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Establishment and analysis to find the optimal inspection plan of redundant systems considering repair time

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Abstract

Optimal time interval between inspections of the redundant systems is raised as a substantial issue to plan a preventive maintenance model for maintenance planners. To have an optimal time period for preventive maintenance of systems, especially complex systems such as redundant systems, two variables of maintenance are mostly connived. Against other studies in the literature of preventive maintenance in which repair time is a negligible factor as an assumption, repair time is considered as a noticeable variable incorporated into the model developed in this paper. Another contribution, the number of facilities, is focused as a significant variable used in real applications. Particularly, systems with complex performance needing technical repair facilities (i.e., technical repairmen, tools, materials, outsourced repair, etc.). In this regard, parallel systems have been analyzed stochastically using the way of preventive maintenance in which repair time would be contemplated as an essential factor in maintenance planning. Using Markov chain, a model based on expected total cost per time is made to demonstrate that a proper time interval achieving lowest possible cost is obtained by taking into account repair time and the number of repair facility. Three models are studied as instances of redundant systems to find the optimal time interval between inspections. These models differ in the number of repair facilities (i.e., one, two and three repair facilities). A sensitivity analysis is done to depict the variability of input variables over optimal the expected total cost per time and time interval between inspections. As a main contribution, the repair time could be an essential factor in maintenance planning, this study contemplates this factor in redundant systems.

Keywords: Preventive maintenance, repair time, repair facility, redundant systems

1-Introduction

1-1-Motivation

In capital-intensive firms such as oil and gas, power, refinery, robotic machines, etc., providing an effective plan for preventive maintenance of equipment and parallel redundancy helps a business be more flexible and has a smooth path to plan its production. That a system can be reliable for a long time depends on what interval time, it will be maintained and how it would be maintained. There are several techniques to ensure system reliability, the most important way is redundancy. Moreover, repairable systems can be repaired and referred to as good as new state and this is another way. Systems that can be constructed like redundant systems are redundant turbines in power systems, redundant pumps in refinery industries, redundant robotic arms in automotive industries etc., as whole units can be repaired and restored to their

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initial status. Designing the optimal preventive maintenance for these parallel systems will minimize the operating cost considering system availability. Both provisions of redundant components and repairing or maintaining components are costly and must be reasonably decided to achieve the optimum decision. In these systems, contemplation of all cost-effective details is supposed to be analyzed. These factors motivate authors to review deeply on redundant systems and study in repair time as a basis factor.

1-2-Literature review

Availability and reliability analysis with discrepant approaches have been studied by many researchers. Maintenance as a way to improve reliability and availability in redundant systems has been evolved and many cost-effective models considering redundancy and maintenance jointly have been constructed to improve system availability and system reliability. A discussion on maintenance optimization has been studied by Barlow and Proschan (1965). Lai and Yuan (1999) developed a maintenance model for a redundant system. In their model, optimal redundant units and the optimal number of units to be repaired have been obtained considering repair and maintenance costs. Laprie et al. (1981) have studied a 2-unit redundant system through a semi-Markov model that the failure rate of the operating unit is directly dependent on the failed unit. Actually, failure rate of the operating unit increases when another unit is under repair. Percy et al. (1997) considered stochastic models and Bayesian methods to set preventive maintenance plans. Various maintenance optimization models and their applications have been analyzed (Dekker 1996). Dekker (1996) approximately reviewed all aspects of the maintenance optimization factors involved in applications. It is commonly used as an appropriate reference for maintenance optimization. Chareonsuk et al. (1997) studied on determining the optimal preventive maintenance intervals for paper production systems. They established a model incorporating multiple criteria for their case study in the paper factory; the expected cost and reliability have been taken into the model, and method called PROMETHEE has been used the relevant problem. For redundant systems, Levitin and Lisnianski in (1999) discussed redundancy and maintenance optimization as a joint model. They studied on a series-parallel system and generalized a joint redundancy and maintenance optimization model for multistate systems where the system and its units have a range of performance levels.

A cost-effective maintenance plan can play a vital role in the productivity and safety of redundant systems. In the literature on redundant systems, planned preventive maintenance has been studied, and new models have been proposed. For instance, Bris et al. (2003) presented a cost function of a maintenance policy under a given availability constraint. They presented a basic maintenance optimization model in which a method is used to select critical objects. Then preventive maintenance plans are separately calculated based on the availability constraint for each selected object in a redundant system. Samrout et al. (2005) introduced a new method using the ant colony optimization to improve the previous study by Bris et al. (2003). They used the ant colony optimization technique to solve the model proposed by Bris et al. (2003), compared the results with the previous study and demonstrated the improvement. Castro and Cavalca (2006) have studied maintenance optimization for manufacturing systems. The authors proposed a model to optimize the availability of a series-parallel production line in the presence of maintenance cost constraints. A maintenance optimization problem has been investigated to find the minimal cost configuration of a multi-state series-parallel system under reliability constraints (Nourelfath and Ait, 2007). The analytical approach has been developed to model the system using the universal moment-generating function and the Markov chain method. Another research by Nahas et al. (2008) proposed an approach to improve the results of the maintenance optimization problem of series-parallel systems studied by Levitin and Lisinianski (1999). An optimization method based on the extended excellent deluge algorithm has been presented to solve the problems (Lai and Yuan 1999). Samrout et al. (2009) modeled maintenance optimization of the systems based on their proportional hazard functions and considering either corrective maintenance or preventive maintenance optimization. An improved particle swarm optimization has been established to minimize preventive maintenance period for a series-parallel system by Wang and Lin (2011). They proposed an efficient meta-heuristic to solve the model developed by Bris et al. (2003). Wang and Tsai (2012) presented a bi-objective model for preventive maintenance optimization of a series-parallel system and solved the model using a hybrid genetic algorithm. In their study, an improvement factor method has been employed to investigate how repairing components can restore the system's reliability. Two objectives consist of total maintenance cost and mean system reliability simultaneously would be optimized. Nourrelfath et al. (2012) modeled imperfect preventive maintenance using the Markov process and the universal moment generating function for a multi-state series-parallel system and solved the established model by a genetic algorithm. A new multi-objective nonlinear mixed-integer model has been presented by Moghaddam, 2013. They developed it for preventive maintenance of multi-workstation manufacturing system in which the rate of occurrence of failure is increasing. Three maintenance actions (i.e., repair, replacement and do nothing) have been incorporated for each workstation in each period during the planning horizon. A multi-objective function consists of total operational costs, overall reliability and the system availability and, it has been solved using a hybrid Monte Carlo simulation and goal programming methods. A new approach to model preventive maintenance has been established through two-stage stochastic programming by Chatwattanasiri et al. (2014). They strictly studied uncertain environment for the preventive maintenance optimization. Their approach was to specify some scenarios based on uncertain future usages. Their problem is divided into two stages which the first stage includes both selection and the number of components, and the second stage is to determine the preventive maintenance plan for the specified configuration in the first stage. Alrabghi and Tiwari (2015) reviewed lots of papers having researched on preventive maintenance optimization based on the simulation method. They showed that discrete event simulation was the most technique to model maintenance systems.

Hajipour and Taghipour (2016) proposed a model to find the optimal inspection interval over a finite planning horizon for k-out-of-n systems. They studied *m* identical components with redundancy in each subsystem and find a non-periodic inspection plan with failure following non-homogeneous Poison process. In their model, there is not any evidence to consider repair time as an essential factor. A new model for inspection intervals of a turbine rotor with failure interaction is presented by Rezaei (2017). Also, both perfect and minimal repairs are considered in the model. The case study provided by Rezaei (2017) does not include repair time as a parameter in the model. Seyedhosseini et al (2018) studied on inspection plan for a two-component system with a hidden failure and a three-mode failure. They provided an optimal inspection interval in which repair time is ignored and developed a simulation algorithm to calculate the expected total cost.

Sharifi and Taghipour (2020) developed a continuous-time discrete-state model for periodic inspection of a k-out-of-n system with non-identical components. They try to find an optimal inspection plan to minimize the total cost of the inspection intervals. In their study, repair time is ignored. Sharifi et al (2021) provided an optimal inspection interval plan for a k-out-of-n load sharing system with a mixed redundancy strategy. They assumed that the component's repair time is less than the inspection interval (i.e., the repair time was ignored). Peng et al (2022) using simulation procedure tried to find the optimal inspection interval for parallel systems regarding minimization of total cost. Yang et al. (2022) studied redundant systems and established a condition-based strategy, and proposed reinforcement learning as a novel approach to reduce the maintenance cost. Their assumption about imperfect repair and replacement of failed components is dynamically decided.In k-out-of-n systems, each subsystem consists of parallel components, Zhao et al (2023) studied on a k-out-of-n system in which a load sharing dependency exists among components of parallel subsystems. They presented a regular inspection policy to minimize long-run expected cost per unit time. In their study repair time would be assumed zero.

Many researchers have used the Markov process and numerical solutions to elaborate a maintenance optimization model (Osaki 1972). Osaki (1972) used the Markov renewal process and the laplace-Stieltjes transform to model a system with two similar units. Billinton and Pan (1998) studied redundant components in a system. The authors tried to demonstrate the optimum maintenance interval by using equations that consist of failure frequency and the failure rate of the whole system. In the studied system, each component operates in its wear-out period, and the failure rate of each component is not constant. The stochastic approach, in the particular Markov process, in the preventive maintenance has been employed to model real applications with various states in pieces of research (Bloch 2001, Flammini et al. 2009, Montoro and Perez 2006, Taghipour et al. 2010, Xu and Hu 2013 and Zequeria and Berenguer 2006). The instantaneous availability of the repairable system has been modeled using the Markov process and considering the optimal preventive maintenance plan (Xu and Hu 2013). Most of the research has investigated steady-state availability, but Xu and Hu (2013) presented a new model for instantaneous availability. A comprehensive investigation of a preventive maintenance plan for various models of the system consisting of active and standby systems with discrepant repair conditions has been studied by Mendes et al. (2014). They used the Markov process and the Laplace transform to model a system with four states (i.e., active redundant systems without component repair, active redundant systems with component repair, standby redundant systems without component repair and standby redundant systems with component repair). They also contemplated downtime cost and restarting the system after failure cost into their model. Models have been presented for two and three components. A cost function consists of four types (i.e., cost of periodic inspection, cost of repair for a component, cost of a system downtime per time and cost of system repair after failure).

Table 1. All studies based on year, redundancy, inspection plan, repair time and repair facility

Authors	Year	Redundancy	Inspection Interval	Repair Time	Repair Facility
Laprie J et al.	1981	Yes	Yes	No	No
David F et al.	1997	No	Yes	No	No
Chareonsuk et al.	1997	No	Yes	No	No
Billinton R & Pan J.	1998	Yes	Yes	No	No
Lai M & Yuan J.	1999	Yes	Yes	No	No
Levitin G & Lisnianski A.	1999	Yes	Yes	No	No
Bloch-Mercier S.	2001	No	Yes	No	No
Bris R et al.	2003	Yes	Yes	No	No
Samrout M et al.	2005	Yes	Yes	No	No
Castro HC & Cavalca KL.	2006	No	Yes	No	Yes
Montoro-Cazorla D & Perez-Ocon R.	2006	Yes	Yes	No	No
Zequeria RI & Berenguer C.	2006	No	Yes	No	No
Nourelfath M & Ait-Kadi D.	2007	Yes	Yes	No	No
Dimitrakos & Kyriakidis.	2008	No	Yes	Yes	No
Nahas N et al.	2008	Yes	Yes	No	No
XU H & HU W.	2008	No	Yes	No	No
Flammini F et al.	2009	Yes	Yes	No	No
Samrout M et al.	2009	No	Yes	No	No
Taghipour S et al.	2010	Yes	Yes	No	No
Wang CH & Lin TW.	2011	Yes	Yes	No	No
Nourelfath M et al.	2012	Yes	Yes	No	No
Wang CH & Tsai SW.	2012	Yes	Yes	No	No
Moghadaddam K.S.	2013	No	Yes	No	No
XU H & HU W.	2013	Yes	Yes	No	No
Chatwattanasiri N et al.	2014	No	Yes	No	No
Mendes A. et al.	2014	Yes	Yes	No	No
Yassin Hajipour & Sharareh Taghipour.	2016	Yes	Yes	No	No
Esmaeil Rezaei.	2017	Yes	Yes	No	No
Seyedhosseini et al.	2018	Yes	Yes	No	No
Mani Sharifi & Sharareh Taghipour.	2020	Yes	Yes	No	No
Mani Sharifi et al.	2021	Yes	Yes	No	No
Yang A et al.	2022	Yes	Yes	No	No
Peng et al.	2022	Yes	Yes	No	No
Zhao et al.	2023	Yes	Yes	No	No

Table 1 summarizes all studies in literature from four points of view; redundancy, inspection plan, repair time and repair facility. As noted in table 1, all studies did not consider repair time and repair facility as noticeable variables that can be a remarkable gap in this study. Thus, it is highlighted that incorporating repair time as a remarkable time into the model and repair facility as a resource in the model are considered in this study.

1-3-Contribution

Due to the stochastic behavior of the failing process in most systems, a common approach to study reliability and availability of the systems are Markov and semi-Markov methods with Laplace transforms (Osaki 1972). To the best of our knowledge, from the literature review, all the above-mentioned research did not consider the repair time as real time. In contrast, in real applications, repair time can be a significant time during the planning horizon. One of the most critical concerns, a maintenance manager is facing with is time-consuming actions during preventive maintenance processes. For instance, consider redundant pumps, boilers, rails, power motors, etc. When one of the redundant components fails, a repairer may deviate hours or days to repair or even to outsource the component and restore it to the system.

Our contributions in this study can be summarized as follows:

- The main contribution of the model proposed in this paper is to study on repair time in the preventive maintenance model using the Markov process.
- Another advantage of the presented model in this study as a minor contribution is the inclusion of the number of repair facility in the redundant systems.

The main contribution of the model proposed in this paper is to study the repair time in the preventive maintenance model using the Markov process. Thus, an analysis is presented for the repair time of the redundant systems. However, more in high technology, the repair facilities (i.e., repairer, tools and materials) can play an outstanding role in maintenance costs.

Models for three components with a various numbers of repair facilities will be modeled and analyzed in this paper. This model can be generalized for n components. The model's objective is to find the optimal preventive maintenance plan (i.e., the optimal time interval between inspections) considering repair time.

As mentioned, two main contributions are presented in this paper. The first one as a major contribution is to incorporate repair time as a remarkable time into the model and the second one is the number of repair facilities as a sensitive parameter which would be decision variable in the model. We used Markov process to model the maintenance plan and made a comprehensive analysis associated to the parameters in the model.

This paper is organized as follows. Section 2 describes the model description, mathematical model, method and assumptions. Section 3 presents some numerical examples, particularly considers the number of repair facilities in various examples. A comprehensive sensitivity analysis is presented in section 4. Section 5 depicts a case study as a tangible instance in real world and finally in section 6, concluding remarks and a summarization of the paper are described.

2-System description

2-1-Model description and assumptions

Modeling a redundant system with repairable components considering repair time and the number of repair facilities are carried out in this paper. When system's behavior in a stochastic environment is studied, the Markov process is a standard method to model the system. The system under consideration is modeled based on the Markov process. The system consists of some known states which the state of the system can change with the known probability and would be characterized by the number of failed components in the time interval between two inspections and the number of failed components in the repair period.

The system, discussed in this paper, consists of n redundant components with r repair facilities. Some assumptions have been considered to construct the model as follows:

- Every T unit of time, the system goes under inspection and repair, T is fixed in the model.
- Each repair facility can just be allocated to a component during repair time.
- Time to failure of each component follows the same exponential distribution with the parameter λ.
- Each repair time period spends t units of time, which means that if we have two repair facilities, in each period, two components can be repaired and it takes t unit of time or if we have two repair facilities and three failed components have been detected, only two failed components can be repaired, and restored to the operating state and one of them must wait for the next repair period.
- Perfect repair is considered in the paper.

- The inspection time of all components is considered negligible compared to the operation time and repair time.

All components of the system can be first inspected, the possible failed component(s) can be detected and considering the maintenance resource, the failed components would be repaired in the period of inspection.

The exponential distribution for the time to failure of each component makes us use the Markov Chain method, and it is well-known to use the Markov Chain method (Mendes et al. 2014) to model the time to failure of a system.

As stated before, systems with n redundant components with r repair facilities are studied in this paper, all components start to work at the same time and once all components fail, the system stops working until the inspection period starts. Every T units of time the system goes to repair operations while the system can continue to work if, at least, one component works. However, the real world, due to the high level of availability, the system can work continuously. The main advantage of this study is to take into account the repair time and maintenance resource as two essential factors in the model because many machines in some industries, such as nuclear, rail, and power are complex and needs to be repaired through special repair facilities and technical repair person. Thus, this resource can have a high cost for the system.

As seen in figure1, the horizontal line denotes the timeline, the T time interval is the system operation time, and the possible failed components are proportional to the number of repair facilities that can be restored during time $[T, T+t]$.

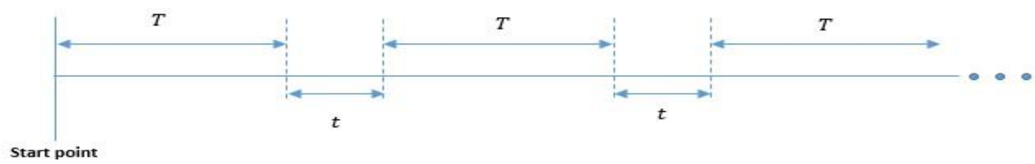

Fig 1. The time to repair and repair time of the model

2-2-Notations

All notations applied in the models are described as follows.

n	The number of redundant components
r	The number of repair facilities
C_r	The cost of a repair facility
C_i	The inspection cost of components per a period
C_d	The cost of downtime of the system per unit of time
C_s	The cost of reactivating the system after failure
T	The operation time of the system
t	The repair time of the possible failed component(s)
λ	Parameter for the exponential distribution of each component
TC	Expected cost function of the system
$J \mid v, w$	
J	State number
v	The number of failed components during operation time (T)
w	The number failed components during repair time (t)
P	Matrix of state transitions probabilities
p_{ij}	The probability from state i to j
Q	Transition part of matrix p

N	Matrix of expected number of times that the process is among states
n_{ij}	The expected number of times that the process is in state j given that it is started from state i
TTF	Time to failure
$MTTF_{ij}$	Mean time to failure between state i and j
$MTTF$	Mean time to failure
ρ_{ij}	Downtime between state i and j
ρ	Downtime
AT	The operation time of the component(s)

2-3-Mathematical model

Three models are presented for a redundant system with three components and one, two and three repair facilities, respectively. To model the system, we need to calculate the transition probabilities. With the transition probabilities for the system presented in the following, we make use the Markov Chain process to form the system states and known probabilities, p_{ij} , where p_{ij} is the probability of moving from state i to state j. Transition probability, the probability that the system is in a state with respect to its last state and along with the cost of the transition help us to calculate the expected cost of the maintenance. More importantly, the possible states of the system without dependency on the time or previous states are drawn through the Markov chain process. This state space would be presented by a diagram shown Fig 2, Fig 3 and Fig 4 for a system with three components with one repair facility, three components with two repair facilities and three components with three repair facilities, respectively. Modeling this configuration with the various repair facilities is intended to show the important effect of maintenance resources on preventive maintenance optimization. A comprehensive analysis is described in section 4.

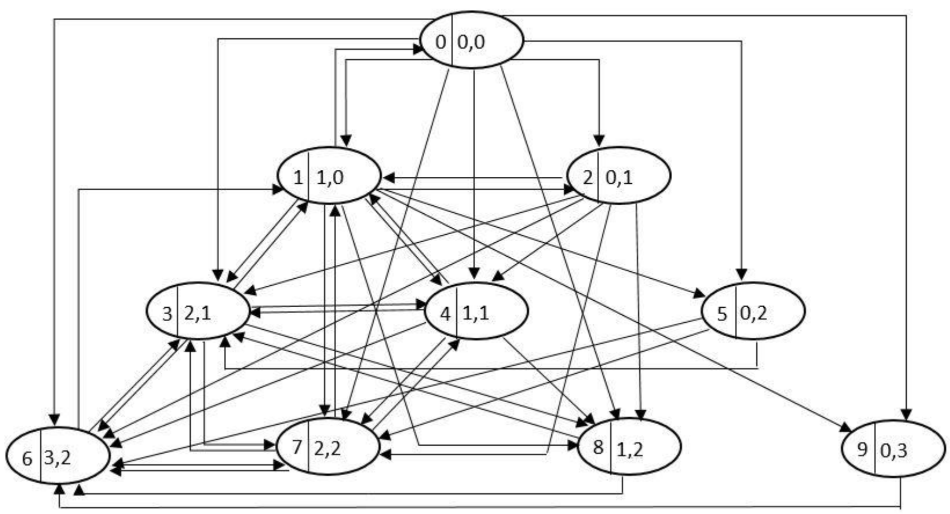

Fig 2. State space diagram of a system with three components and one repair facilities

A 10-by-10 matrix of the transition probability is formed for three models. In the following, we calculate the probability for each possible transition. For instance, the probability p_{01} showing the probability of going from state 0 to state 1 is interpreted as the probability of one component between three components failing while two other components are still working during T, with respect to one repair facility, it is automatically detected and repaired during t and two other components are also working, so no failed component is remained at the end of time period t. This probability can be calculated as $p_{01} = \left[\binom{3}{1}\ (1-e^{-\lambda T})(e^{-\lambda T})^2(e^{-\lambda t})^2\ \right]$.

All probabilities that constitute the 10-by-10 transition probability are calculated as follows.

$p_{00} = \left[\left(e^{-\lambda(T+t)}\right)^3\right]$, $p_{01} = \left[\binom{3}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)^2\left(e^{-\lambda t}\right)^2\right]$, $p_{10} = \left[\left(e^{-\lambda(T+t)}\right)^3\right]$, $p_{02} = \left[\left(e^{-\lambda T}\right)^3\binom{3}{1}\left(e^{-\lambda t}\right)^2\left(1-e^{-\lambda t}\right)\right]$, $p_{20} = 0$, $p_{03} = \left[\binom{3}{2}\left(1-e^{-\lambda T}\right)^2\left(e^{-\lambda T}\right)\left(e^{-\lambda t}\right)\right]$, $p_{30} = 0$, $p_{04} = \left[\binom{3}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)^2\binom{2}{1}\left(e^{-\lambda t}\right)\left(1-e^{-\lambda t}\right)\right]$, $p_{40} = 0$, $p_{05} = \left[\left(e^{-\lambda T}\right)^3\binom{3}{2}\left(e^{-\lambda t}\right)\left(1-e^{-\lambda t}\right)^2\right]$, $p_{50} = 0$, $p_{06} = \left[\left(1-e^{-\lambda T}\right)^3\right]$, $p_{60} = 0$, $p_{07} = \left[\binom{3}{2}\left(1-e^{-\lambda T}\right)^2\left(e^{-\lambda T}\right)\left(1-e^{-\lambda t}\right)\right]$, $p_{70} = 0$, $p_{08} = \left[\binom{3}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)^2\left(1-e^{-\lambda t}\right)^2\right]$, $p_{08} = 0$, $p_{09} = \left[\left(e^{-\lambda T}\right)^3\left(1-e^{-\lambda t}\right)^3\right]$, $p_{90} = 0$, $p_{11} = \left[\binom{3}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)^2\left(e^{-\lambda t}\right)^2\right]$, $p_{12} = \left[\left(e^{-\lambda T}\right)^3\binom{3}{1}\left(e^{-\lambda t}\right)^2\left(1-e^{-\lambda t}\right)\right]$, $p_{21} = \left[\left(e^{-\lambda(T+t)}\right)^2\right]$, $p_{13} = \left[\binom{3}{2}\left(1-e^{-\lambda T}\right)^2\left(e^{-\lambda T}\right)\left(e^{-\lambda t}\right)\right]$, $p_{31} = \left[\left(e^{-\lambda T}\right)^2\left(e^{-\lambda t}\right)^2\right]$, $p_{14} = \left[\binom{3}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)^2\binom{2}{1}\left(e^{-\lambda t}\right)\left(1-e^{-\lambda t}\right)\right]$, $p_{41} = \left[\left(e^{-\lambda(T+t)}\right)^2\right]$, $p_{15} = \left[\left(e^{-\lambda T}\right)^3\binom{3}{2}\left(e^{-\lambda t}\right)\left(1-e^{-\lambda t}\right)^2\right]$, $p_{51} = 0$, $p_{16} = \left[\left(1-e^{-\lambda T}\right)^3\right]$, $p_{61} = 0$, $p_{17} = \left[\binom{3}{2}\left(1-e^{-\lambda T}\right)^2\left(e^{-\lambda T}\right)\left(1-e^{-\lambda t}\right)\right]$, $p_{71} = 0$, $p_{18} = \left[\binom{3}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)^2\left(1-e^{-\lambda t}\right)^2\right]$, $p_{81} = 0$, $p_{19} = \left[\left(e^{-\lambda T}\right)^3\left(1-e^{-\lambda t}\right)^3\right]$, $p_{91} = 0$, $p_{22} = 0$, $p_{23} = \left[\binom{2}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)\left(e^{-\lambda t}\right)\right]$, $p_{32} = 0$, $p_{24} = \left[\left(e^{-\lambda T}\right)^2\binom{2}{1}\left(1-e^{-\lambda t}\right)\left(e^{-\lambda t}\right)\right]$, $p_{42} = 0$, $p_{25} = 0$, $p_{52} = 0$, $p_{26} = \left[\left(1-e^{-\lambda T}\right)^2\right]$, $p_{62} = 0$, $p_{27} = \left[\binom{2}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)\left(1-e^{-\lambda t}\right)\right]$, $p_{72} = 0$, $p_{28} = \left[\left(e^{-\lambda T}\right)^2\left(1-e^{-\lambda t}\right)^2\right]$, $p_{82} = 0$, $p_{29} = 0$, $p_{92} = 0$, $p_{33} = \left[\binom{2}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)\left(e^{-\lambda t}\right)\right]$, $p_{34} = \left[\left(e^{-\lambda T}\right)^2\binom{2}{1}\left(e^{-\lambda t}\right)\left(1-e^{-\lambda t}\right)\right]$, $p_{43} = \left[\binom{2}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)\left(e^{-\lambda t}\right)\right]$, $p_{35} = 0$, $p_{53} = \left[e^{-\lambda(T+t)}\right]$, $p_{36} = \left[\left(1-e^{-\lambda T}\right)^2\right]$, $p_{63} = \left[e^{-\lambda(T+t)}\right]$, $p_{37} = \left[\binom{2}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)\left(1-e^{-\lambda t}\right)\right]$, $p_{73} = \left[\left(e^{-\lambda(T+t)}\right)\right]$, $p_{38} = \left[\left(e^{-\lambda T}\right)^2\left(1-e^{-\lambda t}\right)^2\right]$, $p_{83} = \left[\left(e^{-\lambda(T+t)}\right)\right]$, $p_{39} = 0$, $p_{93} = 0$, $p_{44} = \left[\left(e^{-\lambda T}\right)^2\binom{2}{1}\left(1-e^{-\lambda t}\right)\left(e^{-\lambda t}\right)\right]$, $p_{45} = 0$, $p_{54} = 0$, $p_{46} = \left[\left(1-e^{-\lambda T}\right)^2\right]$, $p_{64} = 0$, $p_{47} = \left[\binom{2}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)\left(1-e^{-\lambda t}\right)\right]$, $p_{74} = 0$, $p_{48} = \left[\left(e^{-\lambda T}\right)^2\left(1-e^{-\lambda t}\right)^2\right]$, $p_{84} = 0$, $p_{49} = 0$, $p_{94} = 0$, $p_{55} = 0$, $p_{56} = \left[\left(1-e^{-\lambda T}\right)\right]$, $p_{65} = 0$, $p_{57} = \left[\left(e^{-\lambda T}\right)\left(1-e^{-\lambda t}\right)\right]$, $p_{75} = 0$, $p_{58} = 0$, $p_{85} = 0$, $p_{59} = 0$, $p_{95} = 0$, $p_{66} = \left[\left(1-e^{-\lambda T}\right)\right]$, $p_{67} = \left[\left(e^{-\lambda T}\right)\left(1-e^{-\lambda t}\right)\right]$, $p_{76} = \left[\left(1-e^{-\lambda T}\right)\right]$, $p_{68} = 0$, $p_{86} = \left[\left(1-e^{-\lambda T}\right)\right]$, $p_{69} = 0$, $p_{96} = 1$, $p_{77} = \left[\left(e^{-\lambda T}\right)\left(1-e^{-\lambda t}\right)\right]$, $p_{78} = 0$, $p_{87} = \left[\left(e^{-\lambda T}\right)\left(1-e^{-\lambda t}\right)\right]$, $p_{79} = 0$, $p_{97} = 0$

$p_{88} = 0$, $p_{89} = 0$, $p_{98} = 0$, $p_{99} = 0$

In a similar way to that of three redundant components with one repair facility, for the three redundant system with two repair facilities, the transition probability matrix is derived as follows. As seen in Fig 3 and Fig 2, 10 states exist in two models and their difference between two models incurs in states that the number of failed components exceed the number of facilities. For instance, in state 3, for the first diagram two components have failed during T and due to one repair facility, just one failed component can be restored to the system during t, while in the second model, as seen in Fig 3, both two failed components can be repaired.

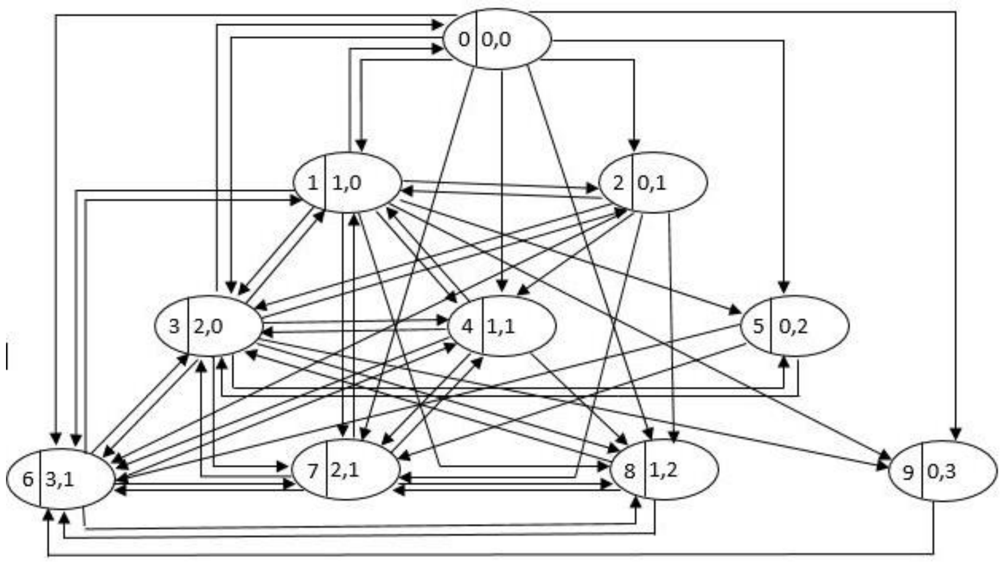

Fig 3. State space diagram of a system with three components and two repair facilities

$p_{00} = \left[\left(e^{-\lambda(T+t)}\right)^3\right]$, $p_{01} = \left[\binom{3}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)^2\left(e^{-\lambda t}\right)^2\right]$, $p_{10} = \left[\left(e^{-\lambda(T+t)}\right)^3\right]$, $p_{02} = \left[\left(e^{-\lambda T}\right)^3\binom{3}{1}\left(e^{-\lambda t}\right)^2\left(1-e^{-\lambda t}\right)\right]$, $p_{20} = 0$, $p_{03} = \left[\binom{3}{2}\left(1-e^{-\lambda T}\right)^2\left(e^{-\lambda T}\right)\left(e^{-\lambda t}\right)\right]$, $p_{30} = \left[\left(e^{-\lambda(T+t)}\right)^3\right]$, $p_{04} = \left[\binom{3}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)^2\binom{2}{1}\left(e^{-\lambda t}\right)\left(1-e^{-\lambda t}\right)\right]$, $p_{40} = 0$, $p_{05} = \left[\left(e^{-\lambda T}\right)^3\binom{3}{2}\left(e^{-\lambda t}\right)\left(1-e^{-\lambda t}\right)^2\right]$, $p_{50} = 0$, $p_{06} = \left[\left(1-e^{-\lambda T}\right)^3\right]$, $p_{60} = 0$, $p_{07} = \left[\binom{3}{2}\left(1-e^{-\lambda T}\right)^2\left(e^{-\lambda T}\right)\left(1-e^{-\lambda t}\right)\right]$, $p_{70} = 0$, $p_{08} = \left[\binom{3}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)^2\left(1-e^{-\lambda t}\right)^2\right]$, $p_{80} = 0$, $p_{09} = \left[\left(e^{-\lambda T}\right)^3\left(1-e^{-\lambda t}\right)^3\right]$, $p_{90} = 0$, $p_{11} = \left[\binom{3}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)^2\left(e^{-\lambda t}\right)^2\right]$, $p_{12} = \left[\left(e^{-\lambda T}\right)^3\binom{3}{1}\left(e^{-\lambda t}\right)^2\left(1-e^{-\lambda t}\right)\right]$, $p_{21} = \left[\left(e^{-\lambda(T+t)}\right)^2\right]$, $p_{13} = \left[\binom{3}{2}\left(1-e^{-\lambda T}\right)^2\left(e^{-\lambda T}\right)\left(e^{-\lambda t}\right)\right]$, $p_{31} = \left[\binom{3}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)^2\left(e^{-\lambda t}\right)^2\right]$, $p_{14} = \left[\binom{3}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)^2\binom{2}{1}\left(e^{-\lambda t}\right)\left(1-e^{-\lambda t}\right)\right]$, $p_{41} = \left[\left(e^{-\lambda(T+t)}\right)^2\right]$, $p_{15} = \left[\left(e^{-\lambda T}\right)^3\binom{3}{2}\left(e^{-\lambda t}\right)\left(1-e^{-\lambda t}\right)^2\right]$, $p_{51} = 0$, $p_{16} = \left[\left(1-e^{-\lambda T}\right)^3\right]$, $p_{61} = \left[\left(e^{-\lambda(T+t)}\right)^2\right]$, $p_{17} = \left[\binom{3}{2}\left(1-e^{-\lambda T}\right)^2\left(e^{-\lambda T}\right)\left(1-e^{-\lambda t}\right)\right]$, $p_{71} = \left[\left(e^{-\lambda(T+t)}\right)^2\right]$, $p_{18} = \left[\binom{3}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)\left(1-e^{-\lambda t}\right)^2\right]$, $p_{81} = 0$,$p_{19} = \left[\left(e^{-\lambda T}\right)^3\left(1-e^{-\lambda t}\right)^3\right]$, $p_{91} = 0$, $p_{22} = 0$, $p_{23} = \left[\binom{2}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)\left(e^{-\lambda t}\right)\right]$, $p_{32} = \left[\left(e^{-\lambda T}\right)^3\binom{3}{1}\left(e^{-\lambda t}\right)^2\left(1-e^{-\lambda t}\right)\right]$, $p_{24} = \left[\left(e^{-\lambda T}\right)^2\binom{2}{1}\left(1-e^{-\lambda t}\right)\left(e^{-\lambda t}\right)\right]$, $p_{42} = 0$, $p_{25} = 0$, $p_{52} = 0$, $p_{26} = \left[\left(1-e^{-\lambda T}\right)^2\right]$, $p_{62} = 0$, $p_{27} = \left[\binom{2}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)\left(1-e^{-\lambda t}\right)\right]$, $p_{72} = 0$, $p_{28} = \left[\left(e^{-\lambda T}\right)^2\left(1-e^{-\lambda t}\right)^2\right]$, $p_{82} = 0$, $p_{29} = 0$, $p_{92} = 0$, $p_{33} = \left[\binom{3}{2}\left(1-e^{-\lambda T}\right)^2\left(e^{-\lambda T}\right)\left(e^{-\lambda t}\right)\right]$, $p_{34} = \left[\binom{3}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)^2\binom{2}{1}\left(e^{-\lambda t}\right)\left(1-e^{-\lambda t}\right)\right]$, $p_{43} = \left[\binom{2}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)\left(e^{-\lambda t}\right)\right]$, $p_{35} = \left[\left(e^{-\lambda T}\right)^3\binom{3}{2}\left(e^{-\lambda t}\right)\left(1-e^{-\lambda t}\right)^2\right]$, $p_{53} = \left[e^{-\lambda(T+t)}\right]$, $p_{36} = \left[\left(1-e^{-\lambda T}\right)^3\right]$,

$p_{63} = \left[\binom{2}{1}(1-e^{-\lambda T})(e^{-\lambda T})\right]$, $p_{37} = \left[\binom{3}{2}(1-e^{-\lambda T})^2(e^{-\lambda T})(1-e^{-\lambda t})\right]$, $p_{73} = \left[\binom{3}{2}(1-e^{-\lambda T})^2(e^{-\lambda T})(e^{-\lambda t})\right]$, $p_{38} = \left[\binom{3}{1}(1-e^{-\lambda T})(e^{-\lambda T})^2(1-e^{-\lambda t})^2\right]$, $p_{83} = \left[(e^{-\lambda(T+t)})\right]$, $p_{39} = \left[(e^{-\lambda T})^3(1-e^{-\lambda t})^3\right]$, $p_{93} = 0$, $p_{44} = \left[(e^{-\lambda T})^2\binom{2}{1}(1-e^{-\lambda t})(e^{-\lambda t})\right]$, $p_{45} = 0$, $p_{54} = 0$, $p_{46} = \left[(1-e^{-\lambda T})^2\right]$, $p_{64} = \left[(e^{-\lambda T})^2\binom{2}{1}(1-e^{-\lambda t})(e^{-\lambda t})\right]$, $p_{47} = \left[\binom{2}{1}(1-e^{-\lambda T})(e^{-\lambda T})(1-e^{-\lambda t})\right]$, $p_{74} = \left[(e^{-\lambda T})^2\binom{2}{1}(1-e^{-\lambda t})(e^{-\lambda t})\right]$, $p_{48} = \left[(e^{-\lambda T})^2(1-e^{-\lambda t})^2\right]$, $p_{84} = 0$, $p_{49} = 0$, $p_{94} = 0$, $p_{55} = 0$, $p_{56} = \left[(1-e^{-\lambda T})\right]$, $p_{65} = 0$, $p_{57} = \left[(e^{-\lambda T})(1-e^{-\lambda t})\right]$, $p_{75} = 0$, $p_{58} = 0$, $p_{85} = 0$, $p_{59} = 0$, $p_{95} = 0$, $p_{66} = \left[(1-e^{-\lambda T})^2\right]$, $p_{67} = \left[\binom{2}{1}(1-e^{-\lambda T})(e^{-\lambda T})(1-e^{-\lambda t})\right]$, $p_{76} = \left[(1-e^{-\lambda T})^2\right]$, $p_{68} = \left[(e^{-\lambda T})^2(1-e^{-\lambda t})^2\right]$, $p_{86} = \left[(1-e^{-\lambda T})\right]$, $p_{69} = 0$, $p_{96} = 1$, $p_{77} = \left[\binom{2}{1}(1-e^{-\lambda T})(e^{-\lambda T})(1-e^{-\lambda t})\right]$, $p_{78} = \left[(e^{-\lambda T})^2(1-e^{-\lambda t})^2\right]$, $p_{87} = \left[(e^{-\lambda T})(1-e^{-\lambda t})\right]$, $p_{79} = 0$, $p_{97} = 0$, $p_{88} = 0$, $p_{89} = 0$, $p_{98} = 0$, $p_{99} = 0$

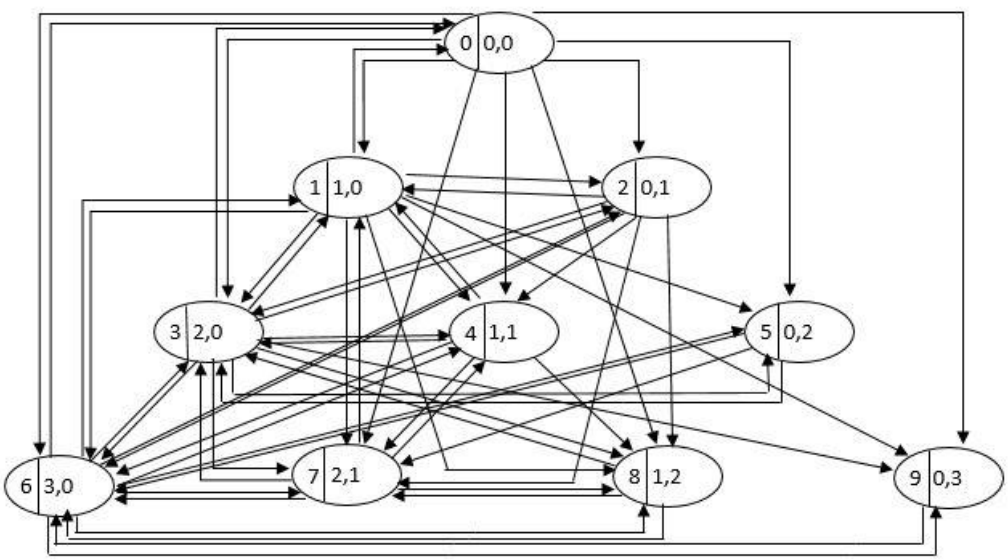

Fig 4. State space diagram of a system with three components and three repair facilities

Figure 4 shows the state space diagram for a system having three redundant components with three repair facilities. Note that if all components are failed during T, due to existing three repair facilities, all components can be repaired during t. Thus, only operating components might be failed during t and the failed one(s) must waited for the next repair period. The transition probabilities are computed same as two previous models and presented as below.

$p_{00} = \left[(e^{-\lambda(T+t)})^3\right]$, $p_{01} = \left[\binom{3}{1}(1-e^{-\lambda T})(e^{-\lambda T})^2(e^{-\lambda t})^2\right]$, $p_{10} = \left[(e^{-\lambda(T+t)})^3\right]$, $p_{02} = \left[(e^{-\lambda T})^3\binom{3}{1}(e^{-\lambda t})^2(1-e^{-\lambda t})\right]$, $p_{20} = 0$, $p_{03} = \left[\binom{3}{2}(1-e^{-\lambda T})^2(e^{-\lambda T})(e^{-\lambda t})\right]$, $p_{30} =$

$\left[\left(e^{-\lambda(T+t)}\right)^3\right]$, $p_{04}=\left[\binom{3}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)^2\binom{2}{1}\left(e^{-\lambda t}\right)\left(1-e^{-\lambda t}\right)\right]$, $p_{40}=0$, $p_{05}=\left[\left(e^{-\lambda T}\right)^3\binom{3}{2}\left(e^{-\lambda t}\right)\left(1-e^{-\lambda t}\right)^2\right]$, $p_{50}=0$, $p_{06}=\left[\left(1-e^{-\lambda T}\right)^3\right]$, $p_{60}=\left[\left(e^{-\lambda(T+t)}\right)^3\right]$, $p_{07}=\left[\binom{3}{2}\left(1-e^{-\lambda T}\right)^2\left(e^{-\lambda T}\right)\left(1-e^{-\lambda t}\right)\right]$, $p_{70}=0$, $p_{08}=\left[\binom{3}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)^2\left(1-e^{-\lambda t}\right)^2\right]$, $p_{08}=0$, $p_{09}=\left[\left(e^{-\lambda T}\right)^3\left(1-e^{-\lambda t}\right)^3\right]$, $p_{90}=0$, $p_{11}=\left[\binom{3}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)^2\left(e^{-\lambda t}\right)^2\right]$, $p_{12}=\left[\left(e^{-\lambda T}\right)^3\binom{3}{1}\left(e^{-\lambda t}\right)^2\left(1-e^{-\lambda t}\right)\right]$, $p_{21}=\left[\left(e^{-\lambda(T+t)}\right)^2\right]$, $p_{13}=\left[\binom{3}{2}\left(1-e^{-\lambda T}\right)^2\left(e^{-\lambda T}\right)\left(e^{-\lambda t}\right)\right]$, $p_{31}=\left[\binom{3}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)^2\left(e^{-\lambda t}\right)^2\right]$, $p_{14}=\left[\binom{3}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)^2\binom{2}{1}\left(e^{-\lambda t}\right)\left(1-e^{-\lambda t}\right)\right]$, $p_{41}=\left[\left(e^{-\lambda(T+t)}\right)^2\right]$, $p_{15}=\left[\left(e^{-\lambda T}\right)^3\binom{3}{2}\left(e^{-\lambda t}\right)\left(1-e^{-\lambda t}\right)^2\right]$, $p_{51}=0$, $p_{16}=\left[\left(1-e^{-\lambda T}\right)^3\right]$, $p_{61}=\left[\binom{3}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)^2\left(e^{-\lambda t}\right)^2\right]$, $p_{17}=\left[\binom{3}{2}\left(1-e^{-\lambda T}\right)^2\left(e^{-\lambda T}\right)\left(1-e^{-\lambda t}\right)\right]$, $p_{71}=\left[\left(e^{-\lambda(T+t)}\right)^2\right]$, $p_{18}=\left[\binom{3}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)\left(1-e^{-\lambda t}\right)^2\right]$, $p_{81}=0$, $p_{19}=\left[\left(e^{-\lambda T}\right)^3\left(1-e^{-\lambda t}\right)^3\right]$, $p_{91}=0$, $p_{22}=0$, $p_{23}=\left[\binom{2}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)\left(e^{-\lambda t}\right)\right]$, $p_{32}=\left[\left(e^{-\lambda T}\right)^3\binom{3}{1}\left(e^{-\lambda t}\right)^2\left(1-e^{-\lambda t}\right)\right]$, $p_{24}=\left[\left(e^{-\lambda T}\right)^2\binom{2}{1}\left(1-e^{-\lambda t}\right)\left(e^{-\lambda t}\right)\right]$, $p_{42}=0$, $p_{25}=0$, $p_{52}=0$, $p_{26}=\left[\left(1-e^{-\lambda T}\right)^2\right]$, $p_{62}=\left[\left(e^{-\lambda T}\right)^3\binom{3}{1}\left(e^{-\lambda t}\right)^2\left(1-e^{-\lambda t}\right)\right]$

$p_{27}=\left[\binom{2}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)\left(1-e^{-\lambda t}\right)\right]$, $p_{72}=0$, $p_{28}=\left[\left(e^{-\lambda T}\right)^2\left(1-e^{-\lambda t}\right)^2\right]$, $p_{82}=0$, $p_{29}=0$, $p_{92}=0$, $p_{33}=\left[\binom{3}{2}\left(1-e^{-\lambda T}\right)^2\left(e^{-\lambda T}\right)\left(e^{-\lambda t}\right)\right]$, $p_{34}=\left[\binom{3}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)^2\binom{2}{1}\left(e^{-\lambda t}\right)\left(1-e^{-\lambda t}\right)\right]$, $p_{43}=\left[\binom{2}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)\left(e^{-\lambda t}\right)\right]$, $p_{35}=\left[\left(e^{-\lambda T}\right)^3\binom{3}{2}\left(e^{-\lambda t}\right)\left(1-e^{-\lambda t}\right)^2\right]$, $p_{53}=\left[\left(e^{-\lambda(T+t)}\right)\right]$, $p_{36}=\left[\left(1-e^{-\lambda T}\right)^3\right]$, $p_{63}=\left[\binom{3}{2}\left(1-e^{-\lambda T}\right)^2\left(e^{-\lambda T}\right)\left(e^{-\lambda t}\right)\right]$, $p_{37}=\left[\binom{3}{2}\left(1-e^{-\lambda T}\right)^2\left(e^{-\lambda T}\right)\left(1-e^{-\lambda t}\right)\right]$, $p_{73}=\left[\binom{2}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)\left(e^{-\lambda t}\right)\right]$, $p_{38}=\left[\binom{3}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)^2\left(1-e^{-\lambda t}\right)^2\right]$, $p_{83}=\left[\left(e^{-\lambda(T+t)}\right)\right]$, $p_{39}=\left[\left(e^{-\lambda T}\right)^3\left(1-e^{-\lambda t}\right)^3\right]$, $p_{93}=0$, $p_{44}=\left[\left(e^{-\lambda T}\right)^2\binom{2}{1}\left(1-e^{-\lambda t}\right)\left(e^{-\lambda t}\right)\right]$, $p_{45}=0$, $p_{54}=0$, $p_{46}=\left[\left(1-e^{-\lambda T}\right)^2\right]$, $p_{64}=\left[\binom{3}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)^2\binom{2}{1}\left(e^{-\lambda t}\right)\left(1-e^{-\lambda t}\right)\right]$, $p_{47}=\left[\binom{2}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)\left(1-e^{-\lambda t}\right)\right]$, $p_{74}=\left[\left(e^{-\lambda T}\right)^2\binom{2}{1}\left(1-e^{-\lambda t}\right)\left(e^{-\lambda t}\right)\right]$, $p_{48}=\left[\left(e^{-\lambda T}\right)^2\left(1-e^{-\lambda t}\right)^2\right]$, $p_{84}=0$, $p_{49}=0$, $p_{94}=0$, $p_{55}=0$, $p_{56}=\left[\left(1-e^{-\lambda T}\right)\right]$, $p_{65}=\left[\left(e^{-\lambda T}\right)^3\binom{3}{1}\left(e^{-\lambda t}\right)^2\left(1-e^{-\lambda t}\right)\right]$, $p_{57}=\left[\left(e^{-\lambda T}\right)\left(1-e^{-\lambda t}\right)\right]$, $p_{75}=0$, $p_{58}=0$, $p_{85}=0$, $p_{59}=0$, $p_{95}=0$, $p_{66}=\left[\left(1-e^{-\lambda T}\right)^3\right]$, $p_{67}=\left[\binom{3}{1}\left(1-e^{-\lambda T}\right)^2\left(e^{-\lambda T}\right)\left(1-e^{-\lambda t}\right)\right]$, $p_{76}=\left[\left(1-e^{-\lambda T}\right)^2\right]$, $p_{68}=\left[\binom{3}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)\left(1-e^{-\lambda t}\right)^2\right]$, $p_{86}=\left[\left(1-e^{-\lambda T}\right)\right]$, $p_{69}=\left[\left(e^{-\lambda T}\right)^3\left(1-e^{-\lambda t}\right)^3\right]$, $p_{96}=1$, $p_{77}=\left[\binom{2}{1}\left(1-e^{-\lambda T}\right)\left(e^{-\lambda T}\right)\left(1-e^{-\lambda t}\right)\right]$, $p_{78}=\left[\left(e^{-\lambda T}\right)^2\left(1-e^{-\lambda t}\right)^2\right]$, $p_{87}=\left[\left(e^{-\lambda T}\right)\left(1-e^{-\lambda t}\right)\right]$, $p_{79}=0$, $p_{97}=0$, $p_{88}=0$, $p_{89}=0$, $p_{98}=0$, $p_{99}=0$

Now, the cost model of the system per unit of time is calculated as a way to measure the performance of the system in the long period. Total maintenance cost per the cycle of the inspection period, as described below, is a base to be optimized in this study.

$$TC=\frac{E[cost\ in\ a\ cycle]}{E[lengh\ of\ the\ cycle]} \tag{1}$$

The cost in a cycle consists of the inspection cost of components and it is independent of the number of failed components, cost of the system downtime, cost of repair facility and cost of the system repair after the failure of the system. Four types of costs are computed based on the states of the system.

The inspection cost of the components consists of monitoring all components and doing regular maintenance action on each component (C_i) that its time is assumed negligible. The cost of the repair facility consists of technical repairer, unique materials and tools involved in performing the repair action on the failed component (C_r); its time is assumed to be significant. The cost of the system repair after failure is defined as a type of cost spent on the system reactivating after it is down (C_s). The final type of cost is related to the loss of the system availability per unit of time (C_d). To compute the last cost, the expected downtime should be calculated based on the system state. As mentioned, C_s is a function of time, which means that the more downtime increase, the more losses increase.

For a system with three redundant components as mentioned above, the system's mean time to failure is calculated as below:

$$MTTF = \frac{1}{\lambda} + \frac{1}{\lambda} + \frac{1}{\lambda} - \frac{1}{2\lambda} - \frac{1}{2\lambda} - \frac{1}{2\lambda} + \frac{1}{3\lambda} = \frac{11}{6\lambda} \tag{2}$$

In some states, the operating components are known and may be smaller than three redundant components. For instance, the transition from state 4, the system with one failed component, to state 6 in which two more components will be failed during T (i.e., that one component has been failed is known at the begging of state 6), $MTTF$ would be calculated as:

$$MTTF_{46} = \frac{1}{\lambda} + \frac{1}{\lambda} - \frac{1}{2\lambda} = \frac{3}{2\lambda} \tag{3}$$

As Mendes et al. [14] presented, for a system with three components, if the inspection interval is relatively small compared to the expected failure time and considering time-to-failure following an exponential distribution, the approximate expected downtime would be as follows.

$$E[\rho] = \frac{AT}{n+1} \quad \text{for} \quad AT \le E[TTF] \tag{4}$$

As noted above, equation (3) is an approximation based on the condition that the inspection interval is less than the expected failure time of the system. As the time interval between inspections increases, the expected downtime becomes longer than the system's mean time to failure and for $AT > E[TTF]$, the expected downtime would be approximated using $(AT - MTTF)$. Generally, the expected downtime for a system would be approximated as below.

$$E[\rho_{ij}] = \max\left\{AT - MTTF_{ij},\ \frac{AT}{n+1}\right\} \tag{5}$$

The above-mentioned equation also depends on states. The model studied in this paper includes two types of times, one is denoted by T must be determined as a decision variable and another type of time is denoted by t must be given as an input variable. A detailed explanation on $E[\rho_{ij}]$ is described state by state as follows. The cost matrix for the above-mentioned models is presented as below:

I. **Model 1:** Three redundant components with one repair facility

$$C = \begin{bmatrix}
C_i & C_i + C_r & C_i & C_i + C_r & C_i + C_r & C_i & C_i + C_r + C_d\rho_{06} + C_S & C_i + C_r + C_d\rho_{07} + C_S & C_i + C_r + C_d\rho_{08} + C_S & C_i + C_d\rho_{09} + C_S \\
C_i & C_i + C_r & C_i & C_i + C_r & C_i + C_r & C_i & C_i + C_r + C_d\rho_{16} + C_S & C_i + C_r + C_d\rho_{17} + C_S & C_i + C_r + C_d\rho_{18} + C_S & C_i + C_d\rho_{19} + C_S \\
0 & C_i + C_r & 0 & C_i + C_r & C_i + C_r & 0 & C_i + C_r + C_d\rho_{26} + C_S & C_i + C_r + C_d\rho_{27} + C_S & C_i + C_r + C_d\rho_{28} + C_S & 0 \\
0 & C_i + C_r & 0 & C_i + C_r & C_i + C_r & 0 & C_i + C_r + C_d\rho_{36} + C_S & C_i + C_r + C_d\rho_{37} + C_S & C_i + C_r + C_d\rho_{38} + C_S & 0 \\
0 & C_i + C_r & 0 & C_i + C_r & C_i + C_r & 0 & C_i + C_r + C_d\rho_{46} + C_S & C_i + C_r + C_d\rho_{47} + C_S & C_i + C_r + C_d\rho_{48} + C_S & 0 \\
0 & 0 & 0 & C_i + C_r & 0 & 0 & C_i + C_r + C_d\rho_{56} + C_S & C_i + C_r + C_d\rho_{57} + C_S & 0 & 0 \\
0 & 0 & 0 & C_i + C_r & 0 & 0 & C_i + C_r + C_d\rho_{66} + C_S & C_i + C_r + C_d\rho_{67} + C_S & 0 & 0 \\
0 & 0 & 0 & C_i + C_r & 0 & 0 & C_i + C_r + C_d\rho_{76} + C_S & C_i + C_r + C_d\rho_{77} + C_S & 0 & 0 \\
0 & 0 & 0 & C_i + C_r & 0 & 0 & C_i + C_r + C_d\rho_{86} + C_S & C_i + C_r + C_d\rho_{87} + C_S & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & C_i + C_r + C_d\rho_{96} + C_S & 0 & 0 & 0
\end{bmatrix}$$

As seen in the matrix above, for instance, downtime cost must be incorporated into all states and their next state is state 6 (i.e., three failed components during T and two failed components during t, the seventh column of the matrix). Thus, their $E\left[\rho_{ij}\right]$ would be calculated as below:

$$E[\rho_{06}] = \max\left(T - MTTF_{06}, \frac{T}{4}\right) + t, E[\rho_{16}] = \max\left(T - MTTF_{16}, \frac{T}{4}\right) + t, E[\rho_{26}] = \max\left(T - MTTF_{26}, \frac{T}{3}\right) + t, E[\rho_{36}] = \max\left(T - MTTF_{36}, \frac{T}{3}\right) + t, E[\rho_{46}] = \max\left(T - MTTF_{46}, \frac{T}{3}\right) + t, E[\rho_{56}] = \max\left(T - MTTF_{56}, \frac{T}{2}\right) + t, E[\rho_{66}] = \max\left(T - MTTF_{66}, \frac{T}{2}\right) + t, E[\rho_{76}] = \max\left(T - MTTF_{76}, \frac{T}{2}\right) + t$$

$$E[\rho_{86}] = \max\left(T - MTTF_{86}, \frac{T}{2}\right) + t, E[\rho_{96}] = T + t$$

The important point to calculate the expected downtime for each one is that time t (repair time) should be added as a fixed number, because state 6 includes three failed components during T and no component could work during t and it is known that the system is down during t. For other states, expected downtime can be calculated in the above-mentioned similar way.

II. Model 2: Three redundant components with two repair facilities

In a similar way to that obtained for the first model, the cost matrix for a system having three redundant components with two repair facilities model is represented as below.

$$C = \begin{bmatrix}
C_i & C_i + C_r & C_i & C_i + 2C_r & C_i + C_r & C_i & C_i + 2C_r + C_d\rho_{06} + C_S & C_i + 2C_r + C_d\rho_{07} + C_S & C_i + C_r + C_d\rho_{08} + C_S & C_i + C_d\rho_{09} + C_S \\
C_i & C_i + C_r & C_i & C_i + 2C_r & C_i + C_r & C_i & C_i + 2C_r + C_d\rho_{16} + C_S & C_i + 2C_r + C_d\rho_{17} + C_S & C_i + C_r + C_d\rho_{18} + C_S & C_i + C_d\rho_{19} + C_S \\
0 & C_i + C_r & 0 & C_i + 2C_r & C_i + C_r & 0 & C_i + 2C_r + C_d\rho_{26} + C_S & C_i + 2C_r + C_d\rho_{27} + C_S & C_i + C_r + C_d\rho_{28} + C_S & 0 \\
C_i & C_i + C_r & C_i & C_i + 2C_r & C_i + C_r & C_i & C_i + 2C_r + C_d\rho_{36} + C_S & C_i + 2C_r + C_d\rho_{37} + C_S & C_i + C_r + C_d\rho_{38} + C_S & C_i + C_d\rho_{39} + C_S \\
0 & C_i + C_r & 0 & C_i + 2C_r & C_i + C_r & 0 & C_i + 2C_r + C_d\rho_{46} + C_S & C_i + 2C_r + C_d\rho_{47} + C_S & C_i + C_r + C_d\rho_{48} + C_S & 0 \\
0 & 0 & 0 & C_i + 2C_r & 0 & 0 & C_i + 2C_r + C_d\rho_{56} + C_S & C_i + 2C_r + C_d\rho_{57} + C_S & 0 & 0 \\
0 & C_i + C_r & 0 & C_i + 2C_r & C_i + C_r & 0 & C_i + 2C_r + C_d\rho_{66} + C_S & C_i + 2C_r + C_d\rho_{67} + C_S & C_i + C_r + C_d\rho_{68} + C_S & 0 \\
0 & C_i + C_r & 0 & C_i + 2C_r & C_i + C_r & 0 & C_i + 2C_r + C_d\rho_{76} + C_S & C_i + 2C_r + C_d\rho_{77} + C_S & C_i + C_r + C_d\rho_{78} + C_S & 0 \\
0 & 0 & 0 & C_i + 2C_r & 0 & 0 & C_i + 2C_r + C_d\rho_{86} + C_S & C_i + 2C_r + C_d\rho_{87} + C_S & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & C_i + 2C_r + C_d\rho_{96} + C_S & 0 & 0 & 0
\end{bmatrix}$$

III. Model 3: Three redundant components with three repair facilities

Like two previous models, the cost matrix of the system having three redundant components with three repair facilities is calculated and represented as below.

$$C = \begin{bmatrix}
C_i & C_i + C_r & C_i & C_i + 2C_r & C_i + C_r & C_i & C_i + 2C_r + C_d\rho_{06} + C_S & C_i + 2C_r + C_d\rho_{07} + C_S & C_i + C_r + C_d\rho_{08} + C_S & C_i + C_d\rho_{09} + C_S \\
C_i & C_i + C_r & C_i & C_i + 2C_r & C_i + C_r & C_i & C_i + 2C_r + C_d\rho_{16} + C_S & C_i + 2C_r + C_d\rho_{17} + C_S & C_i + C_r + C_d\rho_{18} + C_S & C_i + C_d\rho_{19} + C_S \\
0 & C_i + C_r & 0 & C_i + 2C_r & C_i + C_r & 0 & C_i + 2C_r + C_d\rho_{26} + C_S & C_i + 2C_r + C_d\rho_{27} + C_S & C_i + C_r + C_d\rho_{28} + C_S & 0 \\
C_i & C_i + C_r & C_i & C_i + 2C_r & C_i + C_r & C_i & C_i + 2C_r + C_d\rho_{36} + C_S & C_i + 2C_r + C_d\rho_{37} + C_S & C_i + C_r + C_d\rho_{38} + C_S & C_i + C_d\rho_{39} + C_S \\
0 & C_i + C_r & 0 & C_i + 2C_r & C_i + C_r & 0 & C_i + 2C_r + C_d\rho_{46} + C_S & C_i + 2C_r + C_d\rho_{47} + C_S & C_i + C_r + C_d\rho_{48} + C_S & 0 \\
0 & 0 & 0 & C_i + 2C_r & 0 & 0 & C_i + 2C_r + C_d\rho_{56} + C_S & C_i + 2C_r + C_d\rho_{57} + C_S & 0 & 0 \\
C_i & C_i + C_r & C_i & C_i + 2C_r & C_i + C_r & C_i & C_i + 2C_r + C_d\rho_{66} + C_S & C_i + 2C_r + C_d\rho_{67} + C_S & C_i + C_r + C_d\rho_{68} + C_S & C_i + C_d\rho_{39} + C_S \\
0 & C_i + C_r & 0 & C_i + 2C_r & C_i + C_r & 0 & C_i + 2C_r + C_d\rho_{76} + C_S & C_i + 2C_r + C_d\rho_{77} + C_S & C_i + C_r + C_d\rho_{78} + C_S & 0 \\
0 & 0 & 0 & C_i + 2C_r & 0 & 0 & C_i + 2C_r + C_d\rho_{86} + C_S & C_i + 2C_r + C_d\rho_{87} + C_S & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & C_i + 2C_r + C_d\rho_{96} + C_S & 0 & 0 & 0
\end{bmatrix}$$

The expected cost in a cycle can be approximated by:

$$E[cost\ in\ a\ cycle] = \sum_{j \in J} \sum_{i \in I} n_{ij} p_{ij} c_{ij} \tag{6}$$

Where, n_{ij} denotes the expected number of times that the process is in state j given that it is started from state i. It for all i and j incorporate matrix N calculated from the equation below:

$$N = (1 - Q)^{-1} \tag{7}$$

Where Q is the transient part of matrix P.

The length of the cycle means the time of a period consists of the time to repair (T) and repair time(t):

$$E[length\ of\ the\ cycle] = T + t \tag{8}$$

Based on two equations above, the total cost as a function of T to be minimized is shown below:

$$TC(T) = \frac{\sum_{j\in J}\sum_{i\in I} n_{ij} p_{ij} c_{ij}}{T+t} \tag{9}$$

To derivate the optimum point of T, the equation above must be minimized. Some numerical examples are presented to demonstrate the model.

3-Numerical examples and simulation experiments

In this section, numerical examples are given to validate theoretical results in which Optimal inspection interval is determined by minimizing total cost. Then, to evaluate the performance of the policy, the simulation experiments are carried out.

3-1-Numerical examples

The application of the proposed model is illustrated by considering several examples. Numerical examples for the mentioned models are presented and discussed in the sequence below. Based on the three mentioned models, equal parameters are used to compare these models. $\lambda = 0.027$, $C_i = 20000$, $C_r = 20000$, $C_d = 50000$, $C_s = 100000$ and $t = 2$ hours are used as input parameters for three models. All parameter ban be estimated statistically by the method of maximum likelihood.
Fig 5 shows the behavior of the function TC concerning the time interval between two inspections for three models, and the optimal time interval is depicted in the graphs as well.

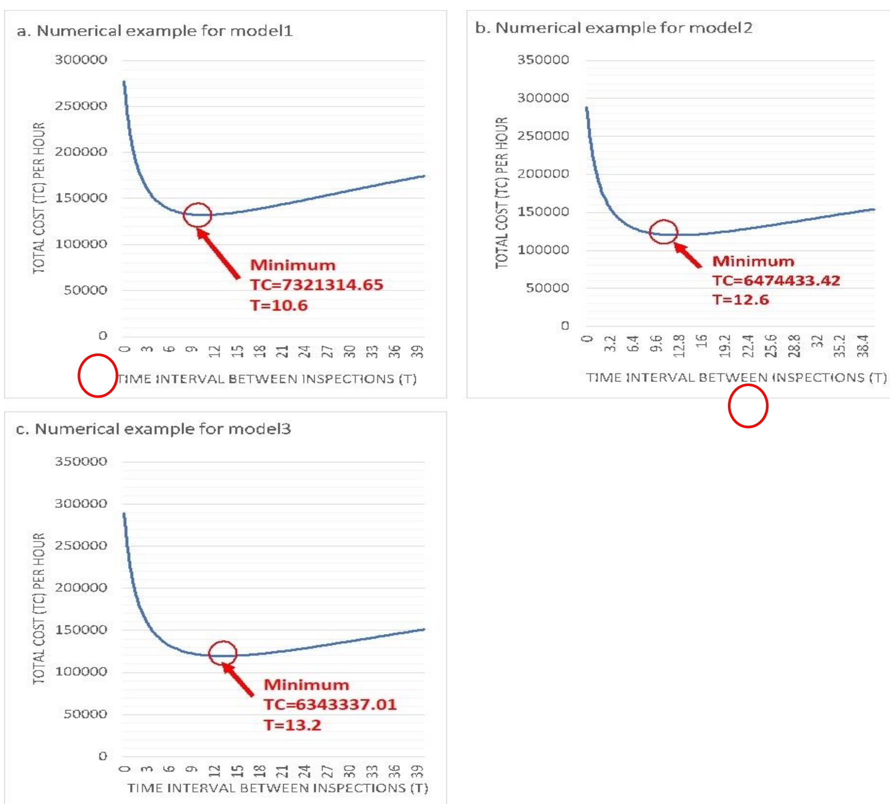

Fig 5. Numerical examples for systems with three redundant components and various number of one repair facilities

The minimum of the expected total cost per unit of time of each example is obtained by utilizing numerical search technique and presented in table 2.

Table 2. A cost comparison for three models and their results.

Model	The number of repair facility	Optimal time interval between inspection	Optimal expected total cost per unit of time	Cost reduction (%)
Model 1	1	10.6 hr	7321314.65	-
Model 2	2	12.6 hr	6474433.42	11.5
Model 3	3	13.2 hr	6343337.01	13.3

According to figure 5 and the results presented in table 2, the best model for a system with three redundant components is model 3. It seems that three repair facilities can be appropriate considering the repair facility cost and other cost types. Model 3 can be used as an applicable model for its cost reduction of approximately 13.3 % in the expected total cost per unit of time in comparison with other models. It can be logical that, due to more maintenance resource for model with three repair facilities, the optimal time interval for model 3 is longer than others.

3-2-Simulation experiments

To evaluate the performance of the policy, simulation experiments under different parameters sets were carried out. The distributions of repair times are exponential distributions which is the same as in our policy. The parameter sets are shown in table 3 and the results of the experiments are shown in table 4.

Table 3. Parameter sets of the scheme

	C_i	C_r	C_d	C_s	λ
Set 1	3	300000	825000	80000	0.01
Set 2	40	250000	900000	70000	0.02
Set 3	350	120000	123000	90000	0.04
Set 4	4800	140000	410000	25000	0.06
Set 5	27000	280000	750000	50000	0.07

Table 4. Total cost per set for model 1

		Set 1	Set 2	Set 3	Set 4	Set 5
Model 1	Optimal expected total cost per unit of time	678485.33	778480.01	922411.00	1895620.32	8462702.34
	Optimal time interval between inspection	7.4	8	9.7	11.6	46.8
Model 2	Optimal expected total cost per unit of time	657890.13	751324.01	901200.60	1563236.01	8188654.26
	Optimal time interval between inspection	9.1	10.3	12	14.1	49.1
Model 3	Optimal expected total cost per unit of time	601967.20	718902.64	798321.76	986543.00	7712145.69
	Optimal time interval between inspection	11.2	14.2	16.1	19.3	54.7

The minimum of the expected total cost per unit of time of each experiment is obtained by utilizing numerical search technique and presented in table 4. According to the results presented in table 4, the best model for a system with three redundant components is model 3. It seems that three repair facilities can be appropriate considering the repair facility cost and other cost types considering various cost values in experiments.

4-Sensitivity analysis

A sensitivity analysis was conducted for model 1 in order to evaluate the performance of the input parameters (i.e., λ, C_i ,C_r , C_d , C_s and particularly t the repair time) and to demonstrate the effect of the inputs on the expected total cost. It is noted that all input were analyzed based on the minimal TC.

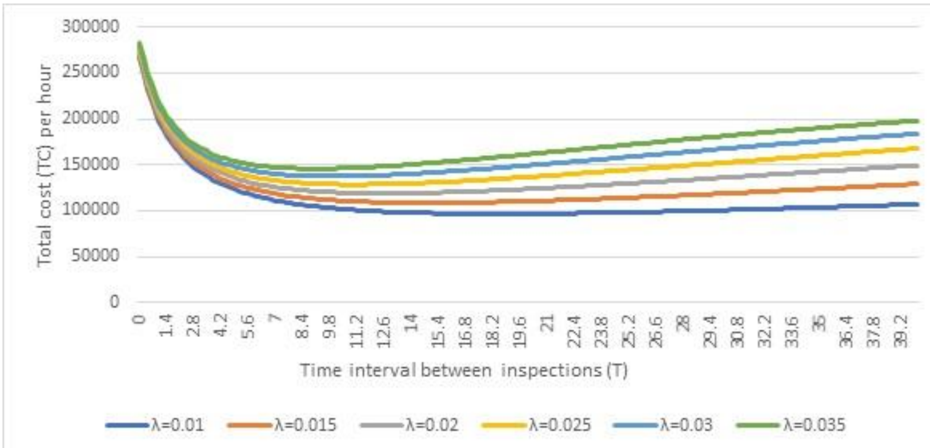

Fig 6. The effect of hazard rate λ on the expected total cost per unit of time for model 1

According to figure 6, the effect of hazard rate (λ) implies that a higher hazard rate increases the expected total cost per hour and decreases the optimal interval between inspections (T). If we divide all figures in sensitivity analysis section preparing the effect of input parameter on TC and optimal T into two areas (after optimal time interval between inspections and before optimal time interval between inspections), it can be stated that the difference between TCs is significant after optimal time interval inspections. It would be logical to say that a higher time interval between inspections can increase the probability of failure in more components, and the probability of a system down can be increased as well.

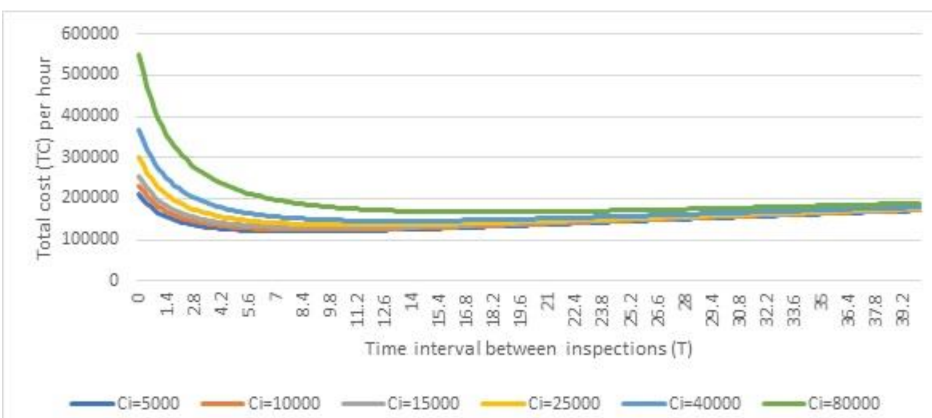

Fig 7. The effect of inspection cost C_i on the expected total cost per unit of time for model 1

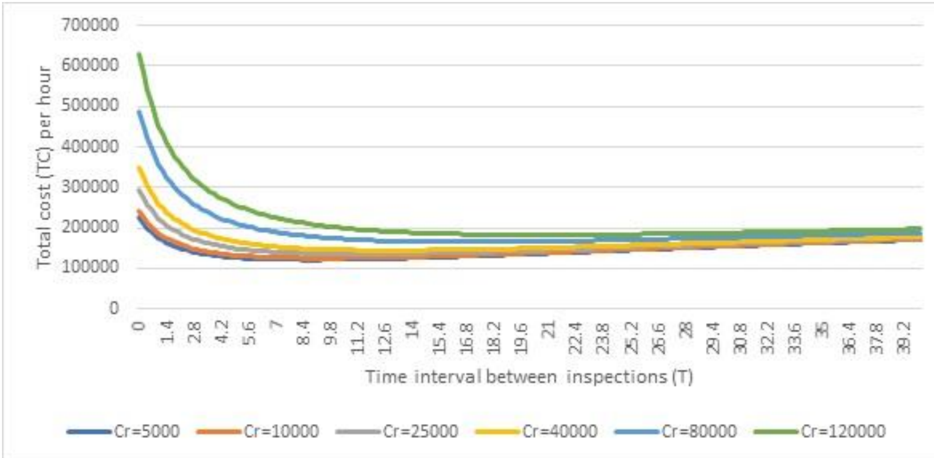

Fig 8. The effect of maintenance resource cost C_r on the expected total cost per unit of time for model 1

Figure 7 shows the effect of inspection cost on components per period. As seen, when the inspection cost increases, both the expected total cost per hour and optimal time interval inspections increases. As seen in Fig 8, like inspection cost, repair facility cost exactly has an effect that increasing in repair facility cost leads to increases in expected total cost and optimal time interval between inspections. In these figures, the difference between TCs before optimal time interval between inspections (T) is more significant than after the optimal T. However, fewer time intervals between inspections (i.e., a greater number of inspection periods) would lead to higher costs spent on inspection and repair facilities.

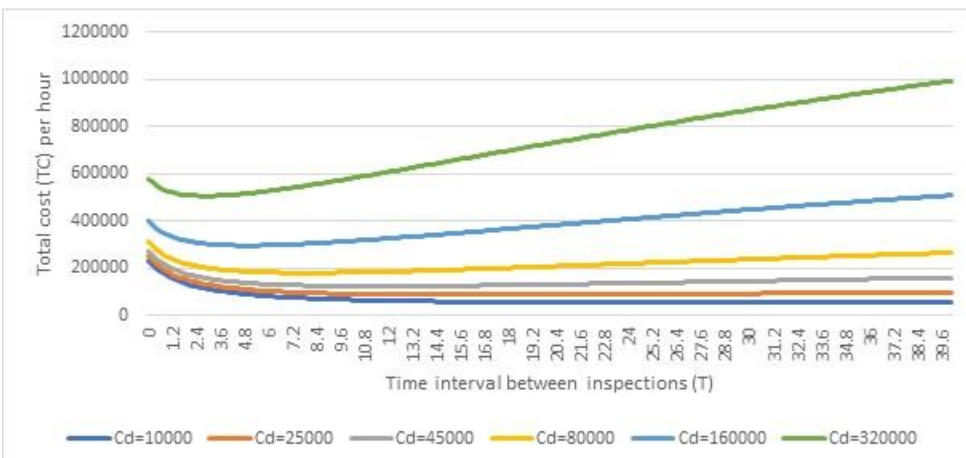

Fig 9. The effect of downtime cost C_d on the expected total cost per unit of time for model 1

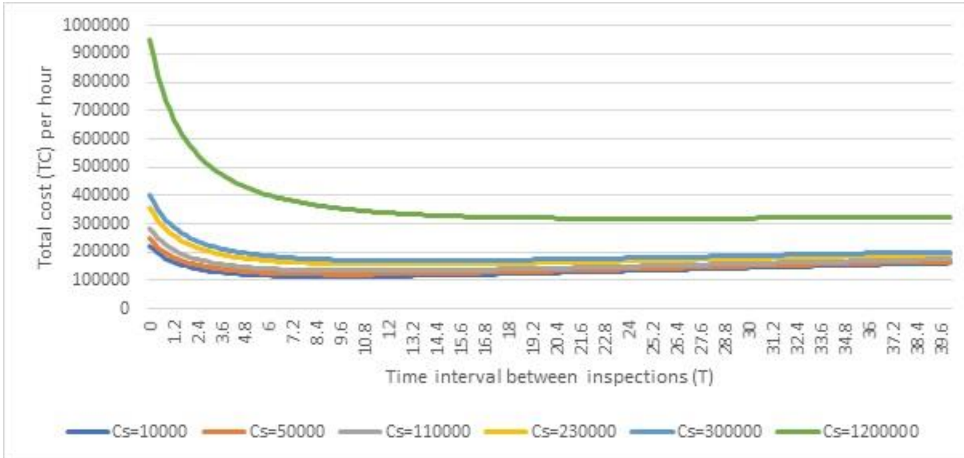

Fig 10. The effect of system reactivating cost C_s on the expected total cost per unit of time for model 1

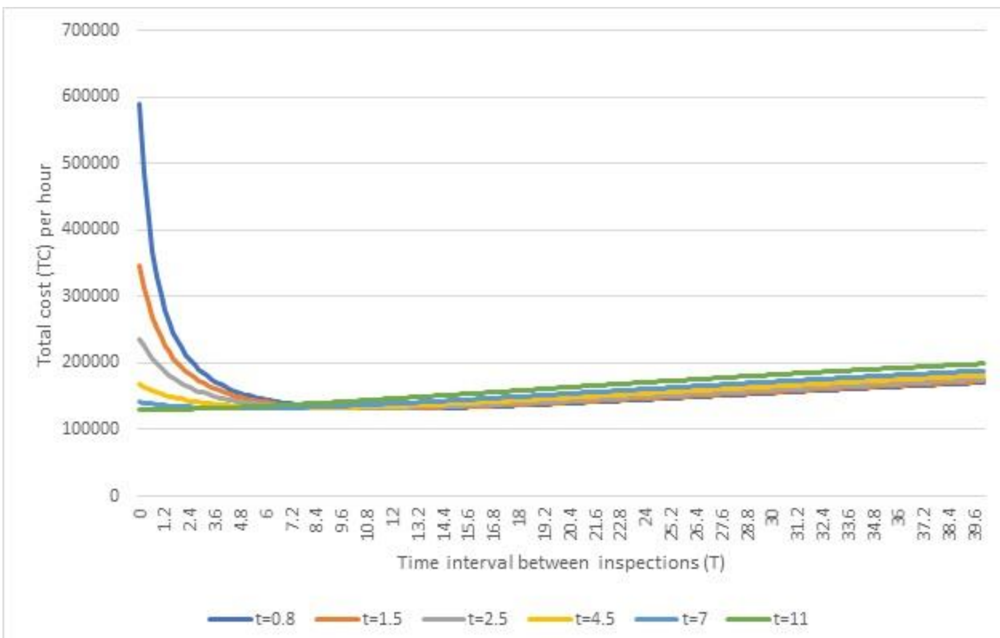

Fig 11. The effect of repair time t on the expected total cost per unit of time for model 1

The effect of downtime cost on TC and optimal T is presented in Fig.9. As shown, increases in the downtime cost would produce increases in the TC and optimal T.

The effect of the cost of the reactivating the system after failure can be seen in Fig.10. It is shown that, like other parameters, higher cost of reactivating will produce a higher TC and optimal T.

One of the contributions considered in this paper is based on repair time (t). Against the research mentioned in the introduction, main advantage of this paper is to incorporate repair time as a noticeable factor in the maintenance model. The last figure, (i.e., figure 11) is related to the variability of repair time and its effect on TC and optimal T. Fig.11 shows that higher t leads to higher optimal TC and T. It is noted that the effect of t conversely produces less TC after optimal T. However, it seems that during t other possible components either work or fail as well.

5-Case study

In power generation industry, pumps can play a vital role and reliable pump would transfer high solid fly ash slurry through long pipelines. With environmental protection, the engineering consultants are supposed to design pump systems that could manage both the flow and high pressure required. Any failure must be handled and downtime would be expensive for this kind of system. The first technique is to consider redundancy and the second one is to plan efficiently maintenance of the pumps. Our study is based on this case study; however, we do not have access right to a real data.

6-Conclusions

This section is divided into four parts; the first one is a brief description of what have been done and the novelty, the second part is related to the main parameters and the way the system is investigated, the third part provided the summary of results and the final part is associated to suggestions for future researches.

6-1-A brief description

Discrete-time Markov chain was used to model an applicable redundant system noticing repair time and the number of repair facilities as two more essential variables. This paper presented three models based on the number of facilities to analyze the reliability and the optimal time interval between inspections based on various costs involved in the system.

6-2-Investigation direction

In every preventive maintenance model, due to a direct relation to maintenance costs, the inspection time interval is an important issue raised from a tradeoff between availability and costs. It is known that long time interval between inspections can decrease costs related to inspection costs and decreases the availability of the system. Still, it might increase downtime cost and repair cost. An appropriate time interval between inspections can be obtained considering system availability and the lowest possible cost.

Using a discrete Markov chain, a model based on the expected total cost per time was established to ensure that a proper time interval achieving the lowest possible cost is obtained by considering repair time and the number of repair facilities. The main advantage of the model proposed in this paper is to incorporate repair time and the number of facilities as substantial variables into the preventive maintenance model. However, in the real world, there exist complex systems in which repairing takes significant time and is costly.

6-3-Results

Three models were studied and numerical examples were solved by search technique in Python to demonstrate the efficiency of the models. A comparison was made between the three models.

- Model 1 consists of three redundant components with one repair facility,
- Model 2 consists of the three redundant components with two repair facilities,
- Model 3 consists of three redundant components with three repair facilities.

The best model for a system with three redundant components is model 3. It seems that three repair facilities can be appropriate considering the repair facility cost and other cost types. Model 3 can be used as an applicable model for its cost reduction of approximately 13.3 % in the expected total cost per unit of time in comparison with other models. It can be logical that, due to more maintenance resource for model with three repair facilities, the optimal time interval for model 3 is longer than others.

6-4-Future research

The model presented in this paper can be studied for other redundant structures, such as cold-standby systems, k-out-of-n systems, and load-sharing systems, as future research. Other extensions to this study would be a study on n redundant components in which n would be considered as a decision variable, and a study would be started on the classified repair facilities. The classified repair facilities can be developed in the repair quality and the time it consumes to restore the failed components.

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