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Reliable location-allocation model for congested systems under disruptions using accelerated Benders' decomposition

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Abstract

This paper aims to propose a reliable location-allocation model where facilities are subject to the risk of disruptions. Since service facilities are expected to satisfy random and heavy demands, we model the congested situations in the system within a queuing framework which handles two sources of uncertainty associated with demand and service. To insure the service quality, a minimum limit reflected in the customers' expected waiting time is considered in the model. We also consider the geographical accessibility of the service network in terms of the proximity of a facility to the potential demands. The model determines the optimal number and locations of facilities and the corresponding customer assignments in such a way as to minimize the fixed installation cost as well as expected traveling, serving and penalty costs. To obtain exact solution of the proposed model, a Benders' decomposition algorithm enhanced by two efficient accelerating methods including valid inequalities and knapsack inequalities is proposed. Numerical results illustrate the applicability of the proposed model as well as the effectiveness of the designed solution procedure.

Keywords: Location-allocation model, disruption, queuing system, service quality, geographical accessibility, Benders' decomposition

1- Introduction

Location-allocation models deal with finding the location of new facilities in some given geographical areas and the allocation of demand nodes to the located facilities in such a way as to minimize total travel time, physical distance, operational or transportation costs or to maximize the market capture or reliability (Boffey et al., 2006; Zarrinpoor and Seifbarghy, 2011).Beginning from the study of Weber (1909), numerous researches have been developed for facility location problem. The exhaustive reviews in this area were provided by Revelle et al.,(2005) and Revelle et al.,(2008).

A common assumption in classical facility location problems is that built facility will remain operational forever. However, in practice there are many types of disruptive events which can make the facility unavailable

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from a time moment to another one. Disruptive events may be originated from different reasons including natural disasters, equipment breakdowns, terrorist attacks, labor strikes, changes in ownership, etc (Snyder and Daskin, 2005). In the occurrence of disruptions, it is impossible to immediately change the network substructure to serve demands. Often, in order to achieve system reliability, mitigation or recourse operation is implemented in such a way as to reassign the demand nodes to other operational facilities much farther than their regularly assigned facilities (Snyder and Daskin, 2005; Aydin and Murat, 2013). Facility disruptions can significantly deteriorate the overall system efficiency and responsiveness. Therefore, it is crucial to design a reliable network that can work properly in both normal and disruptive conditions.

Generally, real service facilities are expected to meet heavy and random demands. Since facilities have not enough capacity to serve all the simultaneous demands immediately, the system may be congested in some situations. To study the probabilistic nature of demand and service of such systems, the incorporation of queuing theory into the facility location has gained much attention recently. Some examples of congested systems include post offices, bank branches, automated teller machines, vending machines, check-in counters in airports, checkout counters in stores, proxy/mirror servers in communication networks, emergency medical service, healthcare facilities and distribution centers in supply chains(Wang et al.,2002;Huang et al.,2005;Aboolian et al.,2009). Precisely, in the disruptive conditions we might expect a higher level of congestion in the system. Consequently, to obtain a more reliable configuration of real service network, congestion must be considered in service facility location and allocation decisions.

Since location and allocation decisions are the most important strategic decisions in numerous contexts such as supply chain planning, transportation infrastructure design and public service systems, different fundamental issues must be considered in the modeling to design an efficient and effective service network design, including reliability aspects, risk of disruptions, congestion, geographical accessibility and corresponding costs. Regarding the literature (See section 2), there is no comprehensive model that considers all the aforementioned aspects. The current research proposes a location-allocation model where facilities are subject to the risk of disruptions. The uncertainty associated with demand as well as service is considered in the model within the queuing system. The geographical accessibility of a service network is considered in terms of the proximity of a facility to the potential customers. To solve the proposed model, a Benders' decomposition algorithm enhanced by two accelerating methods including valid inequalities and knapsack inequalities is proposed. Several generated instances are presented to illustrate the applicability of the developed model as well as the effectiveness of the designed solution procedure.

The rest of the paper is organized as follows. The next section reviews the related literature. Section 3 presents the mathematical model. In Section 4, an accelerated Benders decomposition is developed to solve the model. Section 5 describes several generated instances to illustrate the applicability of the proposed model. Section 6 ends with some conclusions and possible directions for future research.

2- Literature review

The relevant literature in two separate but complementary research streams in the context of facility location problem is reviewed in this section and then the main contributions of this research are presented.

2-1- Facility location problem under disruptions

The earliest study of reliable facility location problem dates back to the work of Drezner (1987), who formulated p-median problem (PMP) under the assumption that some facilities may become inactive, and (p,q)-center problem in which p facilities need to be located and at most q facilities may become inactive. Snyder and Daskin (2005) presented reliability-based formulations of PMP and uncapacitated fixed-charge location problem (UFLP) with equal facility failure probability. Berman et al,. (2007) considered the PMP with independent failure probabilities under the complete information of customers about the operational status of facilities. For site-dependent random facility disruptions, Cui et al. (2010) proposed a compact mixed integer program formulation to study the reliable UFLP and developed both Lagrangian relaxation and continuum approximation methods to solve it. Li and Ouyang (2010) addressed the UFLP under correlated and site-dependent facility disruptions and formulated the proposed model upon the continuum approximation approach. Lim et al., (2010) studied the reliable facility location problem in the presence of random facility disruptions with the option of hardening selected facilities. Shen et al., (2011) proposed both scenario-based stochastic programming and a nonlinear mixed integer programming models for a reliable facility location problem considering unequal failure probabilities. Peng et al., (2011) used the p-robustness criterion to develop a reliable model for designing a logistic network under facility failures. Chen et al., (2011) incorporated inventory management decisions into the facility location design framework assuming that facilities are subject to the equal probabilistic facility failures. The reliable capacitated facility location problem was addressed by Aydin and Murat (2013) who formulated the problem as a two-stage stochastic program. In the context of competitive facility location problem, Wang and Ouyang (2013) proposed game-theoretical models based on continuum approximation approach in which spatial competition and probabilistic facility disruption are considered. Interdependent and correlated failures in facility location problem within a supporting structure framework for an infrastructure system were considered by Li et al., (2013a). The impact of misestimating the disruption probability in the reliable facility location model was studied by Lim et al., (2013). Li et al., (2013b) introduced the reliable PMP and UFLP under heterogeneous facility failure probabilities and facility fortification within a finite budget. An et al., (2014) adopted a two-stage robust optimization approach to study both cases of reliable capacitated and un-capacitated PMP. Alcaraz et al., (2015) presented a set packing formulation of the reliable UFLP and studied certain aspects of its polyhedral properties by identifying a number of clique facets. Yun et al. (2015) proposed a reliable facility location model under independent and identical facility disruptions and assumed that customer does not have complete information about the operational status of facilities. Zhang et al., (2016) studied a reliable location-inventory model in which facilities may fail with heterogeneous disruption probabilities.

2-2- Facility location for congested systems

A facility location problem for congested systems was addressed first by Berman and Larson (1985). Marianov and Serra (1998) presented several maximal coverage models under constrained waiting time and queue length. Several queuing location-allocation models with hierarchical structure were addressed by Marianov and Serra (2001). Wang et al., (2002) studied a facility location problem by considering the queuing system which minimizes the expected total traveling and waiting times. Marianov (2003) presented a location model of congested facilities to maximize expected demands. Shavandi and Mahlooji (2006) presented a fuzzy queuing location-allocation model which maximizes the total covered demands. Syam (2008) developed a multiple server location–allocation model for service system design. Marianov et al., (2008) addressed a competitive facility location model in which customers choose the facilities they patronize, by the travelling and waiting times. Zhang et al., (2009) considered congestion situation in the preventive healthcare facility network design. Aboolian et al., (2009) investigated a multiple server location problem that minimizes the maximum time spent at the server sites, including travelling and waiting times. Zarrinpoor and Seifbarghy (2011) proposed a model for congested systems in which a new entering firm desires to obtain a specific percentage of the market share in such a way as to minimize the total costs. More recently, Rahmati et al.,(2013), Hajipour et al., (2014) and Hajipour et al.,(2016) proposed either bi-objective or multi-objective facility location models with different queuing systems and applied some evolutionary algorithms to solve them.

2-3- Contributions of this research

Regarding the literature, we can conclude that there is no research that considered reliability aspects for service facilities which are expected to satisfy heavy and random demands and due to their limited capacity, the system may be congested in some situations. Although the geographical accessibility of facilities is one of the most important factors in the success of the service network design, no research work considered it in the modeling. Moreover, the un-capacitated models have been gained more attention compared with capacitated ones. However, their applications in the practical contexts have limited due to their unrealistic assumption. The approximation or meta-heuristic algorithms are the main solution procedures in the most of presented research papers and exact solution algorithms have been received less attention.

This paper proposes a location-allocation model for service facilities in which both congestion and reliability are considered concurrently. To cope with congested situations, an *M/G/1* queuing system is investigated which assists to model two source of uncertainty associated with demand and service. The geographical accessibility and service quality restrictions are also considered in the model to design an effective and efficient service network. An accelerated Benders decomposition algorithm is presented which seeks to find exact solution of the proposed model.

3-Problem formulation

In this section, we present the notation and formulation of reliable facility location-allocation model for congested systems under disruptions.

3-1- Notation

The sets, parameters, and decision variables used in the proposed model are defined as follows:

Sets

I Set of demand nodes
J Set of candidate locations for facilities
R Set of assignment levels

Parameters

f_j Fixed installation cost to establish facility at candidate location $j \in J$
c_{ij} Transportation cost from demand node $i \in I$ to facility $j \in J$
s_{ij} Unit cost of serving customer residing at demand node $i \in I$ by facility $j \in J$
d_{ij} Shortest distance between demand node $i \in I$ and facility $j \in J$
d_{max} Maximum acceptable distance for demand nodes to access the service of facilities
h_i Demand rate at demand node $i \in I$
λ_j Total arrival rate at facility $j \in J$
μ_j Service rate of facility $j \in J$
σ_j Variation of service time at facility $j \in J$
w_j Customers' expected waiting time at facility $j \in J$
τ_j Maximum acceptable customers' waiting time at facility $j \in J$
Q Failure probability of each facility
p Maximum number of facilities that can be established

Decision variables

y_j 1 if a facility is located at node $j \in J$, 0 otherwise
x_{ijr} Portion of customers residing at demand node $i \in I$ is assigned to facility $j \in J$ at assignment level $r \in R$

3-2- Formulation

The system under study is represented as a network where arcs are the possible paths between nodes and nodes represent either candidate location for facilities or demand concentrations. The service facilities are expected by system designers to remain operational forever. However, the system may become unavailable due to the disruptive events caused by natural disasters or man-made hazards from a time moment to another one (Snyder and Daskin, 2005; An et al., 2014). To cope with such situations, we assume that each facility may fail independently with an identical failure probability q. In the occurrence of disruption, the facility cannot provide any service and its customers will be reassigned to other operational facilities. We assume that each customer can get service from $R \leq |J|$ facilities.For a customer residing at node i, a "level-r" assignment is used when all its assigned facilities at levels $1, \dots, r-1$ have failed and rth facility can provide service for customer.Therefore, the probability that a customer can receive service at level-r assignment is defined as follows:

$$\mathrm{p(serving\ customer\ at\ level - r\ assignment)} = (1-\mathrm{q})\mathrm{q}^{\mathrm{r}-1} \quad (1)$$

Note that in normal disruption-free situation, a customer will be assigned to level-1 facility. If all R assigned facilities to the customer have failed, the customer gives up service and the system will incur a penalty cost. It is worth mentioning that the probability of incurring penalty cost is q^R. To model the congested situations, we assume that each facility behaves as an *M/G/1* queue implying that the requests for the service appear according to a Poisson process and the service time has a general distribution. It should be noted that the Poisson process is a good representation of the arrival rates of real-world service networks in which there is always a variation around scheduled times. It can also be generalized to other service systems where arrival rates and capacity levels vary significantly over time, as pointed out by many researchers such as Nosek and Wilson (2001), Preater (2002), Green (2006), Zhang et al.(2009) and Zhang et al.,(2012). Moreover, in a general probability distribution for service time, no assumption is made as to the precise form of the distribution. Therefore, results in these cases are applicable to any probability distribution, as pointed out by Gross and Hariss (1988).

Considering the *M/G/1*queuing system, the demand generation rate at each demand node *i*is the Poisson process with average demand rate h_i, thus the demand rate at facility *j* can be written as:

$$\lambda_j = \sum_i \sum_{r\in R} h_i x_{ijr}. \tag{2}$$

Since $\lambda_j$is a linear combination of Poisson processes, it is itself a Poisson process.

According to Gross and Hariss (1988), the customers' expected waiting times at facility$j$ is the following

$$w_j = \left(\frac{1 + CV_j^2}{2}\right)\frac{\rho_j}{\mu_j - \lambda_j} + \frac{1}{\mu_j}. \tag{3}$$

where $\rho_j = \lambda_j/\mu_j$ and $CV_j = \sigma_j\mu_j$. With respect to the definitions of ρ_j and CV_j, we can rewrite the above equation as follows:

$$w_j = \left(\frac{1 + \sigma_j^2\mu_j^2}{2}\right)\frac{\lambda_j}{\mu_j(\mu_j - \lambda_j)} + \frac{1}{\mu_j}. \tag{4}$$

To maintain congestion within acceptable limit, a capacity constraint must be considered in the model. The common approach used to consider the capacity constraints in facility location problem is to force the demand for service at each facility not to exceed the maximum allowable capacity. Such a deterministic capacity constraint is not able to reflect the dynamic nature of congestion in which the number of requests for service is not constant in time and behaves some probabilistic distributions. To deal with congestion, the expected number of requests for service or waiting time can be restricted to be less than some small values (Marianov and Serra, 1998; Marianov and Serra, 2001). Moreover, the quality of service network can be expressed in terms of expected or worst case waiting times or queue length for a specific service (Zhang et al., 2009; Zhang et al., 2012). Therefore, the service quality for facility$j$ is enforced throughrestriction on the level of congestion to ensure that customers' expected waiting times in system do not exceed maximum acceptable level as the following:

$$w_j \le \tau_j y_j. \tag{5}$$

By considering Eq. (4), the above equation can be stated as follows:

$$(1 + \sigma_j^2\mu_j^2)\lambda_j + 2\mu_j(\mu_j - \lambda_j) \le 2\tau_j\mu_j(\mu_j - \lambda_j)y_j \tag{6}$$

The above equation can be rewritten in a simpler form as follows:

$$[(1 + \sigma_j^2\mu_j^2) - 2\mu_j + 2\tau_j\mu_j y_j]\lambda_j \le 2\mu_j^2(\tau_j - 1)y_j \tag{7}$$

As previously mentioned, geographical accessibility is one of the key elements for the success of service networks that can significantly affect the transportation network design. We consider it in the model in terms of the proximity of a facility to the potential demand nodes which specifies customer i should not take more than amaximum acceptable travel distance to access service of facility$j$. Therefore, we have

$$x_{ijr} = 0, \qquad \forall\, j \in \{j | d_{ij} > d_{max}\}. \tag{8}$$

For this service network, we are seeking a model that selects the location of facilities and allocates customers to the constructed facilities with consideration of queuing system and geographical accessibility in such a way as to minimize the fixed installation cost as well as the expected operational costs. By considering the aforementioned assumptions and queuing system representation, the formulation of reliable facility location-allocation model under disruptions can be stated as follows:

$$\text{Min} \sum_{j\in J} f_j y_j + \sum_{i\in I}\sum_{j\in J}\sum_{r\in R} h_i\, c_{ij}(1-q)q^{r-1}x_{ijr} + \sum_{i\in I}\sum_{j\in J}\sum_{r\in R} h_i\, s_{ij}(1-q)q^{r-1}x_{ijr} + \sum_{i\in I} \Pi_i h_i\, q^R \quad (9)$$

s.t.

$$\sum_{j\in J} x_{ijr} = 1\,, \forall i \in I, r \in R, \quad (10)$$

$$\sum_{r\in R} x_{ijr} \le 1\,, \forall i \in I, j \in J, \quad (11)$$

$x_{ijr} \le y_j, \forall i \in I, j \in J, r \in R,$ (12)

$$\sum_{j\in J} y_j \le p \quad (13)$$

$$\left[\left(1+\sigma_j^2\mu_j^2\right) - 2\mu_j + 2\tau_j\mu_j y_j\right]\sum_{i\in I}\sum_{r\in R} h_i\, x_{ijr} \le 2\mu_j^2\left(\tau_j - 1\right)y_j, \forall j \in J, \quad (14)$$

$$x_{ijr} = 0, \qquad \forall i \in I, j \in \left\{j \middle| d_{ij} > d_{max}\right\}, r \in R, \quad (15)$$

$$y_j = 0, 1, \forall j \in J, \quad (16)$$

$$x_{ijr} \ge 0, \forall i \in I, j \in J, r \in R. \quad (17)$$

The Objective function (9) minimizes fixed installation cost, expected traveling cost from customers to the facilities, expected serving cost of customers by constructed facilities and expected penalty cost of unsatisfied demands. Constraint (10) insures that each customer is assigned to exactly one facility at each assignment level. Constraint (11) states no customer can be assigned to the same facility at two or more assignment levels. Constraint (12) specifies that a customer only assigned to the open facilities. Constraint (13) specifies maximum number of facilities that can be established. Constraint (14) ensures the service quality by considering maximum limit reflected in customers' expected waiting time at each service facility. Constraint (15) ensures that customers should not take more than a maximum acceptable distance to access service at facilities. Constraint (16) and (17) enforce the binary and non-negativity restrictions on the corresponding decision variables.

3-3- Linearization of the proposed model

Due to the multiplication of binary and integer variables in constraint (14), the proposed model is non-linear. By considering $\Omega_{ijr} = y_j x_{ijr}$ and M as a reasonably large number, the non-linear constraint can be replaced by

$$\left[\left(1+\sigma_j^2\mu_j^2\right) - 2\mu_j\right]\sum_{i\in I}\sum_{r\in R} h_i\, x_{ijr} + 2\tau_j\mu_j \sum_{i\in I}\sum_{r\in R} h_i\, \Omega_{ijr} \le 2\mu_j^2\left(\tau_j - 1\right)y_j, \forall j \in J, \quad (18)$$

$$\Omega_{ijr} \le x_{ijr}, \forall i \in I, j \in J, r \in R, \quad (19)$$

$$\Omega_{ijr} \le My_j, \forall i \in I, j \in J, r \in R, \quad (20)$$

$$\Omega_{ijr} \le x_{ijr} - M\left(1 - y_j\right), \forall i \in I, j \in J, r \in R, \quad (21)$$

$$\Omega_{ijr} \ge 0, \forall i \in I, j \in J, r \in R. \quad (22)$$

4- Solution procedure

The linearized proposed problem is a mixed integer programming (MIP) and it can be solved by current state-of-the-art MIP solvers. However, applying such solvers in solving large-scale instances requires significant computational time. The structure of proposed problem is a natural candidate to be solved by a well-known partitioning method, Benders decomposition (Benders, 1962). The successful application of this algorithm has been reported in different contexts such as supply chain network design (Pishvaee et al., 2014), scheduling

(Verstichel et al., 2015), hub location problem (Camargo et al., 2008) and facility location (Vatsa and Jayaswal, 2016).

4-1- Benders decomposition algorithm

To implement Benders' decomposition procedure, the dual sub-problem (DSP) and master problem (MP) should be formulated. Let fix the binary variable to the given value$\{\hat{y}_j = y_j\}$. Then, the Benders primal sub-problem (PSP) can be formulated as follows:

$$\text{Min PSP} = \sum_{i\in I}\sum_{j\in J}\sum_{r\in R} h_i\, c_{ij}(1-q)q^{r-1}x_{ijr} + \sum_{i\in I}\sum_{j\in J}\sum_{r\in R} h_i\, s_{ij}(1-q)q^{r-1}x_{ijr} \quad (23)$$

$\text{s.t.}\,(10),(11),(15),(17),(19),(22)$

$$x_{ijr} \le \hat{y}_j \forall i \in I, j \in J, r \in R \quad (24)$$

$$\left[\left(1+\sigma_j^2\mu_j^2\right) - 2\mu_j\right]\sum_{i\in I}\sum_{r\in R} h_i\, x_{ijr} + 2\tau_j\mu_j \sum_{i\in I}\sum_{r\in R} h_i\,\Omega_{ijr} \le 2\mu_j^2(\tau_j - 1)\hat{y}_j \forall j = 1,\dots,J \quad (25)$$

$$\Omega_{ijr} \le M\hat{y}_j, \qquad \forall i,j,r, \quad (26)$$

$$\Omega_{ijr} \le x_{ijr} - M(1-\hat{y}_j), \qquad \forall i,j,r, \quad (27)$$

Let $\zeta_{ir}, \varphi_{ij}, \nu_{ijr}, \xi_{ijr}, \varrho_{ijr}, \phi_{ijr}, \varsigma_{ijr}, \vartheta_j, \psi_{ijr}$and ϖ_{ijr}be the vector of dual variables of the constraints (10), (11), (15), (17), (19), (22) and (24)-(27). The DSP can be stated as follows:

$$\text{Max DSP} = \sum_{i\in I}\sum_{r\in R}\zeta_{ir} - \sum_{i\in I}\sum_{j\in J}\varphi_{ij} - \sum_{i\in I}\sum_{j\in J}\sum_{r\in R}\varsigma_{ijr}\hat{y}_j - 2\sum_{j\in J}\mu_j^2(\tau_j-1)\hat{y}_j\vartheta_j - M\sum_{i\in I}\sum_{j\in J}\sum_{r\in R}\psi_{ijr}\hat{y}_j + M\sum_{i\in I}\sum_{j\in J}\sum_{r\in R}(1-\hat{y}_j)\varpi_{ijr} \quad (28)$$

s.t.

$$\zeta_{ir} - \varphi_{ij} + \nu_{ijr} + \xi_{ijr} + \varrho_{ijr} - \varsigma_{ijr} - \left[\left(1+\sigma_j^2\mu_j^2\right) - 2\mu_j\right]h_i\vartheta_j + \varpi_{ijr} \le h_i c_{ij}(1-q)q^{r-1} + h_i s_{ij}(1-q)q^{r-1}, \qquad \forall i \in I, j \in \{j|d_{ij} > d_{max}\}, r \in R \quad (29)$$

$$\zeta_{ir} - \varphi_{ij} + \xi_{ijr} + \varrho_{ijr} - \varsigma_{ijr} - \left[\left(1+\sigma_j^2\mu_j^2\right) - 2\mu_j\right]h_i\vartheta_j + \varpi_{ijr} \le h_i c_{ij}(1-q)q^{r-1} + h_i s_{ij}(1-q)q^{r-1}, \qquad \forall i \in I, j \notin \{j|d_{ij} > d_{max}\}, r \in R \quad (30)$$

$$-\varrho_{ijr} + \phi_{ijr} - 2\tau_j\mu_j h_i\vartheta_j - \psi_{ijr} - \varpi_{ijr} \le 0, \forall i \in I, j \in J, r \in R, \quad (31)$$

$$\varphi_{ij}, \xi_{ijr}, \varrho_{ijr}, \phi_{ijr}, \varsigma_{ijr}, \vartheta_j, \psi_{ijr}, \varpi_{ijr} \le 0, \forall i \in I, j \in J, r \in R, \quad (32)$$

$$\zeta_{ir}, \nu_{ijr} \text{ free}, \forall i \in I, j \in J, r \in R \quad (33)$$

According to the solution of DSP, the MP can be written as follows:

$$\text{Min MP} = \sum_{j\in J} f_j y_j + \sum_{i\in I}\Pi_i h_i\, q^R + \eta \quad (34)$$

$\text{s.t.}\,(13),(16)$

$$\eta \geq \sum_{i\in I}\sum_{r\in R}\zeta_{ir}^{\ell} - \sum_{i\in I}\sum_{j\in J}\varphi_{ij}^{\ell} - \sum_{i\in I}\sum_{j\in J}\sum_{r\in R}\varsigma_{ijr}^{\ell}\,\hat{y}_j - 2\sum_{j\in J}\mu_j^2(\tau_j - 1)\hat{y}_j\vartheta_j^{\ell} - M\sum_{i\in I}\sum_{j\in J}\sum_{r\in R}\psi_{ijr}^{\ell}\,\hat{y}_j + M\sum_{i\in I}\sum_{j\in J}\sum_{r\in R}(1-\hat{y}_j)\,\varpi_{ijr}^{\ell}, \qquad \forall \ell \in \mathbb{L} \tag{35}$$

$$\eta \geq 0 \tag{36}$$

It should be noted that MP produces an upper bound for the primary problem for each iteration. Constraint (35) represents the optimality cut, $(\zeta_{ir}^{\ell}, \varphi_{ij}^{\ell}, \nu_{ijr}^{\ell}, \xi_{ijr}^{\ell}, \varrho_{ijr}^{\ell}, \phi_{ijr}^{\ell}, \varsigma_{ijr}^{\ell}, \vartheta_j^{\ell}, \psi_{ijr}^{\ell}, \varpi_{ijr}^{\ell})$ indicates the extreme point of dual polyhedron obtained by solving the DSP and $\mathbb{L}$ is the set of extreme points. The DSP is always feasible since the unmet demand is penalized. Therefore, the feasibility cut is not considered in the MP. The MP and DSP are solved iteratively by using the solution of one in the other, until the stopping criterion is met and an optimal solution can be obtained. We employ a small percentage gap, ε, between the best upper (UB) and lower(LB) bounds as a stopping criterion. Let *t* be the iteration index. The scheme of Benders decomposition algorithm is presented in Algorithm1.

Algorithm 1. Benders' decomposition algorithm

$UB^n \leftarrow +\infty$, $LB^n \leftarrow -\infty$,$t \leftarrow 1$, $\mathbb{L} \leftarrow 0$.
terminate←**false**
while (terminate=**false**)**do**
Solve MP considering the following cutting plane

$$\eta \geq \sum_{i\in I}\sum_{r\in R}\zeta_{ir}^{\ell} - \sum_{i\in I}\sum_{j\in J}\varphi_{ij}^{\ell} - \sum_{i\in I}\sum_{j\in J}\sum_{r\in R}\varsigma_{ijr}^{\ell}\,\hat{y}_j - 2\sum_{j\in J}\mu_j^2(\tau_j - 1)\hat{y}_j\vartheta_j^{\ell} - M\sum_{i\in I}\sum_{j\in J}\sum_{r\in R}\psi_{ijr}^{\ell}\,\hat{y}_j + M\sum_{i\in I}\sum_{j\in J}\sum_{r\in R}(1-\hat{y}_j)\,\varpi_{ijr}^{\ell}, \qquad \forall l = 0,1,\dots,t-1,$$

and derive its optimal solution$\{y_j^t\}_{j\in J}$
if$\left(\left(\sum_{j\in J} f_j y_j^t + \sum_{i\in I}\Pi_i h_i\, q^R + \eta^t\right) > LB^n\right)$**then**
Update $LB^n \leftarrow \left(\sum_{j\in J} f_j y_j^t + \sum_{i\in I}\Pi_i h_i\, q^R + \eta^t\right)$
end if
For fixed $\{y_j^t\}_{j\in J}$, solve DSP to obtain $(\zeta_{ir}^{t}, \varphi_{ij}^{t}, \nu_{ijr}^{t}, \xi_{ijr}^{t}, \varrho_{ijr}^{t}, \phi_{ijr}^{t}, \varsigma_{ijr}^{t}, \vartheta_j^{t}, \psi_{ijr}^{t}, \varpi_{ijr}^{t}) \in \mathbb{L}$ and its optimal value $\mathcal{L}^t$
if$\left(\left(\sum_{j\in J} f_j y_j^t + \sum_{i\in I}\Pi_i h_i\, q^R + \mathcal{L}^t\right) < UB^n\right)$**then**

$$UB^n \leftarrow \left(\sum_{j\in J} f_j y_j^t + \sum_{i\in I}\Pi_i h_i\, q^R + \mathcal{L}^t\right)$$

end if
if$UB^n = LB^n (UB^n - LB^n \leq \varepsilon)$**then**
terminate←**true**
else

$$\mathbb{L}^{n+1} = \mathbb{L}^n \cup \{(\zeta_{ir}^{t}, \varphi_{ij}^{t}, \nu_{ijr}^{t}, \xi_{ijr}^{t}, \varrho_{ijr}^{t}, \phi_{ijr}^{t}, \varsigma_{ijr}^{t}, \vartheta_j^{t}, \psi_{ijr}^{t}, \varpi_{ijr}^{t})\}$$

end if

$$t \leftarrow t + 1$$

end while

4-2- Accelerating methods

The standard Benders' decomposition algorithm may fail to converge within a reasonable computational time for many large optimization models (Rei et al., 2009; Pishvaee et al., 2014). This motivated us to explore accelerating methods to improve the convergence of the Benders decomposition algorithm.

4-2-1- Valid inequalities

Cordeau et al., (2006) pointed out that appending valid inequalities into the MP can help to find solutions that are close to the optimal. We drive the following valid inequalities based on the structure of the proposed model as follows:

$$\sum_{j} \mu_j y_j \geq \sum_{i} h_i, \tag{37}$$

$$\sum_{j} y_j \geq 1. \tag{38}$$

Constraint (37) insures that constructed facilities have sufficient capacity to serve the whole demand. Constraint (38) forces the selection of at least one facility to be open.

4-2-2-Knapsack inequalities

Adding knapsack inequalities along with optimality cut will result in a good quality solution from the MP, as declared by Santoso et al., (2005). They pointed out that state-of-the-art solvers such as CPLEX can derive a variety of valid inequalities from the knapsack inequality which expedites the convergence of Benders decomposition algorithm. Let UB^lbe the current best known upper bound, we have

$$UB^l \geq \sum_{j \in J} f_j y_j + \sum_{i \in I} \Pi_i h_i \, q^R + \eta \tag{39}$$

The following knapsack inequality can be added in iteration $l+1$:

$$UB^l - \sum_{i \in I} \sum_{r \in R} \zeta_{ir}^{\ell} + \sum_{i \in I} \sum_{j \in J} \varphi_{ij}^{\ell} \geq \sum_{j \in J} f_j y_j + \sum_{i \in I} \Pi_i h_i \, q^R - \sum_{i \in I} \sum_{j \in J} \sum_{r \in R} \varsigma_{ijr}^{\ell} y_j - 2 \sum_{j \in J} \mu_j^2 (\tau_j - 1) y_j \vartheta_j^{\ell} - M \sum_{i \in I} \sum_{j \in J} \sum_{r \in R} \psi_{ijr}^{\ell} y_j + M \sum_{i \in I} \sum_{j \in J} \sum_{r \in R} (1 - y_j) \varpi_{ijr}^{\ell} \quad (40)$$

The scheme of accelerated Benders decomposition algorithm is presented in Algorithm 2.

Algorithm 2. Accelerated Benders' decomposition algorithm

$\mathrm{UB^n} \leftarrow +\infty$, $\mathrm{LB^n} \leftarrow -\infty$,t $\leftarrow 1$, $\mathbb{L} \leftarrow 0$, $\{y_j^0\}_{j\in J}$
terminate←**false**
while (terminate=**false**)**do**
  Use $\{y_j^0\}_{j\in J}$ to solve DSP and to obtain

$$\left(\zeta_{ir}^t, \varphi_{ij}^t, \nu_{ijr}^t, \xi_{ijr}^t, \varrho_{ijr}^t, \phi_{ijr}^t, \varsigma_{ijr}^t, \vartheta_j^t, \psi_{ijr}^t, \varpi_{ijr}^t\right) \in \mathbb{L}$$

$$\mathbb{L}^{n+1} = \mathbb{L}^n \cup \left\{\left(\zeta_{ir}^t, \varphi_{ij}^t, \nu_{ijr}^t, \xi_{ijr}^t, \varrho_{ijr}^t, \phi_{ijr}^t, \varsigma_{ijr}^t, \vartheta_j^t, \psi_{ijr}^t, \varpi_{ijr}^t\right)\right\}$$

  Add (37), (38) and (40) to MP. Solve MP to obtain $\{y_j^t\}_{j\in J}$
  if$\left(\left(\sum_{j\in J} f_j y_j^t + \sum_{i\in I} \Pi_i h_i\, q^R + \eta^t\right) > \mathrm{LB^n}\right)$**then**
  Update $\mathrm{LB^n} \leftarrow \left(\sum_{j\in J} f_j y_j^t + \sum_{i\in I} \Pi_i h_i\, q^R + \eta^t\right)$
  end if
For fixed $\{y_j^t\}_{j\in J}$, solve DSP to obtain $\left(\zeta_{ir}^t, \varphi_{ij}^t, \nu_{ijr}^t, \xi_{ijr}^t, \varrho_{ijr}^t, \phi_{ijr}^t, \varsigma_{ijr}^t, \vartheta_j^t, \psi_{ijr}^t, \varpi_{ijr}^t\right) \in \mathbb{L}$ and its optimal value $\mathcal{L}^t$
  if$\left(\left(\sum_{j\in J} f_j y_j^t + \sum_{i\in I} \Pi_i h_i\, q^R + \mathcal{L}^t\right) < \mathrm{UB^n}\right)$**then**

$$\mathrm{UB^n} \leftarrow \left(\sum_{j\in J} f_j y_j^t + \sum_{i\in I} \Pi_i h_i\, q^R + \mathcal{L}^t\right)$$

end if
  if$\mathrm{UB^n} = \mathrm{LB^n} (\mathrm{UB^n} - \mathrm{LB^n} \le \varepsilon)$**then**
terminate←**true**
  end if

$$t \leftarrow t + 1$$

end while

5- Computational study

In this section, we present some generated instances to examine the performance of the proposed model and solution approaches. The solution procedures are coded in GAMS23.4 optimization software and all the experiments are performed on an INTEL Core 2 CPU with 2.4 GHz processor and 2 GB of RAM. Note that the termination criteria, ε, is considered 10^{-3} for solution procedures.We consider the randomly generated, 30-node to 100-node network with a symmetric traveling distance matrix in which the demands are randomly generated at each node. It should be noted that the number of nodes indicates the sum of nodes for candidate facilities' locations and demand concentrations. The data ranges and obtained results for these generated instances are presented in Tables 1 and 2.In Table 2, BDA and ABDA determine Benders decomposition algorithm and accelerated Benders decomposition algorithm, respectively.

Table 1. Data used in the generated instances

Parameters	Values
f_j	U[800,2000]
c_{ij}	U[40,150]
w_{ij}	U[10,100]
Π_i	$\max_i\{c_{ij}\}$
q	U[0,1)
h_i	U[1,10]
μ_j	U[60,100]
σ_j	U[0,10]
τ_j	U[10,40]
d_{ij}	U[0,100]
$d_{\max}$	50

Table 2. Computational results for the generated instances

Problem	$\|J\|$	$\|I\|$	p	q	Total cost	BDA		ABDA		CPLEX
						Time (s)	iteration	Time (s)	Iteration	Time (s)
1	20	10	8	0.1	10314.41	1.18	7	0.89	4	0.38
2				0.3	11795.64	2.41	7	3.62	5	1.54
3				0.5	13530.23	6.45	8	6.71	5	5.21
4		12		0.1	12858.01	2.17	7	2.41	5	1.34
5				0.3	14652.43	4.91	6	5.21	5	4.54
6				0.5	16943.87	11.48	7	12.34	7	10.71
7		15		0.1	13967.09	4.12	15	5.23	11	3.92
8				0.3	15032.17	12.71	17	12.62	10	9.87
9				0.5	18046.35	18.94	18	19.23	9	17.02
10	30	18	12	0.1	17879.74	58.83	34	25.04	16	53.65
11				0.3	19280.33	118.14	36	32.61	21	100.87
12				0.5	22765.85	98.84	23	21.31	10	93.32
13		20		0.1	19876.43	167.78	41	43.75	28	151.26
14				0.3	23540.51	314.92	34	87.10	19	298.25
15				0.5	24765.90	387.64	27	94.62	15	342.68
16		25		0.1	24043.63	451.41	30	101.17	18	432.34
17				0.3	26876.15	291.67	25	96.55	10	289.20
18				0.5	28654.42	389.96	33	114.63	27	378.13
19	40	28	16	0.1	27657.76	652.15	41	158.23	33	634.01
20				0.3	29654.43	901.36	35	191.45	21	861.29
21				0.5	32234.54	1592.96	27	207.21	15	1532.83
22		33		0.1	29411.13	1551.52	37	194.81	12	1521.42
23				0.3	32002.21	1713.76	32	232.21	19	1693.17
24				0.5	35415.65	1671.42	34	254.77	31	1621.42
25		37		0.1	31211.53	1796.15	39	323.19	27	1759.55
26				0.3	34213.82	1737.11	26	311.65	19	1707.53
27				0.5	37480.08	1708.29	27	305.42	16	1792.81
28	50	40	20	0.1	32754.32	1859.79	29	524.32	25	-
29				0.3	35431.45	1930.18	39	597.55	28	-
30				0.5	38710.61	2075.34	32	621.30	15	-
31		45		0.1	34754.15	1900.19	27	543.13	12	-
32				0.3	37012.40	1939.76	34	574.97	18	-
33				0.5	39447.96	2234.09	38	683.14	24	-
34		50		0.1	36852.23	2401.66	66	928.58	37	-
35				0.3	39431.04	2293.57	37	757.39	23	-
36				0.5	42321.77	2276.28	24	742.15	19	-

Table 2 shows that the total system cost increases as the potential demand for service network increases. For example, the system in 100-node network (e.g.$|J| = 50\ and |I| = 50$) will incur 69.95% much more cost than the one in 30-node network (e.g.$|J| = 20\ and |I| = 15$). Note that this value is obtained by taking the average of total cost over different q values. Moreover, the system cost significantly increases as q grows. We can see that the system cost under $q = 0.1$ is 23.77% greater than the one under $q = 0.5$ for 30-node network. However, the difference between cost values under different failure probability becomes considerably conspicuous as the number of network nodes increases. The results also confirm the benefit of applying several accelerating methods in the proposed solution procedure such that the accelerated Benders decomposition algorithm becomes significantly more time efficient compared to CPLEX and Benders' decomposition algorithm. On average, the accelerated Benders decomposition algorithm is 3.91 times faster than Benders decomposition algorithm. Regarding the instances that can be solved by CPLEX, we can conclude that the accelerated benders decomposition algorithm is 5.35 times faster than CPLEX. The convergence plot of the accelerated Benders decomposition algorithm is presented in Figure 1, when $|J| = 25, |I| = 15, p = 10$ and $q = 0.3$.

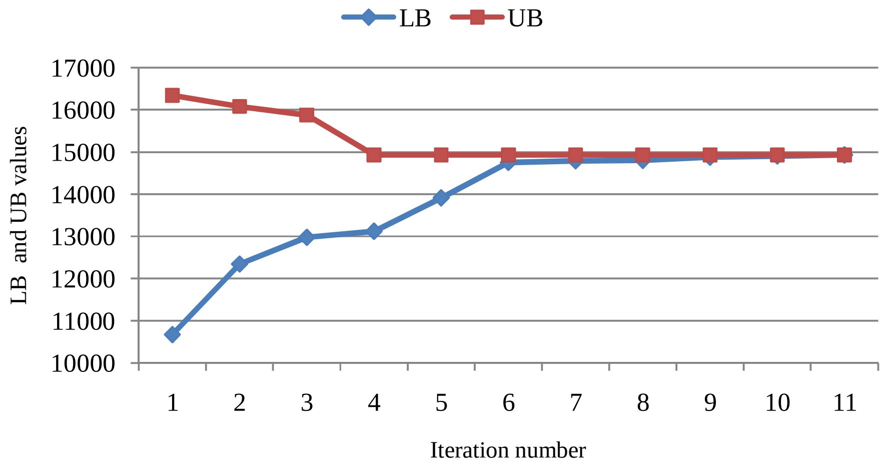

Figure 1. The convergence plot of accelerated Benders decomposition algorithm

To focus on the impact of failure probability on the system performance, we consider a 40-node network with $|J| = 25, |I| = 15$ and $p = 10$. Although some q values here may be unrealistic, they can help reveal trends and model behavior. Note that the case $q = 0$ corresponds to the deterministic facility location problem for congested system without the risk of disruptions consideration, and $q = 1$ corresponds to the case where all facilities fail. Table 3 illustrates how the locations and cost components change as failure probability increases. We see that as q increases, the total number of constructed facilities somehow increases to hedge against the risk of facility disruptions. We also present the percentage of cost components as the functions of failure probability in Figure 2.

Table 3. Results under different failure probability for 40-node network

q	Location	Total cost	Cost components(%)			
			Fixed installation	Traveling	Serving	Penalty
0	2,9,13,18	12171.85	33.5	41.28	25.22	0
0.1	2,9,13,18	13854.61	29.43	42.86	26.46	1.25
0.3	2,9,16,18,23	14928.32	34.83	36.97	20.88	7.32
0.5	2,9,16,18,21,23	18124.54	35.86	32.39	17.24	14.51
0.7	2,5,9,16,18,21,23	21034.17	37.55	26.57	15.05	20.83
0.9	2,3,5,16,18,21,23	24276.92	32.87	23.63	14.31	29.19

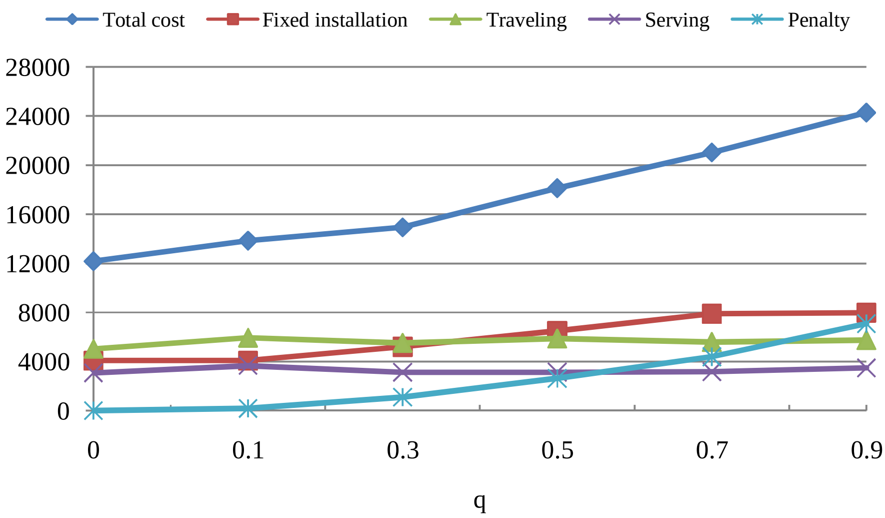

Figure 2. Impact of q on the cost components for 40-node network

Figure 3 shows the effect of geographical accessibility on the system costs for a 40-node network with $|J| = 25, |I| = 15$ and $p = 10$. When maximum acceptable distance increases, it is expected to install more facilities closer to the demand nodes. However, the fixed installation cost has a relatively steady trend due to the consideration of the maximum number of opened facilities in the model. The constructed facilities can serve demand nodes in the much further locations, and thus, traveling and serving costs grow and penalty cost drops significantly, leading to reduction in the total cost. Therefore, to design an efficient service network, geographical accessibility improvement, maximum number of constructed facilities and cost minimization must be considered concurrently.

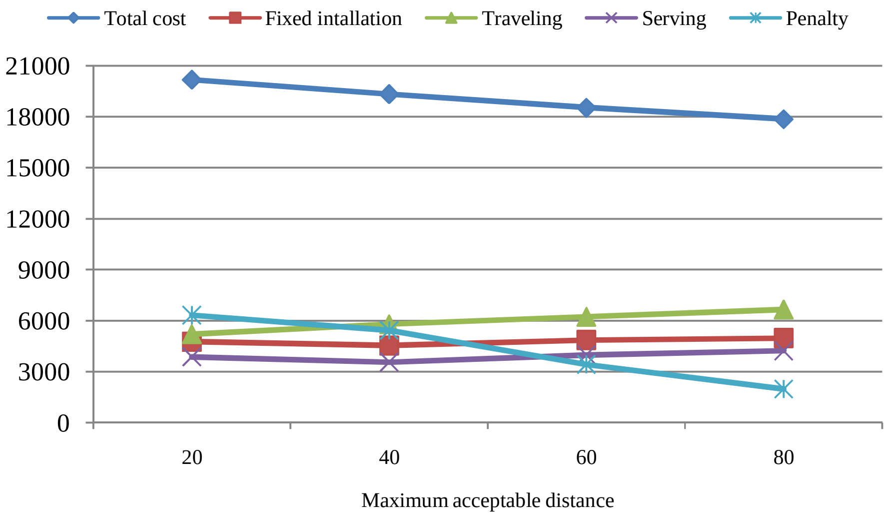

Figure 3. Impact of geographical accessibility on the system cost

The effect of μ_j on the system costs under different failure probabilities is presented in Figure4. To study more precisely how the failure probability affects the system cost, the values of μ_j are considered to be the same for all the candidate locations for facilities.We consider a 40-node network with $|J| = 25, |I| = 15$ and $p = 10$. The cost components is illustrated in Figure 5for the case $q = 0.3$. As it can be seen in Figure 4, the penalty cost decreases significantly despite slight increase in investment cost associated with increasing mean service times, and as a result the total cost decreases. As μ_j increases, the existing facilities can provide service for more potential demand and therefore the serving cost has an increasing trend. The traveling cost is relatively insensitive to the changes of q. Note that when the mean service times exceed 90, the total cost will increase due to the increase in investment cost. Since the mean service time of each facility can be fully controlled by system designer, it is critical to determine the appropriate value of capacity investment for hedging against the risk of disruptions.

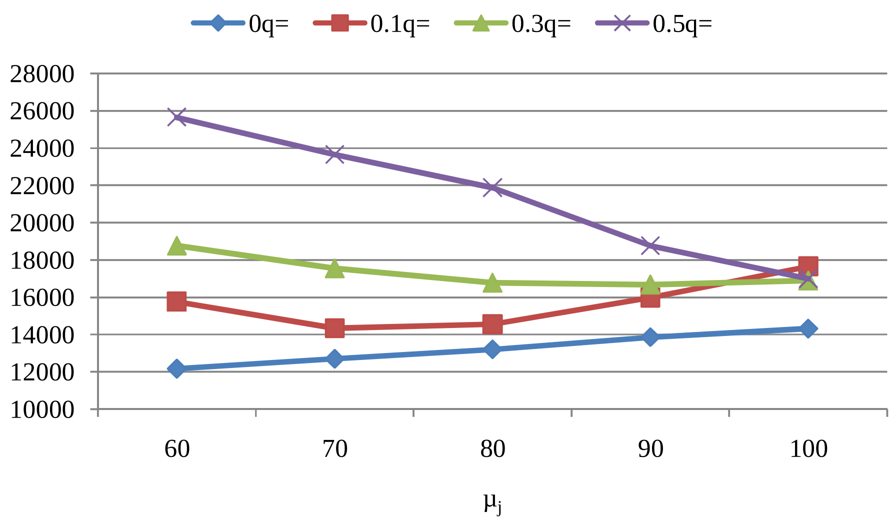

Figure 4. Impact of μ_j on the system cost

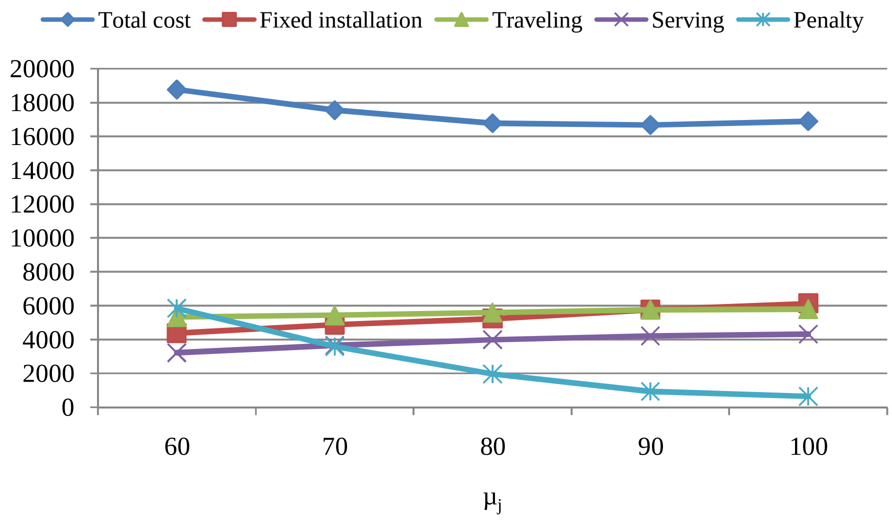

Figure 5. Impact of μ_j on the cost components for $q = 0.3$

6- Conclusions

This paper proposes a reliable location-allocation model to optimize facility location and customer allocations where facilities are subject to the risk of disruptions. Since service facilities are expected to meet random and heavy demands, the congested situations are modeled in service facilities within an *M/G/1* queuing system. In order to insure the service quality, a minimum limit reflected in the customers' expected waiting time is considered in the model. The geographical accessibility of the service network is also considered in terms of the proximity of a facility to the potential demands. The model determines the minimization of the fixed installation cost as well as expected traveling, serving and penalty costs. A Benders' decomposition algorithm enhanced by two efficient accelerating methods including valid inequalities and knapsack inequalities is proposed to solve the model. Based on the results, we can conclude that (i) by considering reliability in the location-allocation model, the total number of constructed facilities somehow increases to hedge against the risk of facility disruptions; (ii) the system costs increases as the failure probability increases. Therefore, system designers should put more weight on the worst disruptive situations to obtain more reliable service networks; (iii) since the mean service time plays a significant role in the system cost and performance, it is crucial to determine its appropriate value for hedging against the risk of disruption; (iv)to improve the geographical accessibility, more facilities must be constructed. However, the system cost may increase due to the significant increase in the fixed installation cost. Therefore, to design an efficient service network, geographical accessibility improvement and cost minimization must be considered concurrently; and (vi) the accelerated Benders decomposition algorithm is significantly more time efficient compared to CPLEX and standard Benders decomposition algorithm.

This paper can be extended in several directions. The system disruptions are characterized by the independent and homogenous failure probability in this paper. It would be interesting to model failure probability with heterogeneous and correlated patterns. Considering other queuing systems like *M/M/1* and *M/M/c* will be another direction for future research. The proposed model can also be extended in the situations in which users have incomplete information about the operational status of service facilities.

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