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Multi-Objective Economic-Statistical Design of VSSI-MEWMA-DWL Control Chart with Multiple Assignable Causes

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Abstract

This paper proposes a multi-objective model for the economic-statistical design of the variable sample size and sampling interval multivariate exponentially weighted moving average control chart by using double warning lines. The Markov chain approach is used to obtain the statistical properties. We extend the Lorenzen and Vance cost function considering multiple assignable causes and multivariate Taguchi loss approach to obtain the expected cost per time unit. The meta-heuristic non-dominated sorting genetic algorithm is used to search for the Pareto optimal solutions. A numerical example is provided to illustrate the solution procedure. Finally, sensitivity analyses for some parameters are given.

Keywords: Multivariate exponentially weighted moving average control chart, variable sample size sampling interval, double warning lines, multi-objective economic-statistical design, non-dominated sorting genetic algorithm.

1-Introduction

Control charts are the most popular statistical tools for monitoring the process to detect changes that may adversely affect the product quality. In modern manufacturing and service industries, there are two or more correlated quality characteristics that affect the quality of a process simultaneously. The correlation between quality characteristics may be ignored through monitoring the quality characteristics separately with using individual univariate control charts. This subject has led to increased interest in using multivariate control charts. In the literature of statistical process control (SPC), some multivariate control charts have been received great attention: Shewhart-type chi-squared (χ^2) control chart developed by Hotelling (1947), multivariate cumulative sum (MCUSUM) control chart presented by Woodall and N cube (1985)and the multivariate exponentially weighted moving average (MEWMA) control chart originating in the work of Lowry et al.(1992). Unlike the χ^2 control chart, MEWMA and MCUSUM control charts take into account the present and past information of the process to provide greater sensitivity to detect small and moderate shifts. Desirable properties of the MEWMA control chart such as the ability to detect small shifts in the process and robustness to the violation of normality assumption in distribution of quality characteristics have been caused that the MEWMA has been received significant attention from researchers. Designing a MEWMA control chart determines the sample size (n_0), sampling interval (h_0), the upper control

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limit (H) and the smoothing parameter (γ). There are three general approaches to design the MEWMA control chart including statistical, economic and economic-statistical designs. Each of these approaches considers different model to obtain the optimal values of the control chart parameters. A statistical design approach considers statistical properties such as probabilities of Type I and Type II errors, in-control and out-of-control average run lengths (ARLs) and average time to signal (ATS). Several researchers have studied statistical design of the MEWMA control chart and used different approaches to calculate the run length of the MEWMA control chart. Lowry et al.(1992) used simulation approach to obtain the ARL. Rigdon (1995) proposed an integral equation to compute the in-control ARL. Runger and Prabhu (1996) extended the Markov chain approximation of univariate control chart to calculate the run length of a multivariate control chart such as the MEWMA. Afterwards, Prabhu and Runger (1997) provided a Markov chain approach to determine the performance of the MEWMA control chart and compared ARL estimation results with the simulation method.

Economic design approach minimizes the expected loss cost of process including sampling costs, defective products, false alarms investigations, corrective action and repairing assignable causes. Two well-known cost models have been widely used to determine the design parameters. The first is the model proposed by Duncan (1956) to economically design an X-bar control chart based on the probability of Type I and Type II errors and the second is Lorenzen and Vance (1986)model based on ARL criterion that could be applied to the most types of control charts in different types of industrial processes. A thorough review of the literature of the economic designs of various control charts is provided by Montgomery (1980). Niaki et al. (2010) and Barzinpour et al. (2013) investigated an economic design of the MEWMA control chart and minimized the Lorenzen and Vance cost function as a part of their work. Due to the low statistical performance of economic approach that may increase false alarms which may cause replicate process adjustments and operator dissatisfaction, Saniga (1989) added probabilities of Type I and Type II errors as constraints to the Duncan's model to satisfy the statistical properties. A review paper in the constrained economic-statistical design of control charts has been written by Celano (2011).

In recent years, the economic-statistical design of MEWMA control chart has been increasingly investigated and due to some complicated optimization model of this chart, different heuristic and meta-heuristic algorithms have been proposed to find near-optimum solutions. Linderman and Love (2000) proposed an economic-statistical model based on Lorenzen and Vance cost function with two statistical constraints including a lower limit and upper limit for in-control and out-of-control ARLs, respectively. They applied Hooke and Jeeves (1961) algorithm to solve their model. Molnau et al. (2001) presented similar approach and used Markov chain to estimate ARL due to the large values of the standard deviations of the estimated ARLs. Niaki et al. (2010) extended the Lorenzen and Vance model to incorporate intangible external costs of applied MEWMA chart and used Taguchi loss approach and a genetic algorithm to find near-optimum solution of the proposed model. Niaki and Ershadi (2012) improved the Linderman and Love’s model and proposed a statistically constrained economic model. They used multivariate Taguchi loss function to estimate external intangible quality costs and developed a Markov chain approach to estimate ARLs and an ant colony algorithm was applied to solve the model. Barzinpour et al. (2013) developed a new approach that combines evolutionary algorithm, particle swarm optimization(PSO), with a search-based method, Nelder–Mead to solve the economic-statistical model of the MEWMA control chart.

In more recent studies, researchers have increasingly devoted attention on improving the efficiency of control charts. They showed that one of the common approaches to improve the efficiency of control charts and provide much faster detection of small and moderate process changes is to use varying design parameters as a function of the current and prior sample results instead of fixed sample rate (FSR).Considering variable design parameters is called adaptive control chart and involves varying the sampling interval (VSI), the sample size (VSS), the sampling interval jointly with the sample size (VSSI), and Variable sampling rate (VSR) that vary all design parameters at the same time. This subject has been investigated extensively in the statistically design of multivariate control charts. Faraz and Saniga (2011) and Mahadik (2012) proposed a VSI scheme for the χ^2 control chart. Recently, Seif et al.(2014) investigated the economic-statistical design of the multivariate T^2control chart with multiple variable sampling interval scheme based on non-dominated sorting genetic algorithm (NSGA-II).In order to statistically design the VSI-MEWMA control chart, Lee (2009)modified the Markov chain approach described in Runger and Prabhu (1996)to appraise the performance of the MEWMA control chart. Lee (2013) used the Markov chain approach to statistically design and obtain the in-control average time to signal (ATS_0) and the value of out-of-control adjusted average time to signal (AATS) of the VSSI-MEWMA control chart with double warning lines(DWLs). He showed that the proposed model reduces the values of the out-of-control AATS for a wide range of shifts in the process mean except for large shifts. Reynolds and Cho (2011) investigated the performance of the VSI feature used in the multivariate Shewhart and MEWMA-type control charts, for simultaneous monitoring of the mean vector and the covariance matrix. Recently, Lee and Khoo (2014) employed the statistically design of VSI-MEWMA control chart by applying Markov chain approach where only two different sampling intervals have been considered.

Most of the previous studies in the scope of MEWMA control chart design are including statistically constrained economic models. It means that they considered cost function as the objective function and statistical properties as the constraints. The results of these models are not efficient enough since the cost function depends on statistical properties in the constraints. Statistical properties and cost may have equal importance in some applications, thus the multi-objective approach that considers cost function and statistical properties as objective functions can help to optimize them simultaneously. A few papers are found in the literature for the multi-objective economic-statistical design of the MEWMA control charts. Amiri et al. (2013) provided two multi-objective economic-statistical approaches including an aggregative and non-aggregative approach in designing the MEWMA control chart based on Lorenzen and Vance cost function with considering Taguchi loss approach and used the genetic algorithm (GA) in each approach to obtain the optimal control chart parameters. They showed that applying these approaches result in more efficient process monitoring, cost reduction and consequently more satisfaction of the management. Note that there are some studies in the area of multi-objective economic-statistical design of other control charts. Safaei et al. (2012) developed a multi-objective model for economic-statistical design of X-bar control chart incorporating with Taguchi loss function. In addition, Safaei et al. (2012) suggested a multi-objective model to design an S control chart for monitoring process variability. Also, economic and statistical design of X-bar and S control charts are extended by Yang et al. (2012) using an improved multi-objective PSO algorithm. Bashiri et al. (2013) provided a multi-objective genetic algorithm for economic-statistical design of X-bar control chart. Faraz and Saniga (2013) investigated multi-objective economic-statistical design of X-bar and S^2 control charts and compared their proposed method with statistical, economic, economic-statistical and heuristic designs. The economic-statistical design of the VP T^2 control chart is considered as a double-objective minimization problem by Faraz et al. (2014) with adjusted average time to signal as a statistical objective and expected cost per hour as an economic objective. They used a multi-objective genetic algorithm to find the Pareto-optimal solution.

Previous studies about adaptive MEWMA control chart only considered its statistical design. In this paper, we look at this problem from an economic-statistical perspective. Moreover in real environments many types of assignable causes may be taken place, so developing economic-statistical model of the MEWMA control chart that incorporates multiple assignable causes is important. No previous work considered economic-statistical design of adaptive MEWMA control chart. Nenes et al. (2014) provided the economic-statistical design of a variable-parameter Shewhart control chart monitoring the mean of the process in the presence of multiple assignable causes. According to these explanations, this paper develops a multi-objective economic-statistical model for the VSSI-MEWMA control chart by considering DWLs and incorporating multiple assignable causes. We extend the Lorenzen and Vance cost function by considering multiple assignable causes and based on multivariate Taguchi loss approach in order to calculate expected cost of the model. Moreover, the Markov chain approach is used to obtain the ATS_0 and AATS of the VSSI-MEWMA-DWL control chart. The meta-heuristic algorithm, NSGA-II algorithm is applied to search for the Pareto optimal solutions of the control chart parameters considering the multi-objective nature of the problem.

The remainder of the paper is organized as follows: Section 2 discusses about the VSSI-MEWMA-DWL control chart. Section 3 describes the proposed multi-objective economic-statistical model of VSSI-MEWMA-DWL. Then, the Lorenzen and Vance cost function for multiple assignable causes are provided and multivariate Taguchi loss approach is briefly reviewed. The Markov chain approach is presented in the last part of this section. Section 4 provides brief description of meta-heuristic algorithm and solution methodology. A numerical example is presented in Section 5. The computation and the sensitivity analyses for some parameters are given in Section 6. Finally, concluding remarks and some suggestions for future researches are given in section 7.

2- VSSI-MEWMA-DWL control chart

Roberts (1959) proposed the univariate EWMA control chart as an alternative to Shewhart control chart. Lowry et al. (1992) developed the MEWMA control chart as an extension of the univariate EWMA. Lowry and Montgomery (1995) showed that this control chart is more efficient in detecting small and moderate shifts in the process since it takes into account the present and past information of the process.

Consider $\mathbf{x}_t$ as a $p \times 1$ vector containing p quality characteristics that is to be monitored simultaneously in time *t* which follows a multivariate normal distribution $N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma}_x)$. While $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}_x$ are the mean vector and the known covariance matrix of the quality characteristics, respectively. Without loss of generality, it is assumed that when the process is in-control, the on target process mean is the zero vectors. The MEWMA vector is defined as follows:

$$\mathbf{z}_t = \gamma(\mathbf{x}_t - \boldsymbol{\mu}) + (1-\gamma)\mathbf{z}_{t-1}, \tag{1}$$

Where γ is a diagonal weight matrix with diagonal elements $0 < \gamma \le 1$ and $\mathbf{z}_0$ represents a zero vector. According to (Lowry et al., 1992), the plotted chart statistics on the MEWMA control chart to decide whether the process is in-control or out-of-control is calculated by Equation (2).

$$T_t^2 = \mathbf{z}_t' \boldsymbol{\Sigma}_z^{-1} \mathbf{z}_t, \tag{2}$$

Where $\boldsymbol{\Sigma}_z$ is the covariance matrix of $\mathbf{z}_t$ and is obtained as follows:

$$\boldsymbol{\Sigma}_z = \left(\frac{\gamma}{2-\gamma}\right)\boldsymbol{\Sigma}_x, \tag{3}$$

Where $\boldsymbol{\Sigma}_x$ is the covariance matrix of $\mathbf{x}_t$. For convenience, it is assumed that $\boldsymbol{\Sigma}_z = \mathbf{I}$ where $\mathbf{I}$ is an identity matrix. The chart alarms an out-of-control state when $T_t^2 > H$ where *H* is the predefined upper control limit selected to obtain a given in-control ARL performance. Lowry et al. (1992) showed that the performance of the MEWMA control chart depends only on a function of distance of the off-target mean vector $\boldsymbol{\mu}_1$ from the on-target process mean vector $\boldsymbol{\mu}_0$ and covariance matrix through the non-centrality parameter. This distance is defined as the square root of the non-centrality parameter by Equation (4).

$$\delta = \sqrt{(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)' \boldsymbol{\Sigma}_z^{-1} (\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)}. \tag{4}$$

Designing a FSR MEWMA control chart involves determining the sample size (n_0), sampling interval (h_0), the upper control limit (*H*) and the smoothing parameter (γ). As modification of the FSR MEWMA, VSSI MEWMA-DWL control chart considers variable sample sizes and variable sampling intervals and two additional warning lines as follows:

- The warning line w_n is utilized to determine the switch between the short sample size n_1 and the large sample size n_2 and $n_1 \le n_0 \le n_2$.
- The warning line w_h for sampling interval is applied to specify the switch between the long and short sampling intervals represented by h_1 and h_2, respectively and $h_2 \le h_0 \le h_1$.

These assumptions and DWL scheme lead to the following three states for the MEWMA-DWL control chart:

(i) $w_h < w_n$: If $T_t^2 < w_h$, both sample size and sampling interval are in safety region. Hence, the pair (n_1, h_1) will be chosen for the next sampling stage. If $w_h < T_t^2 < w_n$, the pair (n_1, h_2) will be selected for the next sampling stage, because sample size is in safety region and sampling interval is in warning region. If $w_n < T_t^2 < H$, the pair (n_2, h_2) will be selected for the next sampling stage because both sample size and sampling interval are in warning region.

(ii) $w_n < w_h$: If $T_t^2 < w_n$, since both sample size and sampling interval are in safety region the pair (n_1, h_1) will be applied for the next sampling stage. If $w_n < T_t^2 < w_h$, the pair (n_2, h_1) will be applied for the next sampling stage. This means that sample size is in warning region while sampling interval is in safety region. If $w_h < T_t^2 < H$, the pair (n_2, h_2) will be applied for the next sampling stage because both sample size and sampling interval are in warning region.

(iii) $w_n = w_h = w$: If $T_t^2 < w$, since both sample size and sampling interval are in safety region, the pair (n_1, h_1) will be used for the next sampling stage otherwise if $w < T_t^2 < H$, the pair (n_2, h_2) will be used for the next sampling stage. This means that both sample size and sampling interval are in warning region.

As mentioned previously, the control chart displays an out-of-control signal when $T_t^2 > H$.

3- Proposed model

The multi-objective optimization approach of economic-statistical design of VSSI-MEWMA-DWL control chart considers two objectives simultaneously including minimization of the expected value of AATS ($\overline{AATS}$) and expected cost per time unit ($E(A)$) with considering a lower limit (ATS_L) for ATS_0 as a constraint. Large value of ATS_0 decrease the false alarm rate as well as unnecessary process adjustments and operator dissatisfaction on control chart performance. In this paper, without loss of generality, the value of $ATS_L = 200$ is used. On the other hand, small value of the expected value of AATS leads to detecting assignable causes as quickly as possible. Minimizing the expected cost is essential to remain in competitive global market and increasing profits. The proposed model is derived as follows:

$$
\begin{aligned}
&Min\ E(A(n_1, n_2, h_1, h_2, \gamma, w_n, w_h, H)) \\
&Min\ \overline{AATS} \\
&\text{subject to} \\
&ATS_0 \geq ATS_L \\
&0 \leq w_n, w_h \leq H \leq H_{\max} \\
&h_{\min} \leq h_2 \leq h_1 \leq h_{\max} \\
&1 \leq n_1 \leq n_2 \leq n_{\max}\ \text{(integers)} \\
&0 < \gamma \leq 1
\end{aligned}
\tag{5}
$$

Most of the previous studies in economic-statistical design of MEWMA control chart assumed that there is one assignable cause in the process which leads to shift in the mean of the process. However, in the real manufacturing environments different types of assignable causes may take place and cause shift in the mean of the process with different magnitudes. In this paper, the magnitude of the shift due to each assignable causes is calculated as follows:

$$\delta_j = \sqrt{(\mu_{1j} - \mu_0)' \Sigma_z^{-1} (\mu_{1j} - \mu_0)} \qquad j = 1, 2, \ldots, s, \tag{6}$$

In which s represents the number of assignable causes and the index of j refers to the j^{th} assignable cause. Thus, we modified the Lorenzen and Vance cost function in order to consider multiple assignable causes with different shift sizes in the mean. In addition, we use Taguchi loss function to estimate external intangible quality costs. The details about extended Lorenzen and Vance cost function and multivariate Taguchi loss approach are given in the next subsections.

3-1- Expected cost function

Suppose that there are multiple assignable causes $(j = 1, 2, ..., s)$ that can take place but in each time just one of them can take place. In other words, the proposed model is designed based on the assignable causes of single occurrence model, since it allows the occurrence of only one assignable cause before a signal. It is assumed that the occurrence times of these s assignable causes are according to independent exponential random variables with corresponding parameters of $\lambda_1, \lambda_2, ..., \lambda_s$ where $\lambda_j = 0.5e^{-0.5\delta_j}$ and δ_j represents the magnitude of the j^{th} assignable cause. The conditional probability of the occurrence time of j^{th} assignable cause is equal to $\frac{\lambda_j}{\lambda}$ for $j = 1, 2, \ldots, s$ where $\frac{1}{\lambda}$ is the expected time for the occurrence of the first assignable cause and $\lambda = \sum_{j=1}^{s} \lambda_j$.

It is assumed that when an assignable cause occurs and changes the mean of the process, the mean remains at the shifted value until the control chart signals and the special cause is found and removed. It will also be assumed that the process returns to the same control state as the starting state after the repair and removing the assignable causes.

So, the modified Lorenzen and Vance cost function for computing the expected cost per hour is defined as

$$E(A) = \frac{E(c)}{E(T)} = \frac{\frac{1}{\lambda}\{c_0 + \sum_{j=1}^{s} \lambda_j c_{1j} [AATS_j + \bar{n}E + r_{1j}T_{1j} + r_{2j}T_{2j}]\} + a_3' ANF}{\frac{1}{\lambda}\{1 + \sum_{j=1}^{s} \lambda_j (AATS_j + \bar{n}E + T_{1j} + T_{2j} + (1 - r_{1j})T_0 ANF)\}}$$
$$+ \frac{\frac{\sum_{j=1}^{s} \lambda_j a_{3j}}{\lambda} + a_1 \overline{ANS} + a_2 \overline{ANI} + \frac{(a_1 + a_2 n_1)}{h_2} \frac{\sum_{j=1}^{s} \lambda_j (\bar{n}E + r_{1j}T_{1j} + r_{2j}T_{2j})}{\lambda}}{\frac{1}{\lambda}\{1 + \sum_{j=1}^{s} \lambda_j (AATS_j + \bar{n}E + T_{1j} + T_{2j} + (1 - r_{1j})T_0 ANF)\}}, \tag{7}$$

Where the parameters of the expected cost function are

c_0	Expected cost of nonconforming items per hour while the process is in-control.
c_{1j}	Expected cost of nonconforming items per hour while the process is out-of-control due to j^{th} assignable cause.
$\bar{n}$	The expected sample size in a cycle time.
E	The expected time required to sample and to plot one item.
T_0	The expected search time to understand that the signal is a false alarm.
T_{1j}	The expected time to detect the j^{th} assignable cause and determine the type of assignable cause.
T_{2j}	The expected time to repair the out-of-control process and perform the corrective action to remove the j^{th} assignable cause.

a_1	The fixed cost of each sample.
a_2	The variable cost per unit sampled.
a_{3j}	Cost of repairing the out-of-control process and performing corrective action to remove the j^{th} assignable cause.
a'_3	The cost of search for false alarm.
$AATS_j$	The expected average time to signal from the occurrence time of j^{th} assignable cause.
ANF	The expected number of false alarms.
$\overline{ANS}$	The expected number of samples until the chart signals.
$\overline{ANI}$	The expected number of inspected items.
r_{1j}	If the process is stopped during the searches of j^{th} assignable cause, it is equal to 0 otherwise it is equal to 1.
r_{2j}	If the process is stopped while the correcting or repairing the out-of-control process due to j^{th} assignable cause, it is equal to 0 otherwise it is equal to 1.

The expected cost per hour $(E(A))$ is achieved through dividing the expected cost $(E(C))$ in a cycle time by the expected cycle time $(E(T))$. The elements of expected cost $(E(C))$ per cycle are

(i) Expected cost of nonconforming items per hour while the process is in-control and out-of-control.

$$\frac{1}{\lambda}\{c_0 + \sum_{j=1}^{s}\lambda_j c_{1j}[AATS_j + \bar{n}E + r_{1j}T_{1j} + r_{2j}T_{2j}]\}. \quad (8)$$

(ii) The expected cost of search when the signal is a false alarm.

$$a'_3 ANF. \quad (9)$$

(iii) The expected cost to repair the assignable causes.

$$\frac{\sum_{j=1}^{s}\lambda_j a_{3j}}{\lambda}. \quad (10)$$

(iv) The expected cost of sampling is given as Equation (11). Based on the DWL method, when the process goes to out-of-control state, if the process is not stopped for searching and repairing the assignable causes, the short sampling interval h_2 and large sample size n_1 are utilized.

$$a_1\overline{ANS} + a_2\overline{ANI} + \frac{(a_1 + a_2 n_1)}{h_2}\frac{\sum_{j=1}^{s}\lambda_j(\bar{n}E + r_{1j}T_{1j} + r_{2j}T_{2j})}{\lambda}. \quad (11)$$

The expected cycle time, $E(T)$, is the sum of expected times when the process is in-control and the expected times when the process is out-of-control which are calculated as follows, respectively:

$$E(\mathrm{I}_{\text{in-control}}) = \frac{1}{\lambda}(1 + \sum_{j=1}^{s}\lambda_j(1 - r_{1j})T_0 ANF), \quad (12)$$

$$E(\mathrm{I}_{\text{out-of-control}}) = \frac{\sum_{j=1}^{s}\lambda_j(AATS_j + \bar{n}E + T_{1j} + T_{2j})}{\lambda}. \quad (13)$$

Equation (12) shows that the average time when the process is in-control state consisting of two parts:

(i) The average time interval that process remains at in-control state $\frac{1}{\lambda}$.

(ii) The expected time to detect the false alarms, $\frac{\sum_{j=1}^{s} \lambda_j (1-r_{1j}) T_0 ANF}{\lambda}$.

Equation (13) denoted that the expected time that process is in the out-of-control state consists of the following four parts:

(i) The average time to signal from the time an assignable cause occurs denoted by $\frac{\sum_{j=1}^{s} \lambda_j (AATS_j)}{\lambda}$.

(ii) The average time to sampling and interpreting the results, $\frac{\sum_{j=1}^{s} \lambda_j \overline{n}E}{\lambda}$.

(iii) The average time to detecting and finding the assignable cause, $\frac{\sum_{j=1}^{s} \lambda_j T_{1j}}{\lambda}$.

(iv) The average time to repairing the out-of-control process and removing the assignable cause, $\frac{\sum_{j=1}^{s} \lambda_j T_{2j}}{\lambda}$.

3-2-Taguchi loss approach

In traditional Lorenzen and Vance cost function, all parameters are estimated based on the internal costs of an organization. In this paper, the multivariate version of Taguchi loss approach that presented by Kapur and Cho (1996) is used to consider the external quality impacts of a produced item and to estimate the external quality cost. The multivariate quality characteristics loss function is given in Equation (14) as

$$L(y_1, y_2, \ldots, y_p) = \sum_{i=1}^{p} \sum_{j'=1}^{i} k_{ij'} (y_i - t_i)(y_{j'} - t_{j'}), \tag{14}$$

in which p represents the number of quality characteristics, $k_{ij'}$ is the constant that represents the correlation between y_i and $y_{j'}$, t_i and $t_{j'}$ are the target values of the quality characteristics y_i and $y_{j'}$, respectively. The expected external cost of each product at the in-control and out-of-control states are denoted by J_0 and J_1, respectively and are obtained as follows:

$$J_0 = \sum_{i=1}^{p} \sum_{j'=1}^{i} k_{ij'}[(\mu_{0i} - t_i) + \sigma_i^2] + \sum_{i=2}^{p} \sum_{j'=1}^{i-1} k_{ij'}[(\mu_{0i} - t_i)(\mu_{0j'} - t_{j'}) + \sigma_{ij'}], \tag{15}$$

$$J_1 = \sum_{i=1}^{p} \sum_{j'=1}^{i} k_{ij'}[(\mu_{1i} - t_i) + \sigma_i^2] + \sum_{i=2}^{p} \sum_{j'=1}^{i-1} k_{ij'}[(\mu_{1i} - t_i)(\mu_{1j'} - t_{j'}) + \sigma_{ij'}], \tag{16}$$

Where μ_i and σ_i^2 are the mean and the variance of y_i and $\sigma_{ij'}$ is the covariance between y_i and $y_{j'}$. Afterwards, by replacing J_0 and J_1 the external costs are achieved by using Equation (17).

$$\begin{aligned} c_0 &= J_0 P + c_0', \\ c_{1j} &= J_1 P + c_{1j}' \qquad j = 1,2,\ldots,s, \end{aligned} \tag{17}$$

In which the cost of producing nonconforming items at the in-control and out-of-control states are represented by c_0' and c_{1j}', respectively. By replacing parameters c_0 and c_{1j} in the Lorenzen and Vance cost function by the ones in Equation (7), the expected cost is calculated.

In order to compute the values of the ATS_0, AATS, $\overline{ANS}$, $\overline{ANI}$ and ANF that are applied in the Lorenzen and Vance cost function, we extend the Markov chain approach proposed by Lee (2013)by considering multiple assignable causes. This method is explained in the next subsection.

3-3-Markov chain approach for VSSI-MEWMA-DWL control chart

The performance measure of the VSSI-MEWMA-DWL control chart could be achieved by the Type I and Type II error rates or equivalently ATS_0 and AATS. In the in-control state, it is desired to have a large ATS_0 to decrease false alarm rate. On the other hand, in the out-of-control state the value of the AATS should be as small as possible because the shift in the process should be detected as quickly as possible. In this paper by considering the multiple assignable causes, the Markov chain approach proposed by Lee(2013)is modified to evaluate the value of the VSSI-MEWMA-DWL control chart as follows:

$$\overline{AATS} = \frac{\sum_{j=1}^{s} \lambda_j AATS_j}{\lambda}, \tag{18}$$

$$\overline{ANS} = \frac{\sum_{j=1}^{s} \lambda_j ANS_j}{\lambda}, \tag{19}$$

$$\overline{ANI} = \frac{\sum_{j=1}^{s} \lambda_j ANI_j}{\lambda}, \tag{20}$$

In which the index of *j* indicates the j^{th} assignable cause (j=1, 2,…, s). The values of ATS_0, ANF, and the values of $AATS_j$, ANS_j, ANI_j related to each assignable cause are calculated according to the Markov chain approach that provided by Lee (2013). The interested readers are referred to Lee (2013)for details.

4- The solution methodology

The model given in Equation (5) is a multi-objective non-linear programming model with some constraints and the objectives should be optimized simultaneously. Since in multi-objective models the objectives may be conflict with each other, it is common that no single solution can optimize all the objectives. One of the practical approaches in many real life applications which can characterize optimal trade-offs among the objectives is Pareto optimal solutions. The meta-heuristic NSGA-II algorithm is a well-known algorithm which optimizes the multi-objective problems and achieves the Pareto optimal solutions.

There are four parameters in the NSGA-II that may affect the solution, including population size (N_{POP}), Crossover percentage (p_C), mutation percentage (p_m) and number of iterations (N_{iter}). Hence, we first optimize the parameters of NSGA-II. To determine the best values of the parameters of the NSGA-II, we consider a range for each parameter as shown in Table (1).

Table 1. The input range of the NSGA-II parameters.

Parameter	Range	Low	Center	High
N_{POP}	50-200	50	125	200
p_C	0.2-0.8	0.2	0.4	0.6
p_m	0.1-0.6	0.1	0.2	0.3
N_{iter}	10-100	10	55	100

Then, we use 2^4 factorial design experiments that is illustrated in table (2). Furthermore, the average cost value of each experiment is reported in the last column of table (2). The best parameters values of N_{POP}, p_C, p_m and N_{iter} are set equal to 50, 0.6, 0.3 and 100, respectively along with the best minimum average cost, which is related to run 15 in table (2).

Table 2. 2^4 factorial designs for NSGA-II parameter tuning

Run	Parameters				Average cost value of each run
	N_{POP}	p_C	p_m	N_{iter}	
1	-1	-1	-1	-1	243786
2	1	-1	-1	-1	286348
3	-1	1	-1	-1	238399
4	1	1	-1	-1	203647
5	-1	-1	1	-1	210369
6	1	-1	1	-1	268453
7	-1	1	1	-1	193645
8	1	1	1	-1	189642
9	-1	-1	-1	1	246389
10	1	-1	-1	1	197853
11	-1	1	-1	1	210397
12	1	1	-1	1	189634
13	-1	-1	1	1	176348
14	1	-1	1	1	214532
15	-1	1	1	1	107663
16	1	1	1	1	214067

In addition, the number of cross-over and mutation in iterations are denoted by $N_{\text{Cross-over}}$ and N_{Mutation} respectively and are calculated as follows:

$$N_{Cross-over} = \left[P_C . N_{pop}\right], \qquad N_{Mutation} = \left[P_m . N_{pop}\right], \tag{21}$$

Where [.] represent the largest integer not greater than the number within the bracket.
The basic steps involved in the proposed Elitist NSGA-II of this paper are explained as follows:

1- **Population initialization:** A chromosome consisting of eight genes $(n_1, n_2, h_1, h_2, \gamma, w_n, w_h, H)$ is generated that each gene represents a decision variable. Then, objective functions and the ATS_0 related to this chromosome are calculated. If the solutions satisfy the ATS_0 constraint, the chromosome is saved as a population member. This procedure continues until the size of the population equals to N_{POP}. An example of two chromosomes that are generated in this step is illustrated in figure (1).

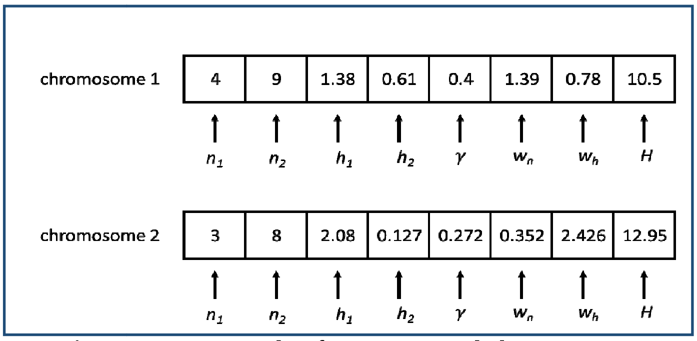

Figure 1. An example of two generated chromosomes

2- **Non-Dominated sort:** In this step, initialized population members have been sorted based on non-domination method. Then, a rank equal to its non-domination level is assigned to each solution. After that, the optimal solutions from a dominant boundary are defined as follows:

Suppose s_1 and s_2 are two arbitrary and variable solutions of population. Hence:

- Solution s_1 dominates s_2 if two objective functions $(E(A)$ and $\overline{AATS})$ of s_1 are equal or less than the objective functions of s_2.
- If there is no other solution in the population that dominates s_1, solution s_1 belongs to the dominant boundary.

3- **Crowding distance:** The crowding distance between the solutions belonging to the same rank is calculated to obtain the most promising solutions.

4- **Selection:** The crowded comparison operator is used to choose the $N_{\text{Cross-over}}$ and N_{Mutation} of the best solutions as parent chromosomes to do crossover and mutation, respectively.

5- **Cross-over:** In this step, $N_{\text{Cross-over}}$ of the best solutions that are achieved in the previous step is chosen as parent chromosomes and are divided to half of $N_{\text{Cross-over}}$ pairs. We assume that in each pair three similar genes of chromosomes are replaced with each other randomly. This step is done for the chromosomes of figure (1) and it is illustrated in figure (2). As illustrated in Figure (2), the second, fourth and eighth genes are replaced with each other, while the other genes are fixed.

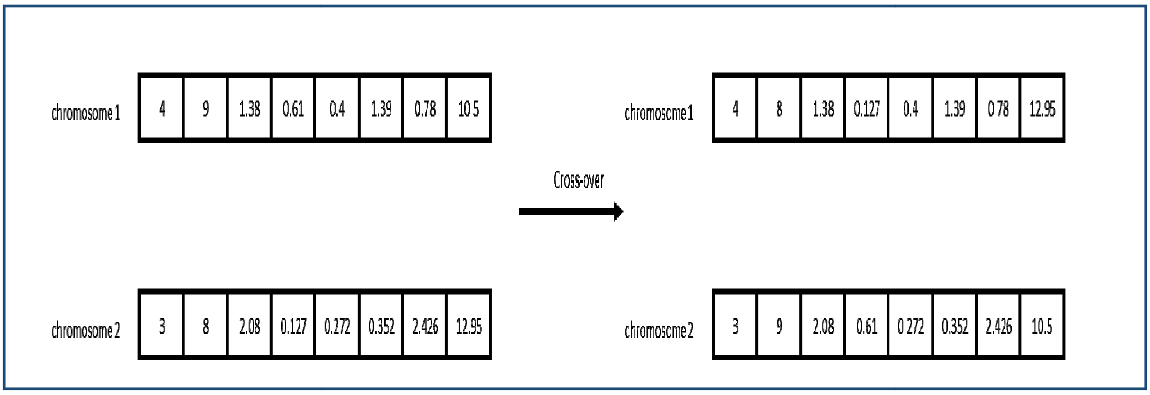

Figure 2. An Example of the Cross-Over on the Chromosomes of Figure (1)

6- **Mutation:** In this step, two genes of each chromosome of $N_{Mutation}$ chromosomes which are chosen in the step 4 are mutated by size *d*. Similar to (Niaki et al., 2010), *d* is obtained by multiplying three factors: (i) constant value denoted by C_m, (ii) length of the feasible range within which the gene generation is performed and (iii) a standardized normal random number. Without loss of generality, in this paper it is assumed that C_m=0.01. In the optimization model, the maximum value of possible upper bound control limit H_{max}, the minimum and maximum values of possible sampling intervals between successive samples, h_{min} and h_{max} and the maximum values of possible sampling size, n_{max} are add to keep chart more practical. In this paper, the values of H_{max}=15, h_{min}=0.01, h_{max}=8, and n_{max}=20 are considered according to Faraz and Saniga (2011) to eliminate the misleading results.

As an example of mutation, the first chromosome of figure (1) is selected for mutation. The procedure is as follows:

In order to do mutation, the fourth and eighth genes are selected randomly. The feasible ranges of these genes are 1 and 13, respectively. We generate two standardized normal random numbers; say 0.1241 and 0.3426, so the value of *d* for the fourth and eighth genes would be as follows.

$$d_{\gamma} = (0.01)(1)(0.1241) = 0.0012$$

$$d_H = (0.01)(13)(0.3426) = 0.0445$$

These values are added to the 0.61 and 10.5, respectively and the values of the fourth and eighth genes are equal to 0.622 and 10.545, respectively.

It worth to mention that if new generated chromosomes after the cross-over and mutation do not satisfy each one of the constraints $n_1 < n_2$, $h_2 < h_1, w_n < H$, or $w_h < H$, the genes of n_1 and n_2, h_1 and h_2, w_n and *H*, or w_h and *H* are replaced with each other until the all constraints are satisfied. Also, if the value of γ after the mutation was more than 1, it is replaced by the standardized random number between 0 and 1. In addition, if the genes of n_1 or n_2 would not be integer after mutation, these parameters are rounded to the nearest integer number.

7- Recombination and selection: Combine the cross-over, mutation and initialized population and sort the extended population based on non-domination method. Choose new population by size of N_{POP} from the sorting fronts starting from the best solutions and use the crowding distance method to ensure diversity and preventing convergence if only some solutions from the front should be chosen for the next generation.
8- **Stopping rule**: Repeat the steps 3 to 7 until a stopping criterion, the number of iterations equals to 50, is met.

The dominant boundary includes all non-dominated optimal solutions of the problem that is called as Pareto set while its image in objective space is named Pareto front.

5- Numerical example

In this section, a numerical example is provided to illustrate the performance of the proposed model. It is assumed that two important quality characteristics of a process are monitored (p=2) and we wish to apply the VSSI-MEWMA-DWL control chart to monitor the process. It will also be assumed that there are five assignable causes in the process $(s = 5)$ that their occurrences lead to the shift in the process mean vector. The first three assignable causes change the mean of the first quality characteristic and the last two assignable causes change the mean of the second quality characteristic. The magnitudes of the shifts are assumed $\delta = 1.87, 2.98, 1.2, 3, 3.31$. As mentioned previously in each time just one of the assignable causes can take place. The fixed and variable costs of sampling are 55 and 6, respectively. Since the out-of-control process is not self-announced, the costs of repairing the out-of-control process and removing the five assignable causes are 6, 9, 10, 8, and 11, respectively. The other required input parameters of the example are represented in table (3).

Table 3. Input parameters of the economic-statistical models

$c'_0 = 10$	$r_1 = [0,1,0,1,0]$	$T_0 = 2$	
$c'_1 = [15,20,25,30,35]$	$r_2 = [1,0,1,0,1]$	$T_1 = [4,8,12,16,20]$	$\mathbf{K} = \begin{bmatrix} 1500 & -1000 \\ -1000 & 8000 \end{bmatrix}$
$E = 0.05$	$a'_3 = 9$	$T_2 = [10,15,20,25,30]$	

As mentioned in the previous sections, the statistical computation is done based on the Markov chain approach proposed by Lee (2013). The parameters of m, m_1, m_2 are the input parameters in Markov chain algorithm for ATS_0 and AATS computations. In this paper, like most practical applications, m = 25 for the in-control state and $m_1 = m_2 = 5$ for the out-of-control state are suggested. Pareto optimal solutions of economic-statistical design of VSSI-MEWMA-DWL control chart are shown using NSGA-II algorithm in Table (4). The Pareto front for $\overline{AATS}$ and E(A)of the multi-objective economic-statistical design is shown in figure (3).

Table 4. Pareto optimal solutions of economic-statistical design of VSSI-MEWMA-DWL control chart

number	n_1	n_2	h_1	h_2	H	w_n	w_h	γ	E(A)	ATS_0	$\overline{AATS}$
1	5	8	2.12	0.78	12.68	1.06	0.53	0.54	40020	372.12	2.34
2	5	8	2.08	0.65	12.56	1.06	0.51	0.52	40036	368.61	2.23
3	5	7	1.82	0.96	12.16	1.04	0.86	0.37	40210	402.22	1.94
4	5	7	1.93	0.93	13.64	1.18	0.91	0.41	40680	426.68	1.69
5	5	7	1.85	0.86	13.58	1.15	0.83	0.28	42864	435.21	1.26
6	5	7	1.79	0.67	13.74	0.86	0.52	0.36	48356	552.14	0.88
7	4	6	1.54	0.52	13.12	0.95	0.59	0.53	54410	394.87	0.59
8	4	6	1.95	0.48	12.96	0.94	0.62	0.54	63534	532.66	0.43
9	4	5	2.05	0.24	11.81	0.94	0.69	0.29	76522	608.14	0.28
10	3	5	1.78	0.18	11.95	0.89	0.35	0.39	96970	395.9	0.19
11	3	5	2.09	0.12	12.85	0.83	0.37	0.53	118604	453.97	0.15
12	3	4	1.65	0.12	12.69	1.15	0.93	0.42	135824	501.12	0.13
13	2	5	1.58	0.14	13.64	1.04	0.59	0.57	164314	695.31	0.11
14	2	5	1.61	0.07	13.68	1.02	0.88	0.33	186456	558.23	0.09
15	2	5	1.24	0.7	12.45	0.93	0.71	0.45	203674	625.9	0.08
16	2	3	1.37	0.06	12.53	1.12	0.96	0.56	225128	539.41	0.08
17	2	3	1.06	0.06	12.84	0.86	0.53	0.37	252682	562.16	0.07

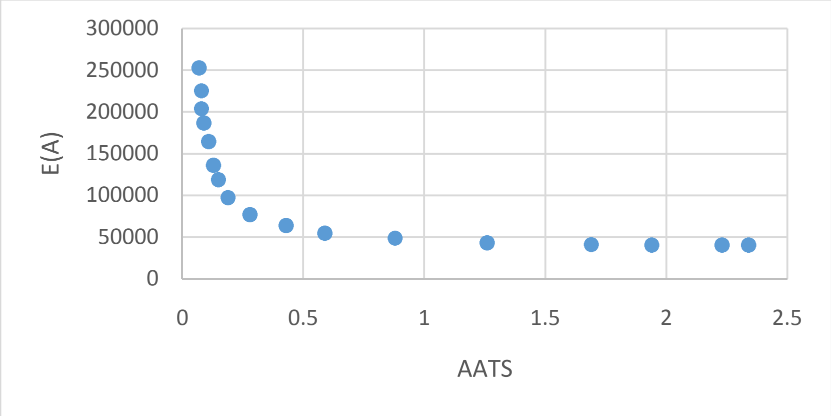

Figure 3. The Pareto-front graph of the proposed model.

6- Comparison of Pareto optimal solutions and sensitivity analyses

6-1- Comparison of Pareto optimal solutions of FSR-MEWMA, VSI-MEWMA-DWL, VSS-MEWMA-DWL and VSSI-MEWMA-DWL control charts

The objective of this section is to compare the effectiveness of the economic-statistical design of the FSR, VSI, VSS, and VSSI-DWL MEWMA control charts. The Pareto optimal solutions of the FSR-MEWMA control chart are obtained by restricting $n_1 = n_2 = n_0$ and $h_1 = h_2 = h_0$ and $w_n = w_h = H$ and are shown in Figure (4). The results are represented in Table (5). Based on the value of the cost function, the first solution in Table (4) and the first solution in Table (5) have the minimum cost value in the Pareto optimal solution of the VSSI-MEWMA-DWL and FSR-MEWMA control chart, respectively. We find the $(68153 - 40020) / 68153 \times 100 = 41.27\%$ increase in terms of the cost by using the solution of the FSR-MEWMA control chart instead of the VSSI-MEWMA-DWL control chart with minimum cost value. The VSS-MEWMA-DWL and VSI-MEWMA-DWL control charts are resulted from$h_1 = h_2$ and $n_1 = n_2$, respectively. tables (6) and (7) show the Pareto optimal solutions of the economic-statistical design of the VSS-MEWMA-DWL and VSI-MEWMA-DWL control charts and the results are shown in figures (5) and (6), respectively. According to the minimum cost value that is provided in tables (6) and (7), the VSSI-MEWMA-DWL control chart reduces the cost rather than the VSS-MEWMA-DWL and the VSI-MEWMA-DWL control charts by $(65353 - 40020) / 65353 \times 100 = 38.76\%$ and $(54104 - 40020) / 54104 \times 100 = 26.03\%$, respectively.

Table 5. Pareto optimal solutions of economic-statistical design of FSR-MEWMA-DWL control chart.

number	n_0	h_0	H	γ	E(A)	ATS_0	$\overline{AATS}$
1	8	1.96	12.96	0.43	68153	430.37	1.82
2	8	1.78	12.82	0.51	76886	456.55	1.27
3	5	2.15	11.63	0.42	86511	465.67	0.85
4	6	1.73	11.85	0.42	101019	590.79	0.62
5	5	1.42	13.05	0.36	121670	422.51	0.4
6	5	0.92	10.83	0.28	154182	569.95	0.27
7	7	1.06	12.76	0.28	188580	650.71	0.21
8	7	1.34	13.56	0.32	215960	423.61	0.16
9	6	0.93	11.68	0.53	261259	485.75	0.13
10	3	0.86	12.96	0.43	296465	536.19	0.11
11	4	0.13	10.74	0.44	338098	743.92	0.09
12	4	0.08	11.08	0.41	402979	597.31	0.09
13	2	0.04	11.53	0.38	401764	669.73	0.09

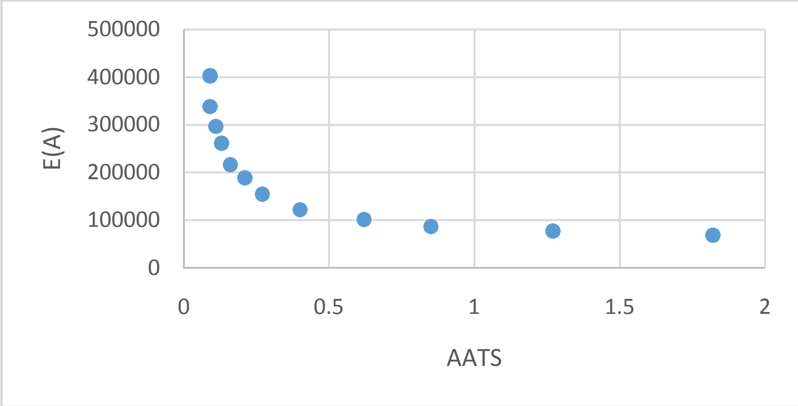

Figure 4. The Pareto-front of FSR MEWMA control chart.

Table 6. Pareto optimal solutions of economic-statistical design of VSS-MEWMA-DWL control chart.

number	n_1	n_2	h	H	w_n	w_h	γ	E(A)	ATS_0	$\overline{AATS}$
1	5	8	1.71	11.69	0.93	0.56	0.37	65353	431.27	3.29
2	5	7	1.65	11.64	0.82	0.64	0.45	74385	470.59	2.46
3	4	6	1.62	13.32	1.41	1.11	0.39	85866	499.22	1.85
4	3	5	1.63	13.65	2.12	1.06	0.43	103419	509.19	1.03
5	3	5	1.76	13.36	1.59	2.29	0.39	131055	646.03	0.69
6	3	4	1.49	13.11	2.29	1.03	0.46	160293	461.99	0.54
7	4	5	1.57	12.75	0.86	1.26	0.36	183566	623.21	0.42
8	5	10	1.74	11.85	0.61	2.14	0.51	222070	711.52	0.33
9	2	5	1.36	13.47	1.67	0.87	0.56	251995	463.2	0.27
10	3	6	1.22	12.26	0.97	1.35	0.61	287384	531.14	0.23
11	5	6	1.69	13.65	2.29	0.45	0.42	325532	586.31	0.23

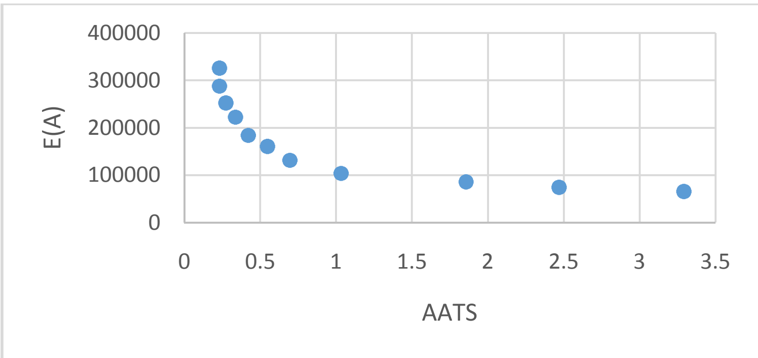

Figure 5. The Pareto-front of VSS-MEWMA-DWL control chart.

Table 7. Pareto optimal solutions of economic-statistical design of VSI-MEWMA-DWL control chart.

number	n	h_1	h_2	H	w_n	w_h	γ	E(A)	ATS_0	$\overline{AATS}$
1	5	1.92	0.86	13.16	1.58	2.27	0.45	54104	501.48	2.03
2	5	1.06	0.72	12.86	1.62	0.96	0.51	57009	676.48	1.51
3	4	2.12	0.36	13.12	1.74	1.13	0.61	64313	772.34	1.06
4	3	2.39	0.54	13.24	2.01	1.04	0.43	72365	502.8	0.71
5	2	3.12	0.56	12.91	1.54	1.28	0.67	84500	576.54	0.52
6	6	1.85	0.37	12.72	1.25	1.58	0.52	101774	636.42	0.34
7	8	3.25	0.65	12.91	1.41	1.56	0.42	128970	883.04	0.23
8	4	1.71	0.31	13.24	1.56	1.09	0.31	157743	708.95	0.18
9	5	3.18	0.95	12.53	1.37	0.24	0.53	180645	794.89	0.16
10	3	2.06	0.67	13.27	1.58	0.82	0.45	218537	685.05	0.13

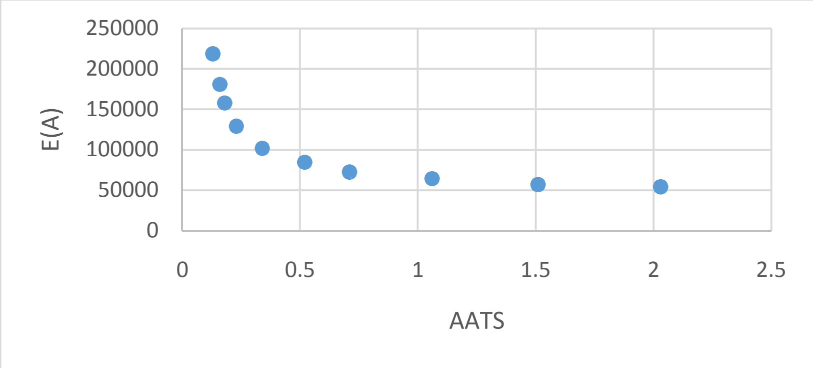

Figure 6. The Pareto-front of VSI-MEWMA-DWL control chart

6-2- Comparison of Pareto optimal solutions of economic-statistical, economic, and statistical design of VSSI-MEWMA-DWL control chart

By considering the minimum value of the $\overline{AATS}$ as the only objective function and solving the model, we can compare the proposed model with statistical design. The first row in Table (8) represents the optimum values of the control chart parameters in the statistical design of the VSSI-MEWMA-DWL control chart. We compare this result with the first row in Table (4) that has minimum value of the AATS. Although the statistical design has the $\overline{AATS}$ less than the proposed model, but it leads to $(52635 - 40020) / 52635 \times 100 = 23.96\%$ more cost value approximately.

Table 8. Optimal solutions of statistical and economic design of VSSI-MEWMA-DWL control chart.

design	n_1	n_2	h_1	h_2	H	w_n	w_h	γ	E(A)	ATS_0	$\overline{AATS}$
Statistical	4	5	2.52	0.53	12.94	1.33	0.48	0.52	52635	567.42	0.59
Economic	3	6	2.58	0.01	13.97	1.49	1.16	0.46	36914	624.03	0.05

Also, to compare the effectiveness of the proposed model with economic design, we solve the model by considering the minimum cost value as the only objective function. The second row in Table (8) represents the optimum values of the parameters in the economic design of the VSSI-MEWMA-DWL control chart. By comparing this result with the last row in Table (4), we conclude that although the economic design has the less cost value; however, $\overline{AATS}$ of the proposed model is $(0.07 - 0.05) / 0.07 \times 100 = 28.57\%$ less than the $\overline{AATS}$ obtained from the economic model.

6-3- Sensitivity analyses of the cost parameters

In this section, we investigate the effects of four cost parameters of the Lorenzen and Vance cost function by using design of experiments. Fixed and variable costs of sampling, the cost of search for the false alarms and the costs of repairing and removing the assignable causes are the four parameters that are considered in sensitivity analyses of the proposed model.Because these parameters are not dependent on the quality characteristics and they are usually uncertain.Table (9) shows the range that is considered for each parameter. Based on these ranges, we consider a 2^4 factorial design that is represented in Table (10). In this table, the values of +1 and -1 are obtained by the following Equation:

$$X_i = \frac{\text{the parameter value-the average value of the parameter}}{\text{the average value of the parameter - the minimum value of the parameter}} \qquad \forall i = 1,2,3,4$$

Table 9. The range of the Lorenzen and Vance's cost parameters.

Parameter	Range	Minimum value	Average value	Maximum value
a_1	10-100	10	55	100
a_2	2-10	2	6	10
a_3'	3-15	3	9	15
a_{31}	3-9	3	6	9
a_{32}	4-14	4	9	14
a_{33}	5-15	5	10	15
a_{34}	6-10	6	8	10
a_{35}	7-15	7	11	15

The 2^4 design experiments are illustrated in table (10). The average cost value of the Pareto optimal solutions of each experiment is represented in table (10). Furthermore, the results of these experiments are shown in table (11) and the Pareto optimal solutions of these 24 runs are illustrated in figure (7). Also, we use the figure (8) as an example to illustrate the sensitivity analyses for average cost value. In this figure, E(A) is considered for the main model and runs 1,2,3,5, and 9. Note that all parameters in run 1 are set in lower bound. Also, all parameters in runs 2, 3, 5, and 9 are set on the lower bound except one parameter which is set on the upper bound.

Table 10. 2^4 factorial designs

Run	Parameters				Average cost value of each run
	X_1	X_2	X_3	X_4	
1	-1	-1	-1	-1	48136
2	1	-1	-1	-1	91667
3	-1	1	-1	-1	69763
4	1	1	-1	-1	56991
5	-1	-1	1	-1	58725
6	1	-1	1	-1	71338
7	-1	1	1	-1	80757
8	1	1	1	-1	124372
9	-1	-1	-1	1	73488
10	1	-1	-1	1	66368
11	-1	1	-1	1	85158
12	1	1	-1	1	149599
13	-1	-1	1	1	80102
14	1	-1	1	1	10733
15	-1	1	1	1	76620
16	1	1	1	1	77650

Table 11. The Pareto optimal solutions of 2^4 factorial design.

Run	n_1	n_2	h_1	h_2	H	w_n	w_h	γ	E(A)	ATS_0	$\overline{AATS}$
	3	4	2.23	0.62	13.68	1.07	0.38	0.55	38819	656.18	2.55
	3	5	2.71	0.63	13.65	1.06	0.62	0.52	38834	749.16	2.54
1	2	4	2.36	0.71	12.96	1.09	0.52	0.51	39003	487.76	2.11
	3	5	2.35	0.68	12.76	1.21	0.79	0.48	39459	559.24	1.84
	2	5	1.92	0.91	13.28	1.35	0.42	0.62	41578	617.37	1.37
	4	5	2.34	0.68	11.97	1.21	0.63	0.55	46905	856.58	0.95
	5	7	1.89	0.59	12.54	1.09	0.43	0.56	52777	687.68	0.64
	3	5	1.75	0.41	13.28	1.09	0.91	0.52	61627	771.43	0.48
	4	6	1.07	0.32	13.09	1.32	0.95	0.53	74226	664.49	0.35
	4	7	2.86	0.93	13.62	1.63	0.92	0.39	37832	493.61	3.41
	4	5	2.23	0.75	13.62	0.97	1.13	0.43	44971	626.27	2.83
2	3	6	1.95	0.96	13.68	1.29	0.85	0.42	50601	447.86	2.47
	2	6	1.96	0.76	13.29	1.38	0.76	0.61	59086	604.14	1.83
	3	7	1.85	1.06	12.85	1.02	0.93	0.54	71165	689.75	1.28
	3	6	2.34	0.79	12.86	2.16	0.98	0.52	90182	449.02	0.86
	4	8	2.26	0.56	11.96	2.19	1.03	0.53	110301	514.89	0.67
	2	5	2.35	0.36	11.89	1.64	0.96	0.38	126316	568.36	0.48
	3	5	2.01	0.23	13.04	1.53	0.36	0.49	152812	788.55	0.27
	4	8	1.77	0.86	12.62	1.78	0.56	0.41	32904	542.46	1.18
	5	9	2.18	0.78	12.21	2.04	0.85	0.62	41697	598.84	0.8
3	6	9	2.69	0.7	12.21	1.11	0.45	0.33	50999	830.86	0.63
	3	9	2.11	0.7	12.21	1.11	0.68	0.5	58404	667.04	0.59
	4	6	2.66	0.77	12.83	1.08	0.46	0.56	70655	748.28	0.46
	4	6	2.51	0.73	13.06	1.64	1.22	0.54	80176	644.55	0.42
	2	7	2.51	0.29	12.69	1.74	0.92	0.54	87579	448.13	0.38
	7	10	1.75	0.25	13.07	0.77	0.82	0.62	96805	604.53	0.33
	3	9	2.66	0.92	13.46	2.12	1.34	0.68	108653	690.17	0.26
	5	8	1.44	1.1	10.98	1.72	1.05	0.42	49300	356.06	2.09
	5	8	1.62	0.96	11.91	2.58	1.13	0.41	49319	408.24	2.08
4	6	7	1.55	0.95	11.57	0.71	1.3	0.59	49533	450.68	1.73
	3	5	2.08	0.78	12.44	0.71	1.01	0.52	50112	625.3	1.5
	2	5	1.57	0.95	12.65	1.78	0.82	0.67	52804	502.09	1.12
	2	4	1.51	0.77	10.33	2.63	0.91	0.36	59569	563.14	0.79
	4	6	1.71	0.42	12.67	3.15	1.04	0.38	67026	606.54	0.54
	4	7	2.06	0.17	12.67	1.78	0.89	0.38	78266	486.94	0.39
	5	8	1.44	1.1	10.98	1.72	1.05	0.42	49300	356.06	2.09
	3	5	2.25	0.64	11.47	0.84	1.08	0.38	47359	400.19	3.49
	3	5	1.84	0.79	12.06	1.04	0.35	0.36	47377	321.28	3.47
5	4	6	2.24	0.98	12.27	0.62	0.47	0.36	47583	360.41	2.89
	5	7	1.81	0.56	11.92	0.82	0.94	0.42	48139	388.18	2.52
	5	7	2.64	0.75	11.99	1.28	0.7	0.53	50725	311.64	2.32
	5	9	1.69	0.85	12.48	0.98	0.49	0.53	57224	349.59	1.61
	2	7	2.09	0.69	11.25	0.8	0.49	0.66	64387	301.13	1.08
	7	9	2.58	0.75	11.78	0.93	0.61	0.38	75184	274.79	0.81
	6	8	0.48	0.68	12.48	1.07	0.53	0.51	90555	299.09	0.59

Table 11. Continued

Run	n_1	n_2	h_1	h_2	H	w_n	w_h	γ	E(A)	ATS_0	$\overline{AATS}$
6	3	4	1.66	0.76	13.38	0.66	0.93	0.52	49319	378.873	3.45
	3	5	2.04	0.83	13.87	1.65	0.76	0.5	49533	379.01	2.86
	3	5	2.52	0.8	12.1	2.44	0.84	0.5	50112	380.66	2.5
	2	6	1.35	0.87	12.1	2.92	0.96	0.58	52804	385.11	1.86
	2	7	1.7	0.88	12.05	1.65	0.82	0.63	59569	405.8	1.29
	5	7	1.6	0.72	13.11	1.27	2.15	0.38	67026	457.79	0.87
	4	6	1.6	0.48	13.3	0.88	1.49	0.58	78266	515.1	0.65
	4	6	1.12	0.38	11.46	0.59	1.04	0.31	94267	601.47	0.47
	4	8	1.72	0.35	14.02	0.44	0.75	0.71	97484	724.44	0.41
7	3	5	2.23	0.58	13.34	1.21	0.64	0.39	53757	344.93	2.51
	2	5	2.13	0.55	13.14	1.16	0.73	0.44	53990	389.12	2.08
	2	4	2.87	0.74	13.67	1.56	0.55	0.53	54622	437.83	1.82
	1	4	2.16	0.56	12.32	1.17	0.62	0.49	71717	511.24	1.13
	1	5	2.08	0.54	12.91	0.78	0.74	0.47	83744	615.77	0.84
	4	5	2.35	0.61	13.67	0.88	0.69	0.64	100865	327.34	0.61
	5	7	1.86	0.52	13.48	0.67	0.66	0.46	104307	322.04	0.53
	5	7	1.93	0.31	12.65	0.75	0.89	0.52	123055	322.15	0.45
	3	5	2.23	0.58	13.34	1.21	0.64	0.39	53757	344.93	2.51
8	3	7	1.84	0.76	11.19	1.32	0.86	0.42	58973	446.72	2.49
	3	6	2.28	0.72	12.14	0.43	0.98	0.37	62139	470.72	1.85
	2	5	1.36	0.72	11.8	0.57	0.84	0.31	78877	531.03	1.20
	4	5	1.8	0.5	12.68	1.15	2.21	0.37	92105	597.5	0.88
	4	6	2.81	0.77	12.9	0.86	1.11	0.35	110933	697.7	0.57
	5	6	2.15	0.24	10.53	1.56	0.36	0.4	140577	396.23	0.39
	2	4	1.76	0.21	12.92	1.08	0.48	0.44	171939	446.72	0.30
	1	4	2.04	0.13	12.08	0.72	0.96	0.26	196903	439.49	0.27
	1	3	2.35	0.17	13.88	0.54	1.04	0.4	206903	487.29	0.25
9	7	10	1.46	0.96	12.22	0.39	0.4	0.43	48918	416.35	3.18
	4	8	1.65	1.08	12.71	0.31	0.34	0.35	49130	468.47	2.64
	4	8	1.58	1.3	11.45	0.35	0.9	0.39	49706	547.02	2.31
	5	7	2.12	1.21	12.06	1.03	0.75	0.45	65262	658.87	1.43
	5	6	1.6	1.16	11.15	0.62	1.16	0.38	76207	477.99	1.06
	3	6	1.54	1.58	11.6	0.43	1.23	0.56	91787	503.67	0.77
	2	5	1.74	1.13	10.46	0.31	1.28	0.7	94919	568.2	0.67
	2	5	2.1	1.28	10.95	0.22	1.96	0.48	111980	639.32	0.57
	7	10	1.46	0.96	12.22	0.39	0.4	0.43	48918	416.35	3.18
10	3	6	1.24	0.67	12.86	1.66	0.91	0.3	52278	619.38	3.72
	3	7	1.31	0.56	13.82	1.24	0.78	0.34	52504	746.53	3.08
	5	7	1.36	1.01	14.06	0.83	2.05	0.25	53118	423.96	2.7
	5	6	2.09	0.84	11.47	0.97	1.03	0.29	55972	513.08	2.08
	4	7	0.62	0.9	14.08	1.15	1.15	0.34	63143	578.82	1.39
	2	6	0.52	0.95	12.64	0.94	0.77	0.46	71047	651.27	0.93
	2	5	1.39	0.92	11.4	1.04	0.92	0.43	82961	760.49	0.7
	1	4	1.16	0.53	11.93	1.19	1.06	0.36	99923	718.16	0.56
	3	6	1.24	0.67	12.86	1.66	0.91	0.3	52278	619.38	3.72

Table 11. Continued

Run	n_1	n_2	h_1	h_2	H	w_n	w_h	γ	E(A)	ATS_0	$\overline{AATS}$
11	5	7	1.72	1.03	12.04	1.08	0.37	0.46	37839	422.54	1.39
	5	8	2.31	0.76	14.78	0.65	0.98	0.39	47951	475.43	0.94
	6	7	1.74	0.73	13.27	0.45	0.81	0.53	58648	555.15	0.74
	4	6	1.67	0.86	11.97	0.32	0.67	0.38	67164	524.26	0.69
	4	5	1.48	1.16	12.52	1.45	0.46	0.51	81253	380.33	0.54
	4	5	2.27	0.94	12.63	0.97	0.58	0.35	92202	341.98	0.48
	3	4	1.21	1.45	11.77	0.73	0.66	0.44	100715	399.32	0.49
	2	5	1.06	0.93	12.18	0.52	0.92	0.5	111325	480.97	0.38
	2	5	2.16	0.95	10.98	1.16	0.69	0.71	124951	348.93	0.36
	2	3	1.33	0.27	12.87	0.94	0.78	0.38	129532	367.67	0.35
12	5	7	1.08	0.84	12.9	1.27	0.61	0.62	78996	455.22	2.13
	5	9	1.04	1.03	13.12	1.18	0.75	0.55	95145	429.89	1.17
	4	8	1.6	0.92	12.75	1.65	0.67	0.38	120570	311.87	0.78
	6	8	0.46	0.83	12.82	0.48	0.6	0.57	147469	280.42	0.62
	2	8	0.65	0.87	13.35	0.67	0.63	0.56	168880	327.44	0.47
	2	7	0.47	0.88	14.2	0.49	0.64	0.43	204304	394.39	0.38
	2	5	1.07	0.82	12.8	0.83	0.59	0.49	231835	286.12	0.31
13	5	6	1.25	0.96	13.16	0.68	0.65	0.42	53320	364.71	2.86
	3	5	1.77	0.85	12.17	0.77	0.97	0.31	53551	425.87	2.37
	2	4	0.51	1.31	12.66	1.08	2.24	0.4	54179	402.17	2.07
	2	5	0.74	0.7	11.42	0.95	1.92	0.28	71135	291.76	1.28
	4	6	1.77	0.61	13.65	0.79	1.58	0.35	83065	262.34	0.95
	4	7	1.56	0.96	13.26	0.54	2.24	0.4	100047	306.33	0.69
	4	7	2.4	0.52	13.34	0.81	1.16	0.56	103461	368.96	0.6
	1	5	1.28	0.49	13.88	1.09	0.76	0.76	122058	413.63	0.51
14	3	5	2.53	1.06	10.82	0.83	0.74	0.49	61921	314.074	2.66
	3	6	2.01	1.5	8.83	0.68	0.61	0.4	65245	366.73	1.97
	2	6	2.85	0.78	10.84	0.56	0.5	0.33	82820	441.72	1.28
	4	7	1.48	0.51	13.52	0.39	0.34	0.22	96710	320.45	0.94
	4	5	0.97	0.54	13.51	0.49	0.44	0.29	116479	621.76	0.96
	4	5	1.02	1.33	12.59	0.56	0.5	0.33	147605	587.17	0.66
	7	9	1.39	0.52	13.03	1.91	1.7	0.57	180536	425.96	0.58
15	4	5	1.22	0.47	11.09	0.99	0.88	0.62	45864	494.66	2.25
	3	5	1.08	0.34	11.15	0.65	0.58	0.64	51740	358.86	1.57
	3	7	1.67	0.48	11.61	0.68	0.61	0.44	58218	322.67	1.05
	3	7	0.89	0.31	11.76	1.69	1.51	0.41	67981	376.78	0.76
	2	6	0.78	0.39	11.75	1.29	0.49	0.4	81878	453.82	0.5
	2	4	1.28	0.44	10.95	1.16	0.34	0.31	103757	508.76	0.34
	1	5	0.66	0.62	11.33	0.76	0.43	0.35	126906	402.3	0.26
16	4	5	2.34	1.08	11.21	0.85	0.57	0.42	61165	386.23	3.45
	4	6	1.32	2.29	13.76	0.56	0.47	0.35	61429	428.67	2.86
	3	6	1.72	0.66	14.47	0.48	0.89	0.23	62148	429.21	2.51
	3	5	1.13	0.92	14.46	0.36	0.6	0.31	65487	372.45	1.93
	2	4	2.33	0.85	13.47	1.07	0.78	0.35	73877	361.74	1.29
	5	7	2.32	0.7	13.03	0.96	0.89	0.59	83124	448.59	0.86
	2	5	1.55	0.91	11.74	0.65	0.99	0.69	97064	523.82	0.65
	1	5	1.12	0.39	11.98	0.49	0.63	0.41	116909	628.23	0.46

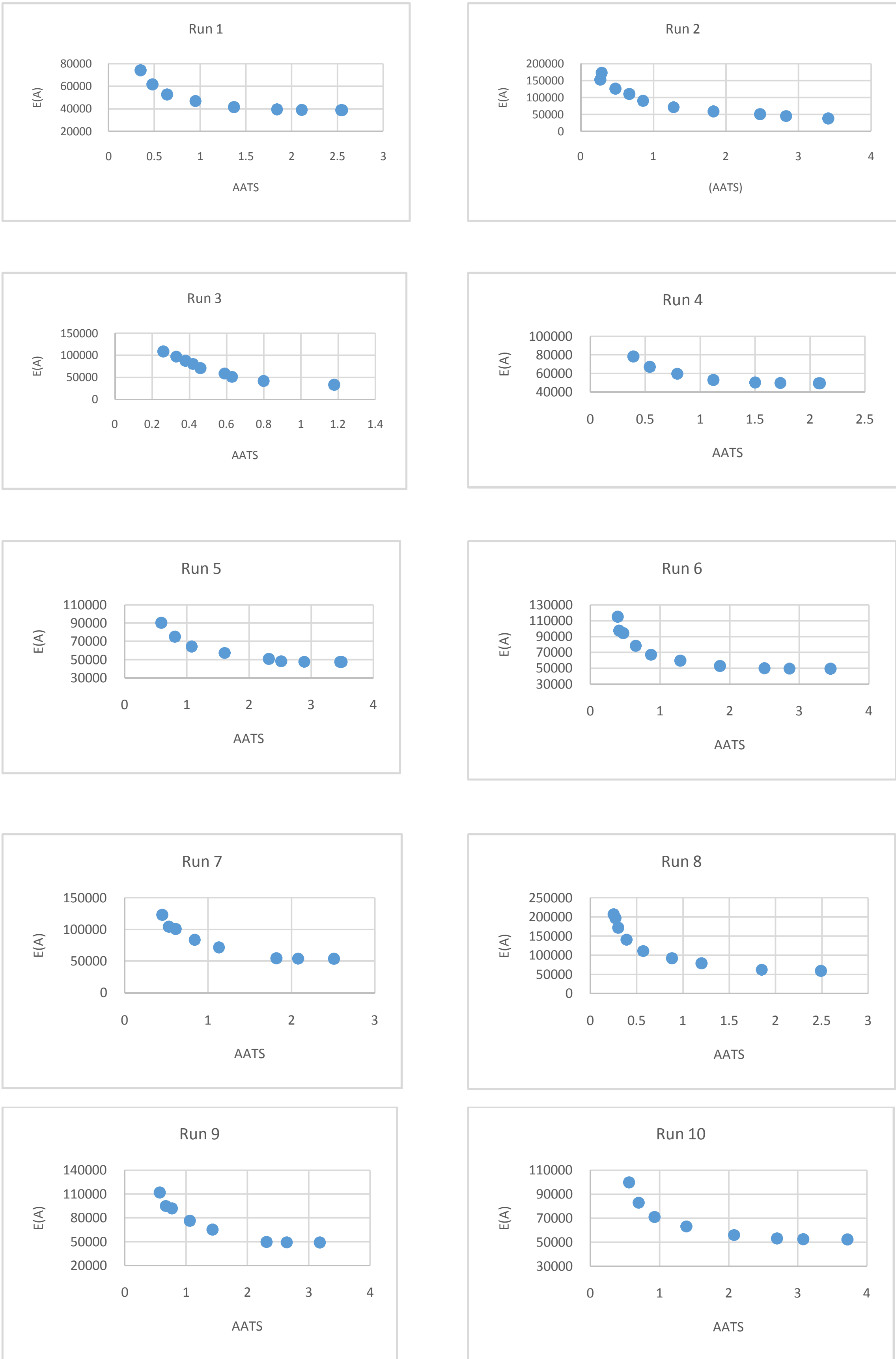

Figure 7. The Pareto fronts of 2^4 factorial design.

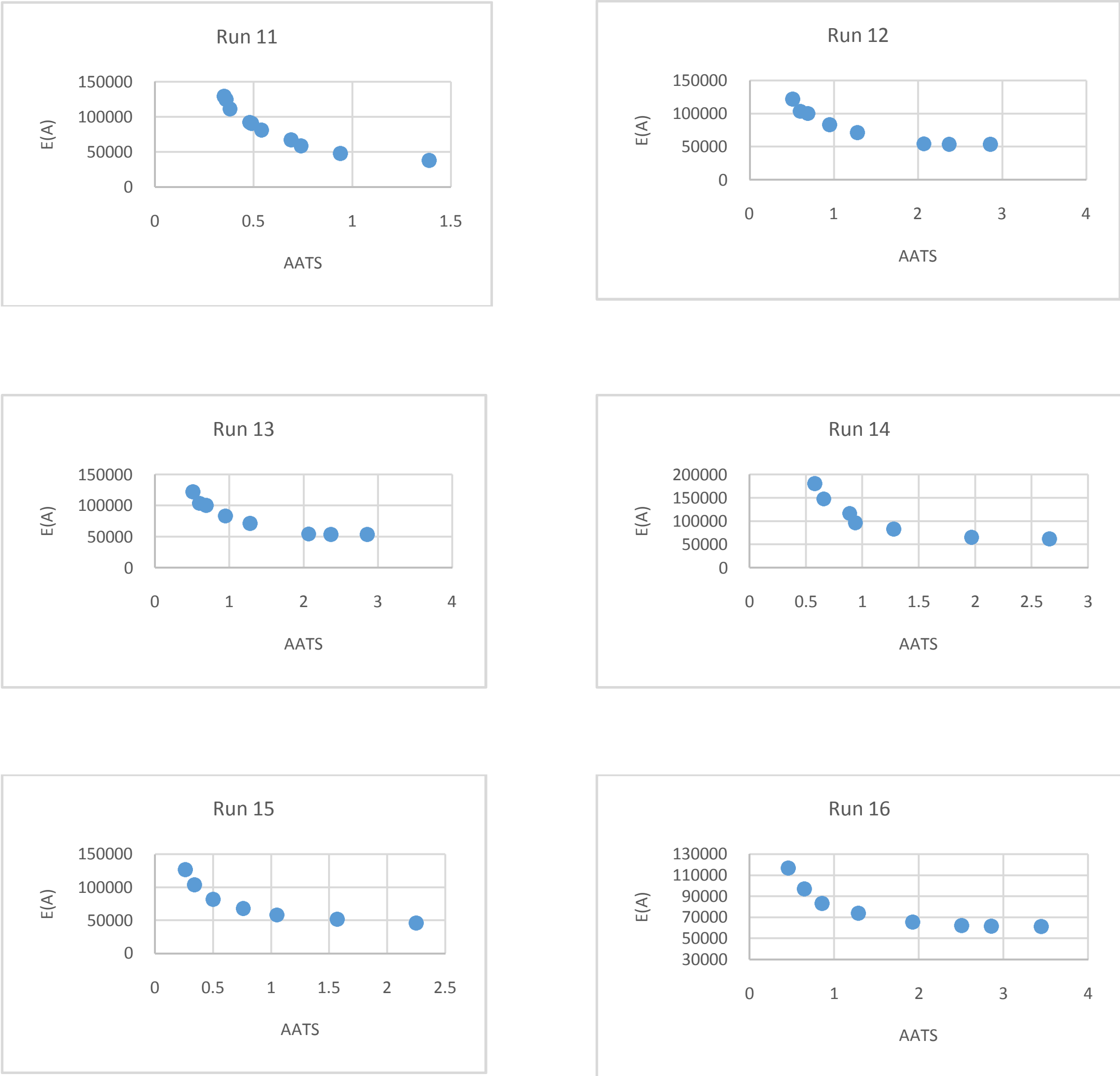

Figure 7. Continued

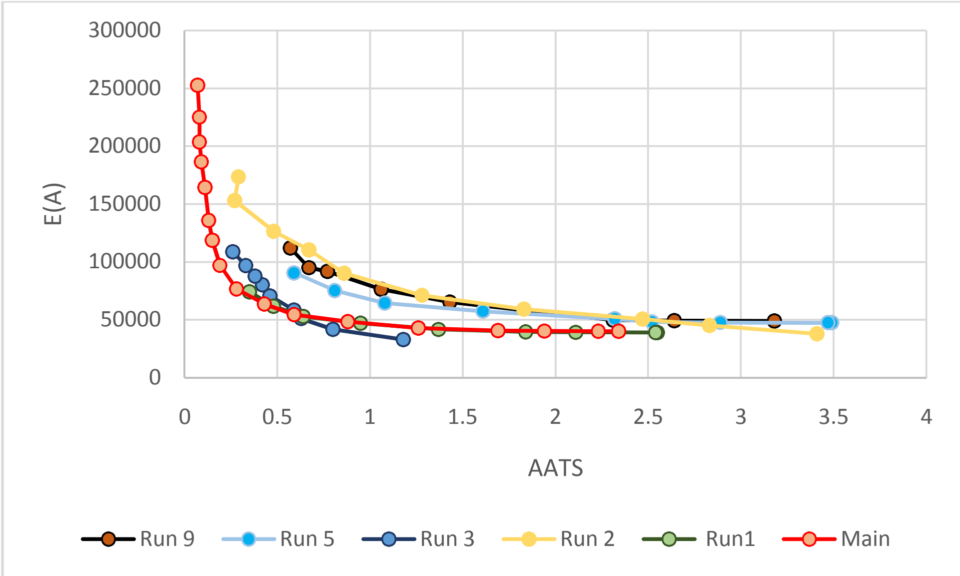

Figure 8. Sensitivity analyses on E(A) and AATS of the proposed model based on some runs in table (11).

As shown in figure (8), when the parameters are set on the corresponding center points (main parameters), the E(A) and AATS results are better than the results of runs in which the upper and lower bounds are considered for the parameters.

7- Conclusion and recommendations for future research

In this paper, we developed a multi-objective model using NSGA-II for the economic-statistical design of VSSI-MEWMA-DWL control chart for monitoring multivariate manufacturing process. We extended the Lorenzen and Vance cost function in order to consider multiple assignable causes that shift the mean of the process. Moreover, multivariate Taguchi loss approach was used to consider external costs as well as internal costs. The main contribution of this paper was proposing a multi-objective model by considering different assignable causes. A numerical example was provided to illustrate the proposed methodology. The obtained solutions defined a Pareto optimal set of solutions which increases the flexibility of the VSSI-MEWMA-DWL control chart in practice and provided a variety of solutions for decision makers and managers. Furthermore, the results showed that economic-statistical design of VSSI-MEWMA-DWL can provide better statistical measures while the cost is not significantly increased. In order to study the effects of different cost parameters on the solutions, some sensitivity analyses were performed. The results showed that underestimating the main parameters of the cost function leads to less expected cost in comparison with overestimation of these parameters. Economic-statistical design of VSSI-MEWMA control chart that considers shift in the mean and covariance matrix simultaneously and adding more objective functions to account for preventive maintenance are suggested for future researches.

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