scenario.

The following paragraph are intended to answer the research questions in the first section:

- ♦ The classification of mental disorders is performed using machine learning (ML) algorithms. In this paper, various ML algorithms are compared, and Random Forest is selected as the optimal algorithm based on its highest Matthews correlation coefficient (MMC) score.

- The assignment of mental patients to therapists is facilitated by a mathematical model, which aims to create optimal matches between patients and therapists while considering time constraints and other relevant factors.
- By establishing optimal matches between therapists and patients and implementing an efficient scheduling system, the effectiveness of therapy sessions can be significantly enhanced. This improvement has the potential to greatly enhance patient treatment outcomes and reduce overall costs for the mental health clinic.

The method proposed in this paper intends to maximize the total utility of the therapy session in a mental clinic with particular attention to the therapist profession. To solve this problem in limited dimensions, we used GAMS software, which has good accuracy and obtained valid answers .

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