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Economical-Statistical Design of One- Sided CCC-r Control chart based on Analytical Hierarchy Process

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Abstract

The cumulative count of a conforming (CCC) control chart is used for high quality processes.The CCC − r chart is an improvement of the CCC chart that is based on the cumulative number of items inspected until observing r non-conforming ones. This paper aims to propose a new approach for manufacturer's decision making according to the criteria among the available options. The objective function of the proposed model is to minimize three criteria simultaneously, including expected cost per hour(C), modified producer risk (PR) and modified consumer risk (CR). The solution method for the proposed model is designed by using AHP technique. A case study is showed in numerical illustration section. In addition, sensitivity analysis is performed to illustrate the impact of input parameters on the optimal solutions of the proposed model.

Keywords: Statistical process control, CCC-r charts, high quality processes, multiple attribute decision making (MADM), AHP technique

1- Introduction

Statistical process control (SPC) is a set of tools for creating stability and improving efficiency through variability reduction in the production process. Zhang et al., (2012) emphasized that traditional control charts do not have required efficiency as a decision making tool and they are not suitable for high quality processes that produce very small fraction of defective products. Many charts have been suggested to control and monitor high quality processes. Many of them are categorized as cumulative charts due to monitor the number of conforming samples before reaching to the first defected nonconforming sample (Zhang et al.2005). One of these charts is known as CCC chart (cumulative count of a conforming control chart) that was designed by Calvin (1983). He presented a control chart by using run length of successive conforming items according to the geometric distribution, which resolved the problem of traditional control charts in generating false alarms in control process of high quality processes. Goh (1987) proposed that for inspecting a process it is better that the number of inspected conforming items is replaced on the chart by the cumulative number of nonconforming items. CCC chart was further studied by Xie et al.,(1998),Tang and Cheong (2004),Liu et al. (2006), Zhang et al.,(2008), Chan and Wu (2009), Chen (2009), Chen and Chen (2001), Acosta-Mejia (2012), Sherbaf Moghaddam (2014). CCC chart is not sensitive to small incremental changes in the nonconforming fraction of the processes and as a result, chart will not show an alarm in such cases.

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One of the weaknesses of these charts is the large average time to signal when the nonconforming fraction increases (Ohta et al., 2001).

A more practical approach for high quality processes in CCC charts is using the extended state, called CCC r chart that considers the cumulative count of conforming items until the detection of r_{th} observation of non conforming item. CCC-r chart was studied by Xie et al.(1998), Ohta et al.(2001), Wu et al.(2001), Kuralmani et al. (2002), Chan et al.(2003), Schwertman (2005), Albers (2010). These charts that follow negative binomial probability distribution and are the extended state of geometric distribution and they have greater efficiency in the high quality processes, but the chart will signal after r, r+1,… nonconforming samples are observed. Therefore, many samples should be inspected in order to reach the first point located on chart and consequently the inspection cost increases. For this reason, in order to minimize the costs, the selection of parameter' r' and other parameters will be important. Optimization models of control charts are developed based on cost or risk. The goal of this paper is to develop a model to optimize these factors simultaneously. If only the economic factors are considered, then the optimal solution can't be applied in high quality systems. Because the basic assumption of this production process is zero defective, so the time to arrive an alarm is so high thus risks should be considered in the model to provide sufficient protection for both producer and consumer.

Many economic studies have been performed for designing optimal control chart. Tang et al.(2000) studied economic statistical design of CCC charts using the approach of primary nonconforming fraction in all control stages of chart with conditional control limits. Utah et al.(2001) determined parameter 'r' and obtained sampling interval of CCC-r chart using the economic model of Lorenzen and Vance (1986).Xie et al.(2001) studied the economic design of CCC chart by analyzing the sensitivity of cost parameters in chart based on Duncan model (1956). Chan et al.(2003) obtained the economic design of CCC charts based on sampling plans and the approach of acceptance risks. Zhang et al.(2011) developed the economic design of time between events (TBE) chart with the aim of maximizing profit at the time unit. Yilmaz and Bornak (2013) used the sensitivity analysis of statistical risks based on the model of Lorenzen and Vance (1986) for economic design of CCC charts. Fallahnezhad and Golbafian (2016) proposed an economic design of CCC-r control charts based on average number of inspected items. They studied several models in order to reduce the cost by selecting suitable parameters of chart and investigated the efficiency of models based on statistical and economical criteria. Fallah Nezhadand Ahmadi Yazdi (2016) proposed a new optimization model for designing acceptance sampling plan based on run length of conforming itemswith the objective of minimizing producer and consumer risks. In this paper, we will develop the economic model of the Xie et al. (2001) for CCC–r chart with considering the factors like C (expected cost per hour) Criteria, PR (modified producer risk) Criteria and CR (modified consumer risk) criteria Simultaneously. In the other word, we propose a Multiple Attribute Decision Making (MADM) method for determining the optimal parameters. In section 2, the economic model is introduced. In section 3, numerical illustration is shown for elaborating solution method and the performance of the proposed model. In section 4, sensitivity analysis is made to analyze the impact of the parameters of the model and finally the paper is concluded in section 5.

2- The proposed modelfor CCC–r chart

2-1- The parameters used in the model

LCL	lower control limit for the CCC-r control chart
h	the time between producing two successive items
C	expected cost per hour
ARL_1	average run length when the process is in control
ARL_2	average run length when process is out of control
ANI_1	average number inspected when process is in control

ANI_2 average number inspected when process is out of control

PR modified producer risk

CR modified consumer risk

S Number of the produced products until the process is in control

C_0 quality cost/hour while the production process is in control

C_1 quality cost/hour while the production process is out of control ($> C_0$)

Y cost per false alarm

W cost to locate and to repair the assignable cause

I the cost of inspecting one item

T_0 expected search time for investigating the false alarm

T_1 expected time to discover the assignable cause

T_2 expected time to repair the process

p_1 expected fraction defective produced when the process is in control

p_2 expected fraction defective produced when the process is out of control

λ 1/mean time when the process is in control

$$\delta_1 = \begin{cases} 1 & \text{if production continues during searches} \\ 0 & \text{if production ceases during searches} \end{cases}$$

$$\delta_2 = \begin{cases} 1 & \text{if production continues during repair} \\ 0 & \text{if production ceases during repair} \end{cases}$$

2-2-The process model

Production cycle of process has started from the beginning of production or after identification operation and eliminating the reason of an out of control condition. The items are inspected one by one at h time interval. We should monitor the number of inspected items until achieving r_{th} nonconforming product on control chart. Assume that at the beginning state of the cycle, even though the process is in control, but we may observe false alarms. The parameter A_1 is the average time that the process is in control. A_1 can be divided into two parts. First part is the average time that the process becomes out of control. Second part is the average time for investigating false alarms when process is in control. If the production process is stopped during the inspection of false alarms, then the value of δ_1 is considered to be zero and the term ST_0 / ANI_1 is time to identify false alarms, that will be added to the time $1/\lambda$ for obtaining A_1.S is equal to the number of produced product until the process is in control and T_0 is the time needed for investigating each false alarm and the ratio S / ANI_1 is equal to the number of false alarms in each cycle.Since we haveconsidered the average number of inspecteditemsin the model, thus the measure ANI is used in the proposed model. So we have:

$$A_1 = 1/\lambda + (1-\delta_1)ST_0 / ANI_1 \ , \tag{1}$$

Also, A_2 is equal to the time duration that the process moves toward out of control state so that we reach to the first true alarm.

$$A_2 = h(ANI_2 - 1) \quad , \tag{2}$$

A_3 is equal to the summation of the expected time of identifying the assignable cause after observing true alarmT_1andthe required time to repair the reason of being in out of control condition and to remove the identified reason as T_2 thus following is obtained,

$$A_3 = T_1 + T_2 \quad , \tag{3}$$

Therefore, the time required for each cycle is obtained from the sum of A_1, $A_2$and A_3:

$$E(T) = 1/\lambda + (1-\delta_1)ST_0 / ANI_1 + h\left(ANI_2 - 1\right) + T_1 + T_2 \tag{4}$$

The expected cost of cycle according to assumed parameters of model can be obtained as follows:

$$E(P) = C_0 / \lambda + C_1(h(\mathrm{ANI}_2 - 1) + \delta_1 T_1 + \delta_2 T_2) + SY / ANI_1 + W + (S + ANI_2)I \quad , \tag{5}$$

When one point on the CCC-r control Chart data is more than the upper control limit (UCL), then process is efficient and there is no need for corrective action. Ttherefore, corrective action is done when one point is located under the lower control limit (LCL). Thus, we consider one-sided CCC-r control chart with LCL in this paper. α is the probability of observing a false alarm when process is in control and β is the probability of not observing any alarm when process is out of control thatfor low sided CCC-r chart, α and β are calculated as follows equations (Ohta et al., 2001):

$$\alpha = \sum_{k=r}^{LCL} \binom{k-1}{r-1} p_1^{\,r} (1-p_1)^{k-r} \tag{6}$$

$$\beta = 1 - \sum_{k=r}^{LCL} \binom{k-1}{r-1} p_2^{\,r} (1-p_2)^{k-r} \tag{7}$$

Average run length when the process is in control (ARL_1) and average run length when process is out of control (ARL_2) can be obtained as follows:

$$ARL_1 = 1/\alpha, \tag{8}$$

$$ARL_2 = 1/(1-\beta) \quad , \tag{9}$$

Since r/p_1, is the mean value of a negative binomial distribution with parameter r and p_1 thus the average number of inspections until a false signal is obtained as follows.

$$ANI_1 = (r / p_1) ARL_1 \quad , \tag{10}$$

Also r/p_2, is the mean value of a negative binomial distribution with parameter r and p_2 and the average number of inspections until a true signal is obtained as follows:

$$ANI_2 = (r / p_2) ARL_2 \quad , \tag{11}$$

Also the expected cost per time unitis calculated as follows.

$$C = E(P) / E(T), \tag{12}$$

Modified producers' risk is probability of observing a signal when process is in control and modified consumers' risk is the probability of not observing a signal when process is out of control based on the number of produced items. These definitions are more applicable because they provide more information about the number of inspected items before observing a signal. Since we have used the average number of inspected items in each cycle, thus the modified producer risk (PR) and the modified consumer risk (CR) of decision making process are obtained as following:

$$PR = 1 / ANI_1 \tag{13}$$

$$CR = 1 - (1 / ANI_2) \ , \tag{14}$$

The objective functions are to minimize C, PR and CR criteria simultaneously and the optimum value of r and LCL should be determined based on the equations (6) or (7) .Therefore, the objective function is as follows:

$$Z = min[C\left(r^*, LCL^*\right), PR\left(r^*, LCL^*\right), CR\left(r^*, LCL^*\right)]$$

Since three different objective functions should be minimized thus a solution algorithm based onMultiple Attribute Decision Making *(*MADM) is employed.

MCDM techniques are used to help the decision makers to evaluate, sort, select candidates based on the analysis expressed by scores, values, preference intensities according to several criteria. These criteria may represent different aspects of the objectives (Colson and De Bruyn, 1989).These processes are generally divided into two branches:

First branch: Multiple Attribute Decision Making (MADM) methods deal with the process of selecting the best alternative in the presence of multiple, usually conflicting, decision criteria. MADM techniques can be applied as an analytical method to analyzed or rank a set of criteria or alternatives (Colson and De Bruyn, 1989).

Second branch: Multi objective methods (MODM-Multi-objective decision making) which are sometimes considered as the extended models of mathematical programming, where several objective functions are considered simultaneously (Colson and De Bruyn, 1989).

According to the different objective functions employed in the proposed model, the MADM method is used to determine the best solution which its application is elaborated in the next section.

3- Numerical illustration

3-1- Solution methodology

The values of input parameters to evaluate the performance of the proposed model is assumed as follows: h =0.3 hours , S =10000 ,δ_1=1 , δ_2=0 , T_0=1hours ,T_1 =5hours ,T_2=8 hours W=60 $, Y= 50 $, I= 4.5 $, C_0= 4 $, C_1= 30 $. Suppose that the process produces about 0.27% nonconforming items when in control.In the other words, The *LCL* is determined using the classical standard false alarm probability level α = 0.0027 based on three-sigma limits.

The values of ANI_1, ANI_2 and C can be obtained based on the input parameters. The effect of the parameter 'r' on the objective functions is shown in the Fig.1.

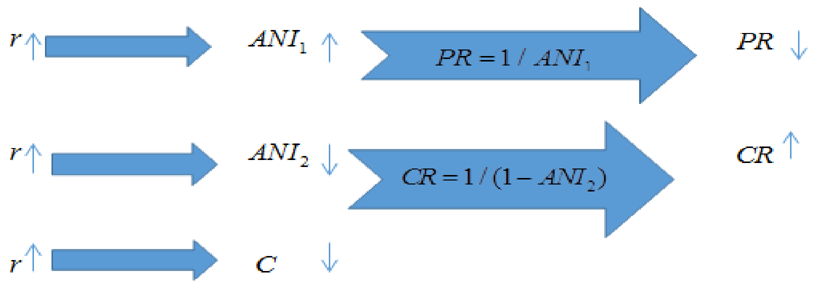

Fig.1. Impacts of increasing the parameter r on the objective functions

It is observed that the values of C and PR increases by increasing the value of the parameter r but the value of CR decreases. Thus the variations C and PR are in the same direction while the variation of CR is not in the same direction with the other objective functions. Therefore the optimal value of r must be selected among the available options.Thus, for decision making, we are using the MADM methods. In this case, AHP (Analytic Hierarchy Process) technique is used. The AHP technique is based on pairwise comparison that can check the different scenarios for Decision-makers (Aksakal and Dağdevire, 2014). AHP model for selecting optimal value ofr is shown in Fig. 2.

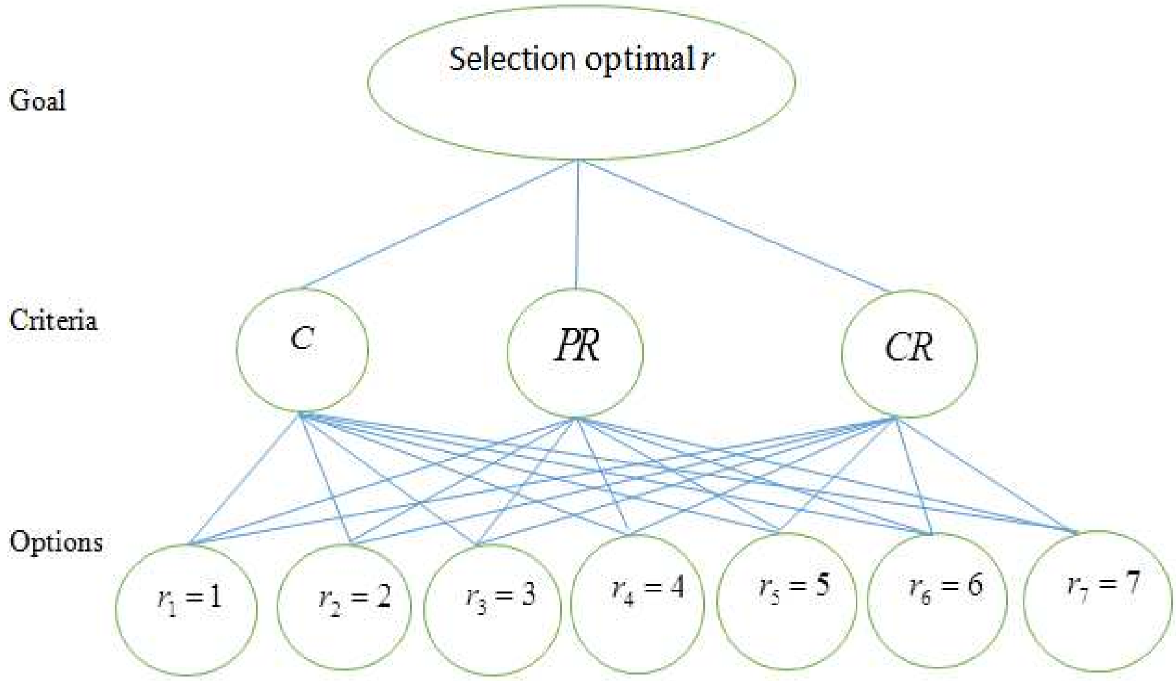

Fig.2 The AHP model for decision making

Decision making steps are as follow:

1- Determine the weights of criteria using pairwise comparisons method.
2- Obtain the values of criteria for different value of r.
3- Pairwise comparisons of criteria for different values of r using the selected framework.
4- Apply the analytical hierarchical process for determining the weight for each value of r.

Step1: Compute the weights of criteria. Pairwise comparison of criteria is performed based on experts' opinion depending on the practical conditions and the policies of decision makers. The results of pairwise comparisons are shown as follows:

C is k times more important than PR $\xrightarrow{\text{Display mode}}$ im(C/ PR) =k

C is k'times more important than CR $\xrightarrow{\text{Display mode}}$ im(C/ CR) =k'

We assume that according to the expert opinion, im(C/PR) =7.5 and im(C/ CR) =2.5. For example, the criterion C is 2.5 times more important than the criterion CR.

Table1. Comparing the relative importance of criteria

	PR	*CR*	*C*
PR	1	0.33	0.13
CR	3	1	0.4
C	7.5	2.5	1

The weights of criteria are obtained as follows:
W_C=0.652, W_{CR}=0.261, W_{PR}=0.087

Step2: Compare the options with respect to criteria. The available options for decision making are r=1, r=2…r=7. The aim of AHP model is to determine the optimal value of r.

First we compare the options with respect to criterion C. For pairwise comparison between options according to experts' opinion, the ratio of cost are classified as follows to obtain the preferences of decision maker in comparison of different values of r based on the cost criterion .

$$
\mathrm{Ex}(r/r') = \begin{cases} 5.5 : if\ 5 \le Cr/Cr' \\ 4.5 : if\ 4 \le Cr/Cr' < 5 \\ 3.5 : if\ 3 \le Cr/Cr' < 4 \\ 2.5 : if\ 2 \le Cr/Cr' < 3 \\ 1.75 : if\ 1.2 \le Cr/Cr' < 2 \\ 1.5 : if\ 1.1 \le Cr/Cr' < 1.2 \\ 1.25 : if\ 1.001 \le Cr/Cr' < 1.1 \\ 1 : if\ Cr/\mathrm{Cr}' < 1.001 \end{cases}
$$

For example:
If
$C_r/C_{r'} = 1.9$, then expected cost per hour (C) of option r is1.75 times more than the option r'.
If$C_r/C_{r'} = 1.09$, then expected cost per hour (C) of option r is1.25 times more than the option r'.
If$C_r/C_{r'} = 1.15$, then expected cost per hour (C) of option r is1.5 times more than the option r'.
Therefore, if we assume p_1=0.001 and p_2=0.01, then the results of pairwise comparisons based on the criterion C are reported in the Table2.

Table 2. Pairwise comparison of the options with respect to the criterion*C*

r \ r'	r_1	r_2	r_3	r_4	r_5	r_6	r_7
r_1	1	1.75	1.75	1.75	1.75	1.75	1.75
r_2	0.57	1	1.25	1.25	1.25	1.25	1.25
r_3	0.57	0.8	1	1	1	1	1
r_4	0.57	0.8	1	1	1	1	1
r_5	0.57	0.8	1	1	1	1	1
r_6	0.57	0.8	1	1	1	1	1
r_7	0.57	0.8	1	1	1	1	1

Also the pairwise comparison between options in other criteria is performed in the same approach framework. The same approach is employed for comparing the options with respect to the criterion CR. Following pairwise comparison was considered by the experts. We have studied a high quality process, so defective parts are rarely produced and the probability of discovering an out of control process is very low because very small values of shift may occur.

$$
\mathrm{R}(r/r') = \begin{cases} 1/2 : if\, 0.99999 \le CRr/CRr' \\ 1/3 : if\, 0.9999 \le CRr/CRr' < 0.99999 \\ 1/4 : if\, 0.9996 \le CRr/CRr' < 0.9999 \\ 1/5 : if\, 0.9990 \le CRr/CRr' < 0.9996 \\ 1/6 : if CRr/CRr' < 0.9990 \end{cases}
$$

Table 3. Pairwise comparison of the options with respect to the criterion *CR*

r \ r'	r_1	r_2	r_3	r_4	r_5	r_6	r_7
r_1	1	0.33	0.33	0.33	0.33	0.33	0.33
r_2	3	1	0.5	0.5	0.5	0.5	0.5
r_3	3	2	1	0.5	0.5	0.5	0.5
r_4	3	2	2	1	0.5	0.5	0.5
r_5	3	2	2	2	1	0.5	0.5
r_6	3	2	2	2	2	1	0.5
r_7	3	2	2	2	2	2	1

Also the options are compared with respect to the criterion PR. Following method is applied for pairwise comparison.

$$
P\left(r/r'\right) = \begin{cases} 4 : if\, 2000000 \le PRr/PRr' \\ 3.5 : if\, 3000 \le PRr/PRr' < 2000000 \\ 3 : if\, 2000 \le PRr/PRr' < 3000 \\ 2.5 : if\, 1500 \le PRr/PRr' < 2000 \\ 2 : if\, 1250 \le PRr/PRr' < 1500 \\ 1.5 : if PRr/PRr' < 1250 \end{cases}
$$

Table 4. Pairwise comparison of the options with respect to the criterion *PR*

r \ r'	r_1	r_2	r_3	r_4	r_5	r_6	r_7
r_1	1	3	4	4	4	4	4
r_2	0.33	1	2.5	4	4	4	4
r_3	0.25	0.4	1	2	3.5	4	4
r_4	0.25	0.25	0.5	1	2	3.5	4
r_5	0.25	0.25	0.28	0.5	1	2	3.5
r_6	0.25	0.25	0.25	0.28	0.5	1	1.5
r_7	0.25	0.25	0.25	0.25	0.28	0.66	1

Step3: After the above steps, AHP technique was used to find the optimal value of r.

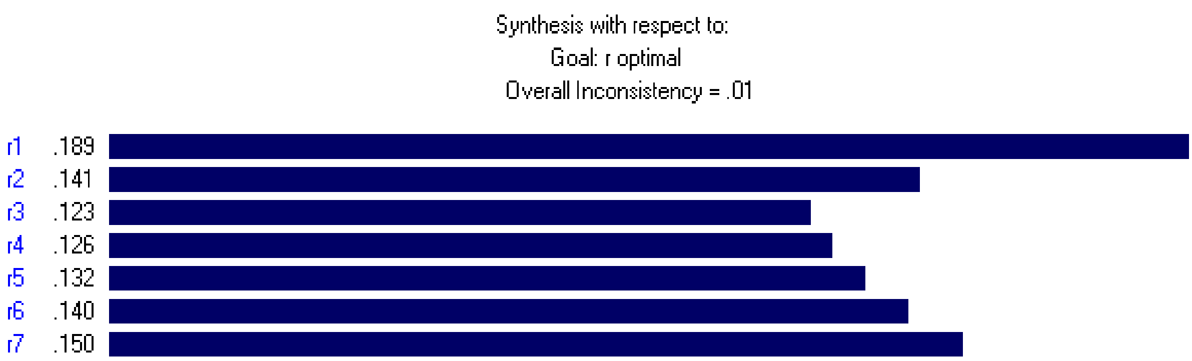

Fig.3. Total value criteria for options

Since the objective function is minimized at r_3, thus r =3 is the optimal option. Also overall inconsistency value is acceptable (less than or equal 0.1) (Aksakal and Dağdevire, 2014).According to the Fig.3, overall inconsistency value is equal to 0.01; therefore,the obtained results are trustable. Also, the proposed approach can be similarly be used to find optimum solution for other scenarios of nonconforming fraction that the results are shown in Table5. Also in Fig.4, the value of each criterion for each option is shown by the bar graph.

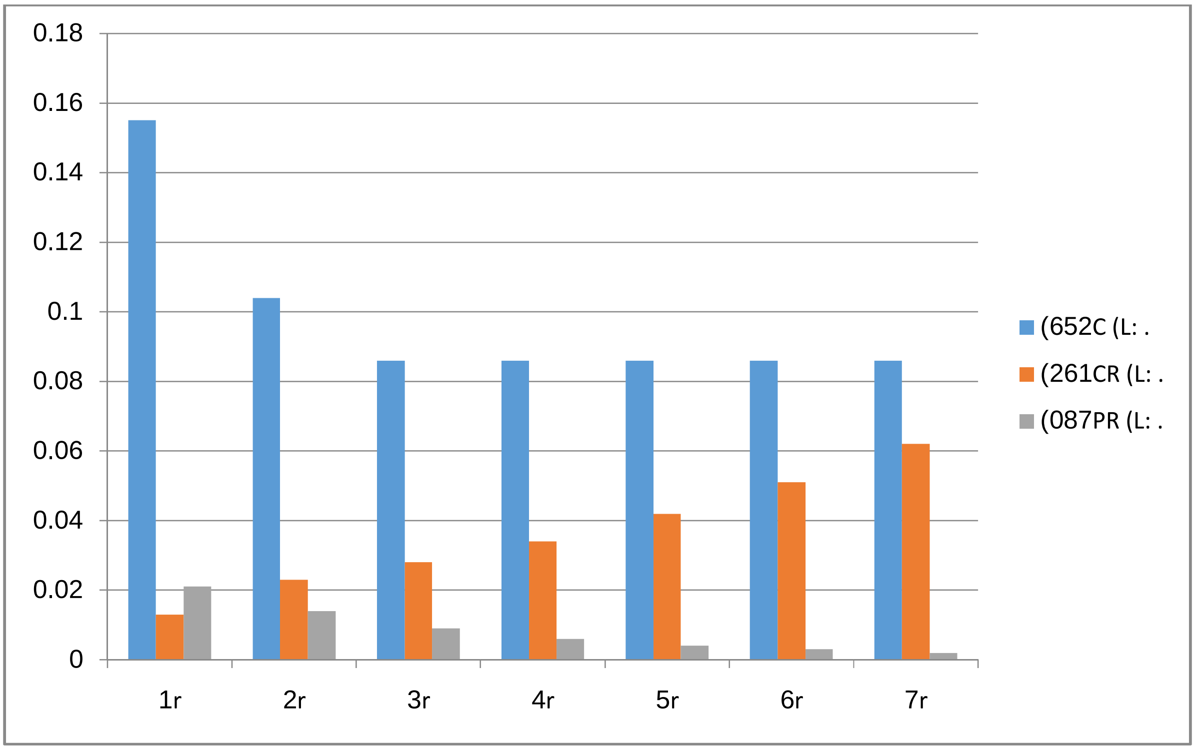

Fig.4.value of each of the criteria in the options

The values of LCL are obtained using producer risk in the equation (6) based on $\alpha = 0.0027$. As can be seen in the Table 5, the value of r=3 is optimal in the cases p_2=0.01, 0.02, 0.03 but when p_2=0.04 then r=4 will be optimal. Thus the optimal value of r increases as the value of p_2 increases.

Table5 Result of the optimal design of the CCC-*r* chart (α = 0.0027)

Scenarios	*r*	*LCL*	ANI_1	ANI_2	*C*	(r*,LCL*)
	1	3	1000000	9999.99	59.8140	
	2	76	2000000000	2000000.00	45.0744	
P_1=0.001	3	272	3000000000000	299999999.99	45.0005	
P_2=0.01	4	566	4.0000e+15	4.0000e+10	45.0000	(3,272)
	5	932	5.0000e+18	5.0000e+12	45.0000	
	6	1354	6.0000e+21	6.0001e+14	45.0000	
	7	1819	7.0000e+24	7.0056e+16	45.0000	
	1	3	1000000	2499.99	103.5158	
	2	76	2000000000	250000.00	45.5950	
P_1=0.001	3	272	300000000000	18749999.99	45.0079	
P_2=0.02	4	566	4.0000e+15	1.2500e+09	45.0001	(3,272)
	5	932	5.0000e+18	7.8125e+10	45.0000	
	6	1354	6.0000e+21	4.6875e+12	45.0000	
	7	1819	7.0000e+24	2.7344e+14	45.0000	
	1	3	1000000	1.1111e+03	173.9760	
	2	76	2000000000	7.4074e+04	47.0072	
P_1=0.001	3	272	3000000000000	3.7037e+06	45.0402	
P_2=0.03	4	566	4.0000e+15	1.6461e+08	45.0009	(3,272)
	5	932	5.0000e+18	6.8587e+09	45.0000	
	6	1354	6.0000e+21	2.7435e+11	45.0000	
	7	1819	7.0000e+24	1.0669e+13	45.0000	
	1	3	1000000	625.0000	267.9270	
	2	76	2000000000	3.1250e+04	49.7540	
P_1=0.001	3	272	3000000000000	1.1719e+06	45.1269	
P_2=0.04	4	566	4.0000e+15	3.9063e+07	45.0038	(4,566)
	5	932	5.0000e+18	1.2207e+09	45.0001	
	6	1354	6.0000e+21	3.6621e+10	45.0000	
	7	1819	7.0000e+24	1.0681e+12	45.0000	

In this subsection, suppose that historical data indicate that the process produces about 5% nonconforming items when out of control ($\beta = 0.05$). Thus we can determine the optimal value of LCL based on the consumer risk in the equation (7) with considering assumed parameters and AHP technique in the subsection 3-1, the result are obtained and denoted in the Table 6. As can be seen in the Table 6, when the parameter p_2 increases, LCL value decreases for the fixed values of r.

Table 6. Result of the optimal design of the CCC-*r* Chart($\beta = 0.05$)

Scenarios	*r*	*LCL*	*ANI₁*	*ANI₂*	*C*	*(r*,LCL*)*
	1	6	1006021.05	10621.57	58.9505	
	2	36	2071275688.78	2843139.60	45.0523	
P_1=0.001	3	83	3253244531335.74	677127562.78	45.0002	
P_2=0.01	4	138	4.5785e+15	1.5535e+11	45.0000	(3,83)
	5	199	6.0771e+18	3.5490e+13	45.0000	
	6	263	7.7670e+21	8.0183e+15	45.0000	
	7	330	9.6801e+24	2.1017e+18	45.0000	
	1	3	1.0030e+06	2.6562e+03	100.1286	
	2	19	2.0363e+09	3.5964e+05	45.4136	
P_1=0.001	3	42	3.1225e+12	4.2069e+07	45.0035	
P_2=0.02	4	70	4.2773e+15	4.8391e+09	45.0000	(3,42)
	5	100	5.5041e+18	5.4336e+11	45.0000	
	6	132	6.8129e+21	6.0986e+13	45.0000	
	7	166	8.2152e+24	6.9286e+15	45.0000	
	1	2	1.0020e+06	1180.90	166.6173	
	2	13	2.0242e+09	1.0676e+05	46.3929	
P_1=0.001	3	28	3.0791e+12	8.1766e+06	45.0182	
P_2=0.03	4	47	4.1800e+15	6.2878e+08	45.0002	(3,28)
	5	67	5.3253e+18	4.6733e+10	45.0000	
	6	89	6.5260e+21	3.5439e+12	45.0000	
	7	111	7.7754e+24	2.6127e+14	45.0000	
	1	2	1.0020e+06	678.1684	251.4764	
	2	10	2.0181e+09	4.5124e+04	48.2937	
P_1=0.001	3	22	3.0606e+12	2.6513e+06	45.0561	
P_2=0.04	4	35	4.1301e+15	1.4424e+08	45.0010	(4,35)
	5	51	5.2407e+18	8.3150e+09	45.0000	
	6	67	6.3840e+21	4.6017e+11	45.0000	
	7	84	7.5682e+24	2.5790e+13	45.0000	

4- Sensitivity analysis

4-1- Sensitivity analysis of pairwise comparisons

In this section, sensitivity analysis of expert's opinion is carried out to investigate their impact on the optimal value of r. The parametersα = 0.0027, P_1=0.001 and P_2=0.01 are assumed for the sensitivity analysis.

Table7. Result of sensitivity analysis of pairwise comparisons (α = 0.0027, P_1=0.001, P_2=0.01)

Scenario	The level of importance	The amount of weight gained	r^*
1	im(C/CR)=3 im(C/PR)=6	W_{PR}=0.111<W_{CR}=0.222<W_C=0.667	3
2	im(C/CR)=3 im(C/PR)=2	W_{CR}=0.182<WPR=0.273<WC=0.545	5
3	im(C/CR)=*3* im(C/PR)=*3*	W_{PR}=W_{CR}=0.200<W_C=0.600	4
4	im(C/CR)=1 im(C/PR)=3	W_{PR}=0.143<W_{CR}=W_C=0.429	3
5	im(C/CR)=3 im(C/PR)=1	W_{CR}=0.143<W_{PR}=W_C=0.429	6
6	im(C/CR)=0.33 im(C/PR)=3	W_P=0.077=<W_C=0.231<W_{CR}=0.692	1
7	im(C/CR)=0.25 im(C/PR)=0.33	WC=0.125<WPR=0.375<WCR=0.500	3
8	im(C/CR)=0.33 im(C/PR)=1	W_C=W_{PR}=0.200<W_{CR}=0.600	3
9	im(C/CR)=0.33 im(C/PR)=0.33	W_C=0.143<W_{PR}=W_{CR}=0.429	4
10	im(C/CR)=0.33 im(C/PR)=0.25	W_C=0.125<W_{CR}=0.375<W_{PR}=0.500	5
11	im(C/CR)=3 im(C/PR)=0.33	W_{CR}=0.077<W_C=0.231<W_{PR}=0.692	7
12	im(C/CR)=1 im(C/PR)=0.33	W_{CR}=W_C=0.200<W_{PR}=0.600	6
13	im(C/CR)=3 im(C/PR)=1	W_{CR}=0.143<W_C=W_{PR}=0.429	6
14	im(C/CR)=1 im(C/PR)=1	W_{CR}=W_C=W_{PR}=0.333	4

According to the table7, almostit can be observedthat when im(C/CR) increases and simultaneously im(C/PR) decreases, then the optimal value of rthe number of r optimal increases .Also, when im(C/PR) increases and im(C/CR) decreases, then the optimal value of decreases, then the optimal value of r decreases.

4-2- Sensitivity analysis of input parameters

In this subsection, asensitivity analysis is performed on the parameters that are used in the economic design for CCC-*r* control chart. In the Table 5, the optimal value of *r*and *LCL* optimal areobtained with considering the criteria C, PR and CR simultaneously. Table 5 shows the variations of the criterion C based on an increase of 50%in the assumed initial parameters.As can be seen, with increasing the parametersh, T1and T_2, the value of C decreases and with increasing the parameters C_1, I and W, the value of C increasesbut with increasing the parameters T_0, C_0 and Y, the variations of C is little and isconsidered to be zero. In the basic mode, assumed production continues during searches and production ceases during repair. In case 9 of Table 5, we have assumed that production ceases during searches and production continues during repair and it is observed that the function C was increased comparing to the basic mode.In case 10, we have assumed that production ceases during searches and production ceases during repair and it is observed that the function C was decreased comparing to the basic mode. In case 11, we have assumed that production continues during searches and production continues during repair and it is observed that the function C was increased comparing to the basic mode. Also, in the case 9, 10 and 11, it is observed that the optimal value of the parameter r is fixed.Also it is observed that both options r=2 and r=3 are optimal in the cases 2 and 15. However r=*3*,is more efficient than *r=2* based on the criterion C. but as mentioned above, with considering criteria C, PR and CR simultaneously, we see that these two options have the same efficiency.

Table 8. Sensitivity analysis for economic design for CCC-*r* chart (α = 0.0027, P_1=0.001, P_2=0.01, W_C=0.652, W_{CR}=0.261, W_{PR}=0.087)

		P_1	P_2	h	T_0	T_1	T_2	δ_1	δ_2	C_0	C_1	I	Y	W	r^*	LCL^*	C^*	%ΔC
Base States		0.001	0.01	0.3	1	5	8	1	0	4	30	4.5	50	60	3	272	45.0004958	
1	h_1	0.001	0.01	0.45	1	5	8	1	0	4	30	4.5	50	60	3	272	40.0003310	-12.5003
2	h_2	0.001	0.01	0.6	1	5	8	1	0	4	30	4.5	50	60	2,	76,	37.5372721,3	-19.8822
															3	272	7.5002484	-20.0005
3	T_{01}	0.001	0.01	0.3	1.5	5	8	1	0	4	30	4.5	50	60	3	272	45.0004958	0
4	T_{02}	0.001	0.01	0.3	2	5	8	1	0	4	30	4.5	50	60	3	272	45.0004958	0
5	T_{11}	0.001	0.01	0.3	1	7.5	8	1	0	4	30	4.5	50	60	3	272	45.0004954	-8.9E-07
6	T_{12}	0.001	0.01	0.3	1	10	8	1	0	4	30	4.5	50	60	3	272	45.0004950	-1.8E-06
7	T_{21}	0.001	0.01	0.3	1	5	12	1	0	4	30	4.5	50	60	3	272	45.0004938	-4.4E-06
8	T_{22}	0.001	0.01	0.3	1	5	16	1	0	4	30	4.5	50	60	3	272	45.0004918	-8.9E-06
9	δ–1	0.001	0.01	0.3	1	5	8	0	1	4	30	4.5	50	60	3	272	45.0004968	2.22E-06
10	δ–2	0.001	0.01	0.3	1	5	8	0	0	4	30	4.5	50	60	3	272	45.0004942	-3.6E-06
11	δ–3	0.001	0.01	0.3	1	5	8	1	1	4	30	4.5	50	60	3	272	45.0004985	6E-06
12	C_{01}	0.001	0.01	0.3	1	5	8	1	0	6	30	4.5	50	60	3	272	45.0004958	0
13	C_{02}	0.001	0.01	0.3	1	5	8	1	0	8	30	4.5	50	60	3	272	45.0004958	0
14	C_{11}	0.001	0.01	0.3	1	5	8	1	0	4	45	4.5	50	60	3	272	60.0004945	24.99979
15	C_{12}	0.001	0.01	0.3	1	5	8	1	0	4	60	4.5	50	60	2,	76,	75.0739809,	40.05847
															3	272	75.0004932	39.99973
16	I_1	0.001	0.01	0.3	1	5	8	1	0	4	30	6.75	50	60	3	272	52.5007448	14.28599
17	I_2	0.001	0.01	0.3	1	5	8	1	0	4	30	9	50	60	3	272	60.0009937	25.00042
18	Y_1	0.001	0.01	0.3	1	5	8	1	0	4	30	4.5	75	60	3	272	45.0004958	0
19	Y_2	0.001	0.01	0.3	1	5	8	1	0	4	30	4.5	100	60	3	272	45.0004958	0
20	W_1	0.001	0.01	0.3	1	5	8	1	0	4	30	4.5	50	90	3	272	45.0004962	8.89E-07
21	W_2	0.001	0.01	0.3	1	5	8	1	0	4	30	4.5	50	120	3	272	45.0004965	1.56E-06

5- Conclusion

In this paper, we proposed a method based on multiple attribute decision making (MADM) methods to deal with the process of designing the CCC-r chart based on the economic model which has been introduced by Xie et al.,(1998). Also the expected cost per hour(C), modified producer risk (PR) and modified consumer risk (CR) are assumed as decision criteria of AHP model then the proposed model was solved by AHP techniques.Numerical illustration was used to demonstratethe solution method.A step by stepsolution method is developed for determining the optimal value of r based on conflicting decision criteria.Sensitivity analysis is done to illustrate the impact of different input parameters on the results of the model. For future studies, we suggested to develop the economic model for CCC-r chart with variable sampling intervals or to develop an optimization economic model for CCC-r chart with the risk constraints.

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