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Interval valued hesitant fuzzy method based on group decision analysis for estimating weights of decision makers

H. Gitinavard [1], A. Makui [1*], A. Jabbarzadeh [1]
[1] *School of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran*
h.gitinavard@gmail.com, amakui@iust.ac.ir, arminj@iust.ac.ir

Abstract

In this paper, a new soft computing group decision method based on the concept of compromise ratio is introduced for determining decision makers (DMs)' weights through the group decision process under uncertainty. In this method, preferences and judgments of the DMs or experts are expressed by linguistic terms for rating the industrial alternatives among selected criteria as well as the relative significance of each criterion. The DMs' opinions are demonstrated by a decision matrix in interval-valued hesitant fuzzy sets (IVHFSs). In addition, the interval-valued hesitant fuzzy positive and negative ideal solutions are defined by the matrix, respectively. Then, the hesitant fuzzy average and worst group scores of the DMs' decision matrix from matrices of interval-valued hesitant fuzzy positive and negative ideal solutions are described based on *n*-dimensional interval-valued hesitant fuzzy Euclidean distance measure. Further, a novel collective index is introduced based on the IVHFS to determine the weight of each DM or expert in the group decision process. Finally, an application example in industrial selection problems is presented about the best site selection for building a new factory to explain the computation process of the proposed soft computing group decision method in detail.

Keywords: Site location selection problem, interval-valued hesitant fuzzy sets, decision makers' weights, multi-criteria group decision making

1- Introduction

Multiple criteria decision making (MCDM) has been applied in different real-world applications, such as strategic planning (Kangas et al., 2001), economic analysis (Melese, 2009), forecasting (Yang et al., 2009), venture capital (Kung and Wen, 2007), supply chain management (Chatterjee and Kar, 2013) and location (Chu and Lai, 2005). When the hesitancy and complexity of the real-world situations is increased, it is difficult that a single decision maker (DM) analyzed all aspects of an industrial decision-making problem (Kim and Ahn, 1999), since the DMs have different specialty fields, knowledge, experience, skills and personality. Therefore, a group of the DMs is established to evaluate the candidate alternatives regarding to select criteria. In addition, DMs could not specify their judgments in these situations. Thus, fuzzy set theory has been known as a powerful tools to address the issue which introduced by Zadeh (1965). In this regard, many articles have extended the theory as interval-valued fuzzy sets (Turksen, 1986; Zadeh, 1975), fuzzy multi-sets (Miyamoto, 2000), intuitionistic fuzzy sets (IFSs) (Atanassov, 1986), hesitant fuzzy set (HFS) (Torra, 2010; Torra and Narukawa, 2009), interval-valued hesitant fuzzy set (IVHFS) (Chen et al., 2013) and etc.

*Corresponding author.
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Also, the fuzzy theory and its developments has been widely used in numerous science fields such as economics (Vis et al., 2013), engineering management (Doria, 2012; Patil and Kant, 2014), pattern recognition (Demirel et al., 2010), artificial intelligence (Greco et al., 2011), industrials (Zhou et al., 2012), and group decision making (Chen, 2000; Chen, 2001; Kahraman et al., 2003; Liu and Wu, 2008; Saghafian and Hejazi, 2005).

Chen (2000) developed the technique for order performance by similarity to ideal solution (TOPSIS) method in which the decision matrix was constructed by linguistic variables and then was converted by triangular fuzzy numbers. Furthermore, this study defined a closeness coefficient to sort the potential alternatives. Chen (2001) proposed a new algorithm to assess the rate of risk in software extension in fuzzy group decision-making analysis conditions. Kahraman et al. (2003) solved the facility location problem by utilizing four fuzzy Multi-criteria group decision-making (MCGDM) approaches. Then, a comparative analysis was considered to indicate the differences between the fuzzy group decision-making approaches. Saghafian and Hejazi (2005) presented a modified TOPSIS method with fuzzy numbers to solve the group decision-making problems. Liu and Wu (2008) applied a fuzzy approach in quality function development to cope with group decision-making process regarding to fuzzy / imprecise information.

According to the focus of researchers to the MCGDM problems, each DM could have various effects on decision-making process regarding to unique characteristics. Hence, it is worthwhile that the weight of each DM is considered separately. Therefore, obtaining the DMs' weights is significant and effective issue in group decision-making analysis. To address this issue, some researchers have focused on specifying the DMs' weights. To cope the issue, French (1956) presented a method to compute the relative significance of the DMs by utilizing the influence relations. Theil (1963) presented a method based on the correlation concepts to provide the DMs' weights. Keeney (1976) and Keeney and Kirkwood (1975) proposed the use of the interpersonal comparison to specify the scales constant values. Mirkin (1979) and Bodily (1979) focused on two methods to compute the DMs' weights by utilizing the eigen-vectors. Brock (1980) proposed a method to estimate the weights of experts by utilizing a Nash bargaining based approach. Martel and Ben Khélifa (2000) presented a procedure to obtain the relative significance of experts by utilizing the individual outranking indexes. Jabeur and Martel (2002) proposed a method to specify the relative importance coefficient of each DM regarding to the idea of Zeleny (1982). Van den Honert (2001) specified the importance of each DM by utilizing the multiplicative AHP and associated SMART model. Xu (2008) compute the DM's weight by applying the separation measures between additive linguistic preference operations. Yue (2011) developed the TOPSIS method to compute the weight of each DM with interval numbers for group decision-making problems.

Among the fuzzy set theories and its application to decision making methods, the interval-valued hesitant fuzzy set (IVHFS) theory is well-known tool in recent years. Indeed, the IVHFSs allows to DMs/experts to assign their judgments by some interval-values membership degrees for an object under a set to decrease the errors in a vagueness condition. In this sake, Zhang (2013) developed a wide range of power aggregation operators based on hesitant fuzzy information to aggregate the opinions of DMs for each candidate regarding to each criterion. Then, the usefulness of these proposed hesitant fuzzy power aggregation operators for group decision making problems are represented based on some practical examples. In addition, Xia et al. (2013) proposed several aggregation relations based on hesitant fuzzy set theory and the quasi-arithmetic means. Moreover, they extended two techniques based on support degrees and the Choquet integral to compute the weight vectors of aggregation arguments. Gitinavard et al. (2015) presented an interval-valued hesitant fuzzy multi-criteria group assessment approach to solve the industrial decision making problems. In their study, the weight of each criterion and DM is determined by proposing the interval-valued hesitant fuzzy entropy method and the interval-valued hesitant fuzzy compromise solution technique, respectively. Tavakkoli-Moghaddam et al. (2015) proposed a novel interval-valued hesitant fuzzy TOPSIS method to compute the weight of each criterion. Moreover, the weight of each DM and the opinions of DMs are considered in procedure of the proposed approach to determine the criteria weight, precisely. Xu et al. (2016) introduced a new version of fuzzy preference structure named incomplete hesitant fuzzy preference relations. In addition, they proposed two goal programming models to determine the collective priority weight of several hesitant fuzzy preference relations based on the defined additive consistency incomplete hesitant fuzzy preference relation and multiplicative consistency incomplete hesitant fuzzy preference relations, respectively.

The survey of the hesitant fuzzy literature in determining the weight of each DM represented that directly focusing on estimating the weight of DMs is poor. Thus, proposing an approach to determine the DMs' weight considering some important factors as preferences DMs' judgment, criteria weight, and the opinions of DMs about the relative importance of each criterion is motivated. Hence, in this paper, to cope with limitations of the DMs’ same weights in the fuzzy set theory, a new soft computing group decision method based on the IVHFS and compromise relation concept is introduced to determine the DMs’ weights for the MCGDM problems in industrial decision-making. Moreover, the criteria’ weights are determined from a non-linear programming concept by applying the opinions of each DM about the relative importance of each criterion. Then, values of interval-valued hesitant fuzzy average group score (IVHF-AGS) and interval-valued hesitant fuzzy worst group score (IVHF-WGS) are provided by using the *n*-dimensional Euclidean distance measure under interval-valued hesitant fuzzy environment. Finally, a novel collective index is presented based on the IVHFS to sort each DM and determine their final weights. In sums, the novelty of this study are explained as follows: (1) propose a soft computing group decision method in an interval-valued hesitant fuzzy setting to compute the DMs’ weight in MCGDM problems; (2) DMs assigned their judgments by some interval-values for a candidate under a set to decrease the errors; (3) the criteria weights are determined based on the extended maximizing deviation method and the preferences DMs’ judgments; (4) the opinion of DMs are expressed by linguistic variables which converted to IVHFS; (5) a novel collective index are presented based on the IVHFS.

The rest of this paper is organized as follows: in section 2, the preliminaries for IVHFSs are defined. In section 3, the proposed soft computing group decision method is illustrated under the IVHFS. In section 4, a numerical example is considered to show the feasibility and capability of the proposed method. Then, the proposed approach is compared with three methods from the recent literature under the six criteria to show the advantages of the proposed approach. Finally, some conclusions are presented in section 5.

2- Basic definitions

In this section, some basic concepts and relations that are utilized in the proposed soft computing group decision method are illustrated as below.

Definition 1: (Chen et al., 2013)**.** Let *X* be a universe of discourse, then the interval-valued hesitant fuzzy set (IVHFS) on *X* is expressed as below:

$$\tilde{E}=\left\{\left\langle x_i,\tilde{h}_{\tilde{E}}\left(x_i\right)\right\rangle \mid x_i\in X, i=1,2,...,n\right\} \tag{1}$$

Where $\tilde{h}_{\tilde{E}}\left(x_i\right)$ is the interval membership degree of an element $x_i\in X$ under set *E*.

Definition 2: (Chen et al., 2013)**.** Consider $\tilde{h},\tilde{h}_1$ and $\tilde{h}_2$ as three interval-valued hesitant fuzzy elements (IVHFEs), then some basic operations are defined as below:

$$\tilde{h}^{\lambda}=\left\{\left[\left(\tilde{\gamma}^{L}\right)^{\lambda},\left(\tilde{\gamma}^{U}\right)^{\lambda}\right] \mid \tilde{\gamma}\in\tilde{h}\right\},\lambda>0; \tag{2}$$

$$\lambda\tilde{h}=\left\{\left[1-\left(1-\tilde{\gamma}^{L}\right)^{\lambda},1-\left(1-\tilde{\gamma}^{U}\right)^{\lambda}\right] \mid \tilde{\gamma}\in\tilde{h}\right\},\lambda>0; \tag{3}$$

$$\tilde{h}_1\oplus\tilde{h}_2=\left\{\left[\gamma_1^L+\gamma_2^L-\gamma_1^L\gamma_2^L,\gamma_1^U+\gamma_2^U-\gamma_1^U\gamma_2^U\right] \mid \tilde{\gamma}_1\in\tilde{h}_1,\tilde{\gamma}_2\in\tilde{h}_2\right\} \tag{4}$$

$$\tilde{h}_1\otimes\tilde{h}_2=\left\{\left[\gamma_1^L\gamma_2^L,\gamma_1^U\gamma_2^U\right] \mid \tilde{\gamma}_1\in\tilde{h}_1,\tilde{\gamma}_2\in\tilde{h}_2\right\} \tag{5}$$

Definition 3: (Chen et al., 2013). The Euclidean and Hamming distance measures for IVHFS are defined as follows, respectively:

$$d\left(h_M,h_N\right)=\sqrt{\frac{1}{2l_{x_i}}\sum_{j=1}^{l_{x_i}}\left(\left|h_M^{\sigma(j)L}\left(x_i\right)-h_N^{\sigma(j)L}\left(x_i\right)\right|^2+\left|h_M^{\sigma(j)U}\left(x_i\right)-h_N^{\sigma(j)U}\left(x_i\right)\right|^2\right)} \tag{6}$$

$$d\left(h_M,h_N\right)=\frac{1}{2l_{x_i}}\sum_{j=1}^{l_{x_i}}\left(\left|h_M^{\sigma(j)L}\left(x_i\right)-h_N^{\sigma(j)L}\left(x_i\right)\right|+\left|h_M^{\sigma(j)U}\left(x_i\right)-h_N^{\sigma(j)U}\left(x_i\right)\right|\right) \tag{7}$$

Definition 4: (Chen et al., 2013). The interval-valued hesitant fuzzy geometric (IVHFG) relation is expressed as below:

$$IVHFG\left(\tilde{h}_1,\tilde{h}_2,...,\tilde{h}_n\right)=\left(\underset{j=1}{\overset{n}{\oplus}}\left(\tilde{h}_j\right)^{\frac{1}{n}}\right)=\cup_{\tilde{\gamma}_1\in\tilde{h}_1,\tilde{\gamma}_2\in\tilde{h}_2,...,\tilde{\gamma}_n\in\tilde{h}_n}\left\{\left[\prod_{j=1}^{n}\left(\gamma_j^L\right)^{\frac{1}{n}},\prod_{j=1}^{n}\left(\gamma_j^U\right)^{\frac{1}{n}}\right]\right\} \tag{8}$$

Definition 5: (Xu and Zhang, 2013). From a non-linear programming model, the following optimal weight vector is obtained that information is considered completely unknown.

$$w_j=\frac{\sum_{i=1}^{m}\sum_{k=1}^{m}\sqrt{\frac{1}{2l}\sum_{\lambda=1}^{l}\left(\left|h_{ij}^{\sigma(\lambda)^L}-h_{kj}^{\sigma(\lambda)^L}\right|^2+\left|h_{ij}^{\sigma(\lambda)^U}-h_{kj}^{\sigma(\lambda)^U}\right|^2\right)}}{\sqrt{\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{k=1}^{m}\sqrt{\frac{1}{2l}\sum_{\lambda=1}^{l}\left(\left|h_{ij}^{\sigma(\lambda)^L}-h_{kj}^{\sigma(\lambda)^L}\right|^2+\left|h_{ij}^{\sigma(\lambda)^U}-h_{kj}^{\sigma(\lambda)^U}\right|^2\right)}\right)^2}} \tag{9}$$

$$w_j^*=\frac{w_j}{\sum_{j=1}^{n}w_j},\quad j=1,2,...,n \tag{10}$$

Where w_j^* is defined the normalized weight vector.

Definition 6: (Jahanshahloo et al., 2009). Normalizing the hesitant fuzzy values (η_{ij}^l and η_{ij}^u) for each potential alternatives ($i=1,...,m$) regarding to each criterion ($j=1,...,n$) is as follows:

$$\eta_{ij}^l=\frac{x_{ij}^l}{\sqrt{\sum_{i=1}^{m}[(x_{ij}^l)^2+(x_{ij}^u)^2]}} \tag{11}$$

$$\eta_{ij}^u=\frac{x_{ij}^u}{\sqrt{\sum_{i=1}^{m}[(x_{ij}^l)^2+(x_{ij}^u)^2]}} \tag{12}$$

where the interval $[\eta_{ij}^l,\eta_{ij}^u]$ is normalized from $[x_{ij}^l,x_{ij}^u]$.

Definition 7: (Tavakkoli-Moghaddam et al., 2015). One of the usual computing of experts' weights is considering the following relations as follows:

$$\lambda_k^L = \frac{\sum_i^m \sum_j^n \mu_{ij}^{Lk}}{\sum_k^K \sum_i^m \sum_j^n \mu_{ij}^{Lk}} \tag{13}$$

$$\lambda_k^U = \frac{\sum_i^m \sum_j^n \mu_{ij}^{Uk}}{\sum_k^K \sum_i^m \sum_j^n \mu_{ij}^{Uk}} \tag{14}$$

$$\lambda_k^f = \frac{\sum_i^m \sum_j^n \left(\mu_{ij}^{Uk} + \mu_{ij}^{Lk}\right)}{\sum_k^K \sum_i^m \sum_j^n \left(\mu_{ij}^{Uk} + \mu_{ij}^{Lk}\right)} \tag{15}$$

Where the final weight of each DM is indicated by $\lambda_k = \left[\lambda_k^L, \lambda_k^U\right]$ and the final DMs' weights is denoted by $\lambda_k^f, \sum_{k=1}^{K} \lambda_k^f = 1$.

3- Proposed soft computing group decision method

In this section, the soft computing group decision method is presented based on the IVHFSs. In this sake, the hierarchical of the proposed method is depicted in Figure 1.

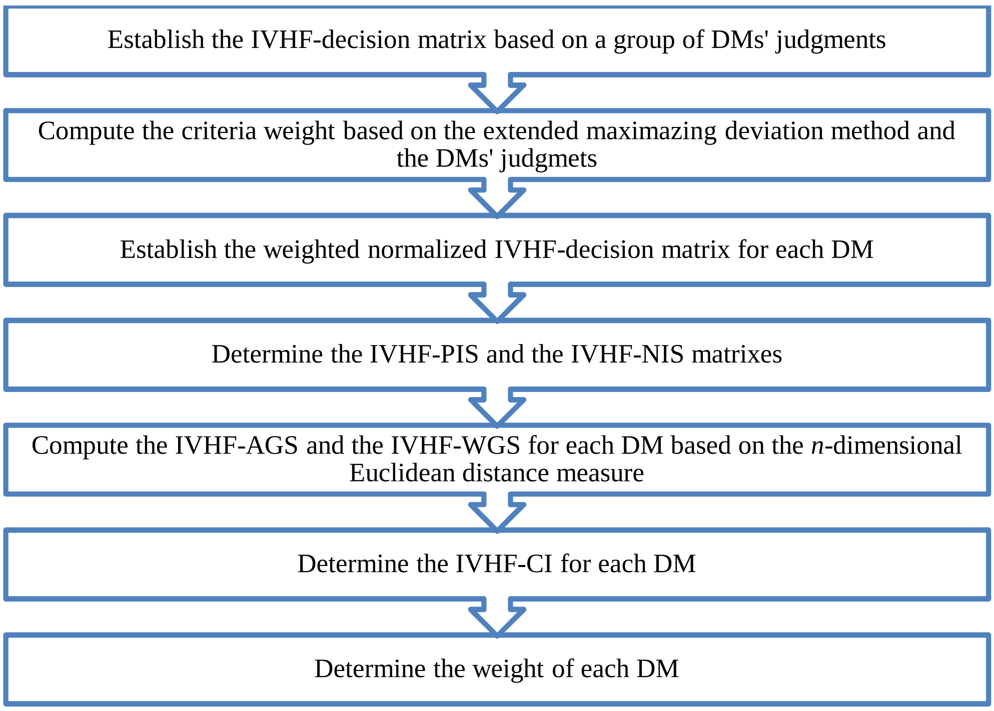

Figure 1. The hierarchical structure of the proposed soft computing group decision method

Step1. Construct a committee of DMs or experts to establish the interval-valued hesitant fuzzy decision matrix (IVHF-decision matrix) by assigning their judgments.

$$G = \begin{matrix} & C_1 & \cdots & C_n \\ A_1 & \left\{\left[\mu_{11}^{L1},\mu_{11}^{U1}\right],\left[\mu_{11}^{L2},\mu_{11}^{U2}\right],...,\left[\mu_{11}^{Lk},\mu_{11}^{Uk}\right]\right\} & \cdots & \left\{\left[\mu_{1n}^{L1},\mu_{1n}^{U1}\right],\left[\mu_{1n}^{L2},\mu_{1n}^{U2}\right],...,\left[\mu_{1n}^{Lk},\mu_{1n}^{Uk}\right]\right\} \\ \vdots & \vdots & \ddots & \vdots \\ A_m & \left\{\left[\mu_{m1}^{L1},\mu_{m1}^{U1}\right],\left[\mu_{m1}^{L2},\mu_{m1}^{U2}\right],...,\left[\mu_{m1}^{Lk},\mu_{m1}^{Uk}\right]\right\} & \cdots & \left\{\left[\mu_{mn}^{L1},\mu_{mn}^{U1}\right],\left[\mu_{mn}^{L2},\mu_{mn}^{U2}\right],...,\left[\mu_{mn}^{Lk},\mu_{mn}^{Uk}\right]\right\} \end{matrix}_{m\times n} \tag{16}$$

Step2. Compute the weight of each criterion (ω_j) based on the extended maximizing deviation method and consider the DMs' opinions for rating the relative importance of each criterion ($\overline{\upsilon}_j$).

$$\omega_j = \frac{\overline{\upsilon}_j . \sum_{i=1}^{m}\sum_{k=1}^{m}\left(\frac{1}{2l}\sum_{\lambda=1}^{l}\left(\left|h_{ij}^{\sigma(\lambda)^L} - h_{kj}^{\sigma(\lambda)^L}\right| + \left|h_{ij}^{\sigma(\lambda)^U} - h_{kj}^{\sigma(\lambda)^U}\right|\right)\right)}{\sqrt{\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{k=1}^{m}\left(\frac{1}{2l}\sum_{\lambda=1}^{l}\left(\left|h_{ij}^{\sigma(\lambda)^L} - h_{kj}^{\sigma(\lambda)^L}\right| + \left|h_{ij}^{\sigma(\lambda)^U} - h_{kj}^{\sigma(\lambda)^U}\right|\right)\right)\right)^2}} \tag{17}$$

$$\upsilon_j = IHVFG\left(\tilde{h}_1,\tilde{h}_2,...,\tilde{h}_k\right) = \left(\overset{K}{\underset{k=1}{\oplus}}\left(\tilde{h}_k\right)^{\frac{1}{k}}\right) = \cup_{\tilde{\gamma}_1\in\tilde{h}_1,\tilde{\gamma}_2\in\tilde{h}_2,...,\tilde{\gamma}_k\in\tilde{h}_k}\left\{\left[\prod_{j=1}^{K}\left(\gamma_k^L\right)^{\frac{1}{k}},\prod_{j=1}^{K}\left(\gamma_k^U\right)^{\frac{1}{k}}\right]\right\} \tag{18}$$

$$\overline{\upsilon}_j = \frac{\mu_j^{Lk} + \mu_j^{Uk}}{2} \quad \forall j \tag{19}$$

Where the normalized weight vector is indicated as follows:

$$\omega_j^* = \frac{\omega_j}{\sum_{j=1}^{n}\omega_j}, \quad j = 1,2,...,n \tag{20}$$

Step3. Construct the IVHF-decision matrix for each DM (G_k). Then, the IVHF-decision matrix is normalized for each DM based on definition 6 and the weighted normalized IVHF-decision matrix is established. In this respect, the weighted normalized IVHF-decision matrix of *k*th DM is represented as follows:

$$G_k = \begin{matrix} & C_1 & C_2 & \cdots & C_n \\ A_1 & \left[\mu_{11}^{Lk},\mu_{11}^{Uk}\right] & \left[\mu_{12}^{Lk},\mu_{12}^{Uk}\right] & \cdots & \left[\mu_{1n}^{Lk},\mu_{1n}^{Uk}\right] \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_m & \left[\mu_{m1}^{Lk},\mu_{m1}^{Uk}\right] & \left[\mu_{m2}^{Lk},\mu_{m2}^{Uk}\right] & \cdots & \left[\mu_{mn}^{Lk},\mu_{mn}^{Uk}\right] \end{matrix}_{m\times n} \tag{21}$$

Step4. Determine the interval-valued hesitant fuzzy positive ideal solution (IVHF-PIS) matrix (δ^*) and interval-valued hesitant fuzzy negative ideal solution (IVHF-NIS) matrix (δ^-).

$$\delta^* = \left(\left[\mu_{ij}^{*L}, \mu_{ij}^{*U}\right]\right)_{m\times n} = \begin{matrix} & C_1 & C_2 & \cdots & C_n \\ \begin{matrix} A_1 \\ \vdots \\ A_m \end{matrix} & \multicolumn{4}{l}{\begin{pmatrix} \left[\mu_{11}^{L*}, \mu_{11}^{U*}\right] & \left[\mu_{12}^{L*}, \mu_{12}^{U*}\right] & \cdots & \left[\mu_{1n}^{L*}, \mu_{1n}^{U*}\right] \\ \vdots & \vdots & \ddots & \vdots \\ \left[\mu_{m1}^{L*}, \mu_{m1}^{U*}\right] & \left[\mu_{m2}^{L*}, \mu_{m2}^{U*}\right] & \cdots & \left[\mu_{mn}^{L*}, \mu_{mn}^{U*}\right] \end{pmatrix}_{m\times n}} \end{matrix} \tag{22}$$

$$\delta^- = \left(\left[\mu_{ij}^{-L}, \mu_{ij}^{-U}\right]\right)_{m\times n} = \begin{matrix} & C_1 & C_2 & \cdots & C_n \\ \begin{matrix} A_1 \\ \vdots \\ A_m \end{matrix} & \multicolumn{4}{l}{\begin{pmatrix} \left[\mu_{11}^{L-}, \mu_{11}^{U-}\right] & \left[\mu_{12}^{L-}, \mu_{12}^{U-}\right] & \cdots & \left[\mu_{1n}^{L-}, \mu_{1n}^{U-}\right] \\ \vdots & \vdots & \ddots & \vdots \\ \left[\mu_{m1}^{L-}, \mu_{m1}^{U-}\right] & \left[\mu_{m2}^{L-}, \mu_{m2}^{U-}\right] & \cdots & \left[\mu_{mn}^{L-}, \mu_{mn}^{U-}\right] \end{pmatrix}_{m\times n}} \end{matrix} \tag{23}$$

Where the elements of the PIS matrix / NIS matrix are computed as follows:

$$\mu_{ij}^{*L} = \frac{1}{K}\sum_{k=1}^{K} \mu_{ij}^{Lk} \tag{24}$$

$$\mu_{ij}^{*U} = \frac{1}{K}\sum_{k=1}^{K} \mu_{ij}^{Uk} \tag{25}$$

$$\mu_{ij}^{-L} = \min_{k}\left\{\mu_{ij}^{Lk}\right\} \tag{25}$$

$$\mu_{ij}^{-U} = \max_{k}\left\{\mu_{ij}^{Uk}\right\} \tag{27}$$

Step5. Calculate the interval-valued hesitant fuzzy average group score (IVHF-AGS) value (α_k) and the interval-valued hesitant fuzzy worst group score (IVHF-WGS) value (β_k) for each DM's decision matrix by utilizing the *n*-dimensional Euclidean distance measure under IVHF-environment.

$$\alpha_k = \left(\frac{1}{2l_{x_i}}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{\lambda=1}^{l_{x_i}}\left(\left|\mu_{ij}^{L\sigma(\lambda)}(x_i) - \delta_{ij}^{*L\sigma(\lambda)}(x_i)\right|^2 + \left|\mu_{ij}^{U\sigma(\lambda)}(x_i) - \delta_{ij}^{*U\sigma(\lambda)}(x_i)\right|^2\right)\right)^{\frac{1}{2}} \quad \forall k \tag{28}$$

$$\beta_k = \left(\frac{1}{2l_{x_i}}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{\lambda=1}^{l_{x_i}}\left(\left|\mu_{ij}^{L\sigma(\lambda)}(x_i) - \delta_{ij}^{-L\sigma(\lambda)}(x_i)\right|^2 + \left|\mu_{ij}^{U\sigma(\lambda)}(x_i) - \delta_{ij}^{-U\sigma(\lambda)}(x_i)\right|^2\right)\right)^{\frac{1}{2}} \quad \forall k \tag{29}$$

Step6. Calculate the value of proposed new interval-valued hesitant fuzzy collective index (IVHF-CI) for each DM (CI_k) as follows:

$$\eta_k = \begin{cases} \left(\dfrac{\alpha_k}{\beta_k}\right)^{\frac{1}{k}} & \forall \alpha_k \le \beta_k \\ 1+\psi_k & \forall \alpha_k \ge \beta_k \end{cases} \quad \forall k \tag{30}$$

$$\psi_k = \left(\max_{k\in\varphi}\left(\frac{\alpha_k}{\beta_k}\right)\right)^{\frac{1}{\max_j w_j}} \qquad \forall\left(\alpha_k \le \beta_k\right) \in \varphi, k \tag{31}$$

$$\tau_k = \left(\alpha_k\right)^{\frac{1}{m}} + \left(1-\beta_k\right)^{\frac{1}{k}} \quad \forall k \tag{32}$$

$$CI_k = \eta_k + \tau_k \quad \forall k \tag{33}$$

Step7. Determine the weight of each DM (ϖ_k) by the following relation:

$$\varpi_k = \frac{CI_k}{\sum_{k=1}^{K} CI_k} \quad \forall k \tag{34}$$

4- Application example

In this section, an application example in industrial selection problems is considered to indicate the capability of the proposed method for estimating the weight (relative importance) of each DM. The application example is about the site selection for building a new factory that has adopted from Wang (2014). In this respect, the proposed soft computing group decision method is compared with the recent TOPSIS method under interval fuzzy environment by Yue (2011) to show corresponding results. The location problem is evaluated by four DM (DM_k, k=1,2,…,4) and is constructed by three potential alternative (A_i, i=1, 2,3) and sixth criteria (C_j, j=1, 2, …, 6) defined as follows:

C_1: Climate condition;

C_2: Regional demand;

C_3: Expansion possibility;

C_4: Transportation availability;

C_5: Labor force; and

C_6: Investment cost.

As demonstrated in Table 1 and Table 2, the relative significance of each linguistic variable for evaluating the importance of criteria and performance rating of possible alternatives are expressed based on IVHFEs, respectively. In addition, the assessment of candidate alternatives defined by preferences of the DMs by linguistic terms that are represented in Table 3, and also these linguistic terms transformed to the IVHFEs and given in Table 4. Similarly, the DMs' judgments about relative significance of selected criteria are expressed as linguistic variables and converted to the IVHFEs. Therefore, the final criteria' weights are computed by utilizing Equations (17)-(20) regarding to DMs' opinions. The corresponding results are provided in Table 5 and Table 6, respectively.

Table 1. Linguistic variables for rating the importance of criteria

Linguistic terms	Interval-valued hesitant fuzzy elements
Very important (VI)	[0.90,0.90]
Important (I)	[0.75, 0.80]
Medium (M)	[0.50, 0.55]
Unimportant (UI)	[0.35, 0.40]
Very unimportant (VUI)	[0.10,0.10]

Table 2. Linguistic variables for the rating of potential alternatives

Linguistic terms	Interval-valued hesitant fuzzy elements
Extremely good (EG)/extremely high (EH)	[1.00,1.00]
Very very good (VVG)/very very high (VVH)	[0.90,0.90]
Very good (VG)/very high (VH)	[0.80, 0.90]
Good (G)/high (H)	[0.70, 0.80]
Medium good (MG)/medium high (MH)	[0.60, 0.70]
Fair (F)/medium (M)	[0.50, 0.60]
Medium bad (MB)/medium low (ML)	[0.40, 0.50]
Bad (B)/low (L)	[0.25, 0.40]
Very bad (VB)/very low (VL)	[0.10, 0.25]
Very very bad (VVB)/very very low (VVL)	[0.10,0.10]

Table 3. The linguistic assessments for rating the potential alternative based on DMs' judgments

Main criteria	Alternatives	k_1	k_2	k_3	k_4
C_1	A_1	MG	MG	G	VG
	A_2	VG	G	G	G
	A_3	F	MG	F	MG
C_2	A_1	G	VG	VG	MG
	A_2	F	MG	F	F
	A_3	VG	VG	VG	G
C_3	A_1	F	MG	MG	MG
	A_2	VG	G	VG	VG
	A_3	MG	MG	G	MG
C_4	A_1	VG	VG	G	G
	A_2	G	G	G	MG
	A_3	G	G	VG	F
C_5	A_1	MG	G	G	VG
	A_2	MG	MG	VG	G
	A_3	VG	G	G	G
C_6	A_1	VVG	VG	VVG	EG
	A_2	F	B	B	MG
	A_3	VVG	VVG	VG	VVG

Table 4. The assessment of alternatives expressed by the IVHFEs

Main criteria	Alternatives	k_1	k_2	k_3	k_4
C_1	A_1	[0.60, 0.70]	[0.60, 0.70]	[0.70, 0.80]	[0.80, 0.90]
	A_2	[0.80, 0.90]	[0.70, 0.80]	[0.70, 0.80]	[0.70, 0.80]
	A_3	[0.50, 0.60]	[0.60, 0.70]	[0.50, 0.60]	[0.60, 0.70]
C_2	A_1	[0.70, 0.80]	[0.80, 0.90]	[0.80, 0.90]	[0.60, 0.70]
	A_2	[0.50, 0.60]	[0.60, 0.70]	[0.50, 0.60]	[0.50, 0.60]
	A_3	[0.80, 0.90]	[0.80, 0.90]	[0.80, 0.90]	[0.70, 0.80]
C_3	A_1	[0.50, 0.60]	[0.60, 0.70]	[0.60, 0.70]	[0.60, 0.70]
	A_2	[0.80, 0.90]	[0.70, 0.80]	[0.80, 0.90]	[0.80, 0.90]
	A_3	[0.60, 0.70]	[0.60, 0.70]	[0.70, 0.80]	[0.60, 0.70]
C_4	A_1	[0.80, 0.90]	[0.80, 0.90]	[0.70, 0.80]	[0.70, 0.80]
	A_2	[0.70, 0.80]	[0.70, 0.80]	[0.70, 0.80]	[0.60, 0.70]
	A_3	[0.70, 0.80]	[0.70, 0.80]	[0.80, 0.90]	[0.50, 0.60]
C_5	A_1	[0.60, 0.70]	[0.70, 0.80]	[0.70, 0.80]	[0.80, 0.90]
	A_2	[0.60, 0.70]	[0.60, 0.70]	[0.80, 0.90]	[0.70, 0.80]
	A_3	[0.80, 0.90]	[0.70, 0.80]	[0.70, 0.80]	[0.70, 0.80]
C_6	A_1	[0.90,0.90]	[0.80, 0.90]	[0.90,0.90]	[1.00, 1.00]
	A_2	[0.50, 0.60]	[0.25, 0.40]	[0.25, 0.40]	[0.60, 0.70]
	A_3	[0.90,0.90]	[0.90,0.90]	[0.80, 0.90]	[0.90,0.90]

Table 5. The linguistic assessment for criteria' weight based on DMs' opinions

C_j	k_1	k_2	k_3	k_4
C_1	I	VI	M	I
C_2	M	I	M	I
C_3	I	M	I	VI
C_4	I	UI	M	M
C_5	M	I	I	M
C_6	M	M	I	I

Table 6. Specify the DMs' judgments about criteria' weight and the final weight

C_j	k_1	k_2	k_3	k_4	$\overline{\upsilon}_j$	ω_j^*
C_1	[0.75, 0.80]	[0.90,0.90]	[0.50, 0.55]	[0.75, 0.80]	0.657138	0.169198
C_2	[0.50, 0.55]	[0.75, 0.80]	[0.50, 0.55]	[0.75, 0.80]	0.549259	0.176777
C_3	[0.75, 0.80]	[0.50, 0.55]	[0.75, 0.80]	[0.90,0.90]	0.657138	0.169198
C_4	[0.75, 0.80]	[0.35, 0.40]	[0.50, 0.55]	[0.50, 0.55]	0.431255	0.069399
C_5	[0.50, 0.55]	[0.75, 0.80]	[0.75, 0.80]	[0.50, 0.55]	0.549259	0.088389
C_6	[0.50, 0.55]	[0.50, 0.55]	[0.75, 0.80]	[0.75, 0.80]	0.549259	0.327038

As demonstrated in Table 7, the weighted normalized IVHF-decision matrix is obtained by considering the Eq. (21) (Step 3), and then the IVHF-PIS / IVHF-NIS matrices are constructed by considering the relations (22)-(27) (Step 4) and are shown in Table 8. In addition, the IVHF-AGS / IVHF-WGS values are computed based on *n*-dimensional IVHF-Euclidean distance measure by Eqs. (28) and (29) (Step 5). Finally, the proposed IVHF-CI has been computed by using Eqs. (30)-(33) (Step 6) to determine the weight of each DM. Thus, the final weight of each DM is assessed by Eq. (34) (Step 7). The results have been shown in Table 9.

Table 7. The weighted normalized IVHF-decision matrix

C_j	A_i	k_1	k_2	k_3	k_4
	A_1	[0.059511, 0.069439]	[0.060346, 0.070404]	[0.069912, 0.079899]	[0.073086, 0.082222]
C_1	A_2	[0.079348, 0.089267]	[0.070404, 0.080462]	[0.069912, 0.079899]	[0.063950, 0.073086]
	A_3	[0.049592, 0.059511]	[0.060346, 0.070404]	[0.049937, 0.059924]	[0.054815, 0.063959]
	A_1	[0.069283, 0.079181]	[0.073029, 0.082159]	[0.075485, 0.084921]	[0.065906, 0.076891]
C_2	A_2	[0.049488, 0.059386]	[0.054772, 0.063901]	[0.047178, 0.056614]	[0.054922, 0.065906]
	A_3	[0.079181, 0.089079]	[0.073029, 0.082159]	[0.075485, 0.084921]	[0.076899, 0.087875]
	A_1	[0.049592, 0.059511]	[0.060346, 0.070405]	[0.054815, 0.063951]	[0.057199, 0.066733]
C_3	A_2	[0.079348, 0.089267]	[0.070404, 0.080462]	[0.073086, 0.082223]	[0.076265, 0.085799]
	A_3	[0.059511, 0.069430]	[0.060346, 0.070405]	[0.063950, 0.073087]	[0.057199,0.066733]
	A_1	[0.028824, 0.032427]	[0.028824, 0.032427]	[0.025221, 0.028824]	[0.030185, 0.034408]
C_4	A_2	[0.025221, 0.028824]	[0.025221, 0.028824]	[0.025221, 0.028824]	[0.025873, 0.030185]
	A_3	[0.025221, 0.028824]	[0.025221, 0.028824]	[0.028824, 0.032427]	[0.021561, 0.025873]
	A_1	[0.029880, 0.034860]	[0.035084, 0.040096]	[0.032122, 0.036711]	[0.036711, 0.041300]
C_5	A_2	[0.029880, 0.034860]	[0.030072, 0.035084]	[0.036711, 0.041300]	[0.032122, 0.036711]
	A_3	[0.039841, 0.044821]	[0.035084, 0.040096]	[0.032122, 0.036711]	[0.032122, 0.036711]
	A_1	[0.150006, 0.150006]	[0.144186, 0.053048]	[0.162210, 0.162210]	[0.154683, 0.154683]
C_6	A_2	[0.083337, 0.100004]	[0.045058, 0.023579]	[0.045058, 0.072093]	[0.092810, 0.108278]
	A_3	[0.150006, 0.150006]	[0.162210, 0.053048]	[0.144186, 0.162210]	[0.139215, 0.139215]

Table 8. The IVHF-PIS and IVHF-NIS

C_j	δ^*		
	A_1	A_2	A_3
C_1	[0.065714, 0.075489]	[0.070904, 0.080679]	[0.053673, 0.063447]
C_2	[0.070926, 0.080788]	[0.051590, 0.061452]	[0.076146, 0.086008]
C_3	[0.055488, 0.065150]	[0.074776, 0.084438]	[0.060252, 0.069914]
C_4	[0.028263, 0.032044]	[0.025384, 0.029164]	[0.025206, 0.028987]
C_5	[0.033449, 0.038242]	[0.032196, 0.036989]	[0.034792, 0.039585]
C_6	[0.152771, 0.129987]	[0.066566, 0.075988]	[0.148904, 0.126120]
C_j	δ^-		
	A_1	A_2	A_3
C_1	[0.059511, 0.082222]	[0.063950, 0.089267]	[0.049592, 0.070404]
C_2	[0.065906, 0.084921]	[0.047178, 0.065906]	[0.073029, 0.089079]
C_3	[0.049592, 0.070405]	[0.070404, 0.089267]	[0.057199, 0.073087]
C_4	[0.025221, 0.034498]	[0.025221, 0.030185]	[0.021561, 0.032427]
C_5	[0.029880, 0.041303]	[0.029889, 0.041300]	[0.032122, 0.044821]
C_6	[0.144186, 0.162210]	[0.045058, 0.108278]	[0.139215, 0.162212]

Table 9. The computational results of Steps 5-7

DM_k	α_k	β_k	η_k	τ_k	CI_k	ϖ_k
k=1	0.034076	0.038966	0.967033	1.314316	2.281349	0.246642
k=2	0.086647	0.127343	0.908228	1.409025	2.317254	0.250524
k=3	0.040086	0.036081	1.026661	1.333095	2.359757	0.255119
k=4	0.038673	0.209397	0.964442	1.326805	2.291247	0.247712

Moreover, the proposed soft computing group decision method is compared with some recent literature methods, which have computed the DMs' weight to indicate the efficiency and feasibility of the proposed approach. In this respect, the survey of the literature shows that the studies of Yue (2011), Gitinavard et al. (2015), and Tavakkoli-Moghaddam et al. (2015) are near to our proposed approach, and thus the application example is solved by these methods to compare the obtained results with the proposed soft computing group decision method. Give the obtained results from the four methods in Table 10.

Table 10. The results of solving the application example by four methods

DM_k	Proposed soft computing group decision method ϖ_k)	Yue (2011) method	Tavakkoli-Moghaddam et al. (2015) method	Gitinavard et al. (2015) method
k=1	0.246642	0.249368	0.248812	0.25982
k=2	0.250524	0.256715	0.247388	0.26213
k=3	0.255119	0.246189	0.253086	0.25121
k=4	0.247712	0.247726	0.250712	0.22683

As demonstrated in Table 11, the proposed soft computing group decision approach is compared with three recent literature methods under the six criteria. The proposed approach in versus the other approaches considered the criteria weights and the preferences DMs' judgments about the criteria importance, which might lead to a precise solution. Computing the DMs' weights by different techniques commonly is caused to different results. It is inappropriate to conclude that specific technique is forceful and effective because every technique has potential and capability underlying the assertion or theory. Hence, the proposed soft computing group decision approach for determining the DMs' weights is suitable to compromise of nearby to the ideal and farther from the negative ideal. Since, the DMs' weights of the proposed decision approaches are generated from the DMs' judgments, a ‘‘biased” or ‘‘false” judgments may cause to a low weight (Yue, 2011). However, the proposed soft computing group decision method for obtaining the DMs' weights is more capable to cope with hesitant / uncertain situations, because the preferences of the DMs are determined through the group decision-making process by some interval-values for an object under a set to decrease the errors. In addition, the criteria weights and the opinions of DMs about the relative significance of each criterion are considered in procedure of determining the DMs' weight.

Table 11. The comparative analysis between the four methods

Approaches	Assign some membership degrees	Based on interval-valued information	Computing the criteria weight	Group decision analysis	Considering the DMs' opinions about the criteria importance	Compromise solution
Yue (2011) method		✓		✓		
Tavakkoli-Moghaddam et al. (2015) method	✓	✓		✓		
Gitinavard et al. (2015) method	✓	✓		✓		✓
Proposed soft computing group decision method	✓	✓	✓	✓	✓	✓

5- Concluding remarks and future researches

In this study, a new soft computing group decision making method has been proposed to specify the decision makers (DMs)' weights for multi-criteria group decision making (MCGDM) which could be implemented to solve industrial selection problems. For this purpose, the weight of each criterion was computed by hybridizing a non-linear programming concept and the opinion of each DM about the relative significance of selected criteria. In addition, the interval-valued hesitant fuzzy positive ideal solution (IVHF-PIS) / interval-valued hesitant fuzzy negative ideal solution (IVHF-NIS) matrices were established. Then, the interval-valued hesitant fuzzy average group score (IVHF-AGS) and the interval-valued hesitant fuzzy worst group score (IVHF-WGS) values were determined by using the *n*-dimensional Euclidean distance measure under interval-valued hesitant fuzzy environment. Also, a novel IVHF-CI based on the compromise ratio concept was introduced to appraise each DM and determine the weight of each DM. For industrial selection problems, the best site selection for building the new factory has indicated the verification and feasibility of the proposed soft computing group decision method. The presented method was compared with three recent literature studies to shows the feasibility of the proposed approach. Moreover, a comparative analysis is established based on six criteria to clear representation of the advantages and merits of each method. The proposed soft computing group decision method has some uniqueness characteristics under hesitant situations. The IVHF-decision matrix and the opinions of the DMs about the relative importance of each criterion were expressed by linguistic variables to help the DMs for assigning the membership degrees. For future direction, the presented method can be enhanced by considering the hierarchical structure in criteria. In addition, the proposed DMs' weight method can be considered in process of a new extended ranking method to help the IVHFS literature for solve the group decision problems, precisely.

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