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Using grey relational analysis for dynamic portfolio selection in Tehran Stock Exchange

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Abstract

In this study, first, a brief survey of various portfolio selection problems is presented to explore the related methodologies, hypotheses, and constraints that are considered in these problems. Among these methods, the grey relational analysis approach is employed to deal with poor information and uncertainties in portfolio selection problems. Return, risk, skewness, and kurtosis are used at the same time as selecting criteria in the portfolio construction. To evaluate the effectiveness of the proposed method, an empirical analysis has done. Therefore, fourteen stocks of various industries like metal, banks, financial institutions, car manufactures, transportation, and petroleum from the thirty largest active companies' index in Tehran Stock Exchange have been randomly selected and all above mention moments have been calculated for each stocks. In this study, the portfolio is restructured dynamically each week based on the ranking of previous week. The result from the analysis indicates that the selected approach has better performance in comparison with the benchmarks in terms of return, standard deviation, and Sharpe ratio.

Keywords: Portfolio selection, grey relational analysis, Tehran stock exchange

1-Introduction

In finance, a "portfolio" is a set of securities constructed by investors to maximize the final wealth. Portfolio selection is evaluating securities to identify and invest in the best one of them. Modern portfolio theory was presented by Markowitz's and Sharpe endeavor. Markowitz introduced mean-variance model, which suggests the "expected return" and "variance" of risky assets for measuring the "investment return" and "investment risk", respectively (Markowitz 1968). The mean-variance model was just an initial idea in optimal portfolio selection. Although, it received a lot of praise, it has been criticized mainly for the dissimilarities of the model with the real-world problems. For instance, the model supposes that the return distribution is normal and it uses only the first and second moments. Researchers developed the basic Markowitz model by adding more constraints, objectives, and hypotheses. They proved that the distribution of return is not always "normal" (Beedles 1984, Aggarwal et al 1989). Since, there are several criteria that affect the portfolio selection problem, it becomes a multi-criteria decision-making (MCDM) problem, instead of a two-criterion problem.

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In this case, Steuer (2013) showed a transition of a two-criterion Markowitz problem (return-risk) to a three-criterion problem. Thus, the efficient frontier has changed from a “line” to a “surface” and made the problem more complex.

To design a roadmap of the research, a brief survey analysis is done based on three keywords, “portfolio optimization”, “portfolio selection”, and “stock selection”. Note that, in a number of literature, “optimization” and “selection” are used interchangeably in the portfolio studies. The results are summarized in Table 1, including criteria, constraints, sample size, solution method, and considered markets.

Investors are always deal with the following questions in portfolio or stock selection.

1) What criteria should be included?
2) What constraints should be included?
3) What is the best solution method?
4) How to evaluate the performance of results?

For answering the first question, it should be declared that diversification in mean-variance model is a tradeoff between return and risk (Naqvi et al 2017). The initial assumption in this model is based on the normality of stock returns distribution. The model assumes that the variance is solely adequate to measure the investment risk. A number of studies confirm that the stock return distribution is not always normal and they recommend to employ higher moments (skewness and kurtosis) in order to measure the investment risk more accurately (Sun & Yan 2003, Liu et al 2013, Škrinjarić 2020, Naqvi et al 2017).

Table 1. Outcome of a bibliography research

Reference	Criteria	Constraint	Market	Sample	Methodology
Xia et al (2000)	Return and risk	Transaction costs	-	6 stock	Genetic Algorithm
Huang et al (2008)	Return	Budget	Taiwan	Electronic stock	Variable Precision Rough Set - moving ARX- Grey systems
Huang et al (2010)	Return	Budget	Taiwan	Electronic stock	GRA - Rough set theory - Fuzzy C-Means clustering
Huang et al (2011)	Return	Number of stock in portfolio	Taiwan	Electronic stock	GRA - Rough set - Clustering - moving ARX
Zhang et al (2012)	Return and risk	Transaction cost	Shanghai	4 stock	Genetic algorithm – Fuzzy
Ghahtarani & Najafi (2013)	Return, Total budget and portfolio beta	Budget, investment rate in each stock and minimum ER rate	Tehran	20 stock	Goal programming
Liu et al (2013)	Final wealth	No short selling and minimum expected return	Shanghai	4 stock	Fuzzy programming - particle swarm optimization algorithm
Abdelaziz & Masmoudi (2014)	Return and Beta	Investment threshold	New York	100 stock based on S&P100	Stochastic goal programming
Hsu (2014)	Semi-variance	Minimum ER, transaction cost and transaction lot	Taiwan	16 stock	DEA - Artificial bee colony algorithm - genetic algorithm
Bayramoglu & Hamzacebi (2016)	Return, beta coefficient, standard deviation and coefficient of variation	-	Turkey	9 stock	GRA
Mashayekhi & Omrani (2016)	Risk and return	-	Tehran	52 stock	Markowitz DEA - Fuzzy – NSGAII
Lwin et al (2017)	VaR asset	Number of stock in portfolio, round lot, class and quantity constraints	-	Two data set (94 and 475 securities)	Non-Parametric
Naqvi et al (2017)	Return, skewness, risk and kurtosis	No short selling	Pakistan	8 stock	Polynomial goal programming
Zou & Xu (2018)	Return and risk	Budget	Shanghai	4 stock	Hesitant Fuzzy
Safitri et al (2020)	Risk	-	Indonesia	5 stock	Non-Linear Programming
Škrinjarić (2020)	Return, standard deviation, coefficient asymmetry and skewness	Transaction cost	Zagreb	5 sector	GRA

Table 1. (Continued)

Reference	Criteria	Constraint	Market	Sample	Methodology
Khedmati & Azin (2020)	Return	Transaction cost, number of stock in portfolio, upper and lower limit of investment	-	5 data set	Clustering
Sini Guo (2020)	Return and variance	Capital gain tax	Shanghai	46 stock	Fuzzy - particle swarm optimization algorithm
Dai & Qin (2021)	Terminal wealth	Number of transaction and transaction lot	-	-	Genetic algorithm

In addition to above explanations, it should be said that this study focuses on market data (return distribution). Most of the previous studies in Tehran Stock Exchange (TSE) includes financial data (like financial ratios; price to earning value, earning per share value, book value to market value and etc.).

In response of the second question, it should be said that the available budget, transactional lot, and investment threshold are the most well-known constraints. Aouni (2018) provides a bibliography research and defined four constraints; cardinality constraints, investment threshold constraints, transaction lot constraints and dependency constraints. In this study, the number of stocks that can be kept in portfolio are considered and the transaction cost is ignored.

To answer the third question, there are several solving methods that can be used. For instance, Xia (2000), Huang (2012), Hsu (2014), and Dai & Qin (2021) used genetic algorithms, Chen, Chang-ho, and Diaz (2014) used Analytical Neural Network (ANN) and Liu (2013) used fuzzy methods.

Some authors proposed the hybrid methods. Zhang (2012) proposed a hybrid method with genetic algorithm and fuzzy, Raei and Jahromi (2012) and Galankashi et al (2020) used the hybrid ANN and fuzzy method, Mashayekhi, and Omrani (2016) proposed a hybrid model of data evolvement analysis (DEA) and genetic algorithm. Since, there is uncertainty in investment problems, methods like statistics, probability, fuzzy mathematics, and grey systems theory are often employed (Kayacan 2010).

Grey systems theory is primarily introduced by Deng in 1989. Liu (2011) provided a summary of the grey theory. He makes a comparison between grey systems and other models like stochastic probability models, rough theory, and fuzzy mathematics in terms of uncertainty. The main difference between mentioned methods is that statistics and probability work with large sample sizes, while grey systems work with small samples. On the other hand, fuzzy methods work with experience data. In another aspect, the statistical and probability methods deal with stochastic information, fuzzy methods deal with cognitive uncertainty, but grey systems deal with poor information. Fuzzy mathematics needs to know membership functions, but grey systems could work with each kind of probability distribution (Liu, Forrest & yang 2013).

In recent years, researchers have proposed the Grey Relational Analysis (GRA) application in solving various problems (Xu 2014, Wang 2016 and, Škrinjarić 2020). GRA is usually used for prediction or decision-making (Xu 2014). It is based on making a comparison between an element which is called Grey Relational Degree (GRD) and a reference series or each acceptable series. It is also appropriate for problems in which there are internal relations between their multiple attributes and criteria (Wang 2016). GRA was basically developed to rank the attributes in terms of their characteristics (Škrinjarić 2020). Yin (2013) performed a library analysis between 1996 till 2010 in order to identify the researches which used the GRA approach and grey prediction in solving problems. For example, Škrinjarić and Sego (2019) declared that grey models could have better performance in asset price prediction. They proved it by an empirical comparison. They used the GM (1, 1) and GM (2, 1) as grey models. The result of their comparison showed that GM (1, 1) provides better performance than the other one. Mohammadipour (2016) used GRA in TSE. He focused on GRA applications in stock prediction and comparing it to the Johnson prediction method. His result showed that there is no significant difference between grey prediction and the Johnson ranking method. He also declares that economic and accounting criteria are the main criteria for investors to construct their portfolios.

With all above explanations, GRA is still a new approach in TSE. Return and risk (first and second moments, respectively) is usually employed in portfolio selection problems. There are a lot of examples that confirm the presence of higher moments (skewness and kurtosis). In this study, by exploring the GRA application, an endeavor is done to give an appropriate response to whether GRA is a helpful tool for portfolio selection in terms of maximizing wealth in TSE market or not? Our contribution is to present an empirical study that considered the four moments (return, risk, skewness, and kurtosis) simultaneously in portfolio construction in TSE.

2-Methodology

GRA is a part of the grey systems theory, and its application is in MCDM problems. "Grey" indicates the uncertain data, values, and systems. The initial idea of GRA as a quantitative analysis is based on correlation and similarity between some elements in a dynamic growth process. The higher similarity will cause a higher GRD and vice versa. For measuring the similarity, a grey relational coefficient is calculated and a reference series is defined. GRA evaluates the similarity of system elements and reference series. It analyzes the uncertain relations between system elements and one member of a reference. GRA can be performed in following steps.

Step 1- Suppose that decision-maker who is an investor in this article wants to rank M alternatives in terms of K criteria. First, the following matrix is constructed, which shows the data of week t.

$$X_t = \begin{bmatrix} x_1(1)_t & \dots & x_1(k)_t \\ \vdots & \ddots & \vdots \\ x_M(1)_t & \dots & x_M(k)_t \end{bmatrix} \tag{1}$$

Rows represent the M alternatives which are stocks. Columns represent criteria that affect the decision-making process. In this paper, return, risk, skewness, and kurtosis are the selected criteria in the decision-making process.

Step 2- Normalize the data of matrix (1). Normalizing data is performed for two reasons. First, it helps dealing with data with largely different scales. Second, it is better to unify the representation of data regardless of whether the larger values are preferable for that data or the smaller values. Huang and Leo (2003) describe three ways to normalize the data. For the case, where the greater value of data is the better, normalization is done by the following equation:

$$y_m(k)_t = \frac{x_m(k)_t - \min\limits_m x_m(k)_t}{\max\limits_m x_m(k)_t - \min\limits_m x_m(k)_t} \tag{2}$$

On the other hand, where the desired data is the smallest one, normalization is done by following equation:

$$y_m(k)_t = \frac{\max\limits_m x_m(k)_t - x_m(k)_t}{\max\limits_m x_m(k)_t - \min\limits_m x_m(k)_t} \tag{3}$$

Moreover, when the value is better to be close to a desired or threshold value, normalization is performed by the following equation:

$$y_m(k)_t = 1 - \frac{|x_m(k)_t - x^*(k)_t|}{\max\limits_m\{\max\limits_m x_m(k)_t - x^*(k)_t, x^*(k)_t - \min\limits_m x_m(k)_t\}} \tag{4}$$

Step 3- The reference series is defined in this step. Since $y_m(k)_t \in [0,1]$, reference serie $y^*(k)_t = (1.1. \dots .1)$ is defined as the most desired reference series. Then investor makes a comparison between each element of the normalized matrix, calculated by one of the equations (2), (3), or (4), and the reference series by the following equation:

$$\Delta y_m(k)_t = |y_m(k)_t - y^*(k)| \tag{5}$$

Step 4- Calculate the grey relational coefficient by following equation:

$$G_m(k)_t = \frac{\Delta_{\min,t} + p\Delta_{\max,t}}{\Delta y_m(k)_t + p\Delta_{\min,t}} \tag{6}$$

where $\Delta_{\min,t} = \min\{\Delta y_m(k)_t\}, m = 1, \dots, M, \forall k$ and $\Delta_{\max,t} = \max\{\Delta y_m(k)_t\}, m = 1, \dots, M, \forall k$. The parameter $p \in [0,1]$ is a design value which scales the obtained coefficients and can be simply considered as $p = 0.5$.

Step 5- GRD will compute by following equation:

$$GRD_{m,t} = \sum_{k=1}^{K} w_k G_m(k)_t \ \forall m \tag{7}$$

After the above steps, the selected stocks are ranked based on the GRD values. Higher GRD means a good stock to invest. Then a comparison is done to evaluate the portfolio performance.

3-Results

For empirical analysis, 14 stocks of various industries selected based on 30 big companies in TSE. In this part, industries in the sector of metal, banks, financial institutions, car manufacture, transportation, and petroleum products were chosen as samples. Then, the final price of each stock in 2019 were gathered daily which is depicted in figure 1.

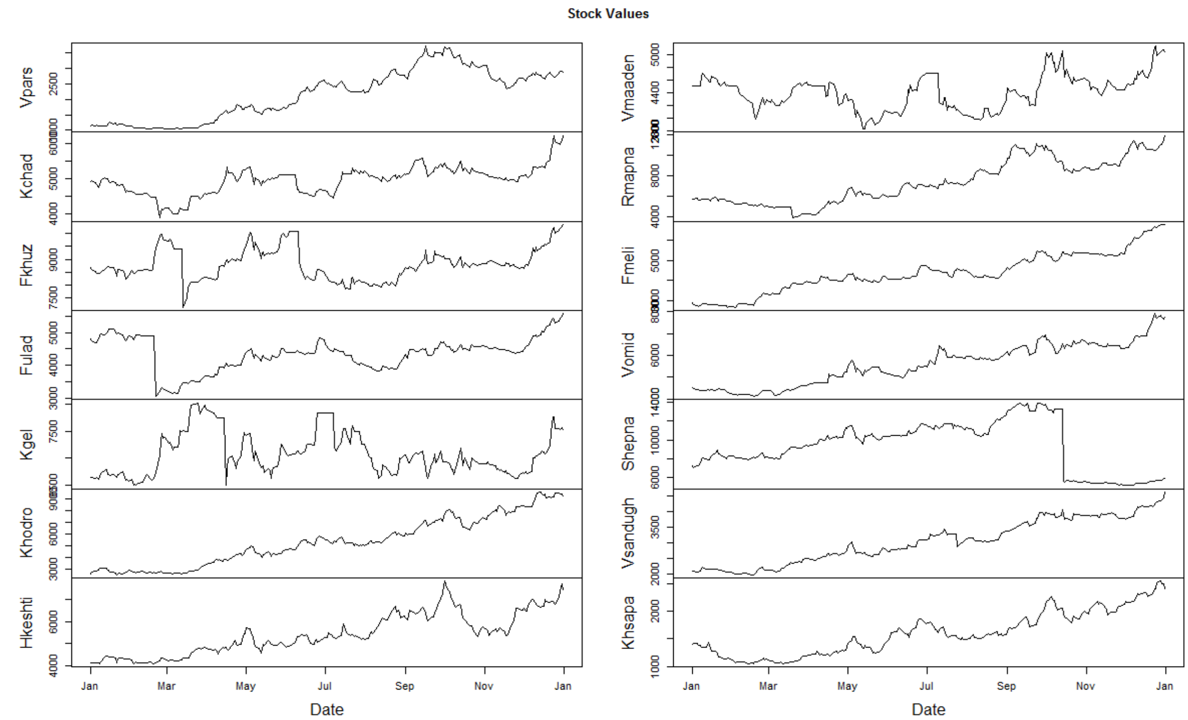

Fig 1. Final price of selected stocks

Return, standard deviation, skewness, and kurtosis are all calculated for each stock weekly. Then, GRD is calculated by equation (7) for each week. Supposing that only 4 stocks can be kept in the portfolio, ranking of the stocks has been done. Five investment strategies are simulated that are given in Table 2. Then, at the beginning of the next week, the portfolio is restructured based on the rankings from the previous week. Thus, the whole investing process is dynamic. Benchmark strategies are defined in order to evaluate the performance of the method.

Table 2. Investment strategies

Name		Short description
GRA	STR1	Equal weights for all 4 moments; First four best stocks by GRA
	STR2	Equal weights for first 3 moments; First four best stocks by GRA
	STR3	Equal weights for first 2 moments; First four best stocks by GRA
Benchmark	STR4	Randomly select four stock; Equally weighted to each stock
	STR5	Randomly select four stock; Equally weighted to each stock

Then portfolio return, portfolio standard deviation, and Sharpe ratio are calculated for each portfolio, and results are given in Table 3. For evaluating the method performance, the ranking has been done based on defined criteria, and it is shown in Table 4. An equally weighted investment strategy is used as a benchmark method for evaluating the GRA results. By supposing that the whole criterion has the same value for investors, the following results are achieved.

Table 3. Portfolio Performance

	Average return (+)	Max return (+)	Min return (+)	SD (-)	Total return (+)	Sharpe (+)
ST1	0.0165	0.1107	**-0.1226**	0,0268	0.8606	-3.1132
ST2	0.0137	0.1169	-0.1260	0.0270	0.7102	-3.2024
ST3	**0.0197**	**0.1417**	-0.1260	0.0285	**1,0241**	**-2.8182**
ST4	0.0096	0.1276	-0.2114	0.0285	0.5018	-3.1740
ST5	0.0108	0.0905	-0.1340	**0.0254**	0.5614	-4.6262

Table 4. Ranking of all simulated portfolio based on performance measures

	Average return (+)	Max return (+)	Min return (+)	SD (-)	Total return (+)	Sharpe (+)
ST1	2	4	1	2	2	2
ST2	3	3	2	3	3	4
ST3	1	1	2	4	1	1
ST4	5	2	4	4	5	3
ST5	4	5	3	1	4	5

- Those strategies which used the GRA achieved the best performance in compare with equally weighted strategy.
- As strategy (3) achieved the best performance in most cases of criteria, so risk and return can be mention as the most important criteria for investors. The comparison between strategy (3) and benchmark strategies is depicted in Fig 2.
- Although the least standard deviation achieved by strategy (5), it has less return in compare with GRA strategies too.

Fig 2. Comparison of strategy 3 with the benchmark strategies

4-Conclusion

This sturdy adopts the GRA approach to consider its application in portfolio selection problems in TSE. The study confirms that it has better performance in compare with benchmark model; equally weighted. It employs a dynamic approach in empirical analysis.

As a suggestion for future work, comparing the GRA with DEA and Fuzzy methods could be more attractive. It should be note that the transaction cost is ignored in this study, so it can be consider in future works to increase the similarity of simulated problems to the real world situation.

References

Abdelaziz, F. B., & Masmoudi, M. (2014). A multiple objective stochastic portfolio selection problem with random Beta. International Transactions in Operational Research, 21(6), 919-933.

Aggarwal, R., Rao, R. P., & Hiraki, T. (1989). Skewness and kurtosis in Japanese equity returns: empirical evidence. Journal of Financial Research, 12(3), 253-260.

Amin, G. R., & Hajjami, M. (2021). Improving DEA cross-efficiency optimization in portfolio selection. Expert Systems with Applications, 168, 114280.

Bayramoglu, M. F., & Hamzacebi, C. (2016). Stock selection based on fundamental analysis approach by grey relational analysis: a case of Turkey. International Journal of Economics and Finance, 8(7), 178-184.

Beedles, W. L. (1984). The Anomalous and Asymmetric Nature of Equity Returns: An Empirical Synthesis. Journal of Financial Research, 7(2), 151-160.

Beshkooh, M., & Afshari, M. A. (2012). Selection of the optimal portfolio investment in stock market with a hybrid approach of hierarchical analysis (AHP) and grey theory analysis (GRA). Journal of Basic and Applied Scientific Research, 2(11), 11218-11225.

Chan, L. K., Karceski, J., & Lakonishok, J. (1999). On portfolio optimization: Forecasting covariances and choosing the risk model. The review of Financial studies, 12(5), 937-974.

Dai, Y., & Qin, Z. (2021). Multi-period uncertain portfolio optimization model with minimum transaction lots and dynamic risk preference. Applied Soft Computing, 107519.

Galankashi, M. R., Rafiei, F. M., & Ghezelbash, M. (2020). Portfolio selection: a fuzzy-ANP approach. Financial Innovation, 6(1), 1-34.

Ghahtarani, A., & Najafi, A. A. (2013). Robust goal programming for multi-objective portfolio selection problem. Economic Modelling, 33, 588-592.

Guo, S., Ching, W. K., Li, W. K., Siu, T. K., & Zhang, Z. (2020). Fuzzy hidden Markov-switching portfolio selection with capital gain tax. Expert Systems with Applications, 149, 113304.

Hsu, C. M. (2014). An integrated portfolio optimization procedure based on data envelopment analysis, artificial bee colony algorithm and genetic programming. International Journal of Systems Science, 45(12), 2645-2664.

Huang, K. Y., & Jane, C. J. (2008, December). An automatic stock market forecasting and portfolio selection mechanism based on VPRS, ARX and grey system. In 2008 IEEE Asia-Pacific Services Computing Conference (pp. 1430-1435).

Huang, K. Y., Jane, C. J., & Chang, T. C. (2010, December). A hybrid model for portfolio selection based on grey relational analysis and RS theories. In 2010 International Computer Symposium (ICS2010) (pp. 1014-1019).

Huang, K. Y., Jane, C. J., & Chang, C. (2011). An enhanced approach to optimizing the stock portfolio selection based on Modified Markowitz MV Method. Journal of Convergence Information Technology, 6(2), 226-239.

Huang, C. F. (2012). A hybrid stock selection model using genetic algorithms and support vector regression. Applied Soft Computing, 12(2), 807-818.

Kayacan, E., Ulutas, B., & Kaynak, O. (2010). Grey system theory-based models in time series prediction. Expert systems with applications, 37(2), 1784-1789.

Khedmati, M., & Azin, P. (2020). An online portfolio selection algorithm using clustering approaches and considering transaction costs. Expert Systems with Applications, 159, 113546.

Liu, S., Forrest, J., & Yang, Y. (2013, November). A summary of the progress in grey system research. In Proceedings of 2013 IEEE international conference on grey systems and intelligent services (GSIS) (pp. 1-10).

Liu, Y. J., Zhang, W. G., & Zhang, P. (2013). A multi-period portfolio selection optimization model by using interval analysis. Economic Modelling, 33, 113-119.

Lwin, K. T., Qu, R., & MacCarthy, B. L. (2017). Mean-VaR portfolio optimization: A nonparametric approach. European Journal of Operational Research, 260(2), 751-766.

Markowitz, H. M. (1968). Portfolio selection. Yale university press.

Mashayekhi, Z., & Omrani, H. (2016). An integrated multi-objective Markowitz-DEA cross-efficiency model with fuzzy returns for portfolio selection problem. Applied Soft Computing, 38, 1-9.

Mills, E. F. E. A., Baafi, M. A., Amowine, N., & Zeng, K. (2020). A hybrid grey MCDM approach for asset allocation: evidence from China's Shanghai Stock Exchange. Journal of Business Economics and Management, 21(2), 446-472.

Naqvi, B., Mirza, N., Naqvi, W. A., & Rizvi, S. K. A. (2017). Portfolio optimization with higher moments of risk at the Pakistan Stock Exchange. Economic research-Ekonomska istraživanja, 30(1), 1594-1610.

Raei, R., & Jahromi, M. (2012). Portfolio optimization using a hybrid of fuzzy ANP, VIKOR and TOPSIS. Management Science Letters, 2(7), 2473-2484.

Safitri, I. N. N., Sudradjat, S., & Lesmana, E. (2020). Stock portfolio analysis using Markowitz model. International Journal of Quantitative Research and Modeling, 1(1), 47-58.

Škrinjarić, T., & Šego, B. (2019). Using grey incidence analysis approach in portfolio selection. International Journal of Financial Studies, 7(1), 1.

Škrinjarić, T. (2020). Dynamic portfolio optimization based on grey relational analysis approach. Expert systems with applications, 147, 113207.

Steuer, R. E., Wimmer, M., & Hirschberger, M. (2013). Overviewing the transition of Markowitz bi-criterion portfolio selection to tri-criterion portfolio selection. Journal of Business Economics, 83(1), 61-85.

Sun, Q., & Yan, Y. (2003). Skewness persistence with optimal portfolio selection. Journal of Banking & Finance, 27(6), 1111-1121.
Vezmelai, A., Lashgari, Z., & Keyghobadi, A. (2015). Portfolio selection using ELECTRE III: evidence from Tehran Stock Exchange. Decision Science Letters, 4(2), 227-236.

Wang, B., Wang, S., & Watada, J. (2011). Fuzzy-portfolio-selection models with value-at-risk. IEEE Transactions on Fuzzy Systems, 19(4), 758-769.

Xia, Y., Liu, B., Wang, S., & Lai, K. K. (2000). A model for portfolio selection with order of expected returns. Computers & Operations Research, 27(5), 409-422.

Xu, G., Guo, P., Li, X., & Jia, Y. (2014). Grey relational analysis and its application based on the angle perspective in time series. Journal of Applied Mathematics, 2014.

Yin, M. S. (2013). Fifteen years of grey system theory research: a historical review and bibliometric analysis. Expert systems with Applications, 40(7), 2767-2775.

Zhang, W. G., Liu, Y. J., & Xu, W. J. (2012). A possibilistic mean-semi variance-entropy model for multi-period portfolio selection with transaction costs. European Journal of Operational Research, 222(2), 341-349.

Zhu, M. (2013). Return distribution predictability and its implications for portfolio selection. International Review of Economics & Finance, 27, 209-223.

Zhou, W., & Xu, Z. (2018). Portfolio selection and risk investment under the hesitant fuzzy environment. Knowledge-Based Systems, 144, 21-31.