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Group Malmquist Productivity Index, focusing on local and global role of groups: A case study of banking industry

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Abstract

Evaluating the performance of the groups of decision-making units (DMUs) in an organization and ranking them is very important because the groups are the basic components of organizations. Also, assessing the performance of groups and finding their strengths and weaknesses affect the future decisions of the organization. Most previous research has been done on the Malmquist productivity index of decision-making units to evaluate productivity changes of them. In practical evaluation problems, although decision units are homogeneous in many organizations, in the sense that they produce similar outputs by receiving the same input sources; However, these units are classified into different groups based on management strategies and environmental conditions. In this paper, we want to develop the Malmquist productivity index for a group of DMU's and examine the productivity changes of groups. We also identify local and global factors that affect changes in group productivity. Finally, to explain the applicability of the proposed index, a case study is presented on the evaluation of bank branches in different regions.
Keywords: Data Envelopment Analysis, group efficiency, Malmquist Productivity Index.

1- Introduction

Data Envelopment Analysis (DEA) is a non-parametric method for evaluating the performance of multi-input and multi-output decision units. In applied evaluation problems, although all decision-making units are homogeneous, in the sense that they produce the same outputs by receiving the same input sources; However, these units can be classified into different groups based on management strategies and environmental conditions. Group performance evaluation means evaluating the performance of a group of decision-making units (DMU's). For example, the branches of a bank in different regions can be considered as groups of decision-making units.

Calculating the productivity index of groups in an organization can provide a valuable information about the factors affecting the productivity growth of the organization, its strengths and weaknesses, and the solution to achieve proper growth. The Malmquist index (MI) is a concept introduced to measure the relative productivity change overtime periods by Caves et al. (1982). Fare et al. (1992) presented the FGLR decomposition of the Malmquist index using constant returns to scale (CRS) technology involving two components: efficiency change and technological change.

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Fare at al. (1994) extended the FGNZ decomposition of the MI using both CRS and variable returns to scale (VRS) technologies and presented the three-component decomposition: pure efficiency change (PEC), scale efficiency change (SEC) and technological change. In this paper, we seek to develop this index for a group of decision units. To extend this index, we first need to find a suitable distance function to evaluate the performance of a group. In the subject literature, most research has been done on evaluating the performance of decision units and measuring their productivity changes. However, few studies have been conducted to examine the performance of a group of decision-making units and evaluate its productivity changes.

Forsund and Hjalmarsson (1979) proposed the technical efficiency of the average unit as a measure of the performance of a group. Afterward, Charnes et al. (1981) examined group analysis instead of individual analysis. They proposed a two-step approach that breaks down DEA performance into two components: unit performance (such as programs, policies, and environmental conditions) and internal management performance. In the proposed method by Charnes et al. (1981), the performance of groups is compared to a co-production boundary. However, the programs of each group cause difficulties to reach the inner group efficiency frontier that the effect of them has not been studied in their method. An application of this method in the banking sector can be found in Grifell and Lovell (1997).

Mirghaderi and Aboumasoudi (2017) also examined the performance of 470 companies in the Tehran stock exchange market during the years 2013 to 2016. They categorized these companies into different industries based on their activities and considered the average efficiency of the companies in an industry as the efficiency of that industry. Ylvinger (2000) showed that the technical efficiency of the average unit is not a good representative of the performance of a group. He examined this issue with an interesting example; He considered a group with some technically efficient units that the average unit was not technically efficient. Then, he added an inefficient unit to the group and observed that the efficiency of the average unit increased. Ylvinger (2000) stated that this is a contradiction because it seems that a group with an inefficient unit has been more efficient than a group with efficient units. Therefore, the average unit performance does not reflect the group.

Using the aggregation of inputs and outputs, Li and Ng (1995) proposed a shadow price model to measure the performance of a group. Bagherzadeh Valami (2009) proposed an index to evaluate the performance of a group, which is defined based on the geometric average of the performance of all units relative to the boundary of that group which this method may be impossible.

Cook and Zhou (2007) proposed the common set of weights method to measure the performance of groups and their units and evaluated the performance of power plants in Canada. They established a multi-objective model and considered its optimal solution as common weights for evaluating the performance of a group and its members. It is noteworthy that in their method, the efficiency of the group is defined as the maximum efficiency of its members. Rezaee and Karimdadi (2015) used the Cook's method to compare the performance of hospitals in different provinces of Iran. Payan and Rahmani (2014) proposed a model of the common set of weights methods. In their method, unlike Cook's method, which defined group performance based on the best performance of its members, they defined group performance as a convex combination of unit performance. Ang et al. (2018) proposed cross efficiency model to evaluate the performance of hotels in China's tourism industry. They stated that the efficiency of all units should be defined based on a common boundary and a common weight. They defined group performance based on integration units into a virtual unit which its input is the sum of its member's inputs and its output is sum of its member's outputs.

Berg et al. (1993) defined the Malmquist index (the same Malmquist productivity index over a period of time) to compare the performance of banks in different countries and used a special unit as the representative unit for each country. Using this index, Berg et al. examined the productivity of banks in Sweden, Finland and Norway. In their method, the performance of the "average" and "largest banks" of the countries was calculated relative to the Swedish frontier. Sweden was considered as the basic technology because its big banks performed better. Berg et al. (1993) Stated that considering a common basic technology would make comparisons between countries easier. Pastor et al. (1997) used the index proposed by Berg and considered the Spanish frontier as the basic technology. They calculated the productivity of banks in different countries based on the " middle" bank, "average" bank and "weighted average" bank of each country; In fact, they used a specific unit for each group (mean, mean, or weighted average) to compare groups.

Since considering a particular unit as a representative of a group can't provide accurate and accurate results about the whole group to managers and decision makers of organizations, so Camanho and Dyson (2006) suggested an index of the Malmquist index type to compare the performance of the two groups that there was no need to select a special representative from the group. Their method has been robust in terms of economic analysis, but it is only suitable for comparing the performance of two groups and is not able to compare several groups with each other. Abbaspour et al. (2009) generalized the introduced index of Camanho and Dyson (2006) for non-discretionary inputs and outputs. Thanassoulis et al. (2015) developed the method of Camanho and Dyson (2006) to compare the performance of the two groups in terms of the lowest cost of using input resources. Boďa and Považanová (2020) propose an extension of the methodology for measuring productivity change based on the Malmquist index that now affords consistent comparisons of productivity change and productivity levels in situations when there are groups of several units monitored in periods 2003 - 2008 and 2010 - 2015. Walheer (2018) propose a new cost Malmquist productivity index to compare groups of DMUs for each output separately. Aparicioa et al. (2021) extend the Camanho and Dyson (2006) one-period Malmquist-type index (CDMI) and the pseudo-panel Malmquist index (PPMI) by Aparicio et al. (2017) and Aparicio and Santín (2018) to evaluate the productivity of public and private schools in some European Union countries. Afsharian et al. (2019) extend the respective DEA approach of Camanho and Dyson (2006) by introducing an index for comparing the performance of the management groups under such a centralized management scenario and decomposing it into the efficiency index and the technological gap index. Fang (2022) expanded metafrontier centralized performance index suggested Afsharian et al. (2019) and proposed a new approach to decompose the meta-frontier centralized performance index into the efficiency index, the technological gap index, and the gap between the frontiers of each group under evaluation groups and the meta-frontier index.

In all these studies, methods have been proposed to evaluate the performance of a group, but little attention has been paid to the productivity changes of a group and the identification of the factors affecting the productivity growth of a group. The group MI is always achievable and usable considering the basic CRS technology. The conditions that exist for our proposed group MI are the same conditions that are assumed in the definition of CRS production technology. According to a case study, group MI can be generalized by considering other technologies as basic technology. The applicability and feasibility of a group MI depend on the conditions defined in each technology and its related mathematical model.

In this paper, we create a suitable distance function to measure the performance of a group, and then introduce the group Malmquist productivity index for evaluating group productivity changes. Also, we identify all intra-group and extra-group factors which affect group productivity changes. The remainder of the paper is organized as follows. Section 2 will discuss the prerequisite concepts for the Malmquist productivity index. In Section 3, the group Malmquist productivity index is introduced and its components are presented. In section 4, a real case study on the branches of a specialized bank of Iran is presented to show the ability and advantage of the suggested group Malmquist productivity index. The conclusion remarks also appear in the final section.

2- Malmquist Productivity Index

This index is defined based on the distance function of a decision-making unit from the frontier of a certain technology. Suppose (x_0^t, y_0^t) and (x_0^{t+1}, y_0^{t+1}) be the input and output vectors of DMU_0 in time periods t and $t+1$. The input-oriented MI is expressed as follows:

$$MI = \left[\frac{D^t(x_0^{t+1}, y_0^{t+1})}{D^t(x_0^t, y_0^t)} \times \frac{D^{t+1}(x_0^{t+1}, y_0^{t+1})}{D^{t+1}(x_0^t, y_0^t)} \right]^{1/2} \quad (1)$$

In this index, the first ratio in bracket shows the changes in the distance of DMU_0 in periods t and $t+1$ with respect to the frontier of period t and the second ratio in bracket shows the changes in the distance of DMU_0 in periods t and $t+1$ with respect to the frontier of period $t+1$. As a result, this index is defined as the geometric mean of these two ratios.

Fare et al. (1992) showed that the Malmquist index decomposed into two components: technology changes (TC) and efficiency changes (EC). In fact, the Malmquist index (1) is converted to the equivalent formula by simple mathematical operations. This decomposition, known as FGLR, is taken from the initials of the author's names.

$$MI = \frac{D^{t+1}(x_0^{t+1}, y_0^{t+1})}{D^t(x_0^t, y_0^t)} \left[\frac{D^t(x_0^{t+1}, y_0^{t+1})}{D^{t+1}(x_0^{t+1}, y_0^{t+1})} \times \frac{D^t(x_0^t, y_0^t)}{D^{t+1}(x_0^t, y_0^t)} \right]^{1/2} = EC \times TC \tag{2}$$

In the above equation, the value outside the bracket shows the ratio of the two distance functions, which indicates technical efficiency changes in periods t and $t+1$. The value inside the bracket indicates technological changes in periods t and $t+1$.

In 1994, the MI was generalized to variable returns to scale technology and provided FGNZ three-component decomposition of MI (Fare et al. 1994). By presenting this decomposition, Scale efficiency change was considered overtime. Thus, technical efficiency changes were divided into two components: pure technical efficiency changes (PEC) and scale efficiency changes (SEC).

$$MI = PEC \times SEC \times TC \tag{3}$$

Where

$$PEC = \frac{D_{VRS}^{t+1}(x_0^{t+1}, y_0^{t+1})}{D_{VRS}^{t}(x_0^{t}, y_0^{t})} \tag{4}$$

$$SEC = \left[\frac{D_{VRS}^{t}(x_0^{t}, y_0^{t})}{D_{CRS}^{t}(x_0^{t}, y_0^{t})} \times \frac{D_{CRS}^{t+1}(x_0^{t+1}, y_0^{t+1})}{D_{VRS}^{t+1}(x_0^{t+1}, y_0^{t+1})} \right] \tag{5}$$

and

$$TC = \left[\frac{D_{CRS}^{t}(x_0^{t+1}, y_0^{t+1})}{D_{CRs}^{t+1}(x_0^{t+1}, y_0^{t+1})} \times \frac{D_{CRS}^{t}(x_0^{t}, y_0^{t})}{D_{CRS}^{t+1}(x_0^{t}, y_0^{t})} \right]^{1/2} \tag{6}$$

It is noteworthy that in all decompositions above, an MI quantity greater than, equal to, or less than 1 means that productivity has grown, remained unchanged, or decreased during periods t and $t+1$. Similar results hold about growth or decrease of individual components in various MI decompositions.

$D_{CRS}^{t+1}(x_0^t, y_0^t)$ is the distance function of the DMU_0 in period t using the constant returns to scale technology in period $t+1$, which is obtained using the following CCR model:

$$D_{CRS}^{t+1}(x_0^t, y_0^t) = min \quad \theta_0$$
$$\begin{aligned} s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij}^{t+1} \le \theta_0 x_{i0}^t, \quad \forall i = 1,...,m \\ & \sum_{j=1}^{n} \lambda_j y_{rj}^{t+1} \ge y_{r0}^t, \quad \forall r = 1,...,s \\ & \lambda_j \ge 0, \quad \forall j = 1,...,n \end{aligned} \tag{7}$$

$D_{VRS}^{t+1}(x_0^t, y_0^t)$ is the distance function of the DMU_0 in period t using the variable returns to scale technology in period $t+1$, which is calculated by adding the convexity constraint $(\sum_{j=1}^{n} \lambda_j = 1)$ to model (7). Other measures in (3) are calculated in a similar manner.

3- Group Malmquist Productivity Index

Groups are the basic components of organizations. Most of organizations have branches which classify in different groups according to the different criteria. For example, consider a bank with a number of branches; the branches of this bank can be classified into different groups based on environmental conditions, different locations, demographic context, economic situation in different regions etc. Therefore, it is necessary to examine the efficiency or productivity of a group of them and to integrate the efficiency of individual units into one measure as the group efficiency measure. Finding a benchmark that reflects the performance of a group of decision-making units is not easy, because, in practical matters, groups do not include members of the same size, some units are large and some are small. Also, the number of units belonging to different groups is not the same. Figure 1 presents a basic structure of the groups of decision-making units.

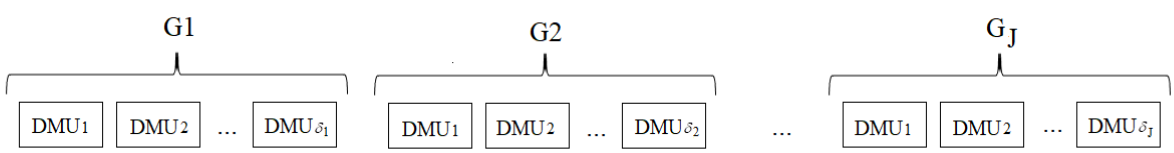

Fig 1. Group structures

In the most previous studies, a special unit was used as a representative of the group. Selecting a representative of a group and evaluating the group performance based on that unit can lead to an incorrect result where appointing a representative of a group may not be accurate. Also, in some of the proposed methods, they used the mean unit as the group representative. Others used the middle unit or the largest unit or unit that performed best to evaluate the performance of a group. It is difficult to answer which of these special units is a good representative of a group. Therefore, it is necessary to create an indicator to evaluate the performance of a group that does not have these drawbacks and shows the effectiveness of the individual performance of each unit on group performance. In this section, we define a new distance function for evaluating group performance and then introduce group Malmquist productivity index for assessing group productivity changes in two time periods.

Consider J groups which each of them has δ_J members. We define the geometric mean of the distance functions of all the units of a group relative to the global frontier of all DMU's as the distance function of a group. The global frontier is created by considering all DMU's of all groups. Therefore, the distance function of group A , including the δ_A units, is expressed as follows:

$$D^{A}(x,y)=\left(\prod_{k=1}^{\delta_A}\left(D_G(x_k,y_k)\right)\right)^{1/\delta_A} \tag{8}$$

According to this definition, the effect of the performance of each unit on the group performance is considered. Also, the impact of the number of members on the group performance evaluation is concerned. The proposed distance function calculates the distance of the units of group relative to the global frontier of all units of all groups.

We know that programs and policies within each group affect the performance of its members. To measure the magnitude of this effect, we break down distance function (8) as follows:

$$D^{A}(x,y)=\left(\prod_{k=1}^{\delta_A}\left(D_G(x_k,y_k)\right)\right)^{1/\delta_A}=\left(\prod_{k=1}^{\delta_A}\left(D_L(x_k,y_k)\right)\right)^{1/\delta_A}\times\left[\frac{\left(\prod_{k=1}^{\delta_A}\left(D_G(x_k,y_k)\right)\right)^{1/\delta_A}}{\left(\prod_{k=1}^{\delta_A}\left(D_L(x_k,y_k)\right)\right)^{1/\delta_A}}\right] \tag{9}$$

The phrase outside the bracket calculates the performance of group A focusing on the local perspective, which is defined as the geometric mean performance of the members relative to the frontier of group A. The phrase inside the bracket is obtained from the ratio of the performance of group A relative to the global frontier to the performance of group A relative to its local frontier. In fact, this ratio measures the effect of the local to global frontier changes in group A.

Now, considering the defined distance function, the Malmquist productivity index of group A can be defined as the geometric mean of the efficiency changes of group A in periods t and $t+1$ relative to the frontier of period t and the efficiency changes of group A in periods t and $t+1$ relative to the frontier of period $t+1$. So, we have:

$$MI_A=\left[\frac{\left(\prod_{k=1}^{\delta_A}\left(D_G^{t}(x_k^{t+1},y_k^{t+1})\right)\right)^{1/\delta_A}}{\left(\prod_{k=1}^{\delta_A}\left(D_G^{t}(x_k^{t},y_k^{t})\right)\right)^{1/\delta_A}}\times\frac{\left(\prod_{k=1}^{\delta_A}\left(D_G^{t+1}(x_k^{t+1},y_k^{t+1})\right)\right)^{1/\delta_A}}{\left(\prod_{k=1}^{\delta_A}\left(D_G^{t+1}(x_k^{t},y_k^{t})\right)\right)^{1/\delta_A}}\right]^{1/2} \tag{10}$$

Similar to the analysis of the Malmquist index in the individual form, this index can be decomposed into two components: group efficiency changes and group technological changes.

$$MI_A=EC_A\times TC_A \tag{11}$$

Where

$$EC_A=\frac{\left(\prod_{k=1}^{\delta_A}\left(D_{G-CRS}^{t+1}(x_k^{t+1},y_k^{t+1})\right)\right)^{1/\delta_A}}{\left(\prod_{k=1}^{\delta_A}\left(D_{G-CRS}^{t}(x_k^{t},y_k^{t})\right)\right)^{1/\delta_A}} \tag{12}$$

and

$$TC_A = \left[\frac{\left(\prod_{k=1}^{\delta_A} \left(D_{G-CRS}^{t}(x_k^{t+1}, y_k^{t+1}) \right) \right)^{1/\delta_A}}{\left(\prod_{k=1}^{\delta_A} \left(D_{G-CRS}^{t+1}(x_k^{t+1}, y_k^{t+1}) \right) \right)^{1/\delta_A}} \times \frac{\left(\prod_{k=1}^{\delta_A} \left(D_{G-CRS}^{t}(x_k^{t}, y_k^{t}) \right) \right)^{1/\delta_A}}{\left(\prod_{k=1}^{\delta_A} \left(D_{G-CRS}^{t+1}(x_k^{t}, y_k^{t}) \right) \right)^{1/\delta_A}} \right]^{1/2} \tag{13}$$

$D_{G-CRS}^{t+1}(x_k^t, y_k^t)$ represents the distance function of the (x_k, y_k) in group A in period t relative to the frontier of the CRS technology in period $t+1$, which is obtained using the following CRS model:

$$\begin{aligned} D_{G-CRS}^{t+1}(x_k^t, y_k^t) = min \quad & \theta_k \\ s.t. \quad & \sum_{j=1}^{J} \sum_{q=1}^{\delta_j} \lambda_q^j (x_{iq}^j)^{t+1} \le \theta_k (x_{ik})^t, \; i = 1, 2, ..., m \\ & \sum_{j=1}^{J} \sum_{q=1}^{\delta_j} \lambda_q^j (y_{rq}^j)^{t+1} \ge (y_{rk})^t, \; r = 1, 2, ..., s \\ & \lambda_q^j \ge 0, \quad \forall q = 1, ..., \delta_j, \; \forall j = 1, ..., J \end{aligned} \tag{14}$$

Considering two CRS and VRS technologies, the group technical efficiency changes can be divided into two indicators: pure efficiency changes of the group and scale efficiency changes of the group. So, we have:

$$MI_A = PEC_A \times SEC_A \times TC_A \tag{15}$$

Where

$$PEC_A = \frac{\left(\prod_{k=1}^{\delta_A} \left(D_{G-VRS}^{t+1}(x_k^{t+1}, y_k^{t+1}) \right) \right)^{1/\delta_A}}{\left(\prod_{k=1}^{\delta_A} \left(D_{G-VRS}^{t}(x_k^{t}, y_k^{t}) \right) \right)^{1/\delta_A}} \tag{16}$$

$$SEC_A = \left[\frac{\left(\prod_{k=1}^{\delta_A} \left(D_{G-VRS}^{t}(x_k^{t}, y_k^{t}) \right) \right)^{1/\delta_A}}{\left(\prod_{k=1}^{\delta_A} \left(D_{G-CRS}^{t}(x_k^{t}, y_k^{t}) \right) \right)^{1/\delta_A}} \times \frac{\left(\prod_{k=1}^{\delta_A} \left(D_{G-CRS}^{t+1}(x_k^{t+1}, y_k^{t+1}) \right) \right)^{1/\delta_A}}{\left(\prod_{k=1}^{\delta_A} \left(D_{G-VRS}^{t+1}(x_k^{t+1}, y_k^{t+1}) \right) \right)^{1/\delta_A}} \right] \tag{17}$$

and

$$TC_A = \left[\frac{\left(\prod_{k=1}^{\delta_A} \left(D_{G-CRS}^{t}(x_k^{t+1}, y_k^{t+1}) \right) \right)^{1/\delta_A}}{\left(\prod_{k=1}^{\delta_A} \left(D_{G-CRS}^{t+1}(x_k^{t+1}, y_k^{t+1}) \right) \right)^{1/\delta_A}} \times \frac{\left(\prod_{k=1}^{\delta_A} \left(D_{G-CRS}^{t}(x_k^{t}, y_k^{t}) \right) \right)^{1/\delta_A}}{\left(\prod_{k=1}^{\delta_A} \left(D_{G-CRS}^{t+1}(x_k^{t}, y_k^{t}) \right) \right)^{1/\delta_A}} \right]^{1/2} \tag{18}$$

$D_{G-VRS}^{t+1}(x_k^t, y_k^t)$ represents the distance function of the (x_k, y_k) in group A in period t relative to the frontier of the VRS technology in period $t+1$, which is obtained using the following VRS model:

$$
\begin{aligned}
D_{G-VRS}^{t+1}(x_k^t, y_k^t) = min \quad & \theta_k \\
s.t. \quad & \sum_{j=1}^{J}\sum_{q=1}^{\delta_j} \lambda_q^j \, (x_{iq}^j)^{t+1} \le \theta_k (x_{ik})^t, \; i = 1, 2, ..., m \\
& \sum_{j=1}^{J}\sum_{q=1}^{\delta_j} \lambda_q^j \, (y_{rq}^j)^{t+1} \ge (y_{rk})^t, \;\; r = 1, 2, ..., s \\
& \sum_{j=1}^{J}\sum_{q=1}^{\delta_j} \lambda_q^j = 1 \\
& \lambda_q^j \ge 0, \quad \forall q = 1, ..., \delta_j, \; \forall j = 1, ..., J
\end{aligned}
\tag{19}
$$

Considering equation (9), each of the components of the Malmquist index can be divided into two parts: local (intra-group performance) and global (extra-group performance). Therefore, we will have:

$$MI_A = PEC_L \times PEC_G \times SEC_L \times SEC_G \times TC_L \times TC_G \tag{20}$$

Where PEC_L and PEC_G represent the local pure efficiency changes and global pure efficiency changes, respectively, and are obtained as follows:

$$PEC_L = \frac{\left(\prod_{k=1}^{\delta_A}\left(D_{L-VRS}^{t+1}(x_k^{t+1}, y_k^{t+1})\right)\right)^{1/\delta_A}}{\left(\prod_{k=1}^{\delta_A}\left(D_{L-VRS}^{t}(x_k^{t}, y_k^{t})\right)\right)^{1/\delta_A}} \tag{21}$$

and

$$PEC_G = \left[\frac{\left(\prod_{k=1}^{\delta_A}\left(D_{G-VRS}^{t+1}(x_k^{t+1}, y_k^{t+1})\right)\right)^{1/\delta_A}}{\left(\prod_{k=1}^{\delta_A}\left(D_{L-VRS}^{t+1}(x_k^{t+1}, y_k^{t+1})\right)\right)^{1/\delta_A}} \times \frac{\left(\prod_{k=1}^{\delta_A}\left(D_{L-VRS}^{t}(x_k^{t}, y_k^{t})\right)\right)^{1/\delta_A}}{\left(\prod_{k=1}^{\delta_A}\left(D_{G-VRS}^{t}(x_k^{t}, y_k^{t})\right)\right)^{1/\delta_A}}\right] \tag{22}$$

SEC_L and SEC_G represent the local scale efficiency changes and global scale efficiency changes, respectively, and are obtained as follows:

$$SEC_L = \left[\frac{\left(\prod_{k=1}^{\delta_A} \left(D^t_{L-VRS}(x^t_k, y^t_k) \right) \right)^{1/\delta_A}}{\left(\prod_{k=1}^{\delta_A} \left(D^t_{L-CRS}(x^t_k, y^t_k) \right) \right)^{1/\delta_A}} \times \frac{\left(\prod_{k=1}^{\delta_A} \left(D^{t+1}_{L-CRS}(x^{t+1}_k, y^{t+1}_k) \right) \right)^{1/\delta_A}}{\left(\prod_{k=1}^{\delta_A} \left(D^{t+1}_{L-VRS}(x^{t+1}_k, y^{t+1}_k) \right) \right)^{1/\delta_A}} \right] \tag{23}$$

and

$$SEC_G = \frac{\left(\prod_{k=1}^{\delta_A} \left(D^t_{G-VRS}(x^t_k, y^t_k) \right) \right)^{1/\delta_A}}{\left(\prod_{k=1}^{\delta_A} \left(D^t_{L-VRS}(x^t_k, y^t_k) \right) \right)^{1/\delta_A}} \times \frac{\left(\prod_{k=1}^{\delta_A} \left(D^t_{L-CRS}(x^t_k, y^t_k) \right) \right)^{1/\delta_A}}{\left(\prod_{k=1}^{\delta_A} \left(D^t_{G-CRS}(x^t_k, y^t_k) \right) \right)^{1/\delta_A}} \times \frac{\left(\prod_{k=1}^{\delta_A} \left(D^{t+1}_{G-CRS}(x^{t+1}_k, y^{t+1}_k) \right) \right)^{1/\delta_A}}{\left(\prod_{k=1}^{\delta_A} \left(D^{t+1}_{L-CRS}(x^{t+1}_k, y^{t+1}_k) \right) \right)^{1/\delta_A}} \times \frac{\left(\prod_{k=1}^{\delta_A} \left(D^{t+1}_{L-VRS}(x^{t+1}_k, y^{t+1}_k) \right) \right)^{1/\delta_A}}{\left(\prod_{k=1}^{\delta_A} \left(D^{t+1}_{G-VRS}(x^{t+1}_k, y^{t+1}_k) \right) \right)^{1/\delta_A}} \tag{24}$$

TC_L and TC_G represent the local technological changes and global technological changes, respectively, and are obtained as follows:

$$TC_L = \left[\frac{\left(\prod_{k=1}^{\delta_A} \left(D^t_{L-CRS}(x^{t+1}_k, y^{t+1}_k) \right) \right)^{1/\delta_A}}{\left(\prod_{k=1}^{\delta_A} \left(D^{t+1}_{L-CRS}(x^{t+1}_k, y^{t+1}_k) \right) \right)^{1/\delta_A}} \times \frac{\left(\prod_{k=1}^{\delta_A} \left(D^t_{L-CRS}(x^t_k, y^t_k) \right) \right)^{1/\delta_A}}{\left(\prod_{k=1}^{\delta_A} \left(D^{t+1}_{L-CRS}(x^t_k, y^t_k) \right) \right)^{1/\delta_A}} \right]^{1/2} \tag{25}$$

and

$$TC_G = \left[\frac{\left(\prod_{k=1}^{\delta_A} \left(D^t_{G-CRS}(x^{t+1}_k, y^{t+1}_k) \right) \right)^{1/\delta_A}}{\left(\prod_{k=1}^{\delta_A} \left(D^t_{L-CRS}(x^{t+1}_k, y^{t+1}_k) \right) \right)^{1/\delta_A}} \times \frac{\left(\prod_{k=1}^{\delta_A} \left(D^{t+1}_{L-CRS}(x^{t+1}_k, y^{t+1}_k) \right) \right)^{1/\delta_A}}{\left(\prod_{k=1}^{\delta_A} \left(D^{t+1}_{G-CRS}(x^{t+1}_k, y^{t+1}_k) \right) \right)^{1/\delta_A}} \times \frac{\left(\prod_{k=1}^{\delta_A} \left(D^t_{G-CRS}(x^t_k, y^t_k) \right) \right)^{1/\delta_A}}{\left(\prod_{k=1}^{\delta_A} \left(D^t_{L-CRS}(x^t_k, y^t_k) \right) \right)^{1/\delta_A}} \times \frac{\left(\prod_{k=1}^{\delta_A} \left(D^{t+1}_{L-CRS}(x^t_k, y^t_k) \right) \right)^{1/\delta_A}}{\left(\prod_{k=1}^{\delta_A} \left(D^{t+1}_{G-CRS}(x^t_k, y^t_k) \right) \right)^{1/\delta_A}} \right]^{1/2} \tag{26}$$

By examining each of these components, all factors affecting the productivity growth of a group can be identified and appropriate measures can be taken to increase the productivity of the group.
Now, a question: how can the group MI be validated?

In the definition of group MI and its components, the following distance functions are used, all of which are calculated from linear models.

$$\begin{aligned}&D_{L-VRS}^{t+1}(x_k^{t+1},y_k^{t+1}),\ D_{L-VRS}^{t}(x_k^{t},y_k^{t}),\ D_{G-VRS}^{t}(x_k^{t},y_k^{t}),\ D_{G-VRS}^{t+1}(x_k^{t+1},y_k^{t+1}),\\&D_{L-CRS}^{t+1}(x_k^{t+1},y_k^{t+1}), D_{L-CRS}^{t}(x_k^{t},y_k^{t}), D_{L-CRS}^{t}(x_k^{t+1},y_k^{t+1}), D_{L-CRS}^{t+1}(x_k^{t},y_k^{t}),\\&D_{G-CRS}^{t}(x_k^{t},y_k^{t}), D_{G-CRS}^{t+1}(x_k^{t+1},y_k^{t+1}), D_{G-CRS}^{t+1}(x_k^{t},y_k^{t}), D_{G-CRS}^{t}(x_k^{t+1},y_k^{t+1})\end{aligned} \tag{27}$$

$D_{G-CRS}^{s}(x_k^{h},y_k^{h})$ represents the distance function of the (x_k, y_k) in group A in period h relative to the global frontier of the CRS technology in period s, which is obtained using the following CRS model:

$$\begin{aligned}D_{G-CRS}^{s}(x_k^{h},y_k^{h}) = min\quad &\theta_k\\ s.t.\quad &\sum_{j=1}^{J}\sum_{q=1}^{\delta_j}\lambda_q^j\,(x_{iq}^j)^s \le \theta_k\,(x_k)^h,\ i=1,2,...,m\\ &\sum_{j=1}^{J}\sum_{q=1}^{\delta_j}\lambda_q^j\,(y_{rq}^j)^s \ge (y_k)^h,\ \ r=1,2,...,s\\ &\lambda_q^j \ge 0,\quad \forall q=1,...,\delta_j,\ \forall j=1,...,J\end{aligned} \tag{28}$$

Where J is the number of groups and δ_J is the members of group J.

When $s=h=t$ or $s=h=t+1$, then $\lambda_k^A=1$, $\lambda_k^j=0$, $\forall j=1,...,J, j\ne A$ and $\theta_k=1$ is the feasible solution for the above model. So, the distance function $D_{G-CRS}^{s}(x_k^{h},y_k^{h})$ exists.

When $s=t$ and $h=t+1$ or vice versa, then $\lambda_k^A=\dfrac{(x_{ik})^h}{(x_{ik})^s}$, $\lambda_k^j=0$, $\forall j=1,...,J, j\ne A,$ $\theta_k=1$ is the feasible solution for the above model. So, the distance function $D_{G-CRS}^{s}(x_k^{h},y_k^{h})$ exists, but it is not necessarily less than or equal to one.

Note that the global frontier is the boundary consisting of all units in all groups and the local frontier is the boundary of the units in group A.

$D_{L-CRS}^{s}(x_k^{h},y_k^{h})$ represents the distance function of the (x_k, y_k) in group A in period h relative to the local frontier of the CRS technology in period s, which is obtained using the following CRS model:

$$\begin{aligned}D_{L-CRS}^{s}(x_k^{h},y_k^{h}) = min\quad &\theta_k\\ s.t.\quad &\sum_{q=1}^{\delta_A}\lambda_q\,(x_{iq}^j)^s \le \theta_k\,(x_k)^h,\ i=1,2,...,m\\ &\sum_{q=1}^{\delta_A}\lambda_q\,(y_{rq}^j)^s \ge (y_k)^h,\ \ r=1,2,...,s\\ &\lambda_q \ge 0,\quad \forall q=1,...,\delta_A,\ \forall j=1,...,J\end{aligned} \tag{29}$$

When $s=h=t$ or $s=h=t+1$, then $\lambda_k=1,\ \lambda_q=0,\ \forall q=1,...,\delta_A, q\neq 0$ and $\theta_k=1$ is the feasible solution for the above model. So, the distance function $D^s_{L-CRS}(x^h_k, y^h_k)$ exists.

When $s=t$ and $h=t+1$ or vice versa, then $\lambda_k=\frac{(x_{ik})^h}{(x_{ik})^s}$, $\lambda_q=0,\ \forall q=1,...,\delta_A, q\neq 0,\ \theta_k=1$ is the feasible solution for the above model. So, the distance function $D^s_{L-CRS}(x^h_k, y^h_k)$ exists, but it is not necessarily less than or equal to one.

$D^s_{G-VRS}(x^h_k, y^h_k)$ represents the distance function of the (x_k, y_k) in group A in period h relative to the global frontier of the VRS technology in period s, which is obtained using the following BCC model:

$$D^{t+1}_{G-VRS}(x^t_k, y^t_k) = min \quad \theta_k$$

$$\begin{aligned} s.t.\quad & \sum_{j=1}^{J}\sum_{q=1}^{\delta_j}\lambda^j_q\,(x^j_{iq})^{t+1} \leq \theta_k\,(x_{ik})^t,\ i=1,2,...,m \\ & \sum_{j=1}^{J}\sum_{q=1}^{\delta_j}\lambda^j_q\,(y^j_{rq})^{t+1} \geq (y_{rk})^t,\ r=1,2,...,s \\ & \sum_{j=1}^{J}\sum_{q=1}^{\delta_j}\lambda^j_q = 1 \\ & \lambda^j_q \geq 0,\quad \forall q=1,...,\delta_j,\ \forall j=1,...,J \end{aligned} \tag{30}$$

where J is the number of groups and δ_J is the members of group J.

When $s=h=t$ or $s=h=t+1$, then $\lambda^A_k=1,\ \lambda^j_k=0,\ \forall j=1,...,J, j\neq A$ and $\theta_k=1$ is the feasible solution for the above model. So, the distance function $D^s_{G-VRS}(x^h_k, y^h_k)$ exists.

$D^s_{L-VRS}(x^h_k, y^h_k)$ represents the distance function of the (x_k, y_k) in group A in period h relative to the local frontier of the VRS technology in period s, which is obtained using the following BCC model:

$$D^s_{L-VRS}(x^h_k, y^h_k) = min \quad \theta_k$$

$$\begin{aligned} s.t.\quad & \sum_{q=1}^{\delta_A}\lambda_q\,(x^j_{iq})^s \leq \theta_k\,(x_k)^h,\ i=1,2,...,m \\ & \sum_{q=1}^{\delta_A}\lambda_q\,(y^j_{rq})^s \geq (y_k)^h,\ r=1,2,...,s \\ & \sum_{q=1}^{\delta_A}\lambda_q = 1 \\ & \lambda_q \geq 0,\quad \forall q=1,...,\delta_A,\ \forall j=1,...,J \end{aligned} \tag{31}$$

When $s=h=t$ or $s=h=t+1$, then $\lambda_k=1,\ \lambda_q=0,\ \forall q=1,...,\delta_A, q\neq 0$ and $\theta_k=1$ is the feasible solution for the above model. So, the distance function $D^s_{L-VRS}(x^h_k, y^h_k)$ exists.

As it was observed, all the distance functions used in calculating the group MI and its components are feasible, so MI always exists.

In conventional evaluation issues, evaluation of the efficiency and productivity of decision-making units has been studied, but little attention has been paid to the evaluation of a group of decision-making units. In real evaluation issues, sometimes the number of units under management is high, so decision-makers prefer to have a group evaluation of their units and also consider the impact of environmental conditions on the performance of units. The Group MI offers a new perspective on unit evaluation that considers the impact of environmental conditions and management strategies on evaluating decision-making units.

4- Case study

In this section, we calculate the proposed group MI on a real-world case study from Maskan Bank of Iran located in Tehran for two time periods 2017-2018 and then analyze the results. Tehran province includes various regions that are different in terms of social and cultural context, facilities and economic conditions. This study relates to eight regions located in Tehran and involves 206 branches. The number of branches in each region is shown in table 1.

It is noted that Maskan Bank is the largest Iranian governmental bank in the housing sector. The results of the case study can be useful for managers to evaluate the productivity changes of the regions also to find out how they can improve the productivity of the regions.

Table 1. Number of branches in each region.

Region	Branch
1	23
2	21
3	27
4	23
5	33
6	27
7	27
8	25

Input and output data

Production analysis is one of the most significant dimensions of bank branch performance (Paradi and Zhu, 2013). In this case study, we measure the performance of bank branches respect to this aspect as shown in Fig 2. Then, branches are considered as producers of services for taking deposits, making loans, and providing other diverse banking services using personnel expenses and location index as inputs.

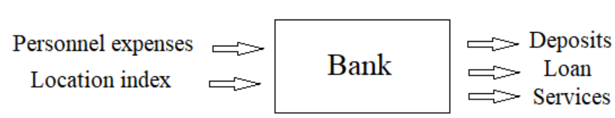

Fig 2. Production perspective in a bank.

The input of human resources has to include all the quantity and quality entities related to the staff of a branch. The input of location has to include all the quantity and quality entities related to the physical location of a branch. The planning and programming department of bank has done a project for this index and they considered all the related factors in the developed location index. The most important factors considered in computing location index are branch customers' specifications, physical location of branch, and branch staff characteristics. We used the data of the location index in our evaluation.

The output of deposit has to include all kinds of methods of gathering money by a branch. The planning and programming department of bank has done a project for this index and they considered a weighted sum of all kinds of accounts considering their values and number of transactions for calculation of the deposit index and we used the data of the deposit index. The output of loans includes all the money gave as all kinds of loans and mortgages by a branch and similar to deposit index some calculations have been done. Finally, the output of services is an index which includes all kinds of services delivered by a branch to its customers.

The descriptive statistics of inputs and outputs for two time periods are given in table 2. Measurement unit of personnel expenses is 1000000 Rials. Other indices have no units because they are normalized. All the values and results are rounded in two digits.

Table 2. Descriptive statistics for the data.

	2017				**2018**			
	Min	**Max**	**Mean**	**STD**	**Min**	**Max**	**Mean**	**STD**
Inputs								
Personnel expenses	1115.09	18396.58	4827.90	2523.21	1384.62	16214.78	4810.22	2718.98
Location index	384.00	1212.00	951.15	153.52	384.00	1212.00	952.09	152.28
Outputs								
Deposits	154.50	7540.00	1372.41	1033.07	86.80	5620.00	1317.76	843.88
Loans	35.07	18300.00	1237.07	1608.93	62.53	16127.00	1217.43	1555.49
Services	35.45	22045.00	1180.84	2058.31	110.60	8472.00	1040.89	1030.39

Results

The results of the group MI and their components for all regions are shown in table 3. $MI > 1$ and $MI < 1$ means the growth and non-growth of productivity of the region, respectively. $MI = 1$ means that no change in the productivity of the region is observed.

Table 3. Group *MI* results and its components.

Regions	PEC_L	PEC_G	SEC_L	SEC_G	TC_L	TC_G	MI
1	0.99	1.07	0.96	1.27	1.08	0.80	1.12
2	1.10	0.98	1.08	1.19	0.87	0.88	1.07
3	1.01	0.99	1.08	1.10	0.87	0.95	0.99
4	1.06	1.04	1.00	1.21	1.06	0.84	1.19
5	1.06	1.02	1.24	0.94	0.86	1.02	1.11
6	1.05	0.83	1.10	0.98	0.71	1.22	0.82
7	1.07	0.91	1.10	1.08	0.86	1.01	1.00
8	1.04	0.81	1.08	0.99	0.72	1.21	0.78

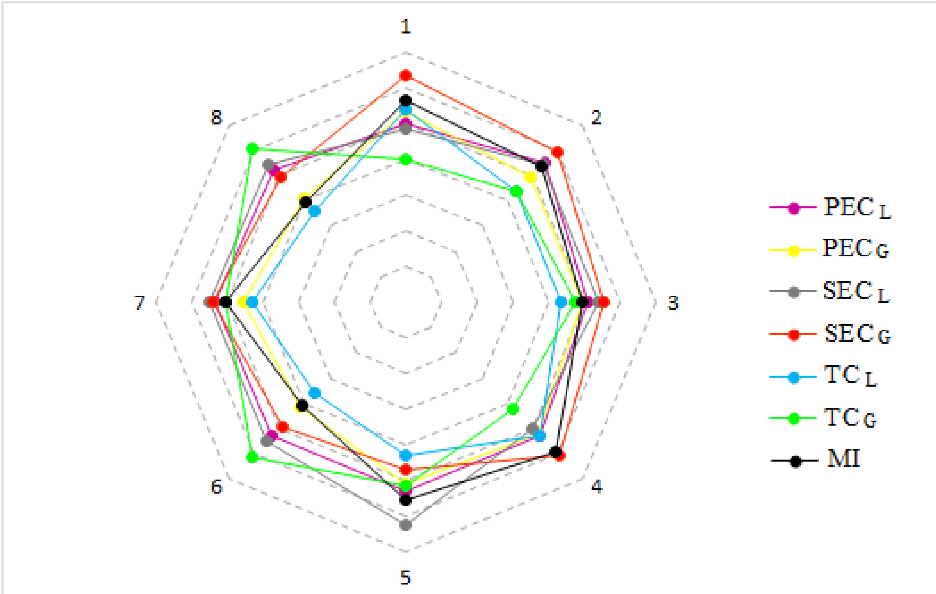

Fig 3. The effect of group *MI* decompositions on group productivity changes.

Table 3 and figure 3 display the effectiveness of each component of the group MI on group productivity changes. Components larger than one indicate their growth, values less than one indicate their regress, and values equal to one mean that there is no change over two time periods. By identifying the impact of each component on productivity growth, managers can take the necessary steps to improve their plans and cover their weaknesses and strengthen their strengths. For this purpose, the concept of each of these components must be described, which we will discuss below.

PEC measures the optimal use of inputs to generate outputs. PEC_L shows the pure efficiency change of a region compared to its performance in the previous period. PEC_G measures the changes in the pure efficiency of a region compared to the pure efficiency changes of the other regions. For example, region 1 has almost no change in the operational efficiency of its branches compared to the performance of its previous period. ($PEC_L = 0.99$). But its pure efficiency has grown well compared to the other groups (seven percent growth recorded in this component).

Although regions 2, 6, 7, and 8 have positive growth in their pure efficiency changes compared to the previous period ($PEC_L > 1$), they still need more effort respect to other groups performance ($PEC_G < 1$). For regions 4 and 5, positive growth is observed in both local and global pure efficiency changes. It means that the method adopted at the branches in these regions is the correct method and should continue in the same way. For region 3, almost no changes have been recorded in the local and global pure efficiencies. The managers of this region and its branches need to make better decisions in order to increase pure efficiency.

SEC reflects the effect of scale changes of the branches of a region on its productivity growth. If the change of this component leads to positive growth in the region's productivity, we can say that changing the size of the branches in that region was the right decision; otherwise, it was wrong decision. Here, regions 2, 3, and 7 have grown in both local and global components. Region 1 had a %4 decrease in local scale efficiency but performed better than the other groups ($SEC_G = 1.27$).

Although region 4 has not changed the scale of its branches, it still has positive scale efficiency changes compared to competitors in other regions. Region 5 has tried to increase its productivity by changing scale efficiency but it requires better measures to changes the size of its branches compared to others.

TC indicates the use of new technologies and its impact on the productivity growth of a region. Regions 1 and 4 have positive growth in technological changes compared to the previous period. Nevertheless, they need to use more new technologies compare to other regions.

Regions 2 and 3 have negative growth in technological changes in both local and global components. Hence, they need to reassess the use of new technologies in their branches. Regions 5, 6, 7 and 8, although having a negative growth compared to their performance in the previous period, but perform better than their competitors in using new technologies.

Finally, from the interaction of all the factors affecting the productivity growth of a region, it can be found that regions 1, 2, 4, and 5 have positive productivity growth. Region 4 has the most productivity growth. After region 4, the maximum productivity growth was obtained for regions 1, 2, and 5, respectively. Regions 3 and 7 have almost no change in productivity growth.

Regions 6 and 8 have productivity decline. Region 8 has the most regression. Twenty-two percent regression in productivity has been measured for this region. Region 6 has an eighteen percent productivity reduction. The managers and decision-makers of this bank must take appropriate measures to increase the productivity of these regions.

5- Conclusion

Today, most organizations are assembled of different groups and divisions. Sometimes, it is necessary to examine the performance of a group of decision-making units instead of evaluating the individual performance of each decision-making unit to analyze the impact of environmental conditions and geographical categories on the performance evaluation of the units. In this paper, we introduced the group Malmquist productivity index to assess the productivity changes of a group of decision-making units. In addition to, we identified local and global factors that cause the growth or non-growth of a group's productivity. Recognizing these factors can be a useful guide for decision-makers and managers of organizations to make the right decisions for non-productive groups.

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