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Investigating unpopularity of utility-based approaches in portfolio optimization; introducing an extension to the UTASTAR method

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Abstract

Despite the passing of more than 30 years from introducing the UTilitès-Additives (UTA) method and its extensive presentation in academic communities, this method is still not very popular among portfolio managers. Many portfolio managers still question the usefulness of the UTA method and prefer to rely on other multi-criteria decision making (MCDM) approaches. Therefore in this study, we examined the features of one of the most popular variants of the UTA methods, called UTASTAR, and on this basis, we have been developed this traditional approach in such a way that it would have more ability to meet the expectations of portfolio managers. In this way, to demonstrate how the proposed method can be applied in practice it is implemented in Tehran stock exchange (TSE) and to validate its efficiency, we designed an experiment, which is a novel approach in operations research but common in psychology and experimental economics. From the experimental results, we can extract that the outstanding features of the proposed method, compared to the original UTASTAR method are as follows: (1) it can provide a more accurate estimation of the portfolio managers' attitude because in addition to the sequential preferences of the alternatives it also considers the relative preferences; (2) it has always feasible solutions although it requires more comparison data and (3) it allows portfolio managers to observe the inconsistency of their decisions and take corrective action if desired.

Keywords: Multi criteria decision making, preference disaggregation analysis, portfolio optimization, behavioral finance

1-Introduction

Portfolio optimization is one of the most important areas of financial management. In simple terms, this problem involves creating a portfolio of stocks in order to maximize the investor's utility. Since the mean-variance model of Markowitz (1952) was introduced as a basic framework for modern portfolio theory, several methodologies have been proposed to support the decision making process in portfolio optimization. Other approaches besides the Markowitz's model include the capital asset pricing model (CAPM), the arbitrage pricing theory (APT) and also various kinds of multi-objective optimization models.

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In the same vein, MCDM approaches are one of the most attractive research areas in which portfolio optimization has been widely taken into account (Abdollahi Moghadam et al., 2019; Amiri et al., 2010; Argyris et al., 2011; Ashraf & Khawaja, 2016; Ballestero et al., 2012; Bilbao-Terol et al., 2012; Bilbao-Terol et al., 2016; Chanvarasuth et al., 2019; Chen & Hung, 2009; Curatola, 2017; De et al., 2018; Emamat et al., 2022; Fellner et al., 2004; Galankashi et al., 2020; Gherzi et al., 2014; Ghosh, 2021; Gladish et al., 2007; Guangul et al., 2021; Ho et al., 2011; Hurson et al., 2012; Joshi et al., 2022; Kocadağlı & Keskin, 2015; Li & Wang, 2020; Mansour et al., 2019; Meghwani & Thakur, 2018; Mokhtar et al., 2017; Ramezani, 2022; Ruiz et al., 2020; Şakar & Köksalan, 2013; Tamiz & Azmi, 2019; Utz et al., 2014; Vezmelai et al., 2015; Xidonas et al., 2009; Xidonas et al., 2010).

In this regard and for further insights Zopounidis et al. (2015) and Almeida-Filho et al. (2020) provided two comprehensive overviews of financial modeling through MCDM approaches, which can be summarized in Figure 1 and Table 1. According to the results of these researches, more than a quarter of the products are in the field of portfolio optimization and by applying a wide range of techniques, it has occupied a significant part of attention.

Since the main purpose of portfolio optimization is to maximize the investor's utility, it would be necessary to create a utility function that represents the investor's judgmental policy and preferences. But as shown in Table 1 Zopounidis et al. (2015) and Almeida-Filho et al. (2020) claimed that, despite the existence of such a fact, the application of utility-based approaches has not been greatly addressed in the literature (less than 3% of the relevant literature has been devoted to this area). In this regard, it should be noted that utility is not an emerging phenomenon and has a historical background. Since Bernoulli's time (1738), it has been approved that the main purpose of the decisions must be to maximize the decision maker's utility (Bernoulli, 1954). However, this finding was neglected for more than 200 years until 1953 when Von Neumann and Morgenstern (1953) again referred to this issue. Therefore, this research field still has significant potential for doing more.

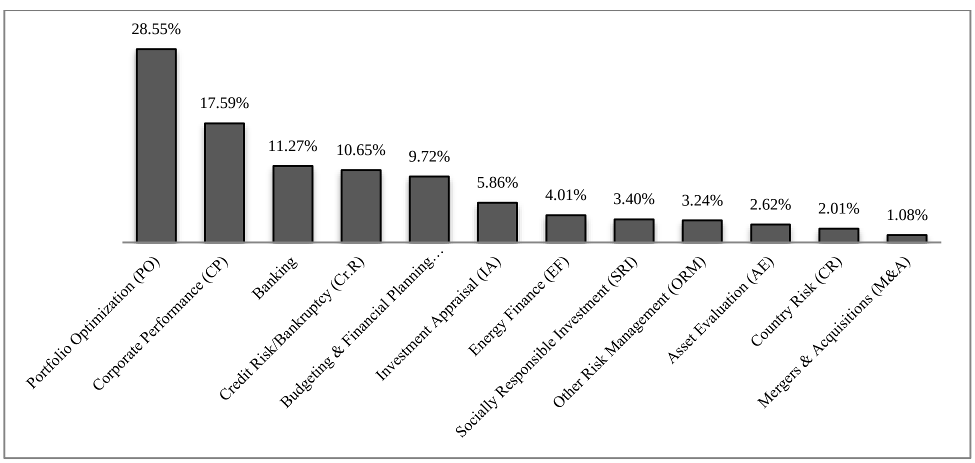

Fig 1. Distribution of MCDM papers on financial areas
Source: Almeida-Filho et al. (2020)

Table 1. MCDM methods and financial areas

Method	PO	CP	Banking	Cr.R	B&FP	IA	EF	SRI	AE	ORM	CR	M&A	Total
MO	74	1	4	2	17	2	8	5	1	3	0	0	117
MOEA	56	0	0	7	8	1	0	0	1	0	0	0	73
GP	36	3	2	4	11	3	1	5	0	0	0	0	65
AHP	6	34	17	22	11	11	6	6	3	7	1	2	126
TOPSIS	5	34	20	16	4	2	3	6	4	5	2	2	103
VIKOR	1	16	9	1	3	2	0	0	2	1	1	1	37
ANP	2	10	8	1	3	3	0	2	1	3	0	1	34
DEMATEL	2	11	2	0	1	0	0	2	2	3	0	1	24
UTADIS	2	1	3	2	1	0	0	0	0	0	2	1	12
COPRAS	0	5	3	0	1	0	0	0	0	0	0	0	9
Additive Model	0	1	0	0	3	2	0	0	0	2	0	0	8
MACBETH	0	1	4	3	0	0	0	0	0	0	0	0	8
SAW	0	4	2	0	1	0	0	0	0	0	0	0	7
MAUT	2	1	1	0	1	2	0	0	0	0	0	0	7
PROMETHEE	0	7	3	4	0	2	2	1	2	1	4	0	26
ELECTRE	1	1	1	5	0	0	2	0	2	0	2	0	14
DRSA	1	6	1	3	1	0	0	0	0	0	1	0	13
Hybrid Fuzzy	43	40	19	19	11	12	3	6	5	2	2	3	165
DEA	1	6	7	2	0	0	1	0	1	0	1	0	19
Hybrid GRA	0	5	1	5	0	0	0	0	0	0	0	0	11
DELPHI	0	4	2	2	0	0	0	0	0	0	0	0	8

Source: Almeida-Filho et al. (2020)/ Distinguished rows refer to utility-based approaches.
MO: Multi Objective / MOEA: Multi-Objective Evolutionary Algorithm / GP: Goal Programming / AHP: Analytic Hierarchy Process / TOPSIS: Technique for Order of Preference by Similarity to Ideal Solution / VIKOR: Vise Kriterijumska Optimizacija Kompromisno Resenje (which means multi-criteria optimization and compromise solution, in Serbian) / ANP: Analytic Network Process / DEMATEL: Decision Making Trial and Evaluation Laboratory / UTADIS: Utilities Additives Discriminants / COPRAS: Complex Proportional Assessment / MACBETH: Measuring Attractiveness by a Categorical-Based Evaluation Technique / SAW: Simple Additive Weighting / MAUT: Multi-Attribute Utility Technique / PROMETHEE: Preference Ranking Organization Method for Enrichment Evaluation / ELECTRE: ELimination Et Choix Traduisant la Realite (which means elimination and choice translating reality, in French) / DRSA: Dominance-based Rough Set Approach / DEA: Data Envelopment Analysis / GRA: Gray Relational Analysis.

Many MCDM methodologies can be used for creating utility functions which among them, preference disaggregation analysis (PDA) is widely used. PDA approach is used to determine an aggregation model of the criteria that represents the preference result, by analyzing the global preferences of the portfolio managers. PDA approach uses common utility decomposition forms to model the portfolio manager’s preferences while using regression-based techniques for estimating the global utility model (Zopounidis et al., 1999). In this situation, the problem is to estimate a utility function (usually an additive one) that is as consistent as possible to the known subjective preferences of the portfolio manager. When this model is approved, it can be used to support a real world financial decision making.

Now the key question is that, despite the advantages mentioned for the PDA approach, why this approach has received less attention from portfolio managers. In response to this question, the reasons can be stated as follows:

1) Most of the PDA approaches only consider the sequential preferences of the alternatives and ignore the relative preferences, thus these methods cannot provide an accurate estimation of the portfolio managers’ attitude.
2) In the PDA approaches, lack of consistency in the subjective preferences which usually occurs in practice, leads to difficulty in the process of finding the feasible solution based on the portfolio managers’ opinion.

3) Regardless of achieving the feasible solution in a particular situation (consistency of the subjective preferences), this solution does not have the sufficient capability to distinguish between the specified criteria weights.

Therefore, to overcome the above limitations, the main purpose of this paper is to introduce a utility-based method that expands the concept of PDA, in making decisions concerning the portfolio optimization problem. To this end, by examining the features of one of the most well-known UTA models, called the UTASTAR method, in the first step, we identified its weaknesses that have reduced the investor willingness to use it and in the second step, by eliminating the objections raised, we have developed the mentioned method and proposed a new method called modified UTASTAR. Compared to the original UTASTAR method, our proposed method has shown better performance and in many cases leads to an increase in the satisfaction of portfolio managers.

The remainder of this paper is organized as follows: In section 2, a brief description of the UTASTAR method is presented, and referring to its limitations our new method is proposed. In section 3, we evaluated these two methods for their usefulness through a comparison based on a designed experiment, which is a novel approach in operations research but common in psychology and experimental economics. Through this section to demonstrate how the proposed method can be applied in practice, it was implemented in the Tehran Stock Exchange (TSE). Eventually, in Section 4, the conclusions of this study are summarized and some directions for future researches are outlined.

2-Description of the original and modified UTASTAR methods

The proposed method in this paper is based on the UTASTAR method which presented by Siskos and Yannacopoulos (1985). Therefore, in this section, first the UTASTAR method is briefly introduced and then, referring to its limitations, our new method is proposed. The aim of the UTASTAR method is to develop an additive utility model for minimizing the dispersion of points all around the monotone ordinal regression curve by introducing a double positive error function (Figueira et al., 2005). Suppose that $G = \{g_1, g_2, \dots, g_n\}$ is a set of criteria to evaluate a set of preordered alternatives $A = \{a_1, a_2, \dots, a_m\}$ in which a_1 is the most and a_m is the least preferred alternative in the ranking list, respectively. Each criterion is defined as a function$g_i: A \to R$, where$g_i(a_k) = x_i^k$. The value of x_i^k is the performance of the alternative a_k over the criteriong_i.

In this situation, based on the weak ordering made by the decision maker, the UTASTAR method attempts to estimate a utility function (usually an additive one) that is as consistent as possible to the known subjective preferences of the decision maker. For this purpose, the UTASTAR method estimates a set of marginal utility functions $u_i: g_i \to [0,1]$ to be aggregated in an additive manner to estimate the global utility associated with each alternative. Finally, alternatives are ranked based on their global utilities.

It should be noted that the formulation of the UTASTAR method involves defining α_i characteristic points and hereafter $\alpha_i - 1$ subintervals$G_i^j = [g_i^j, g_i^{j+1}]$,$j = 1,2, \dots, \alpha_i - 1$, on the criterion g_i, in which g_i^1 and $g_i^{\alpha_i - 1}$ are the minimum and maximum performance levels over the criterion g_i, respectively. Therefore, the marginal utility at a characteristic point g_i^l on criterion g_i is expressed as follows:

$$u_i(g_i^l) = \sum_{j=1}^{l} u_i(g_i^{j+1}) - u_i(g_i^j) = \sum_{j=1}^{l} u_{ij} \tag{1}$$

Where $u_{ij} = u_i(g_i^{j+1}) - u_i(g_i^j) \geq 0$ due to the monotonicity of the criteria.

In this manner, the marginal utility for an alternative a_k whose performance over the criterion g_i is $g_i(a_k) = x_i^k \in [g_i^l, g_i^{l+1}]$ is obtained by linear interpolation between $u_i(g_i^l)$ and $u_i(g_i^{l+1})$ as follows:

$$u_i[g_i(a_k)] = \sum_{j=1}^{l} u_{ij} + \frac{g_i(a_k) - g_i^l}{g_i^{l+1} - g_i^l} . u_{il+1}, \tag{2}$$

Afterward, the global utility of an alternative a_k is obtained by aggregating its marginal utilities, as follows:

$$U[g(a_k)] = \sum_{i=1}^{n} u_i[g_i(a_k)], \tag{3}$$

Therefore, based on the above formulations the marginal utilities for each alternative can be estimated by solving the following linear program:

$$Min\, Z = \sum_{k=1}^{m} [\sigma^+(a_k) + \sigma^-(a_k)]$$

Subject to:

$$\begin{aligned}
&\Delta(a_k, a_{k+1}) \geq \delta \qquad if\, a_k \succ a_{k+1}, && \forall\, k \\
&\Delta(a_k, a_{k+1}) = 0 \qquad if\, a_k \sim a_{k+1}, && \\
&\sum_{i=1}^{n} \sum_{j=1}^{\alpha_i - 1} u_{ij} = 1, && \forall\, i, j \\
&u_{ij} \geq 0, && \forall\, i, j \\
&\sigma^+(a_k) \geq 0, && \forall\, k \\
&\sigma^-(a_k) \geq 0; && \forall\, k
\end{aligned} \tag{4}$$

Where $\sigma^+(a_k)$ and $\sigma^-(a_k)$ representing the overestimation and underestimation error of an alternative a_k , respectively. The first two constraints represent the preorder relations provided by the decision maker, while δ is a small positive number (user specified) to discriminate significantly two consecutive equivalence classes of relationship. The third constraint ensures that the maximum contribution of the criteria in the global utility of the alternatives sum up to one. Finally, the objective function minimizes the deviation of the estimated utility function from the preferential model of the decision maker.

In this way, after obtaining the optimal solution Z^* of this linear program, a post optimality stage is done to identify other optimal or near optimal solutions that could better represent the preferences of the decision maker. These solutions correspond to error values lower than $Z^* + d(Z^*)$, where $d(Z^*)$ is a positive number which is a small proportion of Z^*. So, the error objective is transformed into a new constraint as follows:

$$\sum_{k=1}^{m} [\sigma^+(a_k) + \sigma^-(a_k)] \leq Z^* + d(Z^*), \qquad \forall\, i, j \tag{5}$$

It is worth mentioning that, in the case of non-uniqueness, the mean additive utility functions of those near optimal solutions have been found which maximize the following objective function (Figueira et al., 2005):

$$u_i(g_i^*) = \sum_{j=1}^{\alpha_i - 1} u_{ij}, \qquad \forall\, i, j \tag{6}$$

Now, after a brief overview of the UTASTAR method, we will again address the question raised in the previous section. Why despite the emphasis on the importance of utility in decision making process, utility-based approaches such as the UTASTAR method have not received much attention and welcome from portfolio managers in practice? To answer the above question, this issue can be examined and evaluated from two different aspects: (1) technical aspect and (2) behavioral aspect.

From technical aspect, we must consider that the UTASTAR method based on the first two constraints in equation (4) only considers the sequential preferences of the alternatives and does not take into account the relative preferences, thus a significant part of the decision maker's attitude in the process of evaluation and comparison will be ignored. Besides, based on the objective function in equation (4) and concerning the Allais paradox (1953) and its argument about the violation of the independence axiom, this method is not competent for solving the problem with inconsistency in the decision makers' preferences, so the structure of UTASTAR will led the model to an infeasible solution. Moreover, according to equation (5) a significant part of the problem solving process called the "post optimality stage" only makes sense if the model initially has a feasible solution. If this condition is not met, the application of this part of the model is under question.

From behavioral aspect, the reasons for this reluctance by portfolio managers are a little broader and somehow include all utility-based approaches. These approaches follow the classical school of decision making and have accepted the "rational behavior" of the decision maker as a basic assumption. Based on this assumption, expectations are made from the decision maker, which are generally not compatible with his or her human nature. The most important of these expectations are (1) unbounded rationality, (2) unbounded willpower, (3) unbounded selfishness, (4) freedom in choice, and (5) usage of full information in decision making process. However, it is important to note that this inconsistency with human nature has always attracted the attention of researchers in the field of decision making and psychology and in recent years has led to the emergence of a new school in decision science called "behavioral decision making" to overcome these contradictions (Wright, 2013). In this regard, we have shown the evolution from the expected utility theory (EUT) as the main axis of the classical decision making school to the prospect theory (PT) as the main axis of the behavioral decision making school in figure 2.

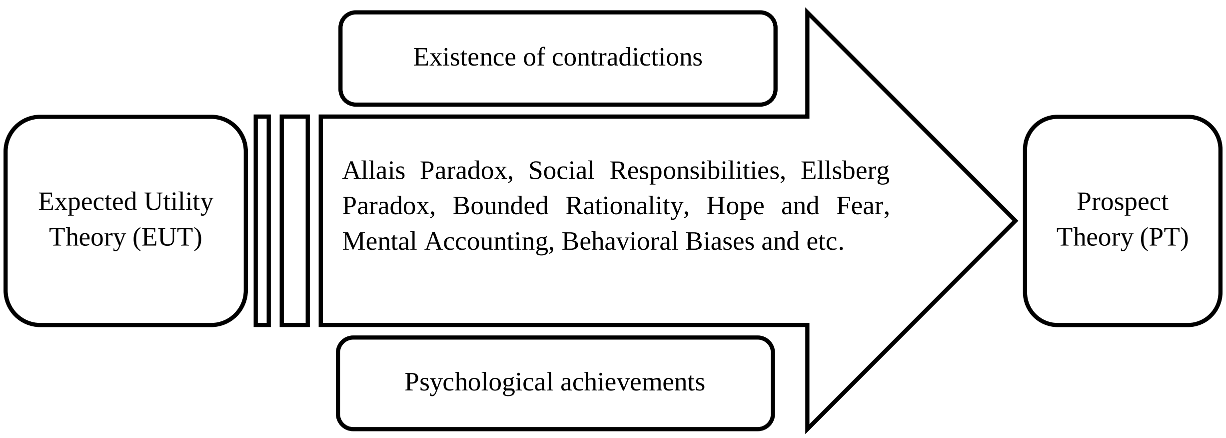

Fig 2. Evolution from the EUT to the PT

According to the above descriptions; now we will examine the key components of the PT, and explain how to use it in order to improve the UTASTAR method as a variant of UTA methods.

In 1979 the original version of the PT is proposed by Kahneman and Tversky (1979). The proposed model has some limitations. So, in 1992, Kahneman and Tversky published a modified version of their theory known as "cumulative prospect theory" which resolves the problems (Tversky & Kahneman, 1992). This version is commonly used in financial analysis and is the version that we will briefly review here.
Consider a gamble as follows:

$$(x_{-m}, p_{-m}; x_{-m+1}, p_{-m+1}; \dots; x_0, p_0; \dots; x_{n-1}, p_{n-1}; x_n, p_n) \tag{7}$$

Where the notation should be read as "gain x_{-m} with probability p_{-m}, x_{-m+1} with probability p_{-m+1}, and so on," while the outcomes are arranged in increasing order; so that $x_i < x_j$ for $i < j$, and $x_0 = 0$. For example, a $50:50$ bet to lose \$100 or gain \$200 would be expressed as$\left(-\$100, \frac{1}{2}; \$200, \frac{1}{2}\right)$. Under EUT, an individual evaluates the above gamble as follows:

$$\sum_{i=-m}^{n} p_i u(x_i) \tag{8}$$

Where $u(.)$ is an increasing and concave utility function.
Under cumulative PT, by contrast, the gamble is evaluated as follows:

$$\sum_{i=-m}^{n} \pi_i v(x_i - z) \tag{9}$$

Where $v(.)$ is an increasing value function with $v(0) = 0$, and π_i is the probability weights.

Based on equation (9) it can be concluded that the four key components of the PT are (1) reference dependence, (2) loss aversion, (3) diminishing sensitivity, and (4) probability weighting.

First, in the PT, decision makers derive utility from gains and losses, measured relative to some reference point rather than from absolute levels of wealth (the argument of $v(.)$ is $x_i - z$, not x_i). Interestingly, our ordinary perceptual systems work similarly: we are more adaptable to changes in attributes such as brightness, loudness, and temperature than we are to their absolute magnitudes.

Second, the value function $v(.)$ captures "loss aversion", the idea that decision makers are much more sensitive to losses than to gains of the same magnitude. Loss aversion is achieved by making the value function sharper in the region of losses than in the region of gains. This can be seen in Figure 3, which plots a typical value function.

Third, as shown in figure 3, the value function is concave in the region of gains but convex in the region of losses. This characteristic of the PT is known as diminishing sensitivity (Sharma et al., 2020), because it implies that, while replacing a \$100 gain (or loss) with a \$200 gain (or loss) has a significant utility impact, replacing a \$1,000 gain (or loss) with a \$1,100 gain (or loss) has a smaller impact. The concavity over gains reveals that decision makers tend to be risk averse over moderate probability gains and also tend to be risk seeking over losses. This motivates the convexity over losses.

The fourth and final key component of the PT is probability weighting. In the PT, decision makers do not weight outcomes by their objective probabilities p_i but rather by transformed probabilities to decision weights π_i. The decision weights are calculated using a weighting function $w(.)$ whose argument is an objective probability. The solid line in Figure 4 illustrates the weighting function proposed by Tversky and Kahneman (1992). As can be seen in comparison with the dotted line, which corresponds to the expected utility benchmark, the weighting function overweighs low probabilities and underweights high probabilities. In cumulative PT, the weighting function is applied to cumulative probabilities (for example, to the probability of gaining at least \$100, or of losing \$50 or more).

At this point, we must also point out that developing application of the PT in financial issues is taking a long time, and it will always be faced with serious challenges because it is not exactly obvious how to apply it (Barberis, 2013). The central idea in the PT is that decision makers derive utility from "gains" and "losses" which are measured relative to a reference point.

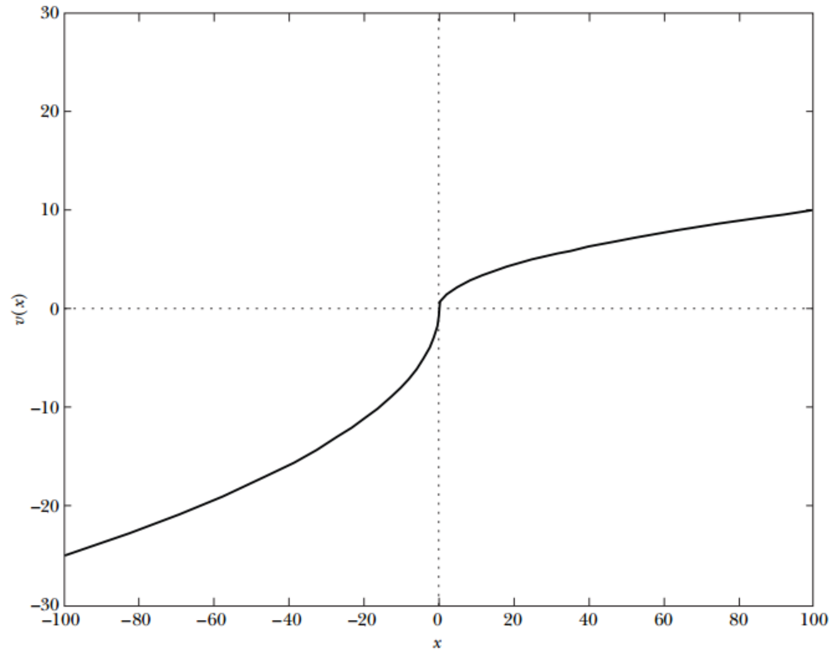

Fig 3. Value function in the PT

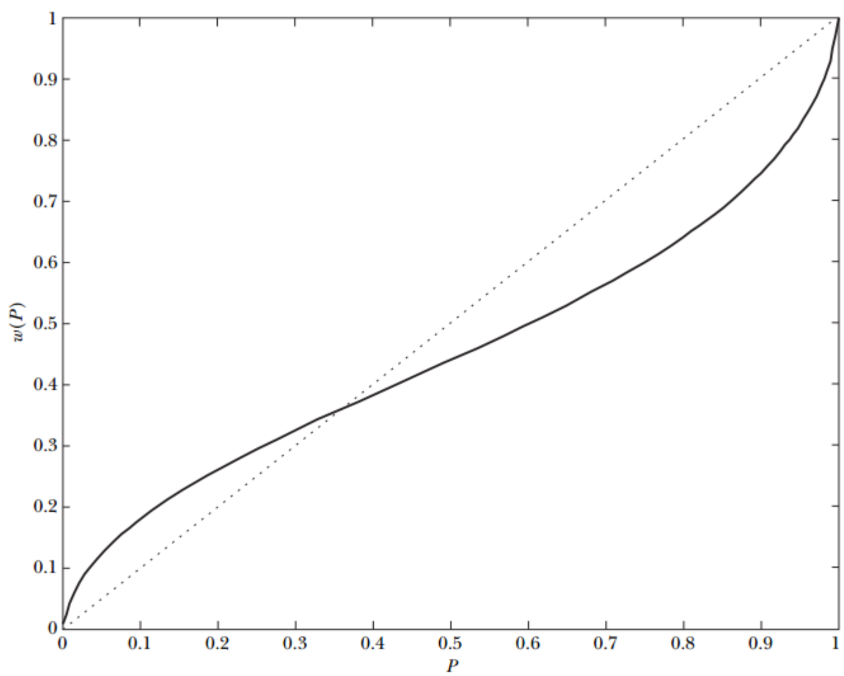

Fig 4. Probability weighting function in the PT

Notes: The graph plots the probability weighting function proposed by Tversky and Kahneman (1992) as a part of cumulative PT, namely $w(p) = p^{\delta}/(p^{\delta} + (1-p)^{\delta})^{1/\delta}$, where p is an objective probability, for two values of δ. The solid line corresponds to $\delta = 0.65$ as the value estimated by the authors from experimental data. The dotted line corresponds to $\delta = 1$, in other words, to linear probability weighting.

In each filed, it is often unclear how to determine precisely what is "gain" or "loss", because Tversky and Kahneman (1992) provide little guidance on how the reference point is defined. A real example from portfolio optimization may help to identify this problem. Suppose that we want to predict what kind of portfolio an investor will hold with the PT preferences. Immediately, we need to identify the investors' desired "gains" and "losses". Thus, to achieve this goal, we must answer some questions as follows:

Do they define "gains" and "losses" in overall wealth, in the value of total stock market, or in the value of specific stocks? If the investor's attitude about "gains" and "losses" is in the value of his or her stock market holdings, does "gain" in the stock market simply mean the positive stock market return? Or does it mean that the stock market return exceeded the risk free rate or the investor's expected return over "gain"? And do the investors think about annual "gains" and "losses" or monthly or even weekly fluctuations?

The lack of a clear answer to the above mentioned questions has led some researchers to avoid addressing this issue. In the same time, other researchers were faced with the challenge of trying to understand how decision makers conceptualize "gains" and "losses" in different fields such as insurance, consumption-savings decisions, industrial organization and labor supply (Crawford & Meng, 2011; Feng et al., 2020; Heidhues & Kőszegi, 2014; Kaluszka & Krzeszowiec, 2012; Kőszegi & Rabin, 2009; Pagel, 2017; Sydnor, 2010). Therefore, according to the best of our knowledge, the best way to deal with this question and the main approach taken by the researchers is to derive the interpretations of the PT under a variety of plausible definitions of "gains" and "losses", and then test these interpretations, both in the laboratory and in the field. Through this process, gradually better theories were developed about the decision makers' perceptions about "gains" and "losses".

Therefore, considering the above mentioned limitations of the PT, we intend to continue this section by providing a new way to define a reference point and identify the decision makers' desired "gains" and "losses". Through this way, we can take two basic actions simultaneously. The first action is to develop the application of the PT and the second action is to overcome the drawbacks of the original UTASTAR method. Hence in our proposed model which is called the "modified UTASTAR", the following steps will be taken to determine the marginal utilities of each alternative.

Step 1: Provide a set of alternatives as a "reference set" and a set of evaluation criteria in order to present to the decision maker. In this step, the analyst identifies m alternatives $\{a_1, a_2, \dots, a_m\}$ and n evaluation criteria $\{g_1, g_2, \dots, g_n\}$ that are used to make a decision.

Step 2: Rank the alternatives in the reference set. In this step, the decision maker arranges the alternatives in the reference set from the best to the worst based on the evaluation criteria.

Step 3: Determine the relative preference of each alternative over the next one in the reference set. In this step, the decision maker presents his or her attitudes toward the relative preference of the consecutive alternatives in the reference set. The resulting vector would be:

$A_P = (p_{12}, p_{23}, \dots, p_{k,k+1})$,

Where $p_{k,k+1}$, $k = (1,2,\dots,m)$ indicates the relative preference of the alternative k over the alternative $k+1$.

Step 4: Determine the reference point as a hypothetical alternative in the reference set where all attribute values correspond to the intermediate level.

$a_R = (g_1^R, g_2^R, \dots, g_n^R)$,

Where

$$a_i{}^R = \left(g_i^R\right) = \left(\underset{k}{\text{median}}\, g_{ik}\right), k \in 1,2,\dots,m \tag{10}$$

Step 5: Determine the relative preference of each alternative over the reference point. In this step, the decision maker presents his or her attitudes toward the relative preference of each alternative over the reference point. The resulting vector would be:

$A_R = (p_{1R}, p_{2R}, \dots, p_{k,R})$,

Where $p_{k,R}$, $k = (1,2,\dots,m)$ indicates the relative preference of the alternative k over the reference point a_R .

Step 6: Find the marginal utilities$\left(u_1(g_1^*), u_2(g_2^*), \dots, u_n(g_n^*)\right)$. The marginal utilities of the evaluation criteria are fully consistent, where for each pair of $U[g(a_k)]/U[g(a_{k+1})]$, we have $\frac{U[g(a_k)]}{U[g(a_{k+1})]} = p_{k,k+1}$ and also for each pair of $U[g(a_k)]/U[g(a_R)]$, we have $\frac{U[g(a_k)]}{U[g(a_R)]} = p_{k,R}$. To satisfy these conditions for all k, we should find a solution where the maximum absolute differences $\left|\frac{U[g(a_k)]}{U[g(a_{k+1})]} - p_{k,k+1}\right|$ and $\left|\frac{U[g(a_k)]}{U[g(a_R)]} - p_{k,R}\right|$ for all k is minimized as follows:

$$min\, max_k \left\{ \left|\frac{U[g(a_k)]}{U[g(a_{k+1})]} - p_{k,k+1}\right|, \left|\frac{U[g(a_k)]}{U[g(a_R)]} - p_{k,R}\right| \right\}$$

Subject to:

$$\begin{aligned} &\sum_{i=1}^{n}\sum_{j=1}^{\alpha_i-1} u_{ij} = 1, && \forall\, i,j \\ &u_{ij} \geq 0, && \forall\, i,j \\ &\sigma^+(a_k) \geq 0, && \forall\, k \\ &\sigma^-(a_k) \geq 0; && \forall\, k \end{aligned} \tag{11}$$

The above formulations can be transformed into the following form:

$$min\, \varphi$$

Subject to:

$$\begin{aligned} &\left|\frac{U[g(a_k)]}{U[g(a_{k+1})]} - p_{k,k+1}\right| \leq \varphi, && \forall\, k \\ &\left|\frac{U[g(a_k)]}{U[g(a_R)]} - p_{k,R}\right| \leq \varphi, && \forall\, k \\ &\sum_{i=1}^{n}\sum_{j=1}^{\alpha_i-1} u_{ij} = 1, && \forall\, i,j \\ &u_{ij} \geq 0, && \forall\, i,j \\ &\sigma^+(a_k) \geq 0, && \forall\, k \\ &\sigma^-(a_k) \geq 0; && \forall\, k \end{aligned} \tag{12}$$

By solving the above formulations, the marginal utilities of the evaluation criteria $\left(u_1(g_1^*), u_2(g_2^*), \dots, u_n(g_n^*)\right)$ and the consistency ratio φ^* are obtained.

The consistency ratio φ^* is an important indicator to show the inconsistency level of pairwise comparisons. A comparison is fully consistent when $p_{ik} \times p_{kj} = p_{iR} \times p_{Rj} = p_{ij}$ for all k. When $p_{ik} \times p_{kj} \neq p_{ij}$ and (or) $p_{iR} \times p_{Rj} \neq p_{ij}$, which means $p_{ik} \times p_{kj}$ and (or) $p_{iR} \times p_{Rj}$ may be higher or lower than p_{ij}, , then there would be inconsistency in pairwise comparisons. The smaller the φ^*, the better the consistency.

As can be seen from the structure of the proposed method in equation (11), unlike all the presented methods based on the preferences of the decision makers, our proposed method, not only considers the sequential preferences of the alternatives but also it take into account the relative preferences of them (not only with each other but also with the reference point), consequently this method can provide a more accurate estimation of the decision makers' attitude. Also, it allows the model to accept the inconsistency in the decision maker's preferences, which is very likely to happen in the real world. This feature makes it possible for the proposed method to always provide a feasible solution. Furthermore, another interesting point about our proposed method is incorporating some of the behavioral characteristics of the decision maker into decision making.

It should be noted that the post optimality stage in our modified UTASTAR method is carried out with a slight difference to the common UTA methods. In this way, the main purpose is to help the decision makers to adjust their preferences and improve the inconsistency among its preferences if desired. Thus, with changing the decision makers' preferences, the value of φ approaches to zero and the consistency will increase among the decision makers' preferences.

3- A comparison based on a designed experiment

According to the technology acceptance model, the user's intention to adopt new technology has two major motivations: perceived usefulness and perceived ease of use (Venkatesh & Bala, 2008). In our view, one of the possible reasons that UTA methods often remain in academic communities is that portfolio managers do not clearly perceive the added value in them (perceived usefulness). Therefore, this situation led us to evaluate the methods discussed in this study based on the answering to the following questions: (1) Are these two methods useful? (2) Which of them is more useful?

Thus, in this section we design an experiment to evaluate these two methods (original UTASTAR and modified UTASTAR) for their usefulness in a real world multi-criteria problem. In this regard, to evaluate their usefulness, based on their algorithm provided in previses section, a software has been developed and installed on a computer in our laboratory and then participants were asked to express their opinion about the reference set which are provided in this experiment. However, before stating the experiment results, it is necessary to explain some of the issues regarding the experiment conditions as follows:

3-1-Participants

In our experiment, ten financial experts who were technically competent and experienced in TSE were invited to make a straightforward but not necessarily easy decision in a real world problem.

3-2-Reference set

The nine stocks were considered from the annual bulletin of TSE published in 2019 to generalize the acquired utility to other stocks.

3-3-Evaluation criteria

The evaluation of these stocks will be based on 13 financial and stock market criteria, which can be classified into four main categories as follows:

g_1: Liquidity Measurement Ratios (two criteria),

- Current Ratio, measured as: $\frac{(Current\ Assets)}{(Current\ Liabilities)}$,
- Quick Ratio, measured as: $\frac{(Current\ Assets - Inventories)}{(Current\ Liabilities)}$,

g_2: Investment and Leverage Ratios (two criteria),

- Debt Ratio, measured as: $\frac{(Total\ Liabilities)}{(Total\ Assets)}$,
- Debt to Equity Ratio, measured as: $\frac{(Total\ Liabilities)}{(Shareholder's\ Equity)}$,

g_3: Operating Performance Ratios (four criteria),

- Inventory Turnover Ratio, measured as: $\frac{(Cost\ of\ Goods\ Sold)}{(Average\ Inventories)}$,
- Working Capital Turnover Ratio, measured as: $\frac{(Net\ Sales)}{(Working\ Capital)}$,
- Fixed Asset Turnover Ratio, measured as: $\frac{(Net\ Sales)}{(Average\ Fixeed\ Assets)}$,

- Total Asset Turnover Ratio, measured as: $\frac{(Net\ Sales)}{(Average\ Total\ Assets)}$,

g_4: Profitability Indicator Ratios (five criteria),

- Net Profit Margin Ratio, measured as: $\frac{(Net\ Income)}{(Net\ Sales)}$,
- Operating Profit Margin Ratio, measured as: $\frac{(Operating\ Profit)}{(Net\ Sales)}$,
- Return on Assets Ratio, measured as: $\frac{(Net\ Income)}{(Average\ Total\ Assets)}$,
- Return on Equity Ratio, measured as: $\frac{(Net\ Income)}{(Average\ Shareholder's\ Equity)}$,
- Return on Capital Employed Ratio, measured as: $\frac{(Net\ Income)}{(Average\ Debt\ \ Liabilities+Average\ Shareholder's\ Equity\ )}$,

In this study due to the nature of the above criteria and in order to facilitate the process of evaluating stocks, these 13 criteria were integrated into the form of four main criteria. Therefore, the simple weighting method and normalization of the extracted data were used. With regard to liquidity measurement ratios, it should be noted that because their values can be analyzed in different views; with a little change in calculation process; we have evaluated their distance from the industry average. Therefore, for the first two criteria, the portfolio managers' preferences are decreasing functions on the criterion's scale, while for the second two criteria these parameters are increasing. This means that lower values in the first two criteria and higher values in the second two criteria lead to greater satisfaction of the portfolio managers.

Table 2. The reference set to be presented to the experts

Stocks	g_1	g_2	g_3	g_4
PSER	1.000	1.000	0.000	1.000
LARP	0.127	0.012	0.074	0.575
FKHZ	0.013	0.024	0.357	0.520
IHTI	0.637	0.005	0.131	0.504
KBLP	0.049	0.011	0.168	0.477
PAKS	0.025	0.047	1.000	0.327
CHAR	0.002	0.039	0.383	0.160
NIRP	0.024	0.003	0.014	0.107
TMAS	0.111	0.010	0.385	0.034

3-4-Application of the methods

Each expert was asked to apply these two methods (original UTASTAR and modified UTASTAR) for ranking and expressing their views on the stocks were provided in the reference set. Afterward, the utility functions for each expert were estimated according to the procedure of these two methods which provided in the previous section. When these utility functions are approved, they used to support a real world financial decision making problem in TSE for 2019 includes the evaluation of 68 stocks. It should be noted that these stocks have been identified as the top assets of the market in the desired time period based on fundamental analysis. Based on equation (10), the reference point in the modified UTASTAR method is determined as follows:

$$a_R = (0.049,0.012,0.168,0.477),$$

Where its attribute values correspond to the intermediate level of each criterion in table 2.

As illustrated in table 3, in each situation (consistency or inconsistency of the subjective preferences) the feasible solution is found through the application of the modified UTASTAR, while the original UTASTAR method lacks this property. This fact is due to the structural modifications made in the UTASTAR method, which is described in detail in the previous section. Also, as presented in Table 4, the answers which are produced through the application of the modified UTASTAR have more capability than the original UTASTAR method in distinguishing between the specified criteria weights. As mentioned earlier, after the experts verified the performance of the instruments they were generalized and used to support a real world financial decision making problem in TSE includes the evaluation of 68 stocks, which the results obtained from this action, are shown in table 5 and table 6.

Table 3. Ranking of the stocks in reference set using the original UTASTAR and modified UTASTAR

Stock		DM_1			DM_2			DM_3			DM_4			DM_5			DM_6			DM_7			DM_8			DM_9			DM_{10}	
	ER	A_P	A_R	ER	A_P	A_R	ER	A_P	A_R	ER	A_P	A_R	ER	A_P	A_R	ER	A_P	A_R	ER	A_P	A_R	ER	A_P	A_R	ER	A_P	A_R	ER	A_P	A_R
PSER	**9**	-	$\frac{1}{3}$	**9**	-	$\frac{1}{4}$	**9**	-	$\frac{1}{4}$	**9**	-	$\frac{1}{3}$	**9**	-	$\frac{1}{4}$	**8**	2	$\frac{1}{3}$	**1**	2	3	**8**	1	$\frac{1}{3}$	**8**	2	$\frac{1}{4}$	**8**	1	$\frac{1}{4}$
LARP	**4**	2	$\frac{1}{2}$	**7**	2	$\frac{1}{3}$	**5**	1	$\frac{1}{2}$	**5**	2	$\frac{1}{2}$	**7**	2	$\frac{1}{3}$	**4**	1	1	**2**	1	2	**3**	2	2	**3**	1	2	**9**	-	$\frac{1}{3}$
FKHZ	**2**	2	3	**2**	3	3	**6**	2	2	**2**	1	3	**4**	3	3	**1**	2	4	**3**	2	3	**1**	1	4	**2**	2	3	**4**	2	3
IHTI	**6**	2	$\frac{1}{2}$	**8**	2	$\frac{1}{3}$	**2**	2	2	**4**	2	2	**6**	1	$\frac{1}{2}$	**6**	2	$\frac{1}{2}$	**4**	1	2	**6**	2	1	**6**	3	2	**6**	2	2
KBLP	**3**	2	1	**5**	2	1	**4**	2	1	**1**	1	1	**5**	2	1	**3**	2	1	**5**	2	1	**4**	1	1	**4**	1	1	**5**	2	1
PAKS	**1**	1	2	**4**	2	2	**8**	3	$\frac{1}{2}$	**3**	2	2	**1**	3	2	**2**	1	2	**6**	2	1	**2**	2	3	**1**	2	2	**2**	2	3
CHAR	**5**	1	2	**1**	1	3	**7**	1	1	**6**	3	2	**3**	1	2	**5**	2	1	**7**	2	$\frac{1}{2}$	**5**	2	1	**5**	2	2	**1**	1	2
NIRP	**7**	2	$\frac{1}{2}$	**3**	1	1	**1**	1	2	**7**	2	$\frac{1}{2}$	**8**	2	$\frac{1}{2}$	**9**	-	$\frac{1}{2}$	**8**	2	$\frac{1}{3}$	**9**	-	$\frac{1}{2}$	**9**	-	$\frac{1}{2}$	**3**	1	$\frac{1}{2}$
TMAS	**8**	2	$\frac{1}{2}$	**6**	1	$\frac{1}{2}$	**3**	2	2	**8**	1	$\frac{1}{3}$	**2**	1	2	**7**	2	$\frac{1}{2}$	**9**	-	$\frac{1}{3}$	**7**	1	$\frac{1}{2}$	**7**	3	$\frac{1}{2}$	**7**	2	$\frac{1}{3}$
FS Original		√			√			×			×			√			√			√			√			√			×	
FS Modified		√			√			√			√			√			√			√			√			√			√	
δ		0.010			0.010			0.010			0.010			0.010			0.010			0.010			0.010			0.010			0.010	
φ^*		0.275			0.319			0.328			0.243			0.016			0.401			0.071			0.183			0.238			0.308	

FS: feasible solution / ER: expert ranking / A_P: the relative preferences of the consecutive alternatives / A_R: the relative preferences of each alternative over the reference point.

Table 4. Weights of the evaluation criteria using the original UTASTAR and modified UTASTAR

Criterion	DM_1		DM_2		DM_3		DM_4		DM_5		DM_6		DM_7		DM_8		DM_9		DM_{10}	
	Original	Modified	Original	Modified	Original	Modified	Original	Modified	Original	Modified	Original	Modified	Original	Modified	Original	Modified	Original	Modified	Original	Modified
g_1	0.146	**0.183**	0.917	**0.110**	-	**0.000**	-	**0.086**	0.010	**0.046**	0.054	**0.235**	0.000	**0.104**	0.121	**0.086**	0.180	**0.106**	-	**0.121**
g_2	0.292	**0.000**	0.047	**0.212**	-	**0.341**	-	**0.075**	0.304	**0.000**	0.000	**0.000**	0.358	**0.000**	0.176	**0.143**	0.113	**0.087**	-	**0.165**
g_3	0.117	**0.567**	0.000	**0.311**	-	**0.342**	-	**0.395**	0.376	**0.923**	0.531	**0.347**	0.046	**0.027**	0.408	**0.334**	0.298	**0.677**	-	**0.390**
g_4	0.446	**0.249**	0.037	**0.367**	-	**0.317**	-	**0.444**	0.310	**0.031**	0.415	**0.419**	0.597	**0.869**	0.295	**0.436**	0.409	**0.130**	-	**0.325**
Total	1.000	**1.000**	1.000	**1.000**	-	**1.000**	-	**1.000**	1.000	**1.000**	1.000	**1.000**	1.000	**1.000**	1.000	**1.000**	1.000	**1.000**	-	**1.000**

Table 5. Evaluation of the stocks in TSE using the original UTASTAR

DM_1										
Stock	g_1	g_2	g_3	g_4	$u_1(g_1)$	$u_2(g_2)$	$u_3(g_3)$	$u_4(g_4)$	$U[g(a_k)]$	Rank
PFAN	0.031	0.037	0.271	0.858	0.882	1.000	0.706	1.000	**1.000**	**1**
CHRZ	0.089	0.016	0.299	0.970	0.650	1.000	0.706	1.000	**0.964**	**2**
SAKH	0.014	0.064	0.740	0.277	0.949	1.000	0.988	0.277	**0.705**	**3**
PAKS	0.025	0.047	1.000	0.327	0.907	1.000	1.000	0.286	**0.704**	**4**
FKHZ	0.013	0.024	0.357	0.520	0.953	1.000	0.706	0.325	**0.694**	**5**
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
KBLP	0.049	0.011	0.168	0.477	0.811	1.000	0.474	0.311	**0.637**	**17**
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
LARP	0.127	0.012	0.074	0.575	0.496	1.000	0.210	0.482	**0.636**	**19**
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
CHAR	0.002	0.039	0.383	0.160	1.000	1.000	0.706	0.144	**0.616**	**29**
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
IHTI	0.637	0.005	0.131	0.504	0.000	1.000	0.369	0.316	**0.502**	**59**
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
NIRP	0.024	0.003	0.014	0.107	0.908	1.000	0.041	0.083	**0.491**	**61**
TMAS	0.111	0.010	0.385	0.034	0.561	1.000	0.706	0.000	**0.481**	**62**
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
PSER	1.000	1.000	0.000	1.000	0.000	0.000	0.000	1.000	**0.470**	**64**
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
BTEJ	0.113	0.012	0.047	0.083	0.555	1.000	0.132	0.056	**0.435**	**67**
YASA	0.231	0.009	0.187	0.122	0.080	1.000	0.527	0.101	**0.432**	**68**

Table 6. Evaluation of the stocks in TSE using the modified UTASTAR

					DM_1					
Stock	g_1	g_2	g_3	g_4	$u_1(g_1)$	$u_2(g_2)$	$u_3(g_3)$	$u_4(g_4)$	$U[g(a_k)]$	Rank
PAKS	0.025	0.047	1.000	0.327	0.907	0.000	1.000	0.181	**1.000**	**1**
ALMR	0.016	0.063	0.816	0.188	0.942	0.000	1.000	0.000	**0.950**	**2**
SAKH	0.014	0.064	0.740	0.277	0.949	0.000	0.971	0.005	**0.932**	**3**
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
FKHZ	0.013	0.024	0.357	0.520	0.953	0.000	0.203	0.857	**0.647**	**8**
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
KBLP	0.049	0.011	0.168	0.477	0.811	0.000	0.110	0.716	**0.500**	**15**
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
LARP	0.127	0.012	0.074	0.575	0.496	0.000	0.048	0.857	**0.427**	**21**
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
CHAR	0.002	0.039	0.383	0.160	1.000	0.000	0.212	0.000	**0.390**	**25**
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
IHTI	0.637	0.005	0.131	0.504	0.000	0.000	0.085	0.812	**0.322**	**39**
PSER	1.000	1.000	0.000	1.000	0.000	0.000	0.000	1.000	**0.320**	**40**
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
TMAS	0.111	0.010	0.385	0.034	0.561	0.000	0.213	0.000	**0.287**	**48**
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
NIRP	0.024	0.003	0.014	0.107	0.908	0.000	0.009	0.000	**0.221**	**64**
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
YASA	0.231	0.009	0.187	0.122	0.080	0.000	0.122	0.000	**0.107**	**67**
JPPC	0.569	0.073	0.021	0.292	0.000	0.000	0.014	0.059	**0.029**	**68**

3-5-Assessing different rankings of the original and modified UTASTAR

After viewing the results, samples that include ten stocks were extracted and presented to the experts. Each expert was asked to rank these stocks in terms of their own understanding and personal preferences and write their order of ranking on the information sheet. Therefore, in this part of the experiment we will have two sets of rankings as follows:

1) Original / Modified UTASTAR ranking (R_1). This is the same ranking that was generated by these two methods for the ten stocks in the samples through the evaluation of 68 stocks.

2) A posteriori ranking (R_2). This is the ranking which generated by each expert for the ten stocks in the samples were extracted after viewing the results.

After taking the posteriori ranking R_2 the final phase involved a correlation assessment. In order to measure the agreement between these two types of rankings, we used the Spearman's rank correlations between each pair of rankings. In this situation ρ_{12} is the correlation between R_1 and R_2. This measurement was used to determine which of these two methods more in line with the experts' preferences is. Obviously, if the value of ρ_{12} is closer to 1, the mentioned method has better performance, and if the value of ρ_{12} is closer to -1, then the reverse is true. It should be noted that if $\rho_{12} = 1$, it implies that the mentioned method presents the same ranking which generated by the experts in the posteriori ranking R_2, and it is fair to conclude that the mentioned method successfully reproduced the preferences of the experts. It is necessary to remember that this fact is the ultimate goal for all methods based on the decision maker's preferences.

Table 7. Assessing different rankings of the original and modified UTASTAR

					DM_1												
					$u_1(g_1)$		$u_2(g_2)$		$u_3(g_3)$		$u_4(g_4)$		$U[g(a_k)]$			R_1	
Stock	g_1	g_2	g_3	g_4	Original	Modified	Original	Modified	Original	Modified	Original	Modified	Original	Modified	R_2	Original	Modified
NGJP	0.030	0.027	0.613	0.319	0.887	**0.887**	1.000	**0.000**	0.839	**0.592**	0.285	**0.155**	0.681	**0.690**	1	1	**1**
SBEH	0.016	0.020	0.106	0.480	0.943	**0.943**	1.000	**0.000**	0.300	**0.069**	0.312	**0.726**	0.636	**0.505**	2	2	**2**
GOST	0.015	0.046	0.487	0.175	0.948	**0.948**	1.000	**0.000**	0.706	**0.250**	0.161	**0.000**	0.616	**0.406**	4	4	**4**
KFRP	0.092	0.010	0.363	0.302	0.638	**0.638**	1.000	**0.000**	0.706	**0.205**	0.282	**0.093**	0.625	**0.329**	5	3	**5**
OFOG	0.063	0.021	0.090	0.036	0.755	**0.755**	1.000	**0.000**	0.255	**0.059**	0.002	**0.000**	0.456	**0.221**	9	10	**9**
DSHP	0.054	0.031	0.039	0.487	0.788	**0.788**	1.000	**0.000**	0.110	**0.025**	0.313	**0.749**	0.589	**0.444**	3	7	**3**
IRGC	0.051	0.011	0.096	0.274	0.802	**0.802**	1.000	**0.000**	0.271	**0.063**	0.275	**0.000**	0.594	**0.235**	8	6	**8**
MOIN	0.073	0.030	0.275	0.213	0.715	**0.715**	1.000	**0.000**	0.706	**0.172**	0.205	**0.000**	0.600	**0.294**	6	5	**6**
SPKH	0.057	0.029	0.176	0.195	0.780	**0.780**	1.000	**0.000**	0.496	**0.115**	0.184	**0.000**	0.575	**0.267**	7	8	**7**
SGAZ	0.081	0.047	0.074	0.104	0.682	**0.682**	1.000	**0.000**	0.207	**0.048**	0.080	**0.000**	0.475	**0.196**	10	9	**10**
ρ_{12}																0.830	**1.000**

As illustrated in table 7, for DM_1 the Spearman's rank correlations related to the modified UTASTAR method is greater than the equivalent value for the original UTASTAR method and this means that the modified UTASTAR method could be better to reflect the expert's preferences. The details of data for each expert are provided in Table 8. According to the results presented in table 8, we can find that the ranking which generated through the application of the modified UTASTAR methods is more in line with the expert's preferences compared to the application of the original UTASTAR method.

Table 8. Correlations between each pair of rankings (ρ_{12}) through the application of original UTASTAR and modified UTASTAR

	DM_1	DM_2	DM_3	DM_4	DM_5	DM_6	DM_7	DM_8	DM_9	DM_{10}
Original	0.830	0.781	-	-	1.000	0.623	0.987	0.975	0.939	-
Modified	**1.000**	**1.000**	**0.929**	**0.942**	**1.000**	**0.786**	**1.000**	**1.000**	**1.000**	**0.936**

Given the nature of the correlation coefficient, six intervals for the value of ρ_{12} are illustrated in Table 9. In this regards, the scores for interval, $0.8 < \rho_{12} \leq 1$ was classified as successful and labeled as "Helped", while interval $-1 < \rho_{12} \leq 0$ was considered as unsuccessful and labeled as "Failed". In addition, other intervals which do not provide clear support in favor of or against these two methods were labeled as "Not Sure". In our experiment, through the application of the modified UTASTAR method, ρ_{12} in 90% of the samples were found in interval $0.8 < \rho_{12} \leq 1$, while this ratio for the original UTASTAR method is 50%. Also, it can be seen in table 9 that our proposed method is capable to respond to 100% of the expert's preferences and reflects their opinions in a desirable way, while this ratio for the original UTASTAR method is 70%.

Table 9. The six intervals and their frequencies of occurrence

	$-1 < \rho_{12} \leq 0$	$0 < \rho_{12} \leq 0.2$	$0.2 < \rho_{12} \leq 0.4$	$0.4 < \rho_{12} \leq 0.6$	$0.6 < \rho_{12} \leq 0.8$	$0.8 < \rho_{12} \leq 1$	Total
	"Failed"	"Not Sure"	"Not Sure"	"Not Sure"	"Not Sure"	"Helped"	
Original	0	0	0	0	2	5	7/10
Modified	0	0	0	0	1	9	10/10

3-6-Assessing the effect of inconsistency

The modified UTASTAR allows a ranking to be calculated even the judgments are inconsistent. This raises the question of whether the modified UTASTAR is helpful when the judgments are highly inconsistent or not? To answer this question, we considered all the experts according to the level of inconsistency found in their judgments. Out of ten experts, all of them report a level of inconsistency in their judgments. Among them, only two experts passed the accepted criterion of$\varphi^* < 0.1$. Since very few experts meeting this criterion, we tested the number of acceptable cases using different levels of φ^* thresholds ranging from 0.10 to 0.30 with increments of 0.10.

Table 10 illustrates the frequency of acceptable cases in two situations of the modified UTASTAR helping or not. Interestingly, these two situations are not clearly different, as indicated below with the statistical tests.

Table 10. Frequencies of consistent and inconsistent cases regarding the usefulness of modified UTASTAR

Threshold	$\varphi^* < 0.10$	$\varphi^* \geq 0.10$	$\varphi^* < 0.20$	$\varphi^* \geq 0.20$	$\varphi^* < 0.30$	$\varphi^* \geq 0.30$
Helped	2	7	3	6	6	3
Not Helped	0	1	0	1	0	1
P-Value	0.200		0.300		1.000	

In this situation because the sample size is small, the data are very unequally distributed among the cells and the expected values in some of the cells are below 5; using the common approaches such as chi-squared test is inadequate for approximation. Therefore, for small, sparse, or unbalanced data, the exact and asymptotic p-values can be calculated through the application of the Fisher's exact test (Daya, 2002). Thus, in our study the Fisher's exact test was used for independency to show that the usefulness of the modified UTASTAR method has no significant relationship with the level of inconsistency in the judgments. This is a remarkable result although the use of φ^* has been widely debated in the literature (e Costa & Vansnick, 2008; Tomashevskii, 2015). Our results experimentally invalidate the threshold of $\varphi^* < 0.1$ and suggest a much higher threshold of acceptance.

3-7-Participants feedback

At the end of the experiment, the experts were asked to provide their feedback about the modified UTASTAR method that they had used during the experiment. The experts were given the following four statements, three statements were related to the perceived usefulness, while one of them was related to the perceived ease of use.

S_1. The modified UTASTAR method was helpful in receipt my point of view.

(For the alternatives in the reference set)

S_2. The modified UTASTAR method helped me in the decision making process.

S_3. I agree with the ranking suggested by the modified UTASTAR method.

(For the alternatives in the sample include ten stocks)

S_4. The modified UTASTAR method was easy to use.

These statements were scored on a Likert scale with seven levels ranging from "strongly agree" to "strongly disagree" with a neutral level in the middle. Based on the positive and negative answers, a binomial test was performed. Table 11 summarizes the feedback received from the experts on these four statements. The results for the modified UTASTAR method were found to be statistically significant at the 0.05 level for all four statements.

Table 11. Participants feedback on the modified UTASTAR method

	Perceived Usefulness			Perceived Ease of Use
	S_1	S_2	S_3	S_4
Negative	1	0	1	0
Neutral	1	1	1	0
Positive	8	9	8	10
P-Value	0.000	0.000	0.000	0.000

Therefore, based on the obtained results it can be concluded that the modified UTASTAR method was perceived to be useful and ease of use from the viewpoint of experts.

4-Conclusions and suggestions for future researches

Today, due to the emergence of a new attitude in the world of decision-making based on the authenticity of human behaviors in solving the problems ahead; designing a utility-based approach for portfolio managers to meet their market expectations is more essential than ever. But it is important to pay attention to the fact that despite the importance of "utility" in the human decision-making process, this matter is also very sensitive. If the portfolio manager feels that there is a significant difference between his or her preferences and the outputs of the relevant models, he or she will hesitate to use them. Thus, in this paper we developed a new variant of the UTA method, called the modified UTASTAR used for portfolio optimization and management. The proposed method is not based on restrictive assumptions concerning the portfolio manager's judgmental policy and preferences. Compared to the common UTA based methods, the modified UTASTAR not only considers the sequential preferences of the alternatives but also take into account the relative preferences of them (not only with each other but also with the reference point), thus this method will be able to provide a more accurate estimation of the portfolio managers' attitude. Also, it allows the model to accept the inconsistency in the preferences of the portfolio managers, which allows the model to always provide a feasible solution. This method in many cases leads to an increase in the level of satisfaction of portfolio managers. In this paper, we designed an experiment to investigate whether the proposed method can really help portfolio managers or not. We observed that the proposed method was helpful. According to the feedback received from the experts, they were significantly agreed with the rankings suggested by the modified UTASTAR method. Furthermore, the modified UTASTAR method was found to be useful in both consistent and inconsistent situations. Another interesting observation was that those experts who did not accept the rankings generated by the modified UTASTAR also asserted that the modified UTASTAR method is ease of use.

An interesting line for future researches can be work on a paradigm in UTA-based methods to facilitate the participation of experts in form of a group in decision making process. Also, another area of interest can be applying this approach for integrating heterogeneous information. Finally, the proposed method can be applied in other practical cases to evaluate its efficiency.

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