Journal of Industrial and Systems Engineering
Vol. 14, No. 2, pp. 32-60
Spring (April) 2022

Design a decision structure for the order promising process in hybrid MTS/MTO environments considering product substitution, a case study

Shahin Sadeghi Ahangar[1] , Masoud Rabbani[1*]

[1]*School of Industrial and System Engineering, College of Engineering, University of Tehran, Tehran, Iran*

shahin.sadeghi@ut.ac.ir, mrabani@ut.ac.ir

Abstract

The importance of order promising process has led manufacturers to use more productive production systems. Optimizing the production system is one of the ways to increase productivity. This issue becomes even more significant when some of the raw materials needed to produce different final products are homogenous. In this paper, a decision structure for the order promising process with product homogeneity and product substitution in a Hybrid Make-To-Stock and Make-To Order environment is studied. For this purpose, a bi-objective mathematical model has been designed and solved by the Lagrangian Relaxation solution method. Despite the extensive studies that have been done in this area, there are few articles that have studied the possibility of substituting the final products by the manufacturer. In order to investigate this gap, product substitution has been studied in this article. Two different types of customers are considered in this model. A case study is also conducted to evaluate the applicability of the proposed model. The results of this article show that the possibility of products substitution will reduce rejected orders and increase system profits. Also, fulfilling orders that are more flexible in terms of product delivery time is a higher priority for the manufacturer than other orders.

Keywords: Order acceptance, order fulfillment, order promising process, hybrid production systems, product substitution, Lagrangian relaxation method

1- Introduction

Order acceptance is one of the most significant decisions for a manufacturing system. When an order is accepted, the manufacturer has to deliver that order completely to the customer (Nguyen, 2016); Sarvestani, Zadeh, Seyfi, and Rasti-Barzoki (2019). This issue has been studied in two general ways in the literature. The goals of these articles were either to maximize revenue from accepting customer orders or minimizing the cost of rejecting those orders (Shabtay, Gaspar, & Kaspi, 2013; Slotnick, 2011).

The order-fulfillment process (OFP) is defined as how a company responds to a customer's order, that is, activities ranging from sales inquiry to product delivery (Shapiro, Rangan, Sviokla, Paul, & Meisel, 1992). Determining when to start OFP is a crucial decision for the owners of the manufacturing company. On the one hand, customers expect them to be responsive and get the product with the right quality and price quickly after submitting their request.

---

*Corresponding author
ISSN: 1735-8272,

On the other hand, starting OFP for a manufacturing company before the customer submits an appeal is risky. This issue may include costs such as holding costs, product spoilage, and the possibility of order cancellation (Li, He, & Wu, 2016).

There are many production systems that companies can use (Yousefnejad & Esmaeili, 2020). Among them are Make-to-Stock (MTS) (Agra, Poss, & Santos, 2018), Make-to-Order (MTO) (Aslan, Stevenson, & Hendry, 2015), Assemble-to-Order (ATO) (ElHafsi, Fang, & Hamouda, 2020), Engineer-to-Order (ETO) (Cannas, Gosling, Pero, & Rossi, 2020), and Configure-to-Order (CTO) (Jansen, Atan, Adan, & de Kok, 2019) environments (Pibernik, 2005). A hybrid production system (HPS) called hybrid MTS-MTO has been used, which can have the characteristics of both systems simultaneously. As depicted in figure 1, the starting and ending points of the production process of products can vary in HPS. Due to the characteristics of HPS, many manufacturing companies are moving towards them (Kalantari, Rabbani, & Ebadian, 2011).

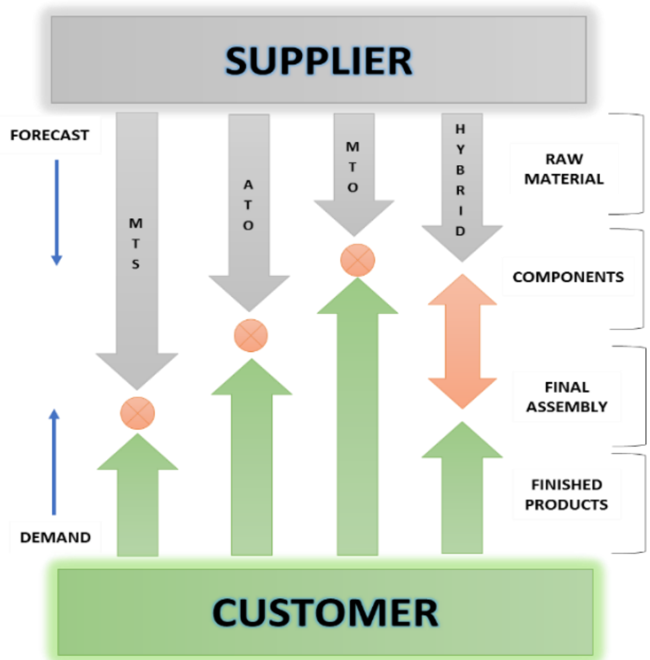

Fig. 1. Comparing the production process in different strategies (Raturi, Meredith, McCutcheon, & Camm, 1990)

Given the increasing competition in the production of various products, choosing the optimal production strategy seems vital (Kim & Min, 2021). In an MTS environment, products have little variety, and the delivery time of the products is short, and the customers' orders are provided from the stored finished products. But, a high-variety of products appears in an MTO environment. Lower holding costs and lower production volumes are the other features of this production system. The combination of these two systems makes optimal use of these two systems (Beemsterboer, Land, & Teunter, 2017; Wang, Qi, Cui, & Zhang, 2019). But inventory control and production planning in an HPS is complex and requires more precision. Due to the high cash flow in companies, even small reductions in cost lead to significant increases in revenue (Abedi & Zhu, 2020). The rest of this paper is organized as follows. In the next section, a related literature review is presented. Section 3 describes the mathematical model, along with its assumptions. Section 4 presents the solution method in detail. In Section 5, numerical results of the model, along with a case study, are described. Section 6 analyzes the sensitivity of the main parameters and their effects on the solving process. Section 7 includes managerial insights and a discussion of different parts of the paper, and the whole paper is concluded in Section 8.

2- Literature review

In this section, a brief review is performed on hybrid MTS/MTO production systems. While both MTO and MTS methods separately have been extensively studied, their combined use has received less attention (Peeters & van Ooijen, 2020). A hybrid MTS-MTO production system is a combination of the features of each make-to-stock and make-to-order production system (Peeters & van Ooijen, 2020). The main purpose of providing hybrid production systems is to use the features of each production system simultaneously to meet customer demand (Jia, Weng, & Fujimura, 2017). A substantial part of the research on the hybrid MTS/MTO production system discusses the decision of which items to produce to stock and which ones to order (Beemsterboer, Land, & Teunter, 2016; Jalali, Ghomi, & Rabbani, 2020). Customer orders may vary and include various models of the company's final products. In this case, meeting customer needs may be more complicated than the homogeneity of the ordered products (Alemany Díaz, Alarcón Valero, Oltra Badenes, & Lario Esteban, 2013).

2-1- Order fulfillment

Order fulfillment models generally consist of two parts. The first part is the decision about the received orders from the customers. The second part is the method of supplying raw materials. The order fulfillment process includes order processing, manufacturing planning and shipping of goods (Laurikainen, 2020). Noroozi, Mazdeh, Heydari, and Rasti-Barzoki (2018) presented a MIP model, which was a combination of accept-reject of the received orders and scheduling the supply chain, in which 3pl shipment was considered. Naderi and Roshanaei (2019) presented a model for accept-reject orders that were combined with scheduling. In this MIP model, production machines were considered in a parallel manner.

Zheng, Chien, and Wu (2018) presented a multi-objective model to meet customer orders. This model was related to the green energy market, and its purpose was to satisfy the demand for solar cells. Sarvestani et al. (2019) presented another model in which the selection from different suppliers was also considered. This model was solved by a meta-heuristic algorithm in a Single-Machine-Environment. Another article was presented to meet the customer orders, in which various contracts were made with customers based on different service-levels. In this paper, the main focus was on comparing different ways of meeting customer demand (Kloos & Pibernik, 2020). An article was also presented to improve the production order fulfillment process by reducing wastes. A textile company was considered as a case study for the mentioned article (Tapia-Leon, Vega-Neyra, Chavez-Soriano, & Ramos-Palomino, 2019).

2-2- Hybrid production system

Rafiei, Rabbani, and Alimardani (2013) presented a model in which a manufacturing company could perform its production process in three forms: make-to-stock, make-to-order, and Hybrid make to stock/order. There were also other issues, such as determining or changing the order time, the size of the warehouse, and the production capacity at each of the decision levels, which are considered in this model. A Dynamic HPS was developed by Yano, Nagasawa, Morikawa, and Takahashi (2019). In this paper, a kind of machine is considered, which is capable of switching between the MTS and MTO production methods. Ellabban and Abdelmaguid (2019) presented a model for an HPS in which a glass tube company was considered as a case study. A simulation model is also developed in this article. Another model was designed for an HPS, which studied a single product under the cap-and-trade environment (Xiong, Feng, & Huang, 2020).

A Capable-to-Promise model was developed for an HPS in which available resources are allocated to customers' future orders. The goal of the proposed model was to minimize the risk of being unreliable due to discrepancies between the real and unused quantities. (Abedi & Zhu, 2020). Cannas et al. (2020) wrote an article to examine engineer-to-order situations in different order-fulfillment strategies based on the degree of the customer. This paper aimed to analyze the existing literature, and then several case studies are developed. A bi-objective optimization model is developed by Bortolini, Faccio, Gamberi, and Pilati (2019) to correctly set the MTO/MTS policy for the sheet metal plate parts in an ATO environment. A mathematical model is also proposed to simultaneously investigate lot sizing and scheduling in a hybrid production system in which a mineral water company is considered as a case study (Akbari, 2020).

2-3- Product Substitution

Disruption is one of the dangers of the supply chain that can cut off product flow, delay product delivery to customers, and reduce supply chain revenue (Khalilabadi, Zegordi, & Nikbakhsh, 2020). Product substitution can be considered as one of the strategies used by manufacturers to deal with problems caused by uncertainty or disruption (Afshar-Bakeshloo, Flapper, Jolai, & Bozorgi-Amiri, 2021; Tsao, Raj, & Yu, 2019). Increasing the service-level, reducing holding costs as well as reducing waste can be named as the results of product substitution (Lang, 2009). Many articles have been written on product substitution. A mathematical model is proposed for the different inventory policies in a hybrid production system considering product substitution (Ahiska, Gocer, & King, 2017). In this context, a model was developed by Tsao et al. (2019) to meet customer orders considering product substitution. The goal of this article was to determine the optimal numbers for ordering quantities and product substitution to maximize the total profit of the company. Jing and Mu (2019) considered product substitution and perishable inventories for a dynamic lot-sizing model.

2-4- Lagrangian relaxation

Lagrangian Relaxation (LR) is mostly used to solve large-scale problems (Van Roy, 1983). This method solves the problems by providing an upper or lower limit for the problem and improving it at each step. In this method, first a set of answers is created and then these answers are improved in each step to obtain optimal solutions. Many extensions have been developed for the LR method. Geoffrion (1974) has reviewed the history of articles presented in this field. LR algorithm can provide optimal solutions for different models in large dimensions in a reasonable time compared to traditional solving methods (Hassannayebi, Zegordi, & Yaghini, 2016). Fathollahi-Fard, Hajiaghaei-Keshteli, Tian, and Li (2020) developed a coordinated water supply and wastewater collection network. The LR method has been used to optimize the mentioned model.

2-5- Research gap

The table (1) shows a comparative review of literature.

Table. 1. A comparative review of the previous articles

Articles	Objective Function		Product Substitution	Uncertainty	Production System		Case Study
	Single	Multi			Single	Hybrid	
Rafiei, Rabbani, Vafa-Arani, and Bodaghi (2017)	□			□	□		□
Ghalehkhondabi and Suer (2018)		□	□	□		□	□
Yousefnejad and Esmaeili (2020)	□		□	□		□	□
Zhang, Guo, Wei, Guo, and Gao (2021)		□	□	□	□		□
This Study		□	□	□		□	□

An overview of the literature review is presented in table 1. In this table, the elements that have been studied in previous papers are classified. The results show that despite the practical research that has been done in this field, there are still some parts of this issue that has been less addressed (Mello, Gosling, Naim, Strandhagen, & Brett, 2017). None of the articles mentioned in this section has examined the ability to satisfy customer orders with a substitute product in an HPS. The following is a list of the most important contributions of this article:

- Consider a decision-making structure for OFP in a Hybrid MTS / MTO production system with a maximum response time for received demands.
- Consider the possibility of fulfilling customer orders with alternative products (product substitution), and including customer dissatisfaction.
- Consider two types of customers (Cloud Clients and Main Clients) with different features.
- Consider the possibility of selecting the optimal suppliers from the existing ones according to their reliability.
- Consider the similarity in the raw materials required to produce some kinds of final products.
- Provide a case study to evaluate the applicability of the model in the real world.

3- Problem description and model formulation

3-1- Problem Description

In this section, a bi-objective mathematical model has been designed. In this case, a manufacturing company is investigated that produces different products in distinct classes. As shown in figure 2, products that differ only in class type have slight differences from each other. So, some of the raw materials needed to produce these products are the same. The products of this company can be provided through MTS- MTO or outsourcing. Supplying through each of the mentioned methods has a limited capacity.

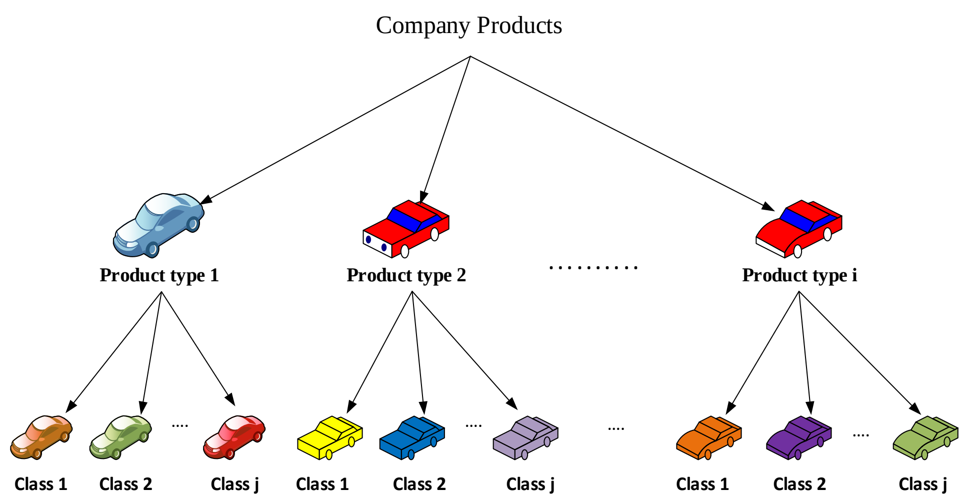

Fig 2. Classification status of the final products of the company

Customer orders are delivered to the company's decision-makers with a maximum response time, which may be accepted or rejected. These orders may be delivered to customers through one or more shipments. Orders that are delivered to customers after the maximum response time will be subject to a delay payment. The company's income will be from selling the final products to customers. Two types of customers have been studied in this model: the main customers and cloud customers. If the orders of the main customers are not met, it will cost the company more penalty, and the resulting dissatisfaction will be higher compared to the cloud ones.

The proposed model has two objective functions. The first objective function maximizes the revenue generated from the selling of products. The products ordered by customers may be satisfied with substitute products. On the one hand, this means that fewer customer orders are rejected, and if a particular product is lacking, it can be replaced with similar ones. On the other hand, it increases customer dissatisfaction. Therefore, to minimize these deviations, another objective function can be defined. The second objective function of the model minimizes the level of customer dissatisfaction. These dissatisfactions include dissatisfaction with order rejection, dissatisfaction with the delay in order satisfaction, and dissatisfaction with order fulfillment with substitute products. Figure 3 illustrates the different scenarios of responding to received orders.

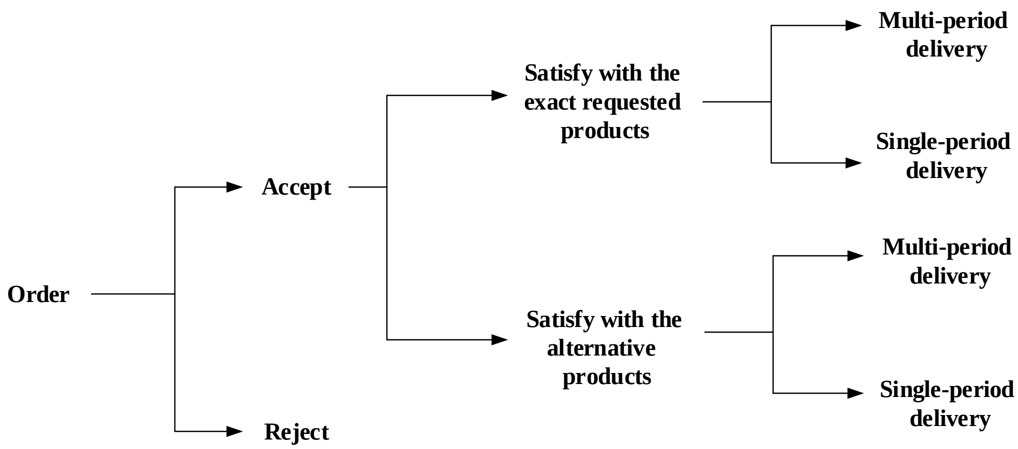

Fig. 3. Different scenarios of dealing with received orders

Outsourcing can be done to procure raw materials and final products. Outsourcing in this model is done at two levels of raw materials and final products. Customer orders are reviewed by the company's managers simultaneously. Also, the number of remaining products from previous periods that have been provided through the Available to Promise (ATP) method will be charged for holding costs at the end of the period. So, in general, this system can be considered as a three-tier supply chain that includes suppliers, manufacturers, and distributors. Other assumptions of the model are as follows:

- Delivery of final products to customers is done by the company.
- The cost of producing products is considered as a mix of a fixed and variable cost.
- Operational risk is considered in this model so that the efficiency and production capacity of machines is defined as a coefficient for each product and in each class and each period.
- Each different type of vehicle has its own cost and capacity.

3-2- Notation

The following sets, variables, and parameters are introduced to formulate this model. Based on these tables, a mathematical model is presented.

Table. 2. Introducing the sets used in the model

Sets	
C	Set of customers
I	Set of final products
J	Set of product classes
N	Set of periods for sending products
M	Set of various raw materials
B	Set of different methods of transporting products
S	Set of various suppliers

Table. 3. Introducing the parameters used in the model

Parameters	
$P_{i,j}$	The selling price of a unit of product i and class j
$d_{c,i,j}^{max}$	Demand for the product i and class j ordered by customer c
$h_{n,i,j}^{ATP}$	Holding cost of a unit of product i and class j in period n, which is prepared as ATP
$C_{c,i,j}^{substitute}$	Payment fee for the replacement of each ordered product with a product of type i and class j to the customer c
$C_{i,j,s}^{outsource}$	Cost of supplying a unit of product i and class j from supplier s by outsourcing method
$C_{i,j}^{ATP}$	Cost of supplying a unit of product i and class j by ATP method
$cvar_{i,j}^{production}$	Variable cost of producing a unit of product type i and class j
$C_{s,m}^{material}$	Cost of supplying a unit of raw material m from supplier s
L_c	Distance of costumer c from the company
C^{accept}	Cost of accepting an order
C^{reject}	Cost of rejecting an order
V_b^{var}	Variable cost of transporting one unit of the product for each unit of distance by type b shipping method
v_b^{fix}	Fixed cost of shipping the product per unit distance by type b shipping method
$\beta_{c,n}$	The coefficient for increasing costs due to delay in sending the customer order in period n for customer c
$BOM_{i,j,m}$	Number and type of raw materials m required to produce one unit of product i of class j
$cap_b^{vehicle}$	Capacity of type b transporters
$ncap_{n,b}^{vehicle}$	Maximum number of available vehicles of type b in period n
$d_{n,c,i,j}^{min}$	Minimum quantity to be delivered from customer c of the product type i and class j in period n
$cap_{m,s,m}^{supply}$	Maximum orderable capacity of type m raw material from supplier s in period n
$cap_{n,i,j}^{production}$	Maximum production capacity of product i of class j in period n
$cap_{n,i,j}^{outsource}$	Maximum supply capacity of product i and class j in period n by outsourcing method
$\omega_{c,n}$	Dissatisfaction coefficient of customer c if the order is met in period n
Ψ_c^1	Coefficient of dissatisfaction caused by rejecting each unit of order for customer c
Ψ_c^2	Coefficient of dissatisfaction caused by the supply of each unit of order from customer c in a delayed manner
Ψ_c^3	Coefficient of dissatisfaction caused by supplying each unit of order for customer c with substitute products
$cap_c^{customer}$	Maximum number of times the products are sent to the customer c
$cap_n^{delivery}$	Maximum number of shipments that the company can handle in period n
$ncap_{c,n}^{delivery}$	Maximum receivable capacity of products ordered in period n by customer c
$\varepsilon_{n,i,j}$	Productivity coefficient of machines in period n to produce the product i and class j
$type_c^{customer}$	Coefficient determining the type of customer c
$cfix_{i,j}^{production}$	Fixed cost of producing product i of class j
M	A large number

Table. 4. Introducing the variables used in the model

Variables	
G_1	Total revenue
F_1	Total holding cost
F_2	Total cost of supplying products through outsourcing
F_3	Total cost of supplying the products through ATP
F_4	Total cost of production by CTP method
F_5	Total fixed cost of rejecting orders
F_6	Total variable cost of sending the products to customers
F_7	Total delay cost
F_8	Total cost required to supply raw materials
F_9	Total fixed cost of sending orders
F_{10}	Total cost of supplying products with alternative products
F_{11}	Total cost of accepting orders
Z_1	The first objective function value
Z_2	The second objective function value
U_1	Dissatisfaction caused by non-fulfillment of orders
U_2	Dissatisfaction caused by fulfilling orders with delays
U_3	Dissatisfaction caused by fulfilling orders with substitute products
$\alpha_{n,s,m}$	The number of raw materials purchased of type m in period n, from supplier s
$Q_{n,i,j}^{WIP}$	Total inventory of the product i and class j, which is available in period n as CTP method
$Q_{n,i,j}^{ATP}$	Inventory of product of type i and class j in period n, which has been prepared by ATP method
$Q_{n,i,j,s}^{outsource}$	The amount of inventory of the product i and class j in the period n which is provided as outsourcing from supplier s
$y_{n,c,i,j}^{WIP}$	The amount of CTP inventory used to respond to the product type i and class j from customer c in period n
$y_{n,c,i,j,s}^{outsource}$	The amount of product type i and class j in period n is sent for customer c, which is outsourced and supplied by supplier s
$y_{n,c,i,j}^{ATP}$	The amount of ATP inventory used to respond to the product type i and class j to customer c in period n
$o_{n,c,i,j}$	Variable represents the total amount of product i and class j in period n sent to customer c
$num_{c,n,b}^{vehicle}$	Number of vehicles type b in period n that carries order of customer c
$x_{c,i,j}^{fullfillment}$	A binary variable that if order of customer c from type i and class j is accepted by the company equals to 1 and otherwise 0
$e_{c,i,j}$	Non-negative auxiliary variable to calculate the number of orders that have been delivered to customer c with a substitute product
λ_c	The binary variable that if one of the orders of customer c is accepted will be equal to 1 and otherwise equal to 0
$\eta_{n,i,j}$	The binary variable, which is equal to 1 if it is produced in period n of product i and class j, and otherwise equal to 0
$W_{n,c,i,j}$	A binary variable that is equal to 1 if the product i and class j is sent to customer c in period n, otherwise it is equal to 0

3-3- Model formulation

3-3-1- Objective functions

$$obj1 = \max\ z1 = G_1 - (F_1 + F_2 + F_3 + F_4 + F_5 + F_6 + F_7 + F_8 + F_9 + F_{10} + F_{11}) \quad (1)$$

$$G_1 = \sum_c \sum_i \sum_j x_{c,i,j}^{fulfillment} \times d_{c,i,j}^{max} \times p_{i,j} \quad (2)$$

$$F_1 = \sum_n \sum_i \sum_j h_{n,i,j}^{ATP} \times (Q_{n,i,j}^{ATP} + \sum_n \sum_i \sum_j (Q_{n,i,j}^{ATP} - (\sum_c y_{n,c,i,j}^{ATP}))) \quad (3)$$

$$F_2 = \sum_i \sum_j \sum_s ((\sum_n Q_{n,i,j,s}^{outsource}) \times c_{i,j,s}^{outsource}) \quad (4)$$

$$F_3 = \sum_i \sum_j ((\sum_n Q_{n,i,j}^{ATP}) \times c_{i,j}^{ATP}) \quad (5)$$

$$F_4 = \sum_i \sum_j (\sum_n Q_{n,i,j}^{WIP} \times cvar_{i,j}^{production}) + (\sum_i \sum_j \left(\sum_n \eta_{n,i,j}\right) \times cfix_{i,j}^{production}) \quad (6)$$

$$F_5 = c^{reject} \times (\sum_c \sum_i \sum_j (1 - x_{c,i,j}^{fulfillment} \times type_c^{customer})) \quad (7)$$

$$F_6 = \sum_c \sum_n \sum_b num_{c,n,b}^{vehicle} \times v_b^{var} \times L_c \quad (8)$$

$$F_7 = \sum_n \sum_c \sum_i \sum_j o_{n,c,i,j} \times \beta_{c,n} \times p_{i,j} \times type_c^{customer} \quad (9)$$

$$F_8 = \sum_s \sum_m ((\sum_n a_{n,s,m}) \times c_{s,m}^{material}) \quad (10)$$

$$F_9 = \sum_b \sum_n \sum_c \sum_i \sum_j w_{n,c,i,j} \times v_b^{fix} \quad (11)$$

$$F_{10} = \sum_c \sum_i \sum_j c_{c,i,j}^{substitute} \times type_c^{customer} \times \frac{\left| x_{c,i,j}^{fulfillment} \times d_{c,i,j}^{max} - \sum_n (y_{n,c,i,j}^{WIP} + \sum_s y_{n,c,i,j,s}^{outsource} + y_{n,c,i,j}^{ATP}) \right|}{2} \quad (12)$$

$$F_{11} = \sum_c (\lambda_c \times c^{accept}) \quad (13)$$

Equation (1) represents the amount of the first objective function. This phrase calculates net profit. Equation (2) calculates the total revenue that comes from selling various products. The holding costs of the products are shown in equation (3). The cost of outsourcing the products is calculated in equation (4). The costs of supplying the products through the ATP are calculated in equation (5). In equation (6), the amount of costs required to produce different products has been calculated through the CTP method. The costs of rejecting orders are stated in equation (7).

Equation (8) calculates the variable cost of sending orders to customers. Of course, in some contracts, the shipping cost may be borne by the customer, which means sending the shipments will be free for the company. The delay cost is also indicated in equation (9). The cost of supplying raw materials is calculated in equation (10). Equation (11) calculates the fixed cost of sending orders. The amount of fine paid by the company due to fulfilling customer orders with substitute products is shown in equation (12). Equation (13) indicates the fixed cost for accepting the orders. These fixed costs generally consist of expenses such as product introduction, advertising sessions, bargaining, support, and after-sales service.

$$obj2 = \min z2 = U_1 + U_2 + U_3 \quad (14)$$

$$U_1 = \sum_c \sum_i \sum_j ((1 - x_{c,i,j}^{fulfillment}) \times type_c^{customer} \times d_{c,i,j}^{max} \times \Psi_c^1) \quad (15)$$

$$U_2 = \sum_n \sum_c \sum_i \sum_j o_{n,c,i,j} \times \omega_{cn} \times type_c^{customer} \times \Psi_c^2 \quad (16)$$

$$U_3 = \sum_c \sum_i \sum_j (\frac{\left| x_{c,i,j}^{fulfillment} \times d_{c,i,j}^{max} - \sum_n (y_{n,c,i,j}^{WIP} + \sum_s y_{n,c,i,j,s}^{outsource} + y_{n,c,i,j}^{ATP}) \right|}{2} \times type_c^{customer} \times \Psi_c^3) \quad (17)$$

Equation (14) calculates the value of the second objective function. The second objective function is a combination of the three variables, U_1, U_2, and U_3. This objective function minimizes the total amount of customer dissatisfaction. These dissatisfactions include dissatisfaction due to non-acceptance of customer orders (equation (15)), dissatisfaction resulting from delayed fulfillment of customer orders (equation (16)), and dissatisfaction resulting from fulfillment of customer orders with alternative products (equation (17)).

3-3-2- Constraints

$$\sum_j \sum_n (y_{n,c,i,j}^{WIP} + \sum_s y_{n,c,i,j,s}^{outsource} + y_{n,c,i,j}^{ATP}) = \sum_j x_{c,i,j}^{fulfillment} \times d_{c,i,j}^{max} \qquad \text{□c, i} \qquad (18)$$

$$o_{n,c,i,j} = y_{n,c,i,j}^{WIP} + y_{n,c,i,j}^{ATP} + \sum_s y_{n,c,i,j,s}^{outsource} \qquad \text{□n, c, i, j} \qquad (19)$$

$$\frac{\sum_i \sum_j o_{n,c,i,j}}{cap_b^{vehicle}} \leq num_{c,n,b}^{vehicle} \qquad \text{□c, n, b} \qquad (20)$$

$$\sum_n \sum_j o_{n,c,i,j} = \sum_j (d_{c,i,j}^{max} \times x_{c,i,j}^{fulfillment}) \qquad \text{□c, i} \qquad (21)$$

$$d_{n,c,i,j}^{min} \times w_{n,c,i,j} \leq o_{n,c,i,j} \qquad \text{□n, c, i, j} \qquad (22)$$

$$\Psi_c^1 + \Psi_c^2 + \Psi_c^3 = 1 \qquad \text{□c} \qquad (23)$$

Constraint (18) states that an accepted order must be met through the ATP, outsourcing, or CTP method. According to this Constraint, a shortage is not allowed in supplying orders. Constraint (19) calculates the amount of product produced by CTP, ATP, and outsourcing methods. Equation (20) calculates the type and number of vehicles required to send orders. Equation (21) indicates that each time an order is sent to customers, not more than the maximum number of requested orders. Equation (22) states that each time a shipment is sent to customers, a minimum amount of $d_{n,c,i,j}^{min}$of the final products must be sent. The sum of the coefficients used to calculate the objective function Z_2 must be equal to 1, which is expressed in constraint (23).

$$\sum_c \sum_i \sum_j (x_{c,i,j}^{fulfillment} \times d_{c,i,j}^{max}) \leq \sum_n \sum_b cap_b^{vehicle} \times ncap_{n,b}^{vehicle} \qquad (24)$$

$$a_{n,s,m} \leq cap_{n,s,m}^{supply} \qquad \forall \text{n, s, m} \qquad (25)$$

$$\sum_i \sum_j (BOM_{i,j,m} \times Q_{n,i,j}^{WIP}) - \sum_s a_{n,s,m} \leq 0 \qquad \forall \text{n, m} \qquad (26)$$

$$\sum_c y_{n,c,i,j}^{WIP} \leq Q_{n,i,j}^{WIP} \qquad \forall \text{n, i, j} \qquad (27)$$

$$\sum_c y_{n,c,i,j}^{ATP} \leq Q_{n,i,j}^{ATP} \qquad \forall \text{n, i, j} \qquad (28)$$

$$\sum_c y_{n,c,i,j,s}^{outsource} \leq Q_{n,i,j,s}^{outsource} \qquad \forall \text{n, i, j, s} \qquad (29)$$

$$\sum_s Q_{n,i,j,s}^{outsource} \leq cap_{n,i,j}^{outsource} \qquad \forall \text{n, i, j} \qquad (30)$$

$$Q_{n,i,j}^{WIP} \leq \varepsilon_{n,i,j} \times cap_{n,i,j}^{production} \qquad \forall \text{n, i, j} \qquad (31)$$

Equation (24) indicates that the capacity of different types of vehicles should deliver customer orders. The full capacity of the transport vehicles used in each period must be greater than the total number of products sent to customers in each period. Constraint (25) shows that the manufacturer is faced with a numerical limitation in ordering raw materials from each supplier. The amount of products produced by the CTP method is calculated in equation (26). Equations (27), (28), and (29) indicate that the number of products which are sent to customers of CTP, ATP, and outsourcing type, should be less than the inventory of finished products of them, respectively. These three limitations determine the system's capacity to accept customer's orders. Equation (30) limits the possibility of providing the products of the company through outsourcing to $cap_{n,i,j}^{outsource}$variable. Constraint (31) states the maximum production limit of the product.

$$\sum_m (BOM_{i,j,m} \times c_{s,m}^{material}) + c_{i,j}^{production} \leq c_{i,j,s}^{outsource} \qquad \forall \text{i, j, s} \qquad (32)$$

$$o_{n,c,i,j} \leq M \times w_{n,c,i,j} \qquad \forall \text{n, c, i, j} \qquad (33)$$

$$w_{n,c,i,j} \leq M \times o_{n,c,i,j} \qquad \forall \text{n, c, i, j} \qquad (34)$$

$$x_{c,i,j}^{fulfillment} \leq \sum_n w_{n,c,i,j} \qquad \forall \text{c, i, j} \qquad (35)$$

$$\sum_j (x_{c,i,j}^{fulfillment} \times d_{c,i,j}^{max}) \leq \sum_n \sum_j \left(Q_{n,i,j}^{ATP} + Q_{n,i,j}^{WIP} + \sum_s Q_{n,i,j,s}^{outsource}\right) \qquad \forall \text{c, i} \qquad (36)$$

$$\sum_n \sum_i \sum_j w_{n,c,i,j} \leq cap_c^{customer} \qquad \forall \text{c} \qquad (37)$$

$$\sum_c \sum_i \sum_j w_{n,c,i,j} \leq cap_n^{delivery} \qquad \forall \text{n} \qquad (38)$$

$\sum_c num_{c,n,b}^{vehicle} \leq ncap_{n,b}^{vehicle}$ $\forall$n, b (39)

$\sum_i \sum_j o_{n,c,i,j} \leq ncap_{c,n}^{delivery}$ $\forall$c, n (40)

$\lambda_c \leq \sum_i \sum_j x_{c,i,j}^{fulfillment}$ $\forall$c (41)

$x_{c,i,j}^{fulfillment} \leq \lambda_c$ $\forall$c, i, j (42)

$Q_{n,i,j}^{WIP} \leq M \times \eta_{n,i,j}$ $\forall$n, i, j (43)

$\eta_{n,i,j} \leq M \times Q_{n,i,j}^{WIP}$ $\forall$n, i, j (44)

The cost of producing a unit of product should not exceed the cost required to outsource that product, which is discussed in equation (32). The limitations for determining the value of the binary variable $w_{n,c,i,j}$ are written in Constraints (33) and (34). According to Constraint (35), accepted orders must be sent within at least one period. If the order is rejected, no delivery will be made by the company for that customer. The total inventory of the final product should not be less than the amount of the accepted order to be able to satisfy the accepted orders, which is mentioned in equation (36). The number of the shipment is sent to each customer is limited in Constraint (37). Equation (38) limits the number of times the product is sent by the system. The number of different types of available vehicles in each period is limited by the Constraint (39). According to Constraint (40), the total number of products sent to customers cannot be more than a certain amount. Constraints (41) and (42) compute the variable λ_c. This binary variable is used to calculate the fixed cost of accepting an order. Finally, equation (43) and (44) compute the auxiliary binary variable $\eta_{n,i,j}$.

3-4- Linearization

The proposed model is a non-linear one due to equation (12) and (17). This model can be converted into a linear one by applying some changes. Equation (45) is the non-linear factor of the proposed mathematical model. By defining the non-negative auxiliary variable $e_{c,i,j}$ and replacing it with the equation (45), according to equation (46), and also defining equations (47) and (48), the non-linear factor can be removed.

$$\left| x_{c,i,j}^{fulfillment} \times d_{c,i,j}^{max} - \sum_n (y_{n,c,i,j}^{WIP} + \sum_s y_{n,c,i,j,s}^{outsource} + y_{n,c,i,j}^{ATP}) \right| \tag{45}$$

$$e_{c,i,j} = \left| x_{c,i,j}^{fulfillment} \times d_{c,i,j}^{max} - \sum_n (y_{n,c,i,j}^{WIP} + \sum_s y_{n,c,i,j,s}^{outsource} + y_{n,c,i,j}^{ATP}) \right| \quad \forall c, i, j \tag{46}$$

$$e_{c,i,j} \geq x_{c,i,j}^{fulfillment} \times d_{c,i,j}^{max} - \sum_n (y_{n,c,i,j}^{WIP} + \sum_s y_{n,c,i,j,s}^{outsource} + y_{n,c,i,j}^{ATP}) \quad \forall c, i, j \tag{47}$$

$$e_{c,i,j} \geq -(x_{c,i,j}^{fulfillment} \times d_{c,i,j}^{max} - \sum_n (y_{n,c,i,j}^{WIP} + \sum_s y_{n,c,i,j,s}^{outsource} + y_{n,c,i,j}^{ATP})) \quad \forall c, i, j \tag{48}$$

4- Solution method

First, the validity of the model is checked in a small size by GAMS software. For this purpose, augmented epsilon-constraint has been used (Aghaei, Amjady, & Shayanfar, 2011; Mavrotas, 2009). Then, in order to evaluate the application of the proposed model, it is examined in the form of a real case study. A Lagrangian Relaxation method has been implemented to solve the problem in large and medium dimensions and reduce the complexity of the problem (Fisher, 1981). The proposed model is first transformed into a single-objective model and then implemented in the LR algorithm. Also, to check the accuracy of the answers generated in this method, first, the responses obtained in the small example are compared with the answers to this solution method. After examining the amount of gap between the answers, it is applied in larger dimensions.

4-1- Defuzzification

In many cases, supplying of items related to the field of suppliers is accompanied by uncertainty. In this model, the maximum amount of raw materials and products that can be provided by suppliers is considered as a fuzzy parameter. So, the parameters $\widetilde{cap_{n,s,m}^{supply}}$, which represents the maximum capacity of suppliers for supplying raw material, and $\widetilde{cap_{n,\iota,J}^{soutsource}}$, which represents the maximum capacity of outsourcing the final products, are considered as fuzzy parameters. However, according to previous information, a pessimistic, optimistic, and a most probable values for these parameters can be regarded as $\left[\widetilde{cap}_{n,s,m}^{p}, \widetilde{cap}_{n,s,m}^{m}, \widetilde{cap}_{n,s,m}^{o}\right] = [40,60,80]$ for $\widetilde{cap_{n,s,m}^{supply}}$ and $\left[\widetilde{cap}_{n,i,j}^{p}, \widetilde{cap}_{n,i,j}^{m}, \widetilde{cap}_{n,i,j}^{o}\right] = [40,60,80]$ for $\widetilde{cap_{n,\iota,J}^{soutsource}}$. So these parameters can be defuzzified by the method introduced by Lai and Hwang (1994). These parameters have been converted to a crisp manner in equation (49) and (50), and the related weights are regarded as $[w_1, w_2, w_3] = \left[\frac{1}{6}, \frac{2}{3}, \frac{1}{6}\right]$.

$$Crisp\left(\widetilde{cap_{n,s,m}^{supply}}\right) = \left([w_1 \times \widetilde{cap}_{n,s,m}^{p}] + [w_2 \times \widetilde{cap}_{n,s,m}^{m}] + [w_3 \times \widetilde{cap}_{n,s,m}^{o}]\right) = 60 \quad (49)$$

$$Crisp\left(\widetilde{cap_{n,\iota,J}^{soutsource}}\right) = \left([w_1 \times \widetilde{cap}_{n,i,j}^{p}] + [w_2 \times \widetilde{cap}_{n,i,j}^{m}] + [w_3 \times \widetilde{cap}_{n,i,j}^{o}]\right) = 60 \quad (50)$$

5- Numerical results

5-1- Model validation (small-size test problem)

In this example, two types of final products are examined in two classes for three customers. There are two types of transportation vehicles, along with five models of raw materials. There are also three different suppliers with various capacities to supply raw materials and products in this test problem. The distance between customers and the factory is 10, 15, and 50 units, respectively, and the capacity of the vehicles is 20 and 50 units of the final product, respectively. Other information is shown in table 5 and table 6.

Table. 5. The initial values of the input parameters

Parameter		Value	
$p_{i,j}$		j_1	j_2
	i_1	75	90
	i_2	120	135
$c_{i,j}^{ATP}$		j_1	j_2
	i_1	45	55
	i_2	70	80
$ncap_{n,b}^{vehicle}$		b_1	b_2
	n_1	3	1
	n_2	3	1
$\omega_{c,n}$		n_1	n_2
	c_1	1	1/5
	c_2	1	1/2
	c_3	1	1/25
$cvar_{i,j}^{production}$		j_1	j_2
	i_1	1	2
	i_2	2	3

Table. 6. The initial values of the input parameters

Parameter		Value			
		i_1, j_1	i_1, j_2	i_2, j_1	i_2, j_2
$h_{n,i,j}^{ATP}$	n_1	0/05	0/2	0/25	0/3
	n_2	0/1	0/15	0/35	0/6
		i_1, j_1	i_1, j_2	i_2, j_1	i_2, j_2
$C_{c,i,j}^{substitute}$	c_1	5	9	10	11
	c_2	6	8	12	10
	c_3	7	7	11	9
		i_1, j_1	i_1, j_2	i_2, j_1	i_2, j_2
$C_{i,j,s}^{outsource}$	s_1	55	70	85	120
	s_2	55	75	90	95
		i_1, j_1	i_1, j_2	i_2, j_1	i_2, j_2
$cap_{n,i,j}^{production}$	n_1	10	10	15	10
	n_2	15	15	5	15
		i_1, j_1	i_1, j_2	i_2, j_1	i_2, j_2
$cap_{n,i,j}^{outsource}$	n_1	15	0	10	30
	n_2	0	15	5	0

The augmented ε-constraint method has been used to solve this model in a small dimension (Mavrotas, 2009). The ε-Contrast method is an efficient solution method for solving multi-objective problems (Steuer, 1986). To implement this method, GAMS software along with CPLEX solver has been executed on an Intel Corei7 PC with 16 GB of RAM and over 2.5 GHz CPU. Other information about the optimal values of the variables and the final answer of each of the objective functions is presented in the following table 7 and table 8. The optimal solutions are also plotted in a Pareto diagram in figure 4. Since the problem has a set of optimal answers and a unique optimal solution cannot be selected for the problem, the value of the variables are considered for the optimal response in the row marked with * sign in table 8.

Table. 7. The optimal values of the variables

Variable	Value	Variable	Value
Z_1	2039/75	Z_2	78/1
F_1	3/4	U_1	64/5
F_2	900	U_2	11/6
F_3	600	U_3	2
F_6	1/75	F_7	0
F_8	788	F_9	4/5

Table. 8. The final answer of each objective function by the ε-Contrast method

Number	Z_1	Z_2
1*	*2039/75	*78/1
2	2026/55	77/7
3	2017/35	77/3
4	2010/95	76/9
5	1997/75	76/5
6	1993/25	76/1
7	1973/75	68/1
Average	2008/48	75/81

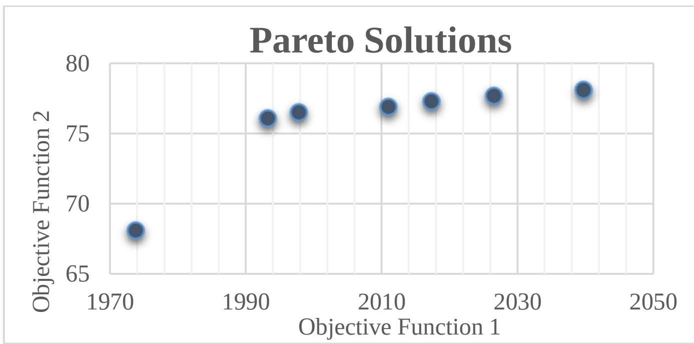

Fig. 4. The optimal solutions to the problem

The problem is solved without any error. It can be concluded that the model is feasible and the validation process has been performed correctly due to the rationality of the obtained answers and the existence of at least one feasible answer.

5-2- Integration of objective functions

For solving this model in larger dimensions, the LR method is implemented. The problem first is converted to a single-objective model. For this conversion, the weighted LP-Metric method has been used (Lupaş, Mache, & Müller, 1995). The proposed model is transformed into a single-objective one by the statements (51) and (52). Parameter $wieght_h$ represents the weight of each objective function, Z_h^* states the optimal value of each objective function. Parameter P represents the degree of importance of each objective function penalty, h indicates the objective function index, and Z^{LP} is the final integrated objective function of the model.

$$\min Z^{LP} = \sum_1^h (wieght_h \times (\frac{Z_h^* - Z_h}{Z_h^*}))^p \tag{51}$$

$$\min Z^{LP} = (wieght_1 \times \left(\frac{2039.75 - Z_1}{2039.75}\right)^1) + (wieght_2 \left(\frac{Z_2 - 68.1}{68.1}\right)^1) \tag{52}$$

The amount of Gap should be examined with the answer obtained from the example discussed in part 5.1. By changing the value of parameter $wieght_h$, different solutions to the problem may be obtained. The values of Z_1^{LP*}, and Z_2^{LP*} shows in table 9. The answers of table 8 and table 9 and the row of average solutions have been used to calculate the amount of Gap. According to the calculated Gap for the first and second objective functions (0% and 3%) in the statement (53) and (54), the LR method can be implemented with an average gap of 1.5%.

Table. 9. The final answers obtained by the weighted LP-metric method

Number	Z_1	Z_2
1	2039/75	68.1
2*	*1973/75	*78.1
Average	2006/75	73/1

$$Gap_1 = \left|\frac{Z_1^{LP*} - Z_1^*}{Z_1^*}\right| = \left|\frac{2006.75 - 2008.48}{2008.48}\right| = 0.0008 \cong 0\% \tag{53}$$

$$Gap_2 = \left|\frac{Z_2^{LP*} - Z_2^*}{Z_2^*}\right| = \left|\frac{73.1 - 75.81}{75.81}\right| = 0.0305 \cong 3\% \tag{54}$$

New solutions to the problem were obtained by implementing the LR solution method. After solving the problem, $Z_1^{LR*} = 1973.75$ and $Z_2^{LR*} = 74.5$ was obtained for the objective functions. According to the obtained answers, this method can be considered as a suitable solution method for this model. The average Gap is about 2.3% (0% and 4.6%) compared to previous answers. For this calculation, the row with * sign from table 9 is used.

5-3- Lagrangian relaxation algorithm

The Lagrangian Relaxation method was first proposed by Fisher (1981). Then, many developments were made on this solution method (Ongsakul & Petcharaks, 2004; Zhao, Luh, & Wang, 1999) and many problems such as railway timetabling problem (Brännlund, Lindberg, Nou, & Nilsson, 1998), and power-generation system scheduling problem (Muckstadt & Koenig, 1977) were optimized by this solution method.

5-3-1- Initialization and Algorithm Performance

Basically, this method decomposes a large problem into smaller sub-problems by relaxing complex constraints via Lagrangian multipliers (Hassannayebi et al., 2016). A Lagrangian multiplier (U_{iter}^{LR}) is considered for each of the relaxed constraints. This problem is solved in 100 iterations ($iter = 100$). At each stage, the Lagrangian multiplier changes by a certain amount ($stepsize$). By changing the Lagrangian multiplier in each iteration, a new bound is generated for the problem, which is stored in a parameter called Bound. The best bound is stored in a parameter called BestBound. At each iteration, if the value of Bound is better than the BestBound ($Bound \leq BestBound$), it replaces the previous BestBound. Also, by changing the coefficients of the objective functions ($wieght_h$) used in the weighted LP-metric method in section 5.2, different optimal solutions can be generated for the problem. The flowchart for solving the problem by LR method is illustrated in figure (5).

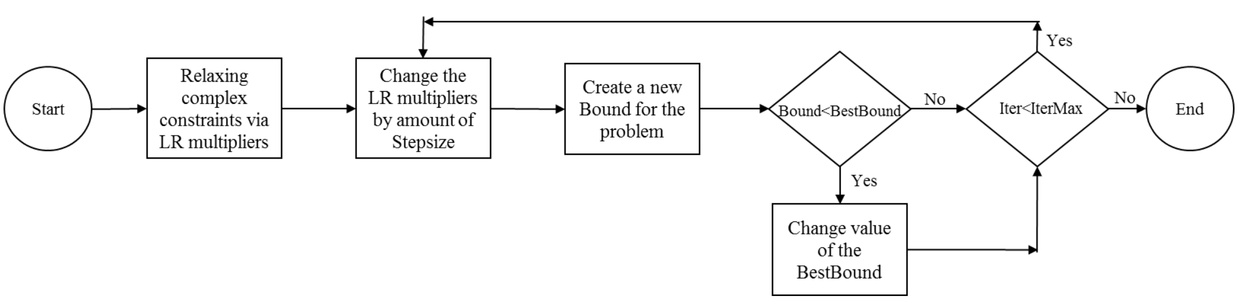

Fig. 5. The flowchart for solving the problem by LR method\

5-3-2- Medium-Size Test Problem

In this example, five customers are considered. Four different types of final products produced in three different classes are also considered for this example. Final products can be sent to customers in four periods and through three different types of vehicles. There are also eight types of raw materials for producing these products, which are supplied by four different suppliers. The cost of accepting orders equals $c^{accept} = 0.3$, and the cost of rejecting orders equals $c^{reject} = 0.2$. The values of some of the most important input parameters of the problem are written in table 10.

Table. 10. Values of some of the input parameters

Parameter	Value	Parameter	Value
$h_{n,i,j}^{ATP}$	Random [0/1,0/7]	$cap_{n,i,j}^{outsource}$	Random [0,30]
$C_{c,i,j}^{substitute}$	Random [1,20]	$cap_{m,s,m}^{supply}$	Random [0,50]
$cap_{n}^{delivery}$	Random [1,20]	$ncap_{n,b}^{vehicle}$	Random [10,50]
$cap_{n,i,j}^{production}$	Random [1,10]	$d_{c,i,j}^{max}$	Random [0,40]

The Lagrangian multiplier was changed at each stage to improve the generated answers. The optimal answers obtained by this algorithm are written in table 11. The quantitative changes in each objective function are illustrated in figures 6 and 7, respectively. The set of optimal solutions is depicted as a Pareto diagram in figure 8 as well.

Table. 11. The optimal answers obtained by the LR method

NO	Objective Function 1	Objective Function 2	NO	Objective Function 1	Objective Function 2
*1	*8480/85	*346/45	10	7133/7	281/05
2	8462/85	328/3	11	6216/55	271/45
3	8448/55	326/65	12	6188/85	269/8
4	8287/25	322/5	13	6169/45	268/7
5	8231/25	314/25	14	6131/95	266/7
6	8118/3	306/85	15	6045/15	262/3
7	7900	295	16	5512/6	254
8	7578/4	286/8	17	5414/8	250/75
9	7155/7	282/8	18	4178/1	248/25

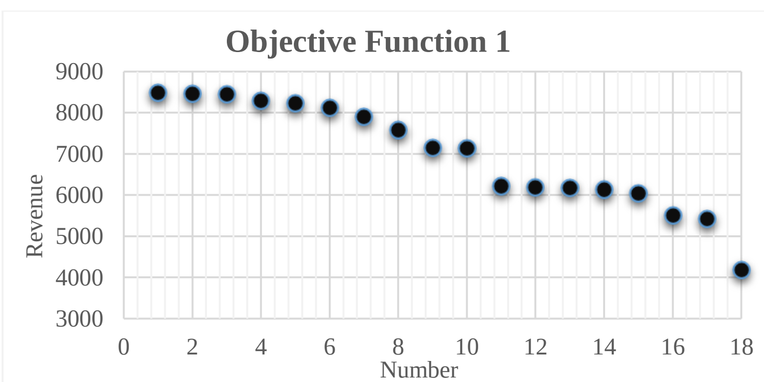

Fig. 6. Optimal answers obtained for the first objective function

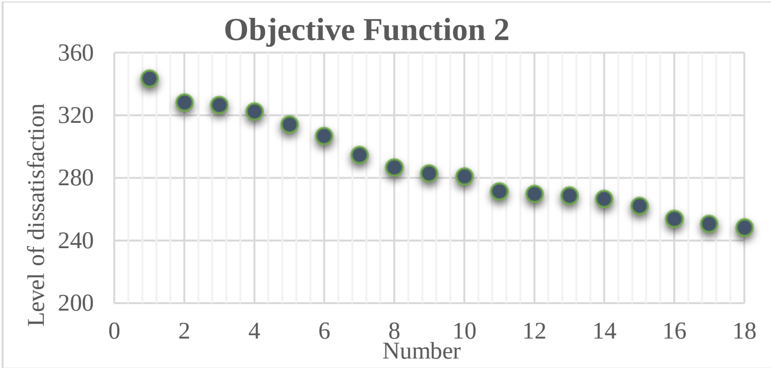

Fig. 7. Optimal answers obtained for the second objective function

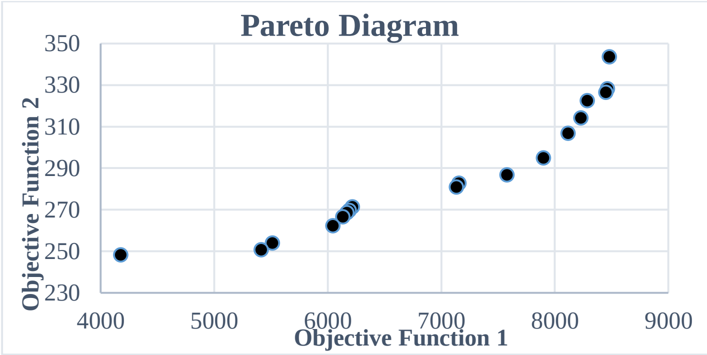

Fig. 8. Pareto diagram of optimal solutions

The first row, which is marked with a * sign, is used to study the variables among the solutions in table 11. As shown in table 12, the value of the first objective function is $Z_1 = 8480.85$. Among the variables that make up this objective function, the variables F_3 and F_8 have the highest value, and the variables F_7 and F_{11} have the lowest value, respectively. The effect of each of these variables on the variable Z_1 is shown in figure 9. The value of the second objective function is $Z_2 = 346.45$. Among the variables that make up this objective function, the variable U_1 has the highest value, and the variable U_2 has the lowest value. The effect of each of these variables on the variable Z_2 is shown in figure 10.

Table. 12. The value of some of the optimal variables of the problem

Variable	Value	Variable	Value
Z_1	8480/85	Z_2	346/45
F_1	7/45	F_8	4938
F_2	1570	F_9	59/4
F_3	4435	F_{10}	1056/5
F_4	217	F_{11}	0/9
F_5	9/8	U_1	237
F_6	100/1	U_2	35/5
F_7	0	U_3	73/95
G_1	20875	-	-

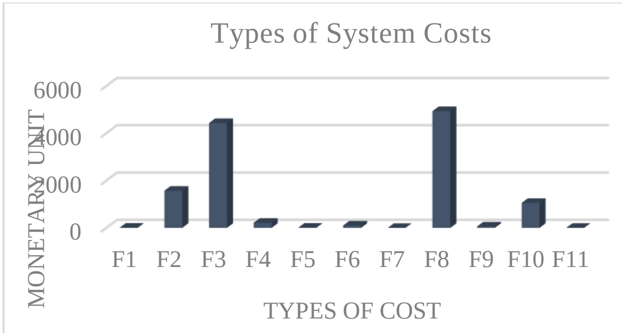

Fig. 9. Types of system costs

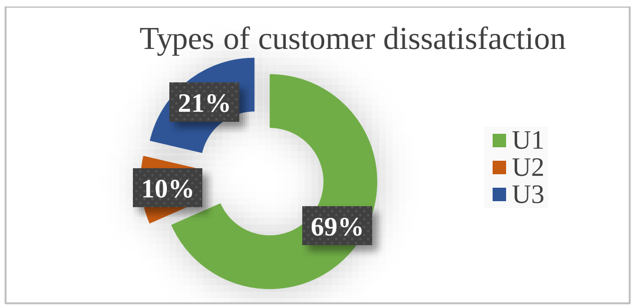

Fig. 10. Types of customer dissatisfaction

In this problem, 48 orders have been sent to the manufacturer, of which approximately 23% of the orders have been accepted. The rest of the orders (about 77% of the orders) have been rejected. The total number of products that must be produced by the company is 175. Among these products, 99 products of these orders (57%) have been delivered to customers, and 76 products (43%) have been delivered to customers as substitute products. The evaluation of the manufacturer's performance against the received orders from customers can be seen in table 13.

Table. 13. The manufacturer's performance against received orders

Variable	Value	Segment	Value	Percent %
Total Number of Orders	48	Accepted Orders	11	22/92
		Rejected Orders	37	77/08
Number of Products Needed to Satisfied	175	Satisfied with Exact Products	99	56/57
		Satisfied with substitute Products	76	43/43

As expected, satisfying the orders of the main customers is a higher priority for the company. According to table 14, it can be seen that out of 77% of the orders that were rejected, about 42% are related to the cloud customers, only about 35% are related to the main ones.

Table. 14. Comparing the status of responding to customer orders

Type/Status	Accepted %	Rejected %
Main Customers	13	35
Cloud Customers	10	42
Total	23	77

5-3-3- Case study

A case study in an LED TV company is studied to investigate the applicability of the proposed model. This company is a manufacturer of various LED TVs in different quality and sizes. The company assembles the TVs after supplying the required raw materials from suppliers. Table 15 shows the final products, along with their total demand and price.

Table. 15. Products information

Type	Class (Inches)	Price ($)	Total Demand
LED (HD)	55"	350	430
	50"	330	430
	43"	300	410
	32"	240	320
LED (Full HD)	55"	460	460
	50"	410	490
	43"	390	430
	32"	360	360
LED (4K)	55"	600	530
	50"	520	480
	43"	480	540
	32"	450	340

This company is located in Isfahan province in Iran and has a number of sales representatives in some cities, each of which is considered as a customer for the company. Each of these sales representatives has a certain amount of demand for each product, which is provided by the manufacturer. Product substitution and outsourcing are also possible for the company. Other information about customers is written in table 16.

Table. 16. Customer information

NO	Sales Representative	Distance (KM)	Type
1	West Azerbaijan	480	Main
2	Kermanshah	290	Cloud
3	Tehran	205	Main
4	North Khorasan	480	Cloud
5	Shiraz	210	Main
6	Hormozgan	425	Cloud
7	Sorth Khorasan	400	Cloud
8	Sistan and Baluchestan	680	Main
9	Kerman	320	Cloud
10	Mazandaran	270	Cloud
11	Yazd	180	Cloud
12	Khuzestan	220	Cloud
13	Gilan	360	Cloud

In this problem, 13 sales representatives have been considered. The customer's location is shown in figure 11. As shown in this figure, the products are shipped to each customer in different periods. Also, ten models of different raw materials are needed to produce these products, which are supplied by five suppliers. Three products are produced in four groups in this company, which are delivered to customers through five types of vehicles, and each period is considered equivalent to five days (one working week). Other assumptions of the problem are also considered as in section 5.3.2.

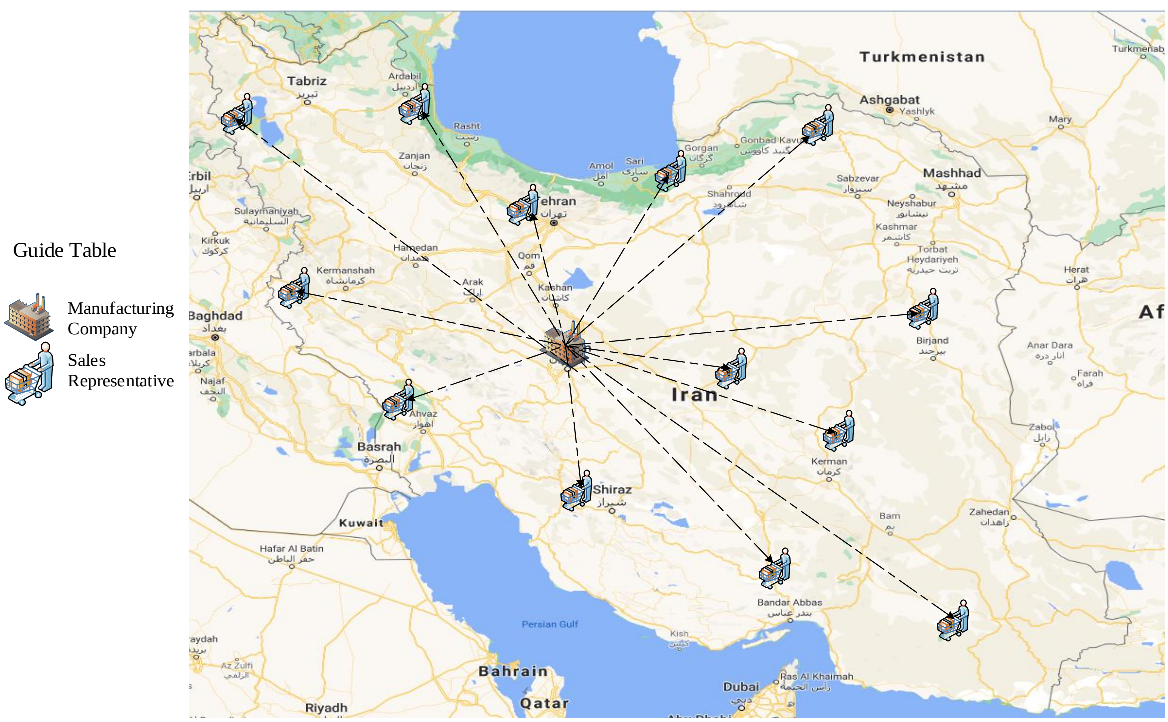

Fig. 11. Location and distance of different customers

After solving the problem by LR solution method, the optimal answers are shown in table 17. A Pareto diagram of the optimal answers can also be seen in figure 12. According to the obtained answers, this problem can be considered profitable.

Table. 17. Optimal answers obtained by the LR method

NO	Objective Function 1	Objective Function 2	NO	Objective Function 1	Objective Function 2
1	161712/9	2211/8	7	156290/2	2015/6
2	159975/7	2178/7	8	152394/1	1980/7
3	159307/6	2131/1	9	145143/7	1777/6
4	157937/3	2108/8	10	144271/2	1750/4
5	157483/7	2072/4	11	139626/2	1717/2
6	156770/5	2031/8	12	137821/0	1684/0

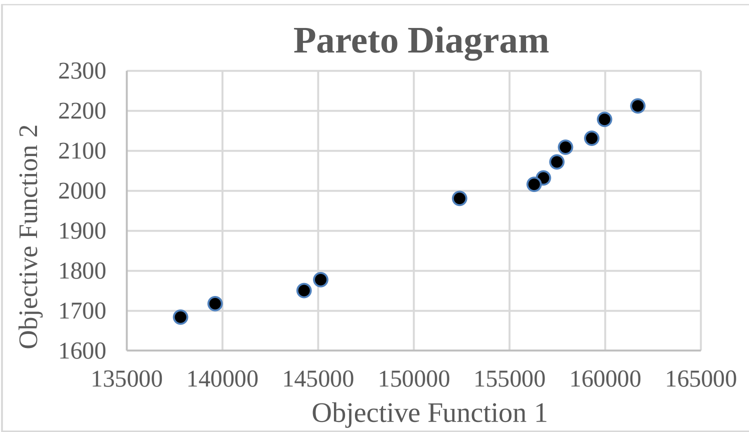

Fig. 12. Pareto diagram of optimal answers

6-Sensitivity analysis

As is clear, increasing or decreasing some of the parameters of the system affects the final answer. Still, some of these parameters have a more significant impact on the final answer. Therefore, it seems necessary to study these changes.

6-1- Solving-time analysis

Among the indices, the index of the number of customers (c) has the most significant impact on the problem-solving time. As the number of customers increases, the solving-time of the problem increases to a large extent. Changes in this index have a more significant effect on other variables than other indices. The change rate of the problem-solving time is examined compared to the original problem-solving time (row marked with a * sign) in table 18. As shown in figure 13, increasing the number of customers impacts problem-solving time significantly. For example, a 50% increase in the number of customers (from 4 to 6) leads to a 495% increase in the solving time. Therefore, it can be concluded from these analyzes that the number of customers is a critical factor for solving-time of the model.

Table. 18. Solving time changes versus the number of customers

Test Problem	Number of Customers	Solving Time (sec)	Change Rate %
1	2	220	10/62
2	3	420	20/28
3	4	1456	70/30
*4	*5	*2071	-
5	6	3705	178/90
6	7	10260	495/41
7	8	21602	1043/07

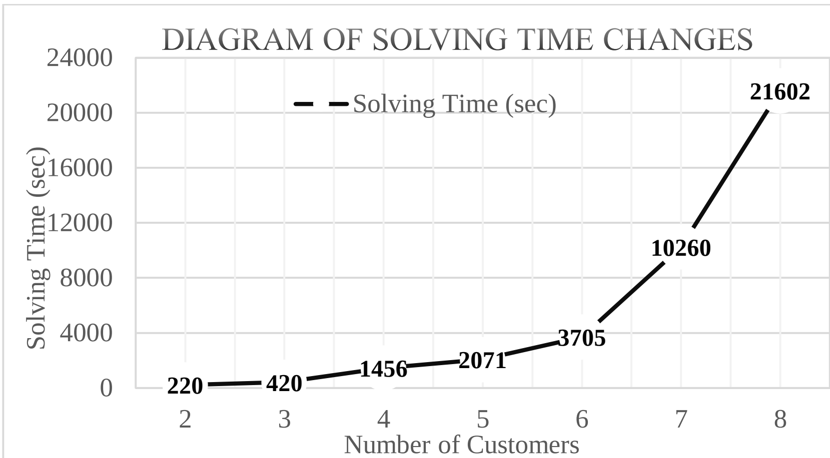

Fig. 13. Diagram of problem-solving time changes

6-2- Objective function analysis

In this section, the generated solutions for each of the objective functions are examined. As is evident in figures 14 and 15, the diagrams of the answers obtained for the objective functions are plotted separately. In these figures, it is assumed that these solutions follow a normal distribution. In this case, the shape of the second objective function is more similar to a normal distribution. Because the points on the graph have less deviation with the hypothetical line drawn. In these shapes, each breakpoint is an optimal answer. Considering the waypoints are broken on the normal line, it can be seen that the degree of uniformity and regularity in the generated answers for the second objective function is higher than the first objective function.

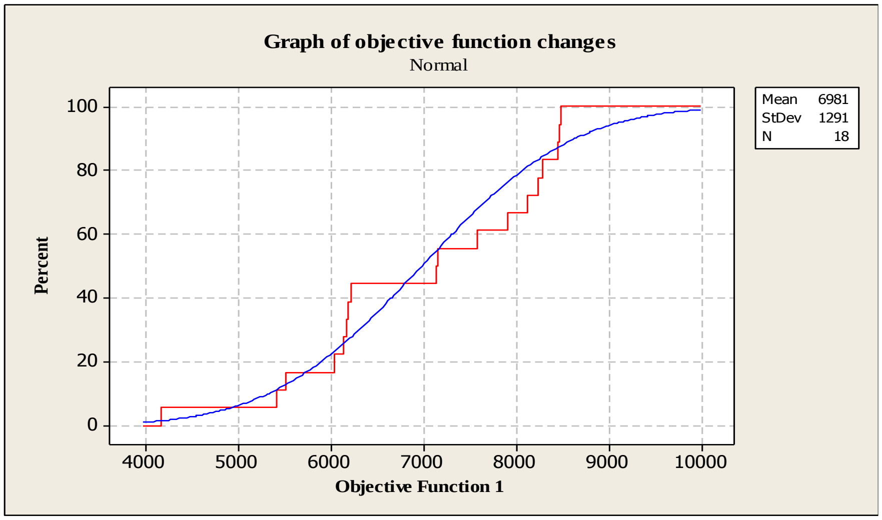

Fig. 14. The answers obtained for the first objective function

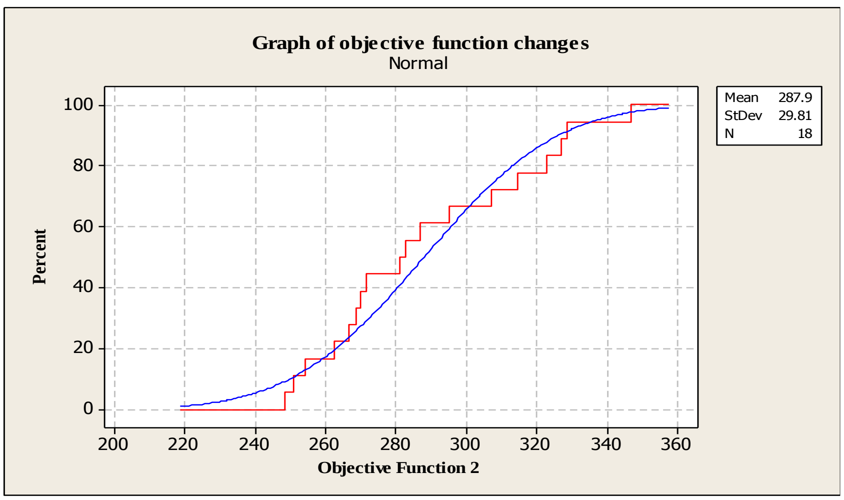

Fig. 15. The answers obtained for the second objective function

6-3- Demand analysis

The revenue of this system is obtained from the sale of products to customers. Therefore, in this model, the demand of customers ($d_{c,i,j}^{max}$) is one of the most vital input parameters and directly affects system revenue. Table 19 examines the impact of changes in customer demand on optimal solutions. As shown in this table, the minimum and maximum values of each of the objective functions change with the variation in demand. With the changes in the demand parameter, the range of changes between the upper and lower limits of the optimal answers has changed by 9.49% (from 51.24% to 41.74%) in the first objective function (Z_1). In the second objective function (Z_2), the range of changes between the upper and the lower limit of the optimal answers has changed by 11.42% (from 33.53% to 22.11%). Therefore, it can be concluded that the second objective function is more sensitive to demand changes. For this reason, if customer demand is uncertain, the upper and lower limits of the second objective function will experience broader changes. A comparison of the changes for each of the objective functions is plotted in figure 16.

Table. 19. Impact of demand changes on the objective functions

Test Problem	Demand Value	Min Value (Z1)	Max Value (Z1)	Change Percentage % (Z1)	Min Value (Z2)	Max Value (Z2)	Change Percentage % (Z2)
1	Random[0,20]	3460/35	7096/6	51/24	159/3	239/65	33/53
2	Random[0,40]	4178/1	8480/85	50/73	248/25	346/45	28/34
3	Random[0,60]	4732/3	8960/65	47/19	417/6	547/45	23/72
4	Random[0,80]	5495/3	9433/1	41/74	504/6	647/8	22/11

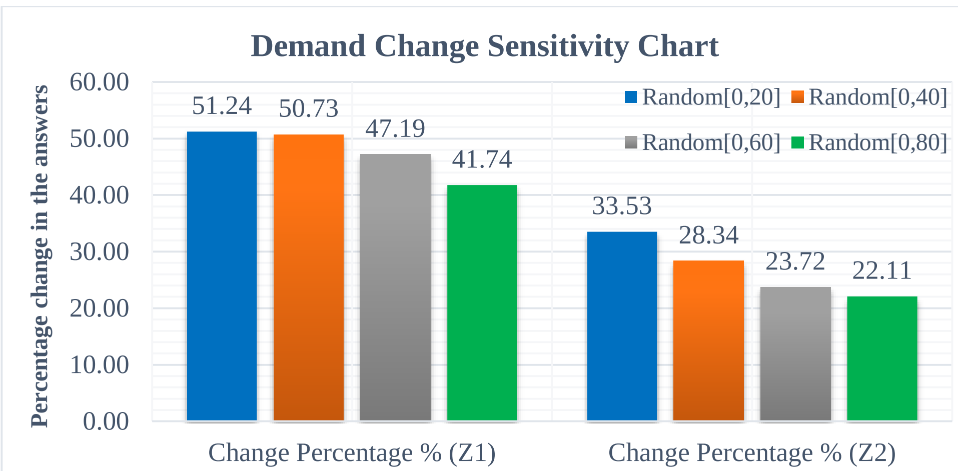

Fig. 16. Demand change sensitivity chart

7- Discussion and managerial insights

Several suggestions can be made to make the system more efficient. Due to the existence of uncertain factors among the input parameters of the problem, it is better for the decision-makers to have background studies about the effects of the occurrence and non-occurrence of each of these uncertainties. It is also recommended that before entering this market, a study on competitors in the market to be done. In a competitive market, where many companies are producing these products, owners should choose a competitive strategy for the company. In this situation, it is necessary to consider a high priority for customer satisfaction rather than financial goals. But if the market conditions are monopolistic and strong competitors in the market cannot be imagined for the mentioned company, it is better to put financial goals in a higher priority. Because, even with customer dissatisfaction, the customer is forced to satisfy their orders, and there is no other manufacturer for this purpose.

The company tends to satisfy the demands that will bring more profit as well as spending fewer resources. Satisfying demands that have easier conditions for receiving their orders will be a priority for the manufacturer. Also, due to the high percentage of the number of rejected orders compared to the number of accepted orders (77% vs. 23%), one of the suggestions that can be offered is to increase the production capacity. Increasing the production capacity of machinery may be a great initial cost for the company, but it can be profitable in the future.

8- Conclusion and future study

Order acceptance is one of the most challenging items that decision-makers may face in a manufacturing company. Increased competition and the expansion of production problems caused by increasing the variety of products and customer demands has led companies to optimize production lines. In particular, many production problems are due to the poor performance of suppliers. Meanwhile, factors such as increasing the company's profit, reduction in costs, and keeping customers satisfied are some crucial issues for company managers. In this paper, a MILP mathematical model is designed for a Hybrid MTS-MTO Production System with product substitution and optimized by the LR solution method. After optimization, the sensitivity of some parameters of the model was analyzed, and their effect on the problem-solving process was investigated. Finally, a case study was conducted to investigate the applicability of the model. It can be concluded that the possibility of products substitution has reduced rejected orders and increased system profits. The manufacturer is more inclined to work with reliable suppliers. The number of customers and orders received has a great impact on the problem solving time. Main customers with more orders were more important to the system. Also, the similarity of some raw materials adds to the complexity of the problem. As a future study, other modes of uncertainty can be considered for this model. Other heuristic or exact solution methods, such as Banders Decomposition or Branch and Cut, can be implemented to solve this model. Different pricing methods can also be considered for the selling of products.

References

Abedi, A., & Zhu, W. (2020). An advanced order acceptance model for hybrid production strategy. *Journal of manufacturing systems, 55*, 82-93.

Afshar-Bakeshloo, M., Flapper, S. D. P., Jolai, F., & Bozorgi-Amiri, A. (2021). A heuristic substitutions policy to control inventories for a hybrid manufacturing/remanufacturing system with product substitutions between three markets considering customers' behavior and remanufacturing limitations. *Journal of Cleaner Production*, 127871.

Aghaei, J., Amjady, N., & Shayanfar, H. A. (2011). Multi-objective electricity market clearing considering dynamic security by lexicographic optimization and augmented epsilon constraint method. *Applied Soft Computing, 11*(4), 3846-3858.

Agra, A., Poss, M., & Santos, M. (2018). Optimizing make-to-stock policies through a robust lot-sizing model. *International journal of production economics, 200*, 302-310.

Ahiska, S. S., Gocer, F., & King, R. E. (2017). Heuristic inventory policies for a hybrid manufacturing/remanufacturing system with product substitution. *Computers & Industrial Engineering, 114*, 206-222.

Akbari, A. (2020). *The Simultaneous Lot-sizing and Scheduling Problem in Process Industries Using Hybrid MTS-MTO Production Systems: An Exploratory Case Study.* NTNU.

Alemany Díaz, M. d. M., Alarcón Valero, F., Oltra Badenes, R. F., & Lario Esteban, F. C. (2013). Reasignación óptima del inventario a pedidos en empresas cerámicas caracterizadas por la falta de homogeneidad en el producto (FHP). *Boletín de la Sociedad Española de Cerámica y Vidrio, 52*(1), 31-41.

Aslan, B., Stevenson, M., & Hendry, L. C. (2015). The applicability and impact of Enterprise Resource Planning (ERP) systems: Results from a mixed method study on Make-To-Order (MTO) companies. *Computers in Industry, 70*, 127-143.

Beemsterboer, B., Land, M., & Teunter, R. (2016). Hybrid MTO-MTS production planning: An explorative study. *European Journal of Operational Research, 248*(2), 453-461.

Beemsterboer, B., Land, M., & Teunter, R. (2017). Flexible lot sizing in hybrid make-to-order/make-to-stock production planning. *European Journal of Operational Research, 260*(3), 1014-1023.

Bortolini, M., Faccio, M., Gamberi, M., & Pilati, F. (2019). MTO/MTS policy optimization for sheet metal plate parts in an ATO environment. *Procedia Cirp, 81*, 1046-1051.

Brännlund, U., Lindberg, P. O., Nou, A., & Nilsson, J.-E. (1998). Railway timetabling using Lagrangian relaxation. *Transportation science, 32*(4), 358-369.

Cannas, V. G., Gosling, J., Pero, M., & Rossi, T. (2020). Determinants for order-fulfilment strategies in engineer-to-order companies: Insights from the machinery industry. *International journal of production economics*, 107743.

ElHafsi, M., Fang, J., & Hamouda, E. (2020). A novel decomposition-based method for solving general-product structure assemble-to-order systems. *European Journal of Operational Research*.

Ellabban, A., & Abdelmaguid, T. (2019). *Optimized Production Control Policy for Hybrid MTS-MTO Glass Tube Manufacturing Using Simulation-Based Optimization.* Paper presented at the 2019 8th International Conference on Industrial Technology and Management (ICITM).

Fathollahi-Fard, A. M., Hajiaghaei-Keshteli, M., Tian, G., & Li, Z. (2020). An adaptive Lagrangian relaxation-based algorithm for a coordinated water supply and wastewater collection network design problem. *Information Sciences, 512*, 1335-1359.

Fisher, M. L. (1981). The Lagrangian relaxation method for solving integer programming problems. *Management science, 27*(1), 1-18.

Geoffrion, A. M. (1974). Lagrangean relaxation for integer programming *Approaches to integer programming* (pp. 82-114): Springer.

Ghalehkhondabi, I., & Suer, G. (2018). Production line performance analysis within a MTS/MTO manufacturing framework: a queueing theory approach. *Production, 28*.

Hassannayebi, E., Zegordi, S. H., & Yaghini, M. (2016). Train timetabling for an urban rail transit line using a Lagrangian relaxation approach. *Applied Mathematical Modelling, 40*(23-24), 9892-9913.

Jalali, M. S., Ghomi, S. F., & Rabbani, M. (2020). A System Dynamics Approach Towards Analysis of Hybrid Make-to-Stock/Make-to-Order Production Systems. *Industrial Engineering & Management Systems, 19*(1), 143-163.

Jansen, S., Atan, Z., Adan, I., & de Kok, T. (2019). Setting optimal planned leadtimes in configure-to-order assembly systems. *European Journal of Operational Research, 273*(2), 585-595.

Jia, Y., Weng, W., & Fujimura, S. (2017). *A hybrid MTS-MTO production model with a dynamic decoupling point for flexible flow shops.* Paper presented at the 2017 IEEE/ACIS 16th International Conference on Computer and Information Science (ICIS).

Jing, F., & Mu, Y. (2019). Forecast horizon for dynamic lot sizing model under product substitution and perishable inventories. *Computers & Operations Research, 110*, 77-87.

Kalantari, M., Rabbani, M., & Ebadian, M. (2011). A decision support system for order acceptance/rejection in hybrid MTS/MTO production systems. *Applied Mathematical Modelling, 35*(3), 1363-1377.

Khalilabadi, S. M. G., Zegordi, S. H., & Nikbakhsh, E. (2020). A multi-stage stochastic programming approach for supply chain risk mitigation via product substitution. *Computers & Industrial Engineering, 149*, 106786.

Kim, E., & Min, D. (2021). A two-stage hybrid manufacturing model with controllable make-to-order production rates. *Journal of manufacturing systems, 60*, 676-691.

Kloos, K., & Pibernik, R. (2020). Allocation planning under service-level contracts. *European Journal of Operational Research, 280*(1), 203-218.

Lai, Y.-J., & Hwang, C.-L. (1994). Fuzzy multiple objective decision making *Fuzzy Multiple Objective Decision Making* (pp. 139-262): Springer.

Lang, J. C. (2009). *Production and inventory management with substitutions* (Vol. 636): Springer Science & Business Media.

Laurikainen, A. (2020). ANALYZING AND DEVELOPING THE ORDER FULFILLMENT PROCESS IN MAKE TO ORDER BUSINESS.

Li, Q., He, Q.-M., & Wu, X. (2016). Timing order fulfillment of capital goods under a constrained capacity. *Annals of Operations Research, 241*(1-2), 431-456.

Lupaş, A., Mache, D. H., & Müller, M. W. (1995). Weighted L p-approximation of derivatives by the method of Gammaoperators. *Results in Mathematics, 28*(3-4), 277-286.

Mavrotas, G. (2009). Effective implementation of the ε-constraint method in multi-objective mathematical programming problems. *Applied mathematics and computation, 213*(2), 455-465.

Mello, M. H., Gosling, J., Naim, M. M., Strandhagen, J. O., & Brett, P. O. (2017). Improving coordination in an engineer-to-order supply chain using a soft systems approach. *Production Planning & Control, 28*(2), 89-107.

Muckstadt, J. A., & Koenig, S. A. (1977). An application of Lagrangian relaxation to scheduling in power-generation systems. *Operations Research, 25*(3), 387-403.

Naderi, B., & Roshanaei, V. (2019). Branch-Relax-and-Check: A tractable decomposition method for order acceptance and identical parallel machine scheduling. *European Journal of Operational Research*.

Nguyen, S. (2016). A learning and optimizing system for order acceptance and scheduling. *The International Journal of Advanced Manufacturing Technology, 86*(5), 2021-2036.

Noroozi, A., Mazdeh, M. M., Heydari, M., & Rasti-Barzoki, M. (2018). Coordinating order acceptance and integrated production-distribution scheduling with batch delivery considering Third Party Logistics distribution. *Journal of manufacturing systems, 46*, 29-45.

Ongsakul, W., & Petcharaks, N. (2004). Unit commitment by enhanced adaptive Lagrangian relaxation. *IEEE Transactions on Power Systems, 19*(1), 620-628.

Peeters, K., & van Ooijen, H. (2020). Hybrid make-to-stock and make-to-order systems: a taxonomic review. *International Journal of Production Research, 58*(15), 4659-4688.

Pibernik, R. (2005). Advanced available-to-promise: Classification, selected methods and requirements for operations and inventory management. *International journal of production economics, 93*, 239-252.

Rafiei, H., Rabbani, M., & Alimardani, M. (2013). Novel bi-level hierarchical production planning in hybrid MTS/MTO production contexts. *International Journal of Production Research, 51*(5), 1331-1346.

Rafiei, H., Rabbani, M., Vafa-Arani, H., & Bodaghi, G. (2017). Production-inventory analysis of single-station parallel machine make-to-stock/make-to-order system with random demands and lead times. *International Journal of Management Science and Engineering Management, 12*(1), 33-44.

Raturi, A. S., Meredith, J. R., McCutcheon, D. M., & Camm, J. D. (1990). Coping with the build-to-forecast environment. *Journal of Operations Management, 9*(2), 230-249.

Sarvestani, H. K., Zadeh, A., Seyfi, M., & Rasti-Barzoki, M. (2019). Integrated order acceptance and supply chain scheduling problem with supplier selection and due date assignment. *Applied Soft Computing, 75*, 72-83.

Shabtay, D., Gaspar, N., & Kaspi, M. (2013). A survey on offline scheduling with rejection. *Journal of scheduling, 16*(1), 3-28.

Shapiro, B. P., Rangan, V. K., Sviokla, J. J., Paul, & Meisel, i. (1992). *Staple yourself to an order*: Harvard Business Review.

Slotnick, S. A. (2011). Order acceptance and scheduling: A taxonomy and review. *European Journal of Operational Research, 212*(1), 1-11.

Steuer, R. E. (1986). Multiple criteria optimization. *Theory, computation and applications*.

Tapia-Leon, R., Vega-Neyra, X., Chavez-Soriano, P., & Ramos-Palomino, E. (2019). *Improving the Order Fulfillment Process in a Textile Company using Lean Tools.* Paper presented at the 2019 Congreso Internacional de Innovación y Tendencias en Ingenieria (CONIITI).

Tsao, Y.-C., Raj, P. V. R. P., & Yu, V. (2019). Product substitution in different weights and brands considering customer segmentation and panic buying behavior. *Industrial Marketing Management, 77*, 209-220.

Van Roy, T. J. (1983). Cross decomposition for mixed integer programming. *Mathematical programming, 25*(1), 46-63.

Wang, Z., Qi, Y., Cui, H., & Zhang, J. (2019). A hybrid algorithm for order acceptance and scheduling problem in make-to-stock/make-to-order industries. *Computers & Industrial Engineering, 127*, 841-852.

Xiong, S., Feng, Y., & Huang, K. (2020). Optimal MTS and MTO Hybrid Production System for a Single Product Under the Cap-And-Trade Environment. *Sustainability, 12*(6), 2426.

Yano, S., Nagasawa, K., Morikawa, K., & Takahashi, K. (2019). A Dynamic Switching Policy with Thresholds of Inventory Level and Waiting Orders for MTS/MTO Hybrid Production Systems. *Procedia Manufacturing, 39*, 1076-1081.

Yousefnejad, H., & Esmaeili, M. (2020). Tactical production planning in a hybrid MTS/MTO system using Stackelberg game. *Operational Research, 20*(3), 1791-1809.

Zhang, Z., Guo, C., Wei, Q., Guo, Z., & Gao, L. (2021). A bi-objective stochastic order planning problem in make-to-order multi-site textile manufacturing. *Computers & Industrial Engineering, 158*, 107367.

Zhao, X., Luh, P. B., & Wang, J. (1999). Surrogate gradient algorithm for Lagrangian relaxation. *Journal of optimization Theory and Applications, 100*(3), 699-712.

Zheng, J.-N., Chien, C.-F., & Wu, J.-Z. (2018). Multi-objective demand fulfillment problem for solar cell industry. *Computers & Industrial Engineering, 125*, 688-694.