Journal of Industrial and Systems Engineering
Vol. 15, No. 1, pp. 328-350
Winter (January) 2023

Optimal bidding in a participative mechanism with bundling and rebidding options

Mahya Hemmati[1], Seyed Mohammad Taghi Fatemi Ghomi[1*], Mohsen Sheikh Sajadieh[1]

[1]*Department of Industrial Engineering, Amirkabir University of Technology, Tehran, Iran*

m_hemmati@aut.ac.ir, fatemi@aut.ac.ir, sajadieh@aut.ac.ir

Abstract

This paper studies a name-your-own-price (NYOP) mechanism in which the retailer allows buyers to participate in the pricing process by submitting bids. Buyers can place both joint and individual bids to purchase products either as a bundle or individually. The retailer utilizes NYOP and posted-price channels simultaneously. The focus of this paper is to assess the impact of adding the posted-price channel and bundling option on buyer behavior and retailer profit. The paper develops a two-stage model where the first stage involves the buyer's decision on participating in NYOP. Moreover, buyers can choose between bidding for a bundle or a single item. Decisions in the second stage depend on the outcome of the first stage. Four distinct purchasing scenarios are formulated to outline the potential ways that buyers can use to purchase products. Furthermore, the buyers' learning effect on their bidding strategy is considered. A dynamic programming approach with backward induction is employed to solve the problem. Moreover, the concavity analysis is used to obtain the solution of each nonlinear subproblem. Then, a solution algorithm based on mathematical analysis is proposed. Results reveal that the frictional costs of the first period have a greater impact on the buyer utility than those of the second period. Moreover, applying the NYOP alongside the posted-price can enhance the retailer's profit. In particular, the retailer can use the NYOP and bundling mechanisms as encouraging tools to attract buyers and increase his profit. Thus, NYOP is a very effective instrument for market penetration.

Keywords: Participative pricing, Name-your-own-price, Bundling, Dynamic programming, Convex optimization, Non-linear programming.

1- Introduction

Nowadays, since competition is extended, sellers have to apply new and efficient marketing methods and pricing strategies to meet the customers' needs more efficiently and surpass rivals. Convenient access to the Internet and the emergence of new pricing mechanisms have led many sellers to use online and offline channels simultaneously. Selling through online channels helps the seller to reach out to a broader set of buyers. Hence, online purchases have a vital role in retail markets.

The Internet enhanced the ability of the interaction between buyers and retailers and deeply changed pricing strategies. Opaque selling is a mechanism in which the retailer conceals product characteristics (e.g.,

*Corresponding author
ISSN: 1735-8272, Copyright c 2023 JISE. All rights reserved

exact location and hotel name in case of hotel reservation) from buyers until the transaction is completed (Feng et al. 2019; Q. Li et al. 2020).

Priceline was the first retailer that applied opaque selling in the hospitality industry and utilized an innovative pricing strategy, name-your-own-price (NYOP) auction.

NYOP is a dynamic pricing mechanism where sellers let buyers participate in the pricing process by submitting bids. Different from traditional pricing, NYOP gives customers high control over the final price.

Under this mechanism, the retailer considers a hidden threshold price and buyers bid based on their knowledge. If the buyer's bid exceeds the threshold, she receives the product and pays the submitted offer (Spann et al. 2018; Wagner and Pacheco 2020). This mechanism is used by a wide variety of retailers and industries to sell goods and services, such as selling event tickets (ScoreBig), electronics (Greentoe), software (Ashampoo), and different types of products on eBay's Best Offer (Zeithammer et al. 2019).

The structure of the NYOP can affect buyer behavior and seller profit. For example, the number of allowable bids is one of the factors that change buyers' and sellers' profit under the NYOP. In the single-bid model, the retailer allows individuals to place only a bid through the NYOP channel, but the repeat-bidding model lets them rebid if their previous bid failed. Though some NYOP retailers such as Priceline restrict buyers to single-bid, they can rebid in the German one (Amaldoss and Jain 2008). When repeat-bidding is allowed, the wise and patient buyers can obtain the threshold via small increments in their bids. Therefore, considering frictional cost, which is disutility that buyers experience during the bidding process such as time, monetary fee, experience, mental effort to obtain optimal offers, etc. is crucial. The vendor can control frictional costs by manipulating the attribute of the NYOP mechanism. Thus, he should balance between the number of the allowable bid and losing the buyer if her previous bid does not meet the threshold price (Joo et al. 2012).

Though most of the NYOP retailers supply more than one product, most of the papers restrict the bidding process to a single item. For instance, in the hospitality industry, many retailers such as Priceline sell airline tickets, hotel rooms, and car rentals. Moreover, many of the passengers may need more than one category of these products (for example, airline ticket and hotel room) or more than one item of the specific category (for example, two airline tickets). some retailers such as Ashampoo sell their products as a bundle and some of them like Priceline, Greentoe, etc., also have the potential to sell items in bundles. Hence, considering the bundling strategy can be attractive to buyers and affect the profitability of the NYOP mechanism.

This paper investigates the theoretical and managerial implications of bundling policy in the NYOP mechanism where repeat-bidding is allowed. Moreover, this is the first work that considers the profitability of applying NYOP with a bundling option parallel to posted-price channels. The buyer decides whether to join the NYOP or the posted-price channel, if she decides to place bids, she can choose to bid for a single item or the bundle. Hence, this paper assesses the impact of various selling mechanisms on the buyers' bidding strategy. Furthermore, the profitability of the retailer is investigated. In other words, this paper aims to find how bundling pricing affects buyers' bidding strategy in NYOP? When do buyers prefer to bid through NYOP rather than buy from the posted-price? Whether applying NYOP is profitable for the retailer? What is the effect of frictional costs on buyer behavior?

The paper has the following structure: Section 2 reviews the related literature. Section 3 describes the mathematical models and solution procedures. Section 4 develops a solution algorithm. Section 5 conducts several sensitivity analyses. Finally, Section 6 concludes the paper and recommends some areas for future studies.

2- Related literature

This study is closely related to the literature on bundling, NYOP, kind of participating pricing mechanisms, and opaque channels.

The pricing policy is a powerful tool for retailers to sell products that has been studied based on different assumptions in a considerable number of studies. Bundling and new pricing methods as operational tools can affect buyers purchasing behavior (Cao et al. 2019). Best Buy used the bundling strategy to sell the extra inventory of Apple products and bundled the products with gift cards to increase sales (Cao et al. 2019). One of the applications of bundling and NYOP is in the transportation and tourism industry. Airline tickets for travel from Chicago to Miami are bundled with returns from Miami to Chicago. Moreover, these tickets can be sold as a bundle with a hotel and a car rental (Vamosiu 2018).

Cao et al. (2019) considered the influence of retail bundling on a supply chain where the retailer ordered the product before demand uncertainty was resolved. The retailer could sell the item alone or as a bundle with a secondary product. They showed that the retailer could take advantage of bundling by mitigating the demand uncertainty. Jena and Ghadge (2022) investigated product bundling in a duopoly supply chain, including two manufacturers and a single retailer with different power-balance and advertising efforts. Lin et al. (2020) studied pricing and bundling of two-competing platforms and investigated the effect of the installed base, mixed-bundling, and competition. Hemmati et al. (2021) studied bundling for complementary products in a two-stage supply chain under a consignment stock agreement. They showed that the firms benefit from economies of scale under bundling. Chen et al. (2021) considered the effect of the degree of product interrelatedness on the supply chain members' decisions, including mixed-bundling, partial mixed-bundling, and pure bundling strategies. Bucarey et al. (2021) developed pricing problems and algorithms for single-minded buyer behavior with the bundle where the buyer only bought the bundle if and only if its total price was less than the budget.

In the NYOP literature, most researchers considered a single item. Gupta and Abbas (2008) studied multiple substitution products where rebidding is allowed. They analyzed the model from the buyer perspective and derived an upper bound for the customer's gain. Amaldoss and Jain (2008) proposed a single-bid model for multiple items where the buyers have three bidding strategies: 1) single bidding in which buyers bids individually for each item, 2) joint bidding where buyers can submit a joint bid for a package of items, and 3) mixed bidding that lets buyers use both previous bidding strategies. He showed some buyers offer more in the joint bidding format, so the firms' profit can increase. Moreover, joint bidding can be more profitable than mixed bidding. Furthermore, he expressed that joint bidding decreases the probability of a mismatch between bid and threshold. Sabbaghnia et al. (2022) studied NYOP considering social responsibility in which firms donate to enhance business image, and customers can donate through a modified NYOP scheme. They showed that this scheme increases consumer participation and total profit.

Many NYOP retailers such as Priceline restrict bidders to a single-bid. However, the customers could submit a second offer using a different username and credit card. The retailer considers charging fees for any new bid to prevent buyers from obtaining the threshold price by incrementing their offer in small steps Bernhardt and Spann (2010). Several works have studied the repeat-bidding phenomenon in which buyers bid on a single item. Although most of the works that allow the buyers to rebid expressed that it is more profitable than single-bid, Fay (2009) showed that firms could have more gain by restricting customers to single-bid. In other words, the design of the optimal NYOP mechanism and the profitability of this strategy is still an open question. Although most researchers assume a fixed threshold, Hinz et al. (2011) suggested an adaptive and transparent threshold through a repeat-bidding setting can increase seller profit.

Fay (2004) considered a partial-double-bid scenario and found repeat-bidding may be more profitable than single-bid for the firm under some conditions. Hann and Terwiesch (2003) studied frictional costs where buyers are allowed to rebid and tried to obtain the bidding strategy, which maximizes the profit using a dynamic programming approach. Spann et al. (2004) compared the single-bid scenario with repeat-bidding and analyzed the effect of willingness-to-pay and frictional costs on the buyers' bidding strategy. Spann and Tellis (2006) analyzed buyers' behavior where the retailer let buyers rebid and examined whether buyers treat rational. Joo et al. (2012) focused on the impact of the buyers' bidding pattern on their gain in the presence of posted-price retailers. The results indicated that the time of bidding and the shape of the bid function are factors affecting the buyer's gain. Liu et al. (2016) considered the impact of information on buyers' expectations and willingness-to-pay where buyers allow rebidding. The results showed both buyers' experience and environmental information could affect buyers' bidding strategy. Levina et al. (2015) assumed buyers are allowed to submit a limited number of bids and studied the effect of buyers' collaboration on social networks.

Though most of the researchers considered NYOP as the only sales channel, selling through the NYOP in the presence of posted-price is another stream studied in the literature. Fay (2009) studied a competitive environment and showed NYOP could dominate posted-price. Moreover, he showed prohibiting repeat-bidding may make more gains for the retailer. Shapiro (2011) analyzed the profitability of adding a posted-price channel to the NYOP where buyers are risk-averse. Huang et al. (2017) studied an environment in which sellers decide whether to use a direct channel or an NYOP channel of a third party. R. R. Chen et al. (2014) developed a two-period model to investigate the impact of different selling strategies through opaque channels on two competing service providers. They studied the effect of capacity constraint on the pricing strategies and showed that posted-price is more beneficial for a provider, while the buyers can get more gain through NYOP. Anderson and Xie (2014) focused on a retailer using NYOP, posted-price, and traditional channels to analyze the impact of opaque selling in market segmentation. Li et al. (2016) investigated a closed-loop supply chain containing remanufacturing and pricing. Nosoohi (2022) studied transparent NYOP and posted-price channels for vertically differentiated products in which the quality level is considered.

Feng et al. (2018) proposed a collaborative game in which two providers service to leisure and business customers. They compared the profitability of applying traditional single-channel, traditional and posted-price channels, and traditional and NYOP channels. They found dual-channel has advantages over single-channel. Zeithammer et al. (2019) considered a dual-channel model and used incentive-compatible experiments to analyze the effect of friction costs on buyer behavior and buyer entry. Whereas the above papers restrict buyers to single-bid, Cai et al. (2009) studied different combinations of single-bid, double-bid, single-channel, dual-channel, where the capacity could be limited. The results showed that the double-bid case can dominate the single-bid in both the single-channel and dual-channel.

2-1- Research gap

Previous studies considered the NYOP mechanism from two aspects; 1) the behavior and bidding strategies of buyers, 2) the optimal design and the profitability for retailers. The current paper follows the first stream and studies the optimal customers' bidding strategy literature (Abbas and Hann (2010), Joo et al. (2012), Hann and Terwiesch (2003), Spann et al. (2004), Bernhardt and Spann (2010)). Table 1 summarizes the most relevant studies to the current paper and highlights the position of the paper in the literature.

Table 1. A summary of the related literature

Authors	Year	NYOP	Posted-price	Re-bidding	Multiple products	Bundling	Dual channel
Amaldoss and Jain	2008	✓	-	-	✓	✓	-
Gupta and Abbas	2008	✓	-	✓	✓	-	-
Cai et al.	2009	✓	✓	✓	-	-	✓
Hinz et al.	2011	✓	-	✓	-	-	-
Li et al.	2016	✓	✓	-	-	-	✓
Huang et al.	2017	✓	✓	-	-	-	✓
Feng et al.	2018	✓	✓	-	-	-	✓
Zeithammer et al.	2019	✓	✓	-	-	-	✓
Cao et al.	2019	-	✓	-	✓	✓	-
Chen et al.	2021	-	✓	-	✓	✓	-
Lin et al.	2020	-	✓	-	✓	✓	✓
Hemmati et al.	2021	-	✓	-	✓	✓	-
Bucarey et al.	2021	-	✓	-	✓	✓	-
Sabbaghnia et al.	2022	✓	✓	-	-	-	-
Jena and Ghadge	2022	-	✓	-	✓	✓	-
Nosoohi	2022	✓	✓	-	✓	-	✓
Hemmati et al.	2023	✓	-	-	✓	✓	-
This study	2023	✓	✓	✓	✓	✓	✓

Table 1 shows that most papers that studied bundling considered it as a promotion tool used by retailers to attract more customers and considered a posted-price mechanism in which the retailer sets the final price. However, in the current paper, the buyers set the final price by submitting bids. Amaldoss and Jain (2008) is the only paper that studied bundling under NYOP. However, they limited customers to a single bid and considered a single NYOP channel. According to the literature and policies used by NYOP retailers such as Ashampoo and Priceline (a hospitality retailer), bundling and rebidding are two main features of the NYOP mechanism. Moreover, in practice, customers always have outside options and can purchase products from posted-price channels. Furthermore, most papers that examine NYOP along with posted-price only considered a single item and limited the customers to a single bid (Zeithammer et al. 2019; Nosoohi 2022; Sabbaghnia et al. 2022). Hemmati et al. (2023) examined bundling of two products in an NYOP channel where customers are restricted to a single bid.

Thus, the current paper seeks to contribute to the related literature in two important ways. First, the previous study limited the buyers to a single bid for the bundle. We developed and numerically tested the model where repeated bidding is possible, and the customers can choose between a single item and bundle in each stage. Second, bidding for the bundle through the NYOP channel is considered where the buyer can purchase from posted-price channels simultaneously. Furthermore, we drive closed-form solutions, develop an algorithm to determine buyers' optimal bids, and propose several managerial insights.

3- Problem definition and mathematical model

A market in which the retailer sells two products through NYOP and posted-price channels simultaneously is considered. The behavior of buyers who learn about her needs relatively close to the date of the service is modeled. Buyers know the retailer announces the result of their bid with a delay. Hence, they can bid up to twice and there is a risk of not receiving the goods. The waiting time and the other disutility that the buyer may experience when using the NYOP are considered as a frictional cost. Therefore, a two-period environment which is in line with other researchers (Cai et al. 2009; Caldentey and Vulcano 2007; Etzion et al. 2006) is studied, so that the buyer first places bid in the NYOP channel and if her bid is rejected she can join the posted-price channel or submit another bid.

The buyer wants to book two items, for example, an airline ticket and a hotel room. She can purchase the items as a bundle or individual. In the first period, she specifies whether she wants to place a joint bid or a single bid. She prefers to buy both items or nothing. In other words, from the buyer's point of view, the value of buying one item is negative, whereas the value of buying nothing is zero, and the value of buying both items is positive. For better intuition, If a buyer can book a hotel room without buying an airline ticket, she can not travel and gains negative value. Hence, the buyer does not select the purchasing options leading to obtaining one product at the end of two periods. Thus, the decisions that the buyer may make are as follows:

Case 1: In the first period, join the NYOP, bid for the bundle and then submit another bid for the bundle if the first bid is rejected.

Case 2: In the first period, bid for the bundle through the NYOP channel and then buy from the posted-price channel if she fails.

Case 3: In the first period, bid for a single item and then two items are bought from the posted-price channel if the first bid is rejected; otherwise, purchase a single item from the posted-price channel.

Case 4: Buy from the posted-price channel in both periods.

Please note that other cases that the buyer may obtain only one product are not profitable for her; hence she never chooses them.

The buyer does not know the exact value of the threshold set by the retailer and she does not have any knowledge about the retailer's decisions making process. However, she knows that the retailer draws the threshold price for a single item from a random distribution and believes that the threshold is uniformly distributed over the normalized interval [0,1] (Anderson and Xie 2014; Abbas and Hann 2010; Hann and Terwiesch 2003; Fay 2009; Amaldoss and Jain 2008; Spann et al. 2004; Bernhardt and Spann 2010). Moreover, similar to Fay (2009), we assume the willingness-to-pay for each buyer is 1 for one unit of product. Considering a common willingness-to-pay simplifies the analysis and does not ruin the generality of the problem.

The notation used in the proposed model is as follows:

k	Period index, $k = 0,1$
WTP_i	Buyer's willingness-to-pay for the individual item
WTP_j	Buyer's willingness-to-pay for the bundle
P_k	A random buyer's bid at period k
T	The threshold price
B	Price of the item through the posted-price channel
c_k	Buyer's frictional costs at period k

Case 1

When the buyer bid for a bundle, the joint distribution of the buyer's belief is triangular. The probability density function of a symmetric triangular distribution, which is the sum of two independent Uniform random variables on $[a,b]$ is considered. Thus, the expected utility of buyer who can submit a maximum of two bids and decided to bid for a bundle is:

$$\max_{P_0,P_1} u_{bN} = (WTP_j - P_0)\,p(P_0 \geq T) - c_0 + [1 - p(P_0 \geq T)][(WTP_j - P_1)\,p(P_1 \geq T \mid T > P_0) - c_1] \tag{1}$$

The buyer learns about the threshold during the bidding process and updates her belief about the threshold price. In other words, the buyer learns from the failure that the threshold price is more than her offer. The Bayesian rule is used to update the buyer's belief and obtain the success probabilities. The above Bellman equation is solved according to the range of possible bids in each stage. The rational buyer never bid more than $2b$ for the bundle. Figure 1 shows these ranges.

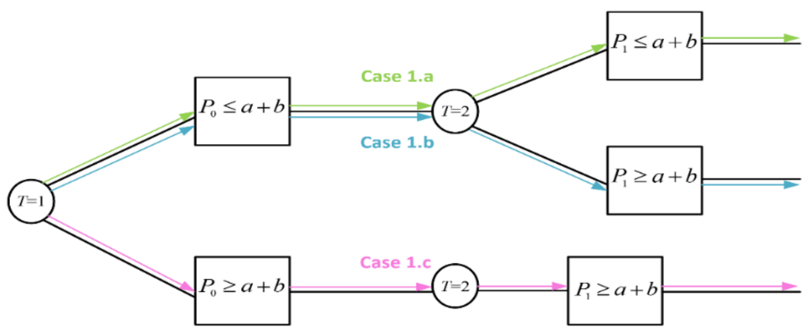

Fig 1. The possible ranges for the bids in each period in case1

Taking $M = \dfrac{y-2a}{(b-a)^2}$ and $N = \dfrac{2b-y}{(b-a)^2}$. Hence, the maximum value of equation (1) can be obtained by solving $u_{bN} = \max(0, u_{bN0}, u_{bN1}, u_{bN2})$, where

$$\max_{P_0,P_1} u_{bN0} = (WTP_j - P_0)\int_{2a}^{P_0} M\,dy - c_0 + \Big(1 - \int_{2a}^{P_0} M\,dy\Big)\Big[(WTP_j - P_1)\Big(\int_{P_0}^{P_1} M\,dy \Big/ \Big(\int_{P_0}^{a+b} M\,dy + \int_{a+b}^{2b} N\,dy\Big)\Big) - c_1\Big] \tag{2}$$

$$\max_{P_0,P_1} u_{bN1} = (WTP_j - P_0)\int_{2a}^{P_0} M\,dy - c_0 + \Big(1 - \int_{2a}^{P_0} M\,dy\Big)\Big[(WTP_j - P_1)\Big(\int_{P_0}^{a+b} M\,dy + \int_{a+b}^{P_1} N\,dy \Big/ \Big(\int_{P_0}^{a+b} M\,dy + \int_{a+b}^{2b} N\,dy\Big)\Big) - c_1\Big] \tag{3}$$

$$\max_{P_0,P_1} u_{bN2} = (WTP_j - P_0)\Big(\int_{2a}^{a+b} M\,dy + \int_{a+b}^{P_0} N\,dy\Big) - c_0 + \int_{P_0}^{2b} N\,dy\Big[(WTP_j - P_1)\Big(\int_{P_0}^{P_1} N\,dy \Big/ \Big(\int_{P_0}^{2b} N\,dy\Big)\Big) - c_1\Big] \tag{4}$$

As noted earlier, the Uniform threshold distribution over the normalized interval [0,1] is considered, i.e., $a = 0$ and $b = 1$, and the distribution of the buyer's belief is triangular, over [0,2]. Parameters u_{bN_0}, u_{bN_1}

, and u_{bN_2} indicate the buyer's expected utility in cases 1.a, 1.b, and 1.c respectively. In Case 1.a the buyer bids less than $a+b=1$ in both periods ($P_0 \leq 1$ and $P_1 \leq 1$). In Case 1.b she bids less than $a+b=1$ in the first period, and more than $a+b=1$ in the second period ($P_0 \leq 1$ and $P_1 > 1$); in case 1.c bids more than $a+b=1$ in both periods ($P_0 > 1$and $P_1 > 1$). Figure 1 depicts the cases. Backward induction to recursively evaluate equations (2), (3), and (4) is applied.

Proposition 3.1. In Case 1.a where the buyer can bid less than 1 for the bundle in both periods, the optimal bidding strategy is $P_1 = 1$ and $P_0 = min\left(1, \frac{2(c_1+1)}{3}\right)$.

Proof: See Appendix A for further details.

Proposition 3.2. In Case 1.b where the buyer bids less than 1 in the first period and more than 1 in the second period for the bundle, the optimal offer in the second period is $P_1 = 2 - \sqrt{-3P_0^2 + 6}/3$. In this case, the optimal solution in the first period is $P_0 = 1$ if $c_1 \geq -2 + \sqrt{186}/6$ and $P_0 = \min(r_{bN1,0}^3, 1)$ if $0 \leq c_1 \leq -2 + \sqrt{186}/6$.

Proof: See Appendix B for further details.

Proposition 3.3. In Case 1.c) The buyer bids more than 1 for the bundle in both periods, and the optimal solution is $P_1 = 2 + \sqrt{3}(P_0 - 2)/3$, where $P_0 = \max(1, 2 + 2c\sqrt{3} + 9c - \sqrt{93c^2 + 36\sqrt{3}c^2 + 92\sqrt{3} + 414}/23)$.

Proof: See Appendix C for further details.

In Case 1, the retailer's expected utility is:

$$u_{rN} = p(P_0 \geq T)P_0 + [1 - p(P_0 \geq T)][p(P_1 \geq T | T > P_0)P_1] \tag{5}$$

Hence, according to the buyer's bid, the retailer's expected utility is equal to one of the following equations:

$$u_{rN0} = P_0 \int_{2a}^{P_0} M dy + P_1 \int_{P_0}^{P_1} M dy \tag{6}$$

$$u_{rN1} = P_0 \int_{2a}^{P_0} M dy + P_1 \left(\int_{P_0}^{a+b} M dy + \int_{a+b}^{P_1} N dy \right) \tag{7}$$

$$u_{rN2} = P_0 \left(\int_{2a}^{a+b} M dy + \int_{a+b}^{P_0} N dy \right) + P_1 \int_{P_0}^{P_1} N dy \tag{8}$$

Case 2

At first, the buyer places a bid for the bundle; if the bid is rejected, the buyer abandons the NYOP auction and buys both items from the posted-price. There are no frictional costs to buy from a posted-price channel. Moreover, purchase from the posted-price is optimal if $B \leq WTP$. Hence, the buyer's expected utility is:

$$\max_{P_0, P_1} u_{bNp} = (WTP_j - P_0)\, p(P_0 \geq T) - c_0 + 2[1 - p(P_0 \geq T)](WTP_i - B) \tag{9}$$

Same to Case1, the range of possible offers in the first period is considered. Hence, in the first period, the buyer bids less than $a+b=1$ in Case 2.a, and she bids more than $a+b=1$ in case 2.b. Figure 2 illustrates the cases.

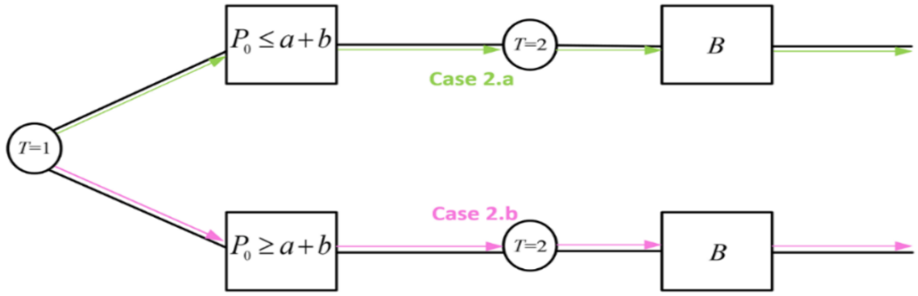

Fig 2. The possible ranges for the bids in case 2

Thus, the maximum value of equation (9) is obtained finding the optimal solution of $u_{bNp} = max(0, u_{bNp0}, u_{bNp1})$, where

$$\max_{P_0,P_1} u_{bNp0} = (WTP_j - P_0)\int_{2a}^{P_0} Mdy - c_0 + 2(1 - \int_{2a}^{P_0} Mdy)(WTP_i - B)a \tag{10}$$

$$\max_{P_0,P_1} u_{bNp1} = (WTP_j - P_0)(\int_{2a}^{a+b} Mdy + \int_{a+b}^{P_0} Ndy) - c_0 + 2(WTP_i - B)\int_{P_0}^{2b} Ndy \tag{11}$$

Proposition 3.4. In Case 2.a where the buyer submits a bid that is less than 1 for the bundle in the first period if the bid fails to meet the threshold, she purchases items at a posted-price *B*, and the optimal offer is $P_0 = \min(1, 4B/3)$.

Proof: See Appendix D for further details.

Proposition 3.5. In Case 2.b, where the buyer bids more than 1 for the bundle in the first period and purchases items at a posted-price *B* if she fails, the optimal offer is $P_0 = \max(1, (2B + 4 - 2\sqrt{(B-1)^2 + 1.5})/3)$

Proof: See Appendix E for further details.

When the buyer's offer does not meet the threshold, the buyers purchase from the posted-price channel. Hence, the retailer's expected gain is:

$$u_{rNp} = P_0 p(P_0 \ge T) + 2B(1 - p(P_0 \ge T)) \tag{12}$$

Thus, according to the buyer's bid, the expected utility of the retailer as follows:

$$u_{rNp0} = P_0 \int_{2a}^{P_0} Mdy + 2B(1 - \int_{2a}^{P_0} Mdy) \tag{13}$$

$$u_{rNp1} = P_0(\int_{2a}^{a+b} Mdy + \int_{a+b}^{P_0} Ndy) + 2B\int_{P_0}^{2b} Ndy \tag{14}$$

Case 3

The buyer initially bids for a single item in the first period and then purchases at posted-price. If the first bid is rejected, the buyer purchases two items at the posted-price; otherwise, she purchases an item. As noted earlier, when the buyer bids for a single item, the distribution of the buyer's belief is Uniform. Hence, the buyer's expected utility is:

$$\max_{P_0} u_{bNp2} = (WTP_i - P_0)\frac{P_0 - a}{b - a} - c_0 + (WTP_i - B)\frac{P_0 - a}{b - a} + 2(WTP_i - B)\frac{b - P_0}{b - a} \tag{15}$$

Proposition 3.6. The buyer's optimal bid of case 3 in which the buyer bid for a single item and then purchases from posted-price is $P_0 = \frac{B}{2}$.

Proof: See Appendix F for further details.
According to the buyer's bid, the retailer's expected utility is:

$$u_{rNp2} = P_0 \frac{P_0 - a}{b - a} + B \frac{P_0 - a}{b - a} + 2B \frac{b - P_0}{b - a} \tag{16}$$

Case 4

The buyer does not participate in NYOP and buys both items at posted-price. Thus, the buyer's expected utility is:

$$u_{bp} = 2(WTP_i - B) \tag{17}$$

In this case, the retailer's expected utility is:

$$u_{rp} = 2B \tag{18}$$

According to the proposed propositions and mathematical proof in appendices, the following algorithm is applied to find the optimal case applied by the buyer to purchase the product.

Step 1: Set $a = 0, b = 1, WTP_i = 1$, and $WTP_j = 2$.

Step 2: Set $P_0 = min\left(1, \frac{2(c_1+1)}{3}\right)$ and $P_1 = 1$. Find the value of the u_{bN_0} and u_{rN_0} applying equations (2) and (6).

Step 3: If $c_1 \geq -2+\sqrt{186}/6$ set $P_0 = 1$, otherwise, set $P_0 = min(r_{bN1,0}^3, 1)$. Set $P_1 = 2-\sqrt{-3P_0^2+6}/3$ and calculate u_{bN_1} and u_{rN_1} utilizing equations. (3) and (7).

Step 4: Set $P_0 = \max(1, 2+2c\sqrt{3}+9c-\sqrt{93c^2+36\sqrt{3}c^2+92\sqrt{3}+414}/23)$ and $P_1 = 2+\sqrt{3}(P_0-2)/3$ then find the value of u_{bN2} and u_{rN2} applying equations (4) and (8).

Step 5: Find $u_{bN}^{opt} = \max(u_{bN0}, u_{bN1}, u_{bN2})$ **.** If $u_{bN}^{opt} = u_{bN0}$ set ${P_0}^{bN} = min\left(1, \frac{(2c_1+2)}{3}\right)$, ${P_1}^{bN} = 1$ and $u_{rN}^{opt} = u_{rN0}$. If $u_{bN}^{opt} = u_{bN1}$ set P_0^{bN} equal to P_0 obtained in Step 3, $P_1^{bN} = 2-\sqrt{-3P_0^2+6}/3$ and $u_{rN}^{opt} = u_{rN_1}$. Otherwise, set $P_1^{bN} = 2+\sqrt{3}(P_0-2)/3$, $P_0^{bN} = \max(1, 2+2c\sqrt{3}+9c-\sqrt{93c^2+36\sqrt{3}c^2+92\sqrt{3}+414}/23)$, and $u_{rN}^{opt} = u_{rN_2}$.

Step 6: Set $P_0 = min(1, \frac{4B}{3})$ and calculate u_{bNp_0} and u_{rNp_0} using equations. (10) and (13).

Step 7: Set $P_0 = \max(1, (2B+4-2\sqrt{(B-1)^2+1.5})/3)$ and calculate u_{bNp_1} and u_{rNp_1} applying equations (11) and (14).

Step 8: Set $P_0 = \frac{B}{2}$ and calculate u_{bNp_2} and u_{rNp_2}utilizing equations (15) and (16).

Step9: Find $u_{bNp}^{opt} = \max(u_{bNp0}, u_{bNp1}, u_{bNp_2})$**.** If $u_{bNp}^{opt} = u_{bNp_0}$ set ${P_0}^{bNP} = min(1, \frac{4B}{3})$ and $u_{rNp}^{opt} = u_{rNp_0}$. If $u_{bNp}^{opt} = u_{bNp_1}$ set $P_0 = \max(1, (2B+4-2\sqrt{(B-1)^2+1.5})/3)$ and $u_{rNp}^{opt} = u_{rNp_1}$; Otherwise, set $P_0 = \frac{B}{2}$ and $u_{rNp}^{opt} = u_{rNp_2}$.

Step 10: Calculate u_{bp} and u_{rp} using equations (17) and (18).

Step 11: Find $u_b^{opt} = \max(u_{bN}^{opt}, u_{bNp}^{opt}, u_{bp})$. If $u_b^{opt} = u_{bN}^{opt}$ set $P_0^* = {P_0}^{bN}$, $P_1^* = {P_1}^{bN}$and $u_r = u_{rN}^{opt}$. If $u_b^{opt} = u_{bNp}^{opt}$ set $P_0^* = {P_0}^{bNp}$ and $u_r = u_{rNp}^{opt}$. Otherwise, set $u_r = u_{rp}^{opt}$.

4- Numerical experiments

This section conducts numerical experiments. The algorithm's solutions are compared to the solutions of subproblems obtained via GAMS 24.9.1. Remark that only one of the subproblems can occur at a time (owing to the customer's single choice), and a rational buyer selects the option that maximizes her profit. Consequently, the maximum solution of the subproblems achieved through GAMS can be compared with the solution generated by the developed algorithm. The algorithm relies on the mathematical characteristics of the model and utilizes calculated closed-form solutions. It enhances time efficiency compared to GAMS.

The parameters of the problem are taken according to Fay (2009). The model undergoes validation through sensitivity analysis and comparison with benchmark cases. The problem is executed with various parameter values to ensure logical solutions are produced by the model and to analyze both buyer's and vendor's behavior.

Here, the impact of bidding for a bundle and applying the NYOP parallel to the posted-price are studied. In other words, the goal of these experiments is to examine whether it is beneficial for the buyers if they have the option to buy a bundle? When they prefer to purchase from the NYOP channel versus the posted-price channel? Is it profitable for the retailer to provide an NYOP along with a posted-price?

At first, the behavior of the buyer is studied where a monopoly retailer sells items only at NYOP. Hence, the buyer submits a bid for the bundle in each period (cases 1.a, 1.b, and 1.c). Table 2 shows the effect of frictional costs on the buyer's bidding behavior.

Table 2. Buyer's bidding strategy and her utility in Case 1

c_0	c_1	U_b	P_0^*	P_1^*	c_0	c_1	U_b	P_0^*	P_1^*	c_0	c_1	U_b	P_0^*	P_1^*
	0.00	0.70	0.92	1.38										
	0.05	0.67	0.97	1.41		0.05	0.62	0.97	1.41					
	0.10	0.64	1.01	1.43		0.10	0.59	1.01	1.43		0.10	0.54	1.01	1.43
	0.15	0.62	1.04	1.44		0.15	0.57	1.04	1.44		0.15	0.52	1.04	1.44
	0.20	0.60	1.06	1.46		0.20	0.55	1.06	1.46		0.20	0.50	1.06	1.46
0	0.25	0.57	1.09	1.47	0.05	0.25	0.52	1.09	1.47	0.1	0.25	0.47	1.09	1.47
	0.30	0.55	1.11	1.49		0.30	0.50	1.11	1.49		0.30	0.45	1.11	1.49
	0.35	0.54	1.13	1.50		0.35	0.49	1.13	1.50		0.35	0.44	1.13	1.50
	0.40	0.52	1.15	1.51		0.40	0.47	1.15	1.51		0.40	0.42	1.15	1.51
	0.45	0.50	1.17	1.52		0.45	0.45	1.17	1.52		0.45	0.40	1.17	1.52
	0.50	0.48	1.20	1.54		0.50	0.43	1.20	1.54		0.50	0.38	1.20	1.54

From table 2 it is obvious that buyers with higher frictional costs obtain lower utility. Moreover, the frictional costs of the first period have a greater effect on the buyer's utility than that of in the second period. When c_0 increases by 0.05, for the same c_1, the utility decreases by 0.05. However, when c_1 increases by 0.05, for the same c_0, the utility decreases at most 0.03. It is due to that buyers always incur frictional costs in the first period but they incur these costs in the second period if their first bid is rejected. Furthermore, the frictional costs in the first period do not affect submitted bids in both periods. However, these costs in the second period influence the offers in both periods. In particular, as the buyer's frictional costs of the second period increase, the submitted bid increases.

When the buyer's frictional costs of the second period are low ($c_1 < 0.1$), the buyers choose Case 1.b and bid less than 1 in the first period and more than 1 in the second period. An increase in the buyer's frictional costs of the second period ($c_1 \geq 0.1$), leads the buyer to choose Case 1.c in which she can gain more by bidding greater than 1 in both periods. Moreover, the buyer never uses Case 1.a. Thus, the variation in frictional costs is an instrument to segment customers.

Table 3 indicates the maximum buyer's utility and her optimal bidding strategy where she can purchase from both NYOP and posted-price channels.

Table 3. Bidding strategy and buyer's and retailer's utilities

Parameters		c_1									
		0.1					0.2				
c_0	B	u_b	P_0^*	P_1^*	Optimal policy	u_r	u_b	P_0^*	P_1^*	Optimal policy	u_r
0	1.00	0.64	1.01	1.43	Case 1	0.98	0.60	1.06	1.46	Case 1	1.02
	0.95	0.64	1.01	1.43	Case 1	0.98	0.60	1.06	1.46	Case 1	1.02
	0.90	0.64	1.01	1.43	Case 1	0.98	0.62	1.11	-	Case 2	1.38
	0.85	0.66	1.08	-	Case 2	1.34	0.66	1.08	-	Case 2	1.34
	0.80	0.70	1.04	-	Case 2	1.30	0.70	1.04	-	Case 2	1.30
	0.75	0.75	1.00	-	Case 2	1.25	0.75	1.00	-	Case 2	1.25
	0.70	0.80	0.93	-	Case 2	1.20	0.80	0.93	-	Case 2	1.20
	0.65	0.86	0.87	-	Case 2	1.14	0.86	0.87	-	Case 2	1.14
	0.60	0.93	0.80	-	Case 2	1.07	0.93	0.80	-	Case 2	1.07
	0.55	1.00	0.73	-	Case 2	1.00	1.00	0.73	-	Case 2	1.00
	0.50	1.07	0.67	-	Case 2	0.93	1.07	0.67	-	Case 2	0.93
	0.45	1.15	0.60	-	Case 2	0.85	1.15	0.60	-	Case 2	0.85
	0.40	1.24	0.20	-	Case 3	0.76	1.24	0.20	-	Case 3	0.76
0.05	1.00	0.59	1.01	1.43	Case 1	0.98	0.55	1.06	1.46	Case 1	1.02
	0.95	0.59	1.01	1.43	Case 1	0.98	0.55	1.06	1.46	Case 1	1.02
	0.90	0.59	1.01	1.43	Case 1	0.98	0.57	1.11	-	Case 2	1.38
	0.85	0.61	1.08	-	Case 2	1.34	0.61	1.08	-	Case 2	1.34
	0.80	0.65	1.04	-	Case 2	1.30	0.65	1.04	-	Case 2	1.30
	0.75	0.70	1.00	-	Case 2	1.25	0.70	1.00	-	Case 2	1.25
	0.70	0.75	0.93	-	Case 2	1.20	0.75	0.93	-	Case 2	1.20
	0.65	0.81	0.87	-	Case 2	1.14	0.81	0.87	-	Case 2	1.14
	0.60	0.88	0.80	-	Case 2	1.07	0.88	0.80	-	Case 2	1.07
	0.55	0.95	0.73	-	Case 2	1.00	0.95	0.73	-	Case 2	1.00
	0.50	1.02	0.67	-	Case 2	0.93	1.02	0.67	-	Case 2	0.93
	0.45	1.10	0.60	-	Case 2	0.85	1.10	0.60	-	Case 2	0.85
	0.40	1.20	-	-	Case 4	0.80	1.20	-	-	Case 4	0.80
0.1	1.00	0.54	1.01	1.43	Case 1	0.98	0.50	1.06	1.46	Case 1	1.02
	0.95	0.54	1.01	1.43	Case 1	0.98	0.50	1.06	1.46	Case 1	1.02
	0.90	0.54	1.01	1.43	Case 1	0.98	0.52	1.11	-	Case 2	1.38
	0.85	0.56	1.08	-	Case 2	1.34	0.56	1.08	-	Case 2	1.34
	0.80	0.60	1.04	-	Case 2	1.30	0.60	1.04	-	Case 2	1.30
	0.75	0.65	1.00	-	Case 2	1.25	0.65	1.00	-	Case 2	1.25
	0.70	0.70	0.93	-	Case 2	1.20	0.70	0.93	-	Case 2	1.20
	0.65	0.76	0.87	-	Case 2	1.14	0.76	0.87	-	Case 2	1.14
	0.60	0.83	0.80	-	Case 2	1.07	0.83	0.80	-	Case 2	1.07
	0.55	0.90	-	-	Case 4	1.10	0.90	-	-	Case 4	1.10
	0.50	1.00	-	-	Case 4	1.00	1.00	-	-	Case 4	1.00
	0.45	1.10	-	-	Case 4	0.90	1.10	-	-	Case 4	0.90
	0.40	1.20	-	-	Case 4	0.80	1.20	-	-	Case 4	0.80

Results show that the posted-price can influence the number of buyers bidding at NYOP. As table 3 shows, when $B > 0.85$, buyers bid more than 1 in both periods (Case 1.c). However, when $B \leq 0.85$, buyers prefer to buy from the posted-price at least in one period (Cases 2, 3, and 4). Figure 3 shows as B increases, the buyer obtains less surplus. In particular, when B decreases from 1 to 0.4, the buyer's utility increases by about 100 percent. Hence, the posted-price significantly affects buyer behavior and her decision on whether to place a bid at NYOP.

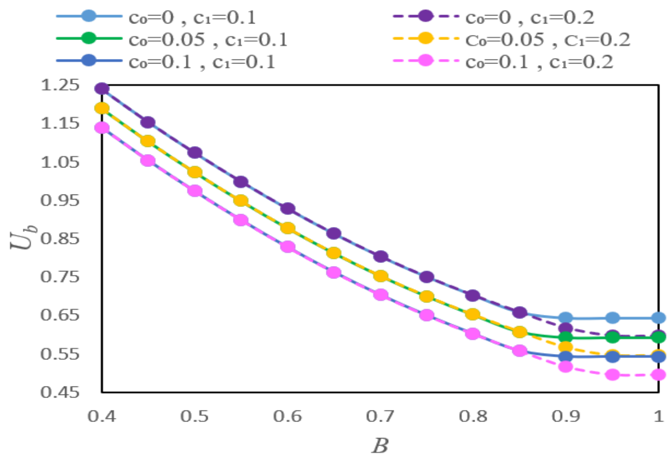

Fig 3. Effect of posted-price and frictional costs on the buyer's utility

It is obvious from figure 3, the buyers with high frictional costs obtain less profit. Hence, buyers with low frictional costs have more incentive to participate in NYOP. Additionally, figure 3 shows, when $B \leq 0.85$, variations in c_1, which is the frictional cost of the second period, do not affect the buyer's bidding strategy and utility. It is due to when B is not very high, the buyer prefers to purchase from the posted-price channel and does not submit bids in both periods (Cases 2, 3, and 4). However, frictional costs of the first period (c_0) can affect buyers' decisions about bid at NYOP. In particular, table 3 shows when $c_0 = 0$, for all values of B, buyers bid at NYOP at least in one period, but increase in c_0 causes buyers to just purchase from posted-price. Therefore, the buyer manages the trade-off between posted-price and frictional costs to maximize her surplus. In particular, the NYOP targets buyers with low frictional costs and high posted-prices, and the posted-price channel targets the buyers with high frictional costs and low posted-prices.

Figure 4 and table 3 show the buyer's optimal offer in the first period in Cases 1, 2, and 3. With an increase in *B*, initially, P_0 increases and then would be constant. At first, P_0 is very low ($P_0 = 0.2$). It is due to when $B = 0.4$ and the buyer has no frictional costs to place a bid ($c_0 = 0$), she prefers to use Case 3, and bids for a single item in the first period and then purchases from the posted-price channel. With an increase in *B*, the buyer's bidding strategy changes and the bid significantly increases ($0.6 \leq P_0 \leq 1.08$). In this situation, the buyer uses case 2, and bids for the bundle in the first period; purchases from the posted-price channel if the bid is rejected.

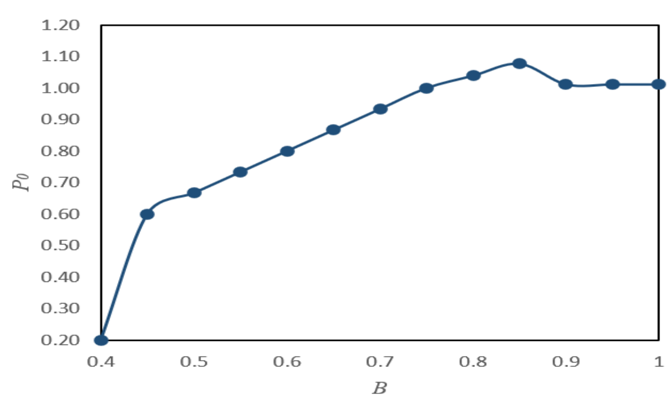

Fig 4. Effect of posted-price on the buyer's bid in the first period

Figure 4 indicates when $B \leq 0.85$ as B increases, the amount of bids increases. However, when $B > 0.85$, the bid first decreases and then would be constant. It is due to when $B \geq 0.9$, the buyer prefers to place a bid in both periods. Hence, the buyer submits a lower bid in the first period because she has the chance to submit a greater bid in the second period. Hence, posted-prices have a significant effect on the buyer's offer and the design of offering rules. Furthermore, B may dramatically influence the retailer's utility.

Figure 5 and table 3 show that when B increases, initially the retailer's utility increases and then decreases. It is due to when B is low, it is more profitable for the buyer to buy from posted-price. With an increase in B, the margin obtained through the posted-price channel increases. Moreover, when $c_0 = 0.05$, for $B \geq 0.45$ at least in one period, buyers are persuaded to purchase from the NYOP retailer; where they can buy items at a price lower than the posted-price. Hence, applying NYOP helps the retailer to attract more buyers and enhances his profit. When $c_0 = 0.1$, the purchasing policy change happens at $B = 0.6$ and shows increase in the fractional cost of the first period can affect the buyer's tendency to only use posted-prices. However, the frictional cost of the second period does not affect her tendency. When $B > 0.85$, there is a reduction in the retailer's profit. It is due to that buyers select NYOP in both periods. Hence, bids of some buyers fail to meet the threshold and they can not purchase the product at all.

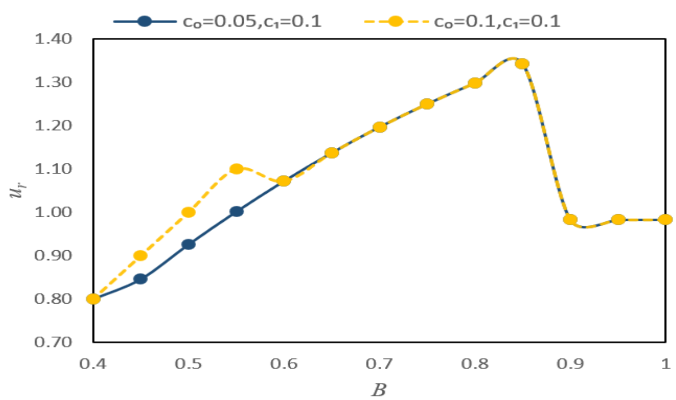

Fig 5. Effect of posted-price on the retailer's utility

It is obvious from table 3 and figure 5 the retailer can obtain more profit when the value of posted-price is such that the buyer decides to submit a bid for a bundle in the first stage and then purchase from posted-price. Thus, setting the appropriate posted-price can make the maximum profit for the retailer. Furthermore, the retailer can obtain more profit by manipulating the frictional costs. Moreover, the results show selling items only through NYOP, especially when the rivals can sell items through posted-price, is not a profitable strategy for the retailer. In this situation, the retailer only can make money when the rivals set posted-prices high enough. Our finding supports Cai et al. (2009) results and indicates it is better for NYOP retailers to have posted-price channels. Furthermore, this is in line with the Priceline policy which has used NYOP parallel to the posted-price channel.

The current study helps the operations managers to better analyze buyers' behavior that directly affects supply chain profit. It should be noted that under the NYOP mechanism, the buyers determine the final price, so the retailer's profit is completely related to the buyers' behavior and their bidding strategy. Accurate analysis of buyers' behavior helps retailers to determine optimal supply chain structure and identify factors that can affect the buyer's frictional cost such that the retailer profit is maximized. The results indicate that if the retailer only applies the NYOP channel, he would lose some of his customers especially when there are some outside options for the customers to buy the product.

5- Conclusion and future research

This paper studied NYOP, an interactive pricing setting, to obtain the buyer's bidding strategy which is the main concern in the NYOP mechanism and retailer's profit. To do this, the theoretical and managerial implications of employing the NYOP channel along with posted-price channels were investigated where joint bidding is allowable. A two-stage model was developed, in which buyers select whether they want to participate in NYOP in the first stage. Moreover, they specify whether they want to bid for a bundle or a single item. The buyers' decisions in the second stage are dependent on the outcome of the first stage. Moreover, the effect of buyers learning when rebid was considered. Four different scenarios that buyers may use to purchase items were developed. The results indicated that for high posted-prices, buyers submit a bid greater than one for the bundle in both periods. However, as posted-prices decreases, buyers prefer to buy from this channel at least in one period. Moreover, for most posted-prices, variation in frictional costs of the second period does not affect the buyer's bidding strategy. However, high frictional costs of the first period cause buyers to bid at NYOP at least in one period. Hence, rational buyers manage the trade-off between posted-price and frictional costs to choose the bidding strategy, and the retailer can make more profit by setting the appropriate posted-price. Furthermore, the analysis showed selling items only in NYOP, especially when the rivals can sell items through posted-price, is not profitable for the retailer. In sum, the results showed the design of the market has a considerable effect on the buyers' behavior and the retailer's utility. There are several avenues for future research. The retailer can use a dynamic threshold with other distributions. This paper considered a retailer, but the competition between retailers can be present. Studying different types of products, such as complementary products, and considering risk-averse buyers are other extensions.

References

Abbas, A. E., & Hann, I. H. (2010). Measuring risk aversion in a name-your-own-price channel. *Decision Analysis*, *7*(1), 123–136.

Amaldoss, W., & Jain, S. (2008). Joint Bidding in the Name-Your-Own-Price Channel : A Strategic Analysis. *Management Science Publication*, *54*(10), 1685–1699.

Anderson, C. K., & Xie, X. (2014). Pricing and market segmentation using opaque selling mechanisms. *European Journal of Operational Research*, *233*(1), 263–272.

Bernhardt, M., & Spann, M. (2010). An Empirical Analysis of Bidding Fees in Name-your-own-price Auctions. *Journal of Interactive Marketing*, *24*(4), 283–296.

Bucarey, V., Elloumi, S., Labbé, M., & Plein, F. (2021). Models and algorithms for the product pricing with single-minded customers requesting bundles. *Computers and Operations Research, 127*.

Cai, G., Chao, X., & Li, J. (2009). Optimal reserve prices in name-your-own-price auctions with bidding and channel options. *Production and Operations Management*, *18*(6), 653–671.

Caldentey, R., & Vulcano, G. (2007). Online auction and list price revenue management. *Management Science*, *53*(5), 795–813.

Cao, Q., Geng, X., Stecke, K. E., & Zhang, J. (2019). Operational role of retail bundling and its implications in a supply chain. *Production and Operations Management*, *28*(8), 1903–1920.

Chen, R. R., Gal-Or, E., & Roma, P. (2014). Opaque distribution channels for competing service providers: Posted price vs. name-your-own-price mechanisms. *Operations Research Publication*, *62*(4), 733–750.

Chen, T., Yang, F., & Guo, X. (2021). Optimal bundling in a distribution channel in the presence of substitutability and complementarity. *International Journal of Production Research*, *59*(4), 1145–1165.

Etzion, H., Pinker, E., & Seidmann., A. (2006). Analyzing the simultaneous use of auctions and posted prices for online selling. *Manufacturing & Service Operations Management*, *8*(1), 68–91.

Fay, S. (2004). Partial-Repeat-Bidding in the name-your-own-price channel. *Marketing Science*, *23*(3), 407–418.

Fay, S. (2009). Competitive reasons for the Name-Your-Own-Price channel. *Marketing Letters*, *20*(3), 277–293.

Feng, B., Liu, W., & Mao, Z. (2018). Use of opaque sales channels in addition to traditional channels by service providers. *International Journal of Production Research*, *56*(10), 3369–3383.

Feng, B., Mao, Z., & Li, H. (2019). Choices for competing service providers with heterogeneous customers : Traditional versus opaque sales modes. *Omega*, *102133*.

Gupta, A., & Abbas, A. E. (2008). Repeat bidding on internet-based multiple-item "name-your-own-price" auctions. *IEEE Transactions on Engineering Management*, *55*(4), 579–589.

Hann, I. H., & Terwiesch, C. (2003). Measuring the frictional costs of online transactions: The case of a name-your-own-price channel. *Management Science*, *49*(11), 1563–1579.

Hemmati, M., Ghomi, S. M. T. F., & Sajadieh, M. S. (2023). Separate and bundling selling strategies for complementary products in a participative pricing mechanism. *Computers & Industrial Engineering*, *177*(January), 109018.

Hemmati, M., Mirzapour Al-e-Hashem, S. M. J., & Fatemi Ghomi, S. M. T. (2021). Heuristic analyses of separate and bundling sales for complimentary products under consignment stock policy. *Computers and Industrial Engineering*, *157*.

Hinz, O., Hann, I. H., & Spann, M. (2011). Price discrimination in E-commerce? An examination of dynamic pricing in name-your-own price markets. *MIS Quarterly: Management Information Systems*, *35*(1), 81–98.

Huang, X., Soˇsi´c, G., & Kersten, G. (2017). Selling Through Priceline ? On the Impact of Name-Your-Own-Price in competitive market. *IISE Transactions*, *49*(3), 304–319.

Jena, S. K., & Ghadge, A. (2022). Product bundling and advertising strategy for a duopoly supply chain: a power-balance perspective. *Annals of Operations Research*, 1729–1753.

Joo, M., Mazumdar, T., & Raj, S. P. (2012). Bidding Strategies and Consumer Savings in NYOP Auctions. *Journal of Retailing*, *88*(1), 180–188.

Levina, T., Levin, Y., McGill, J., & Nediak, M. (2015). Strategic Consumer Cooperation in a Name-Your-Own-Price Channel. *Production and Operations Management*, *24*(12), 1883–1900.

Li, J., Wang, Q., Yan, H., & Zhu, and S. X. (2016). Optimal Remanufacturing and Pricing Strategies Under Name-Your-Own-Price Auctions and Stochastic Demand. *Asia-Pacific Journal of Operational Research*, *33*(1), 1–22.

Li, Q., Tang, C. S., & Xu, H. (2020). Mitigating the Double-Blind Effect in Opaque Selling : Inventory and Information. *Production and Operations Management*.

Lin, X., Zhou, Y. W., Xie, W., Zhong, Y., & Cao, B. (2020). Pricing and Product-bundling Strategies for E-commerce Platforms with Competition. *European Journal of Operational Research*, *283*(3), 1026–1039.

Liu, J., Dai, R., David, X., & Li, Y. (2016). Information revelation and customer decision-making process of repeat-bidding name-your-own-price auction. *Decision Support Systems*, *90*, 46–55.

Nosoohi, I. (2022). Posted price and name-your-own-price in a product line design problem. *Journal of Retailing and Consumer Services*, *64*, 102836.

Sabbaghnia, A., Heydari, J., & Razmi, J. (2022). Participative pricing and donation programs in a socially concerned supply chain. *Corporate Social Responsibility and Environmental Management.*, 1–19.

Shapiro, D. (2011). Profitability of the name-your-own-price channel in the case of risk-averse buyers. *Marketing Science*, *30*(2), 290–304.

Spann, M., Skiera, B., & Schäfers, B. (2004). Measuring individual frictional costs and willingness-to-pay via name-your-own-price mechanisms. *Journal of Interactive Marketing*, *18*(4), 22–36.

Spann, M., & Tellis, G. J. (2006). Does the internet promote better consumer decisions? The case of name-your-own-price auctions. *Journal of Marketing*, *70*(1), 65–78.

Spann, M., Zeithammer, R., Bertini, M., Haruvy, E., Jap, S. D., Koenigsberg, O., … Thomas, M. (2018). Beyond Posted Prices: the Past, Present, and Future of Participative Pricing Mechanisms. *Customer Needs and Solutions*, *5*(1), 121–136.

Vamosiu, A. (2018). Optimal bundling under imperfect competition. *International Journal of Production Economics*, *195*, 45–53.

Wagner, R. L., & Pacheco, N. A. (2020). Name-your-own-price as participative pricing strategy: a review of the literature from 2001–2017. *Journal of Strategic Marketing*, *28*(7), 583–600.

Zeithammer, R., Stich, L., Spann, M., & Häubl, G. (2019). Consumer Behavior in Bidding Markets with Participation Costs. *Available at SSRN 3474406*.

Appendix A

Proof of proposition 3.1. According to equation (2), the buyer's expected utility in the second period is:

$$\max_{P_1} u_{bN\,0,1} = (WTP_j - P_1)(\int_{P_0}^{P_1} Mdy \Big/ (\int_{P_0}^{a+b} Mdy + \int_{a+b}^{2b} Ndy)) - c_1 \quad \text{(A1)}$$

In the above equation $a = 0$, $b = 1$, and $WTP_j = 2$ is substituted. As noted in Section 3, in Case 1.a, the acceptable range for both P_0 and P_1 is $[0, 1]$. Within this range, equation (A1) is continuous and differentiable in P_1. Taking the second-order derivative of $u_{bN\,0,1}$ with respect to P_1 for the given value of P_0 yields:

$$\partial^2 u_{bN\,0,1} \big/ \partial P_1^2 = (6P_1 - 4)\big/(P_0^2 - 2) \quad \text{(A2)}$$

If $P_1 > 2/3$, the equation (A2) is negative. So, $u_{bN\,0,1}$ is concave on this interval. The F.O.C yields the following two real roots: $r^1_{bN\,0,1} = (2 + \sqrt{3P_0^2 + 4})\big/3$, $r^2_{bN\,0,1} = (2 - \sqrt{3P_0^2 + 4})\big/3$. Note that $0 \le P_0 \le 1$, so $r^2_{bN0,1}$is not positive and is not an acceptable root. Though $r^1_{bN0,1}$is smaller than 2, which is the upper bound of P_1, is greater than 1. Hence, it is not in the acceptable interval of case 1.a. The sign of the first-order derivative and the value of its possible roots yields $u_{bN\,0,1}$ is non-decreasing when $0 \le P_0 \le 1$ and $0 \le P_1 \le 1$. Hence, when $1 > P_1 > 2/3$, the maximum value of $u_{bN\,0,1}$ is $P_1 = 1$. If $P_1 < 2/3$, $u_{bN\,0,1}$ is convex and non-decreasing. So, the maximum solution over this region is $P_1 = 2/3$. In sum, because $u_{bN\,0,1}$ is continuous on the interval [0,1], the maximum value of $u_{bN\,0,1}$ is $P_1 = 1$. Figure A1 demonstrating $u_{bN\,0,1}$ for a given value of P_0.

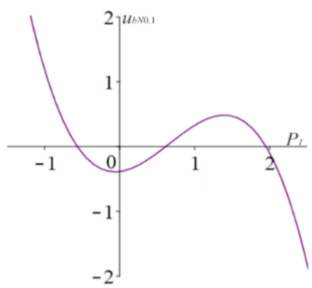

Fig A1. The relation between $u_{bN\,0,1}$ and P_1

Substituting the maximum solution obtained in period two in equation (2) gives:

$$\max_{P_0} u_{bN\,0,0} = (WTP_j - P_0)\int_{2a}^{P_0} Mdy - c_0 + (1 - \int_{2a}^{P_0} Mdy)((P_0^2 - 1\big/P_0^2 - 2) - c_1) \quad \text{(A3)}$$

Equation (A3) is continuous and differentiable in P_0. Taking the second-order derivative of $u_{bN\,0,0}$ gives:

$$\partial^2 u_{bN\,0,0} \big/ \partial P_0^2 = -3P_0 + c_1 + 1 \quad \text{(A4)}$$

If $P_0 > (c_1 + 1)/3$, (A4) would be negative and (A3) is concave in P_0. Using F.O.C, two following real roots are obtained: $r^1_{bN0,0} = 0$, $r^2_{bN0,0} = 2(c_1 + 1)/3$. Where $r^2_{bN0,0}$can be on the interval $[(c_1 + 1)/3, 1]$. So, $r^2_{bN0,0}$is the maximum solution. Note that c_1 is nonnegative and $r^2_{bN0,0}$ is always greater than $(c_1 + 1)/3$. Besides, the first-order derivative shows that $u_{bN\,0,0}$ it is non-decreasing over $[0, (2c_1 + 2/3)]$. Hence, if $r^2_{bN0,0}$ is greater than 1, the maximum value of $u_{bN\,0,0}$ is $P_0 = 1$. If $0 \le P_0 \le (c_1 + 1)/3$, $u_{bN\,0,0}$ is convex and non-decreasing. Hence, the maximum value of $u_{bN\,0,0}$ over this region is $P_0 = c_1 + 1/3$. But, as noted earlier, $u_{bN\,0,0}$ is continuous and non-decreasing over the interval $[0, (2c_1 + 2)/3]$. So, it is concluded that the solution

$P_0 = \min\left(1, (2c_1+2)/3\right)$ maximizes $u_{bN_{0,0}}$ on the interval [0,1]. For better intuition, consider figure A2 which shows $u_{bN_{0,0}}$. (a) Shows the situation that $(2c_1+2)/3 \le 1$ and the optimal solution is $P_0 = (2c_1+2)/3$ (b) shows the situation in which $(2c_1+2)/3 \ge 1$ and the optimal solution is $P_0 = 1$.

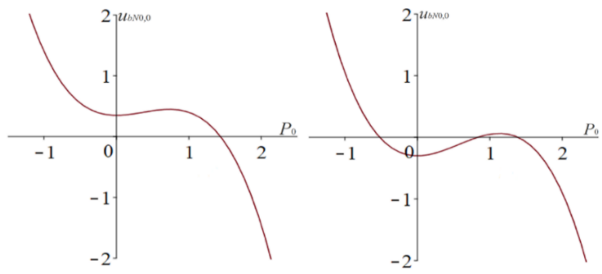

Fig A2. The relation between $u_{bN_{0,0}}$ and P_0

Appendix B

Proof of proposition 3.2. According to equation (3), the buyer's expected utility function in the second period is:

$$\max_{P_1} u_{bN1,1} = (WTP_j - P_1)\left(\left(\int_{P_0}^{a+b} Mdy + \int_{a+b}^{P_1} Ndy\right) \Big/ \left(\int_{P_0}^{a+b} Mdy + \int_{a+b}^{2b} Ndy\right)\right) - c_1 \quad \text{(B1)}$$

In the above equation $a = 0$, $b = 1$, and $WTP_j = 2$ is substituted. If $0 \le P_0 \le 1$ and $1 \le P_1 \le 2$, $u_{bN_{1,1}}$ would be continuous and differentiable in P_1. Taking the second-order derivative of $u_{bN_{0,1}}$ with respect to P_1 for the given value of P_0 gives:

$$\partial^2 u_{bN1,1} \big/ \partial P_1^2 = 6(2-P_1)\big/(P_0^2-2) \quad \text{(B2)}$$

Since $0 \le P_0 \le 1$ and $1 \le P_1 \le 2$, equation (B2) is negative. So, $u_{bN_{1,1}}$ is convex over this region. Using F.O.C gives: $r^1_{bN1,1} = 2 + \sqrt{-3P_0^2+6}\big/3$, $r^2_{bN1,1} = 2 - \sqrt{-3P_0^2+6}\big/3$. According to $0 \le P_0 \le 1$, $r^1_{bN1,1}$ is greater than 2. So, $r^1_{bN1,1}$ is not in the acceptable range. In particular, it is greater than the buyer's willingness-to-pay for the bundle, and the buyer never bid $r^1_{b1,1}$. Moreover, $1 < r^2_{bN1,1} < 2$ is an acceptable root and maximizes $u_{bN_{1,1}}$ over the noted region of Case 1.b (i.e., $0 \le P_0 \le 1$ and $1 \le P_1 \le 2$). Figure B1 depicts $u_{bN_{1,1}}$, for a given value of P_0 and c_1.

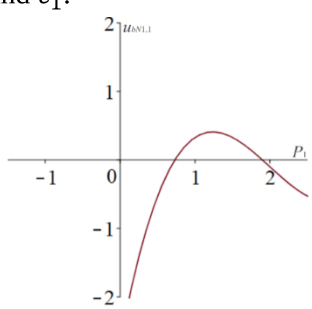

Fig B1. The relation between $u_{bN_{1,1}}$ and P_1

Substituting $P_1 = r^2_{bN1,1}$ in equation (3) gives:

$$\max_{P_0} u_{bN1,0} = (WTP_j - P_0)\int_{2a}^{P_0} Mdy - c_0 + \left(1 - \int_{2a}^{P_0} Mdy\right)\left(\left(2\sqrt{-3P_0^2+6}\big/9\right) - c_1\right) \quad \text{(B3)}$$

Here, the second-order derivative is complicated, so the maximum value of (B3) is obtained by analyzing the behavior of the function. The sign of the first-order derivative of $u_{bN_{1,0}}$ is considered.

$$r^1_{bN1,0}=0\ ,\ r^2_{bN1,0}=18(c_1+2)+2\sqrt{-12c_1^2-48c_1+14}\Big/31\ ,\ r^3_{bN1,0}=18(c_1+2)-2\sqrt{-12c_1^2-48c_1+14}\Big/31$$

If $c_1\geq-2+\sqrt{186}/6$, $r^1_{bN1,0}=0$ is the only real root of (B3). $u_{bN_{1,0}}$ is continuous and 0 is not a double root. Sign analysis gives $u_{bN_{0,1}}$ is non-increasing before $P_0=0$ and non-decreasing after $P_0=0$. Hence, when $0\leq P_0\leq 1$ and $c_1\geq-2+\sqrt{186}/6$, the optimal point is $P_0=1$. Figure B2 demonstrates $u_{bN_{1,0}}$ where the optimal solution is $P_0=1$.

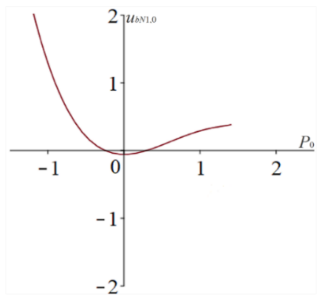

Fig B2. The relation between $u_{bN_{1,0}}$ and P_0

If $0\leq c_1\leq-2+\sqrt{186}/6$, the maximum number of real roots of (B3) is three. Analyzing the sign of the first-order derivative of $u_{bN_{1,0}}$ gives that $u_{bN_{1,0}}$ is non-increasing before 0, non-decreasing on the interval $[0,r^3_{bN1,0}]$, non-increasing on the interval $[r^3_{bN1,0},r^2_{bN1,0}]$, and non-decreasing after $r^2_{bN1,0}$. As noted earlier, in Case 1.b, $0\leq P_0\leq 1$ and $r^2_{bN1,0}$ is always greater than 1. Hence, $r^2_{bN1,0}$ is out of the acceptable domain. However, $r^3_{bN1,0}$ can be on the interval $[0,1]$. As noted above, equation (B3) is continuous, non-decreasing on $[0,r^3_{bN1,0}]$, and non-increasing on $[r^3_{bN1,0},r^2_{bN1,0}]$. Hence, the maximum solution is at $min(r^3_{bN1,0},1)$. For better understanding, consider figure B3 (a) that shows (B3) in which $r^3_{bN1,0}>1$ and the optimal solution is $P_0=1$. Moreover, figure B3 (b) illustrates $u_{bN_{1,0}}$ where $r^3_{bN1,0}$ is on the interval $[0,1]$, and the optimal solution is $P_0=r^3_{bN1,0}$.

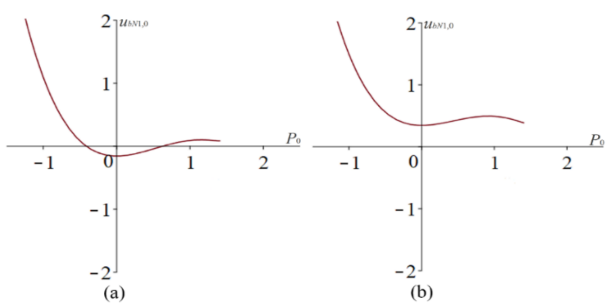

Fig B3. The relation between $u_{bN1,0}$ and P_0

Appendix C

Proof of proposition 3.3. Both P_0 and P_1 are greater than 1 and the buyer's expected utility function in the second period is:

$$\max_{P_1} u_{bN2,1} = (WTP_j - P_1)(\int_{P_0}^{P_1} Ndy \Big/ \int_{P_0}^{2b} Ndy) - c_1 \qquad \text{(C1)}$$

In the above equation $a = 0$, $b = 1$, and $WTP_j = 2$ is substituted. When $1 \le P_0 \le 2$, equation (C1) is continuous and differentiable in P_1. Taking the second-order derivative of $u_{bN\,2,1}$ with respect to P_1 for the given value of P_0, gives:

$$\partial^2 u_{bN2,1} \big/ \partial P_1^2 = 6(P_1 - 2)\big/(P_0 - 2)^2 \qquad \text{(C2)}$$

Because $1 \le P_1 \le 2$, equation (C2) is negative and $u_{bN\,2,1}$ is concave. Using F.O.C yields: $r^1_{bN2,1} = 2 + (P_0 - 2)\big/\sqrt{3}, r^2_{bN2,1} = 2 - (P_0 - 2)\big/\sqrt{3}$. As $1 \le P_0 \le 2$, so $r^2_{bN2,1} \ge 2$. Paying attention to the buyer's willingness-to-pay for a bundle and the range of P_1, the buyer never bid $r^2_{bN2,1}$. When $1 \le P_0 \le 2$, $r^1_{bN2,1}$ is on the interval $[1, 2]$. Thus, $r^1_{bN2,1}$ is the maximum solution of $u_{bN\,2,1}$ when both P_0 and P_1 are more than 1. Figure C1 depicts $u_{bN\,2,1}$ and its optimal solution i.e. $P_1 = 2 + \sqrt{3}(P_0 - 2)\big/3$.

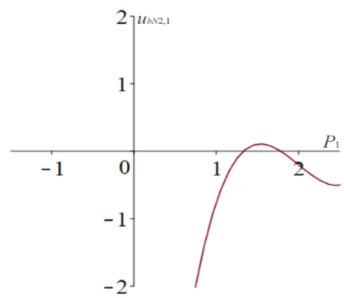

Fig C1. The relation between $u_{bN\,2,1}$ and P_1

Substituting $P_1 = r^1_{bN2,1}$ in equation (4) yields:

$$\max_{P_0} u_{bN2,0} = (WTP_j - P_0)(\int_{2a}^{a+b} Mdy + \int_{a+b}^{P_0} Ndy) - c_0 - \int_{P_0}^{2b} Ndy((2\sqrt{3}(P_0 - 2)\big/9) + c_1) \qquad \text{(C3)}$$

$u_{bN\,2,0}$ is continuous and differentiable in P_0. The second-order derivative of $u_{bN\,2,0}$ with respect to P_0 is taken to find the optimal value of the buyer's bid in the first period.

$$\partial^2 u_{bN2,0} \big/ \partial P_0^2 = (3 - 2\big/\sqrt{3})(P_0 - 2) - c_1 \qquad \text{(C4)}$$

Since $P_0 \le 1$, (C4) is negative and $u_{bN\,2,0}$ is concave. Using F.O.C yields:

$$r^1_{bN2,0} = 2 + \frac{2c\sqrt{3} + 9c + \sqrt{93c^2 + 36\sqrt{3}c^2 + 92\sqrt{3} + 414}}{23},\ r^2_{bN2,0} = 2 + \frac{2c\sqrt{3} + 9c - \sqrt{93c^2 + 36\sqrt{3}c^2 + 92\sqrt{3} + 414}}{23}$$

The buyer never bids $r^1_{bN2,0}$ because it is greater than her willingness-to-pay, i.e. $r^1_{bN2,0} > 2$. Taking $E = 93c^2 + 36\sqrt{3}c^2$, so we can reformulate $r^2_{bN2,0}$ as, $r^2_{bN2,0} = 2 + (\sqrt{E} - \sqrt{E + 92\sqrt{3} + 414})\big/23$. Since $r^2_{bN2,0} < 2$, the optimal solution is at $\max(1, r^2_{bN2,0})$. Figures C2 (a) and (b) depict equation (C3) where $r^2_{bN2,0} > 1$ and $r^2_{bN2,0} \le 1$ respectively.

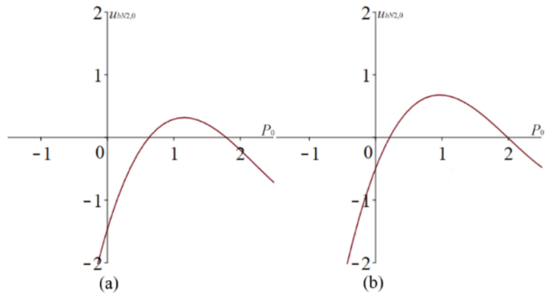

Fig C2. The relation between $u_{bN_{2,0}}$ and P_0

Appendix D

Proof of proposition 3.4. Taking the second-order derivative of equation (10) with respect to P_0 yields:

$$\partial^2 u_{bNp0} / \partial P_0^2 = -3P_0 + 2B \tag{D1}$$

If $P_0 > 2B/3$, (D1) is negative and u_{bNp_0} is concave. But if $P_0 < 2B/3$, (D1) is positive and u_{bNp_0} is convex. Using F.O.C gives: $r^1_{bNp0} = 0$, $r^2_{bNp0} = 4B/3$. Hence, if $P_0 > 2B/3$, the solution is $r^2_{bNp0} = 4B/3$ because of $4B/3 > 2B/3$. If $0 < P_0 < 2B/3$, u_{bNp_0} is convex and the solution is $P_0 = 2B/3$. In sum, because u_{bNp_0} is continuous in P_0 on the interval $[0,1]$ and the sign analysis of the first-order derivative shows u_{bNp_0} is non-decreasing on the interval $[0, 4B/3]$, the maximum solution is $P_0 = min(1, 4B/3)$. For better understanding, consider figure D1, which (a) shows u_{bNP_0} when the optimal offer is 1, and (b) shows u_{bNp_0} when the optimal offer is $4B/3$.

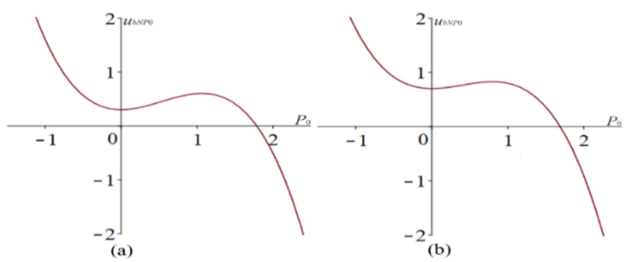

Fig D1. The relation between u_{bNP_0} and P_0

Appendix E

Proof of proposition 3.5. Taking the second-order derivative of equation (11) with respect to P_0 yields:

$$\partial^2 u_{bNp1} / \partial P_0^2 = -4 + 3P_0 - 2B \tag{E1}$$

If $P_0 < 2\,(2 + B)/3$, (E1) is negative and u_{bNp_1} is concave over this region. Using F.O.C we have:

$r^1_{bNp1} = (2B + 4 + 2\sqrt{(B-1)^2 + 1.5})/3$, $r^2_{bNp1} = (2B + 4 - 2\sqrt{(B-1)^2 + 1.5})/3$. The buyer never bid r^1_{bNp1} because $r^1_{bNp1} > 2$ and r^1_{bNp1} is greater than the buyer's willingness-to-pay. As noted in Section 3, in Case

2.b), we have $P_0 > 1$. Moreover, $r^2_{bNp1} < (4+2B)/3 < 2$ and u_{bNp_1} is concave on this interval. Hence, the maximum value of u_{bNp_1} on $[0,(4+2B)/3]$ is at $\max(1, r^2_{bNp1})$.The first-order derivative shows u_{bNp_1} is non-increasing over the interval $[r^2_{bNp1}, r^1_{bNp1}]$. Hence, If $2 \geq P_0 > (4+2B)/3$, u_{bNp_1} is non-increasing and convex. So, the maximum solution is $(4+2B)/3$ over the interval $[(4+2B)/3, 2]$. In sum, since u_{bNp_1} is continuous and non-increasing over $[r^2_{bNp1}, (4+2B)/3]$, the maximum solution of u_{bNp_1} over $[1,2]$ is at $\max(1, r^2_{bNp1})$. Figure E1(a) depicts $u_{bNp_1}$when the optimal offer is $P_0 = r^2_{bNp1}$, and (b) shows u_{bNP_1} when the optimal offer is $P_0 = 1$.

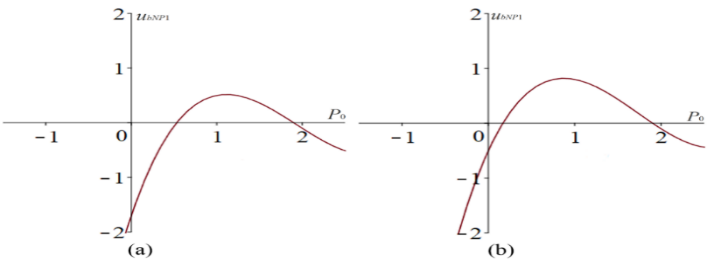

Fig E1. The relation between u_{bNP_1} and P_0

Appendix F

Proof of proposition 3.6. Taking the second-order derivative of equation (15) with respect to P_0, yields:

$$\partial^2 u_{bp2} / \partial P_0^2 = -2 \tag{F1}$$

u_{bNp_2} is concave in P_0 and has a unique optimal point. Using F.O.C, the optimal solution is: $r_{bp2} = B/2$.