Comparing the performance of GARCH \((p,q)\) models with different methods of estimation for forecasting crude oil market volatility

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Abstract

In recent years, empirical literature used widely GARCH models to characterize crude oil price volatility. Because this augmenting attention, six univariate GARCH models and two methods of estimation the parameters for forecasting oil price volatility are examined in this paper. Based on obtained results, the best method for forecasting crude oil price volatility of Brent market is determined. All the examined models in this paper belong to the univariate time series family. The four years out-of-sample volatility forecasts of the GARCH models are evaluated using the superior predictive ability test with more loss functions. The results show that GARCH \((1,1)\) can outperform all of the other models for the crude oil price of Brent market across different loss functions. Four different measures are used to evaluate the forecasting accuracy of the models. Also two methods of estimation the parameters of GARCH models are compared for forecasting oil price volatility. The results suggest that in our study, maximum likelihood estimation (MLE) gives better results for estimation than generalized method of moments (GMM).

Keywords: GARCH \((p,q)\), volatility, efficiency, crude oil price, Brent market

1- Introduction

We should constantly adapt to the changes of the market. This requires gathering market variables in order to set the strategic direction for business and management. As increasing the volatility, the risk of the market variables increases. It is important to monitor volatilities of the market variables such as the price. Volatility is a measure for variation of the price of crude oil over time. History of volatility is derived from time series of past prices.

Volatility is defined as the up-and-down movement of the market. It's usually measured by the standard deviation from the expectation. Since the beginning of the current debt crisis, the word ‘volatility’ has become even more prevalent in the lexicon of financial services professionals, the media and the public.

Oil is the world economy’s most important source of energy and is therefore critical to our economy. The price of crude oil plays an important role in the world economy. Recent fluctuations in oil prices volatility have caused great concern among governments (Wei et al. 2010). Thus, accurate modeling and forecasting of crude oil volatility have a considerable interest to energy researchers and policy makers (Kang et al. 2009).

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As one of the most important econometric papers, Engle (Breusch et al. 1982) introduced autoregressive conditional heteroskedasticity (ARCH) which is a powerful tool of describing the history conditional volatilities. As an extension, (Bollerslev 1986) proposed a generalized autoregressive conditional heteroskedasticity (GARCH) model which was more widely employed. A large number of models were proposed by various researchers which could be regarded as variants of GARCH; such as integrated GARCH (IGARCH), exponential GARCH (EGARCH) in (Nelson 1991), asymmetric power GARCH (APARCH) in (Ding et al. 1993) and fractionally integrated GARCH (FIGARCH) in (Bailie et al. 1996) and (Wang et al. 2011).


Multivariate GARCH models have another estimation models such as Vec and BEKK (Baba, Engle, Kraft and Kroner) and DCC (Dynamic Conditional Correlation model).

Huge researches has been done to evaluate the forecasting performance of different volatility models, especially GARCH-class ones, regarding oil markets (Wei et al. 2010). Table 1 illustrates some of the researches in this topic.

Table 1. Researchers that evaluate the forecasting performance of different volatility GARCH models, regarding oil markets

<table>
<thead>
<tr>
<th>Model</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modeling energy price dynamics: GARCH versus stochastic volatility</td>
<td>Modeling energy price dynamics: GARCH versus stochastic volatility(Chan &amp; Grant 2016)</td>
</tr>
<tr>
<td>Forecasting carbon futures volatility using GARCH models with energy volatilities (Joon &amp; Cho 2013)</td>
<td>Forecasting carbon futures volatility using GARCH models with energy volatilities (Joon &amp; Cho 2013)</td>
</tr>
<tr>
<td>A nonparametric GARCH model of crude oil price return volatility (Hou &amp; Suardi 2012)</td>
<td>A nonparametric GARCH model of crude oil price return volatility (Hou &amp; Suardi 2012)</td>
</tr>
<tr>
<td>International evidence on crude oil price dynamics: Applications of ARIMA-GARCH models (Mohammadi &amp; Su 2010)</td>
<td>International evidence on crude oil price dynamics: Applications of ARIMA-GARCH models (Mohammadi &amp; Su 2010)</td>
</tr>
</tbody>
</table>

Volatility in crude oil futures: A comparison of the predictive ability of GARCH and implied volatility models (Agnolucci 2009)

Forecasting volatility of crude oil markets (Kang et al. 2009)

Estimating the commodity market price of risk for energy prices (Alberg et al. 2008)

Modeling oil price volatility (Kumar & Narayan 2007)

Energy risk management and value at risk modeling (A & Shavvalpour 2006)

Modeling and forecasting petroleum futures volatility (Sadorsky 2006)

Estimating oil price ‘Value at Risk’ using the historical simulation approach (Cabedo & Moya 2003)

Alaska North Slope crude oil price and the behavior of diesel prices in California (Adrangi et al. 2001)

As explained above, there are several studies about forecasting volatility that are improved over the years, and also there are some approaches for estimating the parameters of models. Also some approaches to evaluate the forecasting performance of different volatility GARCH models, regarding oil markets, are developed separately. But it seems that there isn’t any paper to compare the performance of GARCH (p,q) models with different methods of estimation the parameters for forecasting crude oil market volatility with some new functions.

In this paper, we use wide range sample size of Brent crude oil price in different GARCH (p,q) models with two different forecasting methods (MLE & GMM), and evaluate the performance of the models with the RMSE, MAE, Theil-U and Linex loss functions.

The rest of the paper is organized in the following sections; Section 2 explains the data and section 3 is devoted to explain the used methodologies; models, estimation and evaluation. Section 4 explains the results as an implementation and finally we offer our conclusions and ideas for further developments in Section 5.
2- Data

Daily price data (in US dollars per barrel) of Brent from Jan 2002 until Jun 2015 are extracted from (eia.gov) of U.S. Energy information administration, we use. The data of the last four years from 2012 until Jun 2015 are used to evaluate the out-of-sample volatility forecasts.

In the years between 2007 and 2009 (The observation number in figures between 1500-2000), the global financial crisis greatly affected the world economy, and the price of crude oil fluctuated tremendously from about USD 30 to USD 145 per barrel and in the last years, the price of oil has caused deep fluctuation.

In this paper, $P_t$ denote the price of crude oil on day $t$. All daily sample prices are converted into a daily nominal percentage return series for crude oil, i.e., $r_t=100 \ln \left( \frac{P_t}{P_{t-1}} \right)$ for $t=1,2,...,T$, in which $r_t$ is the returns for crude oil at time $t$, $P_t$ is the current price, and $P_{t-1}$ is the previous day's price. Following (Sadorsky 2006) and (Kang et al. 2009), daily actual volatility (variance) is assessed by daily squared returns ($r_t^2$). The advantage of this change is values obtained are comparable with no unit.

$$
\text{Return} = 100 \left( \ln \frac{P_{t+1}}{P_t} \right) = 100 \left( \ln P_{t+1} - \ln P_t \right)
$$

Table 2 provides the descriptive statistics of the return series. The Brent sample price displays some statistical characteristics, such as mean of the data is 72.45616, and with 95% confidence is between 71.40326 and 73.50907.

<table>
<thead>
<tr>
<th>Number of data points (n)</th>
<th>3415</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (AVERAGE(Y))</td>
<td>72.45616</td>
</tr>
<tr>
<td>Sample standard deviation (STDEV(Y))</td>
<td>31.3822</td>
</tr>
<tr>
<td>Std error of the mean (STDEV(Y)/SQRT(n))</td>
<td>0.537017</td>
</tr>
<tr>
<td>Lower 95% limit for mean</td>
<td>71.40326</td>
</tr>
<tr>
<td>Upper 95% limit for mean</td>
<td>73.50907</td>
</tr>
<tr>
<td>Mean absolute error (MAE)</td>
<td>27.02328</td>
</tr>
<tr>
<td>Sum of Squared Errors (SSE)</td>
<td>3362253</td>
</tr>
<tr>
<td>Mean Squared Error (SSE/(n-p))</td>
<td>984.8426</td>
</tr>
<tr>
<td>RMSE (square root of MSE)</td>
<td>31.3822</td>
</tr>
<tr>
<td>Critical t-value (95%, n-p d.f.)</td>
<td>1.960659</td>
</tr>
</tbody>
</table>

The graphical representation of the prices, returns and daily volatility for Brent crude oil is given in Fig. 1-3. The below figures show the daily prices, return and volatility for Brent crude oil during the period from May 20, 1992, to Jun 22, 2015.
Fig. 1. The dynamics of prices for crude oils.

Fig. 2. The dynamics of returns for crude oils.

Fig. 3. The dynamics of daily volatility for crude oils.
3- Methodology

3-1- Models framework

- ARCH model

Autoregressive conditional heteroskedasticity (ARCH) models are used to characterize and model observed time series. ARCH-type models are sometimes considered to be part of the family of stochastic volatility models. An ARCH model was proposed by Engle (1982). This procedure is as follows:

$$\sigma_n^2 = \gamma V_L + \sum_{i=1}^{m} \alpha_i \epsilon_{n-i}^2$$  \hspace{1cm} (2)

$V_L$: long-run average variance rate
$\gamma$: Weight assigned to $V_L$

$$\gamma + \sum_{i=1}^{m} \alpha_i = 1$$  \hspace{1cm} (3)

We use the Bayesian information criterion (BIC) to select the parameter of Arch model. Schwarz criterion (also SBC, SBIC) is a criterion for model selection among a finite set of models; the model with the lowest BIC is preferred. It partly based on the likelihood function.

- GARCH model

If an autoregressive moving average model is assumed for the error variance, the model is a generalized autoregressive conditional heteroskedasticity model (Johansen and Zivot 2009).

In most models that implement arch model, $m$ should be selected largely causing an increase in the amount of calculations leading to GARCH (p,q). Based on the work of Engle, the most popular volatility model is the GARCH model proposed by (Bollerslev 1986)which showed that the GARCH(1,1) specification worked well in most applied situations, and Sadorsky (2006) also demonstrated that the GARCH(1,1) model was a good fit for crude oil volatility. The standard GARCH(1,1) model for daily returns is given by

$$\sigma_n^2 = \omega + \sum_{i=1}^{p} \alpha_i \epsilon_{n-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{n-j}^2$$  \hspace{1cm} (4)

It assigns some weight to the long-run average variance rate, $V_L$. Since weights must sum to 1, $\gamma + \alpha + \beta = 1$. Setting $w = \gamma V_L$

$$V_L = \frac{w}{1 - \alpha - \beta}$$  \hspace{1cm} (5)
The more general, GARCH (p, q) calculate $\sigma^2$ from the most recent p observations on $u^2$ and the most recent q estimates of variance rate. GARCH (1,1) is the most popular of the GARCH model (Chan & Grant 2016).

This is usually used for the purposes of estimating the parameters. For a stable GARCH (1, 1) process, it requires to $\alpha + \beta < 1$, otherwise the weight applied to $V_t$ is negative.

In this paper, we use GARCH (1,1), GARCH (2,2), GARCH (3,1), GARCH (3,2), GARCH(3,3), to estimate the volatility. In GARCH (1,1) needs to estimate $\gamma, \alpha, \beta$ and in GARCH (2,2) needs to estimate $\gamma, \alpha_1, \beta_1, \alpha_2, \beta_2$, and with increasing in p and q, increase the number of parameters (Zivot 2008).

3-2-The estimation of parameters

In this paper, we discuss univariate Standard GARCH Model and Maximization Likelihood Estimator and Generalized Method of Moments.

- MLE (Maximization Likelihood Estimator)

By definition, the likelihood function for a statistical model is described as:

$$L^* = \prod_{n=1}^{N} f \left( y_n | y_{n-1}, y_{n-2}, ..., y_1, \theta_1, \theta_2, ..., \theta_k \right)$$

or:

$$\ln L^* = \sum_{n=1}^{N} \ln f \left( y_n | y_{n-1}, y_{n-2}, ..., y_1, \theta_1, \theta_2, ..., \theta_k \right)$$

Where:
- $L^*$ is the likelihood function.
- $\ln L^*$ is the log-likelihood function.
- $f ()$ is the conditional probability density function.
- $\ln f ()$ is the natural log of the conditional probability density function.
- $y_n$ is the value of the time series at time n.
- $y_{n-1}, y_{n-2}, ..., y_1$ are the past values of the time series at time n.
- $\theta_1, \theta_2, ..., \theta_k$ are the parameters of the statistical model.

-GMM (Generalized Method of Moments)

The idea of generalized method of moments (GMM) was firstly introduced by (Properties et al. 1982) and it is nowadays used for model parameters estimation and test model specification. The GMM framework is applied in various fields of study such as finance, macroeconomics, agricultural economics, environmental economics and labor economics.

Unlike the MLE estimation, GMM estimation does not stand on the basis of the joint probability distribution of the data known as the likelihood function. The GMM is useful in circumstances when the distributional assumption is not precisely known or the likelihood function is difficult for numerical evaluation. Let us outline the assumption for GARCH(1,1) GMM estimation

$$E(\left(\sigma_t^2 - \sigma_l^2\right)S_t) = 0,$$  \hfill (8)

Where $S_t = (y_{t-1}, \sigma_t^2)$. Under this assumption the parameters are estimate by choosing vector $\theta^* = (\omega, \alpha, \beta)$ that minimizes
\[(g(\theta; I_{t-1}))^\dagger \hat{S}(g(\theta; I_{t-1}))\]

(9)

Where

\[
g(\theta; I_{t-1}) = \left[ T^{-1} \sum_{t=1}^{T} (e_t^2 - \sigma_{t}^2) s_t \right]^{-1}
\]

(10)

and the matrix \( \hat{S} \) is positive-definite weighting matrix constructed as a Newey-West estimator (Hammad et al. 2012).

### 3.3-Evaluating Performance

According to Yu (2002), four measures are used to evaluate the forecast accuracy, namely, the root mean square error (RMSE), the mean absolute error (MAE), the Theil-U statistic and the LINEX loss function. They are defined by

- **Root Mean Squared Error (RMSE)**

\[
RMSE = \sqrt{\frac{1}{T} \sum_{i=1}^{T} (\hat{\sigma}_i^2 - \sigma_i^2)^2}
\]

(11)

- **Mean Absolute Error (MAE)**

\[
MAE = \frac{1}{T} \sum_{i=1}^{T} |\hat{\sigma}_i^2 - \sigma_i^2|
\]

(12)

- **Theil U Statistics**

\[
Theil-U = \frac{\sum_{i=1}^{T} (\hat{\sigma}_i^2 - \sigma_i^2)^2}{\sum_{i=1}^{T} (\hat{\sigma}_{i-1}^2 - \sigma_i^2)^2}
\]

(13)

- **LINEX Loss Function**

\[
LINEX = \frac{1}{T} \sum_{i=1}^{T} \left[ \exp(-a(\hat{\sigma}_i^2 - \sigma_i^2)) + a(\hat{\sigma}_i^2 - \sigma_i^2) - 1 \right]
\]

(14)

Where \( \alpha \) in the LINEX loss function is a given parameter.

The RMSE and MAE are two of the most popular measures to test the forecasting power of a model. Despite their mathematical simplicity, however, both of them are not invariant to scale transformations. Also, they are symmetric, a property which is not very realistic and inconceivable under some circumstances (Jiang et al. 2015).

In the Theil-U statistic, the error of prediction is standardized by the error from the random walk forecast. For the random walk model, which can be treated as the benchmark model, the Theil-U statistic equals 1. Of course, the random walk is not necessarily a naive competitor, particularly for many economic and financial variables, so that the value of the Theil-U statistic close to 1 is not necessarily an indication of bad forecasting performance.

However, Several authors, such as (Armstrong & Fildes 1995), have advocated using U-statistic and close relatives to evaluate the accuracy of various forecasting methods. One advantage of using U-statistic is that it is invariant to scalar transformation. The Theil-U statistic is symmetric.

In the LINEX loss function, positive errors are weighed differently from the negative errors. If \( \alpha > 0 \), for example, the LINEX loss function is approximately linear for \( \hat{\sigma}_i^2 - \sigma_i^2 > (\text{‘over-predictions’}) \) and
exponential for $\sigma_t^2 - \sigma_i^2 < 0$ (‘under-predictions’). Thus, negative errors receive more weight than positive errors.

In the context of volatility forecasts, this implies that an under-prediction of volatility needs to be taken into consideration more seriously. Similarly, negative errors receive less weight than positive errors when $a < 0$.

In this paper, four values for $\alpha$ are used, namely, 20, 10, -10 and -20. Obviously, $\alpha = -10, -20$ penalize over-predictions more heavily while $\alpha = 10, 20$ penalize under-predictions more heavily. BF also adopts asymmetric loss functions to evaluate forecasting performance. An important reason why the LINEX function is more popular in the literature is because it provides the analytical solution for the optimal prediction under conditional normality, while the same argument cannot be applied to the loss functions used by BF.(Yu 2002)

4- Empirical results

4-1-Autocorrelation with different estimation methods

We calculate autocorrelation before and after the use of GARCH models for testing the estimation methods. First, estimate the parameters for calculate the GARCH (p,q) models with Maximum Likelihood Methods (MLE) from historical data and choose parameters that maximize the chance (likelihood) of the data occurring.

To evaluate the models while doing a good job in explaining the data, we use a scientific test known as the Ljung-Box statistics. For $K = 15$, zero autocorrelation can be rejected with 95% confidence, when the L-B statistics is greater than 25.

The tables show that the L-B statistic for $u_i^2$ is about 845.87; it means that there is a strong evidence of autocorrelation. After using of GARCH model, The L-B statistic for $u_i^2/\sigma_i^2$ has been largely removed by the GARCH model and there isn’t any autocorrelation. In GARCH (3,3) and GARCH (1,1), the L-B statistic is less than 25, i.e., there is no autocorrelation with 95% confidence(Andersen et al. 2006). Table 3 and table 4 show the results.

| GARCH (3,3) | 845.86621 | 8.73597 |
| GARCH (1,1) | 845.86621 | 13.82939 |
| GARCH (2,2) | 845.86621 | 26.89808 |
| GARCH (3,1) | 845.86621 | 46.43306 |
| GARCH (3,2) | 845.86621 | 49.54064 |

After that, we use GMM model to estimate the parameters, results of Ljung-Box test, is shown at table 4.
Table 4. Ljung-Box test for GMM estimation

<table>
<thead>
<tr>
<th>Ljung-Box test GMM</th>
<th>Autocorrelation for $U_i^2$</th>
<th>Autocorrelation $U_i^2/\sigma^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GARCH (1,1)</td>
<td>845.86621</td>
<td>8.99563</td>
</tr>
<tr>
<td>GARCH (2,2)</td>
<td>845.86621</td>
<td>52.75826</td>
</tr>
<tr>
<td>GARCH (3,1)</td>
<td>845.86621</td>
<td>79.05080</td>
</tr>
<tr>
<td>GARCH (3,2)</td>
<td>845.86621</td>
<td>92.51678</td>
</tr>
<tr>
<td>GARCH (3,3)</td>
<td>845.86621</td>
<td>35.78042</td>
</tr>
</tbody>
</table>

It seems that in general, both MLE and GMM models remove autocorrelations, but the performance of MLE is better.

4.2-Evaluation measures

We calculate monthly volatility and use past data to estimate last year’s data. Figure 4 shows the graphical presentation of monthly volatility.

According to (Yu 2002), Four measures are used to evaluate the forecast accuracy, namely, the root mean square error (RMSE), the mean absolute error (MAE), the Theil-U statistic and the LINEX loss function.

We calculate sum of the difference, between real data and forecasted data as errors. These errors are used for evaluating functions.

Table 5 and table 6, gives values of the evaluated performance function and the ranking for a variety of GARCH (p,q) fitted to the oil price. For pure ARCH (m) models, an ARCH (5) is chosen by BIC for both series.
### Table 5. Comparing the forecasting performance with RSME, MAE, Theil-U functions

<table>
<thead>
<tr>
<th></th>
<th>C Value</th>
<th>Rank</th>
<th>MAE Value</th>
<th>Rank</th>
<th>Theil – U Value</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>GARCH(1,1)</td>
<td>0.000832874</td>
<td>1</td>
<td>0.000605823</td>
<td>1</td>
<td>1.618506755</td>
<td>1</td>
</tr>
<tr>
<td>GARCH (3,3)</td>
<td>0.00089043</td>
<td>2</td>
<td>0.000658887</td>
<td>2</td>
<td>1.710259282</td>
<td>2</td>
</tr>
<tr>
<td>Arch</td>
<td>0.001609145</td>
<td>3</td>
<td>0.001511245</td>
<td>3</td>
<td>6.041511659</td>
<td>3</td>
</tr>
<tr>
<td>GARCH (3,2)</td>
<td>0.013961789</td>
<td>4</td>
<td>0.01393306</td>
<td>4</td>
<td>420.4793797</td>
<td>4</td>
</tr>
<tr>
<td>GARCH (3,1)</td>
<td>0.014429072</td>
<td>5</td>
<td>0.014359728</td>
<td>5</td>
<td>449.0961869</td>
<td>5</td>
</tr>
<tr>
<td>GARCH (2,2)</td>
<td>0.040409509</td>
<td>6</td>
<td>0.039835803</td>
<td>6</td>
<td>3522.327175</td>
<td>6</td>
</tr>
</tbody>
</table>

### Table 6. Comparing the forecasting performance with Linex (α) function

<table>
<thead>
<tr>
<th></th>
<th>Linex α = -20</th>
<th>Linex α = -10</th>
<th>Linex α = 10</th>
<th>Linex α = 20</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Value</td>
<td>Rank</td>
<td>Value</td>
<td>Rank</td>
</tr>
<tr>
<td>GARCH (1,1)</td>
<td>0.00013719</td>
<td>1</td>
<td>0.00003449</td>
<td>1</td>
</tr>
<tr>
<td>GARCH (3,3)</td>
<td>0.00015667</td>
<td>2</td>
<td>0.00003940</td>
<td>2</td>
</tr>
<tr>
<td>ARCH</td>
<td>0.00052323</td>
<td>3</td>
<td>0.00013013</td>
<td>3</td>
</tr>
<tr>
<td>GARCH (3,2)</td>
<td>0.04290721</td>
<td>4</td>
<td>0.01021920</td>
<td>4</td>
</tr>
<tr>
<td>GARCH (3,1)</td>
<td>0.04602733</td>
<td>5</td>
<td>0.01093772</td>
<td>5</td>
</tr>
<tr>
<td>GARCH (2,2)</td>
<td>0.44072894</td>
<td>6</td>
<td>0.0943397</td>
<td>6</td>
</tr>
</tbody>
</table>

For GARCH (p,q) models, in all measures GARCH(1,1) has minimum value errors of function, After that, there are GARCH (3,3), ARCH, GARCH (3,2), GARCH (3,1), GARCH (2,2), respectively. Rapid change in the data, leading to an increase in the volatility and the error measures of evaluation functions.

### 5- Conclusion

Our purpose is to test, “why GARCH(1,1) with MLE estimation of its parameters, is the most popular model between GARCH (p,q) models.”

This paper examined six univariate models for forecasting oil price volatility. All the models examined in this paper belong to the univariate time series family. We use Arch, GARCH(1,1), GARCH(2,2), GARCH(3,1), GARCH(3,2) and GARCH(3,3) models for forecasting oil price volatility of the BRENT market. One of the important models considered here is the GARCH (1,1) model.

After comparing the forecasting performance in out-of-sample volatility of all models, it was found that the GARCH (1,1) model is superior, according to the RMSE, MAE, Theil-U and Linex loss functions, we know, Linex loss functions are an asymmetric index and have high flexibility in evaluating the accuracy of forecasting.
GARCH (1,1) model do perform better than the others. After that GARCH (3,3), Arch, GARCH(3,2), GARCH(3,1) and GARCH(2,2) models, respectively, have minimum loss in all the functions. These results are according to Jun Yu (2001) and Eric Zivot (2008) studies.

Finally, we compare two methods of estimation the parameters of GARCH models, for forecasting oil price volatility of the BRENT market, our results suggest that maximum likelihood estimation (MLE) gives better estimates than generalized method of moments (GMM) to GARCH volatility forecasting methods.

In practical aspects, If we want to use GARCH (p,q) models, GARCH (1,1) model is the best model for forecasting and MLE is better than the other method for estimation of the parameters of GARCH (p,q) models.

For future research, those that work on multivariate and bivariate GARCH models should use different estimation parameter models to get new result and specifications of GARCH models. In this paper, we have used 3415 sample sizes, leading to a wide range. For the future studies, is suggested that various sample sizes is preferred to be applied for the models for the behavior of those models to be revealed.

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