Journal of Industrial and Systems Engineering
Vol. 9, special issue on location allocation and hub modeling, pp 40-52
Winter (February) 2016

A Multi-objective continuous covering location model

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Abstract

This paper presents a multi-objective continuous covering location problem in fuzzy environment. Because the covering radius is assumed to be uncertain, this paper uses the possibility concept. Since uncertainty may cause risk of uncovering customers, the problem is formulated as a risk management model. The presented model is an extension of the discrete covering location model to continuous space. Two variables, namely selecting zone variable and covering variable are introduced for extending the discrete model to the continuous one. In the model, a facility is located in a zone with a predetermined radius from its center and is determined by the selecting zone variable. Allocating a customer to a facility is shown by a covering variable. Also, the paper introduces the possibility of covering, based on distance between the customers and the facilities. Two objectives are considered in the model; the first is the possibility of covering by each facility and the second is the risk cost of the uncovered customers. Fuzzy programming is applied for converting the model to a single objective one. Finally, a numerical example with sensitivity analysis is expressed to illustrate the presented model.

Keywords: continuous covering location problem (CCLP); risk management; fuzzy covering radius; multi objective problem.

1- Introduction

The covering problem aims to locate a set of new facilities in a manner that the customers can receive service by each facility that its distance to customer is equal or less than a predefined value. This critical value is called coverage. Church and ReVelle (1974) are the one of the first researchers that modeled the maximization covering problem. The covering problem is divided into two problems; the total covering and the partial covering based on covering all or some of the demand points. The total covering problem is modeled by Toregas (1971). Up to the present time many developments have been occurred about the total covering and the partial covering problems in solution techniques and assumptions. The covering problem has many applications such as: the design of switching circuits, data retrieving, assembly line balancing, airline staff scheduling, locating defend networks, distributing products, warehouse locating and location of emergency service facility (Francis et al. 1992).Some researchers such as Church and ReVelle (1974), Schilling et al. (1993), Owen and Daskin (1998) and Drezner and Wesolowsky (1999) investigated network covering problems.

The total covering problem cannot cover all location problems in real world, because in many problems budget constraint and other constraints do not let us cover all the points and there is a risk of having unsatisfied customers. In this paper the partial covering model is investigated.

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ISSN: 1735-8272,

Mirchandani and Francis (1990) provided a covering model in discrete space. Assuming that there are n demand points indexed by I, and k candidate locating points indexed by j and P_i is the penalty of not covering demand point i, the model P_1 is written as the following.

$$P_1 : min \sum_{i=1}^{n} P_i W_i \tag{1}$$

st:

$$\sum_{j=1}^{k} a_{ji} V_j + W_i \geq 1, \qquad \forall i = 1,2, \dots, n \tag{2}$$

$$W_i \in \{0,1\}, V_j \in \{0,1\}, \qquad \forall i = 1,2, \dots, n \;\; and \; \forall j = 1,2, \dots, k$$

Where,

a_{ji}: is *1* if the candidate locating point *j* can cover the demand point *i* , otherwise is *0*
(a_{ji}'s constitute the covering matrix)
V_j: is *1* if a facility is located at the candidate locating point *j*, otherwise is *0*
W_i: is *1* if the demand point *i* is not satisfied, otherwise is *0*

Equation (1) is the objective function consisting of the penalty costs. Constraint set (2) guarantees that W_i is *1* if $a_{ji}V_j$ is zero, it means that the demand point *i* is not covered. The above model and other related covering location models have been investigated only in discrete space; while there are situations that might occur in continuous space. In this paper, we introduce a continuous covering location problem and for more adopting on real world, we consider the model in uncertain conditions by considering the fuzzy covering radius and providing the final problem as a risk management model. We are interested in finding location of k facilities in continuous space in order to serve the customers at *n* demand points so that the total cost of uncovered customers is minimized and the possibility of coverage by each facility is maximized.

Basic information underlying a facility location problem includes demand levels, travel time or cost for supplying the customers, location of the customers, presenceor absence of the customers, and price for the commodities. Uncertainty may occur in one or several of these parameters. In this condition, we make a decision under risk and we can apply methods for dealing with the problem (Laporte *et al.*, 2015). Investigating risk is one of the main topics in location models. The main factors which lead to risk could be categorized as uncertain parameters such as production, demand, supplies, processing, transportation, inventory, capacity, cost, interest rate and etc. Robust optimization, stochastic programming, chance-constrained models and fuzzy approaches are applied for considering uncertainty in location models.

The reminder of the paper is organized as follows; in section 2 a literature review about the covering location model in fuzzy environmentand risk management in the location models are provided. In section 3 we present Continuous Covering Location Problem (*CCLP*). The final multiobjective model and fuzzy programming for converting the multiobjective model to the single objective one is provided in section 4. In section 5 and section 6 a numerical example with sensitivity analysis is given to illustrate the usability of the presented model. Finally, Section 7 draws the conclusions and future works.

2- Literature Review

Several researchers have investigated the covering location model in fuzzy environment. Li *et al.* (2002) considered two fuzzy versions of the well-known problem of determining the smallest circle that would cover a given finite set of points in the plane when the locations of points are not precise but fuzzy. The first was modeled as a possibility-constrained mathematical program and the second, as a necessity-constrained one. Berman *et al.* (2003) considered the concept of a gradual coverage by introducing two distances on a network. A demand point is fully covered if it is within the lower distance, not covered at all if it is beyond the larger distance, and, if its distance is between the two distances, a level of coverage is determined using a decay function. Perez *et al.* (2004) claimed that in real applications facility locations can be full of linguistic vagueness that can be appropriately modeled using networks with the fuzzy values which describe nodes. Chiang *et al.* (2005) developed the fuzzy set-covering model using auxiliary 0-1. Chiang *et al.* (2005) proposed a set-covering model using the concept of fuzzy set theory to define fuzzy covers. Huang *et al.* (2006) analyzed a linear feature covering problem with distance constraints, and characterized the problem by a fuzzy multi objective optimization model. Shavandi and Mahlooji (2006) utilized fuzzy theory to develop a queuing maximal covering location–allocation. Araz *et al.* (2007) considered a multi-objective maximal covering location model. The model addresses the issue of determining the best base locations for a limited number of vehicles so that the service level objectives are optimized. The objectives of the model are maximization of the population covered by one vehicle, maximization of the population with backup coverage and minimization of the total travel distance in locations more distant than a prespecified distance standard for all zones. Ni (2008) considered the edge covering problem under fuzzy environment, and formulated three models which are expected minimum weight edge cover model, α-minimum weight edge cover model, and the most minimum weight edge cover model. Batanovic *et al.* (2009) investigated a class of maximum covering location problems in networks in uncertain environments. They assumed that relative weights of demand nodes are either deterministic or imprecise. Sirbiladze *et al.*(2009)introduced a new criterion for minimal fuzzy covering problems, which is the minimal value of the average misbelieve contained in the possible alternatives.

Several researches have been concentrated on considering risk in facility location problem. Guillen *et al.* (2005) considered the design and retrofit problem of a supply chain consisting of several production plants, warehouses, markets and the associated distribution systems. They constructed a two-stage stochastic model in order to take the effects of the uncertainty in the production scenariointo account. Snyder *et al.* (2007) proposed a stochastic version of the location model with risk pooling which optimizes location, inventory and allocation decisions under random parameters described by discrete scenarios. The goal of their model was to find solutions to minimize the expected total cost of the system among all scenarios. They presented a Lagrangian relaxation based exact algorithm for the model. Ozsen *et al.* (2008) introduced the capacitated warehouse location model with risk pooling. The model provided a logistics system in which a single plant shipped one type of product to a set of retailers, each with an uncertain demand. Also, the model was solved by a Lagrangian relaxation solution algorithm. Azaron *et al.* (2008) developed a multi-objective stochastic programming approach for supply chain design under uncertainty. Demands, supplies, processing, transportation, shortage and capacity expansion costs were all considered as uncertain parameters. They used the goal attainment technique to obtain the Pareto-optimal solutions. Afterwards, Wagner *et al.* (2009) considered a location-optimization problem where the classical incapacitated facility location model was recast in a stochastic environment with several risk factors that made demand at each customer site probabilistic and correlated with demands at the other customer sites. They considered “Value-at-Risk” (VaR) measure and designed a branch-and-bound algorithm to solve the problem. You *et al.* (2009) proposed a two-stage stochastic linear programming approach within a multi-period planning model. Furthermore, they developed an algorithm based on the multi-cut L-shaped method in order to solve the resulting large scale industrial size problems. Mete and Zabinsky (2010) developed a stochastic optimization approach for the problem of storage and distribution of medical supplies to be used for disaster management under a wide variety of possible disaster types and magnitudes. Cui *et al.* (2010) investigated reliable facility location models considering unexpected failures with site dependent probabilities, as well as possible customer reassignment. They proposed a compact mixed integer program formulation which was solved using a custom-designed Lagrangian relaxation algorithm. Liu et al. (2010) presented a location model that assigns online demands to the capacitated regional warehouses currently serving in-store demands in a multi-channel supply chain. The model explicitly considered the trade-off between the risk pooling effect and the transportation cost in a two-echelon inventory/logistics system. They formulated the assignment problem as a non-linear integer programming model. A strategic supply chain management problem was studied by Peng et al. (2011) to design reliable networks that perform as well as possible under normal conditions, while also performing relatively well when disruptions strike. They presented a mixed-integer programming model whose objective was to minimize the nominal cost while reducing the disruption risk using the *p*-robustness criterion which bounds the cost in disruption scenarios. Chen et al. (2011) presented a multi-criteria decision analysis for environmental risk assessment with regard to avoiding and eliminating damages and loss under natural disasters in international airport projects. They used the ANP to demonstrate one of its utility modes in decision making support to location selection problems, which aims to an evaluation of different projects from different locations.

A facility location model with fuzzy random parameters and its swarm intelligence approach was studied by Wang and Watada (2012). A VaR based fuzzy random facility location model was built in which both the costs and demands were assumed to be fuzzy random variables. The model was inherently a two-stage mixed 0–1 integer fuzzy random programming problem. A hybrid modified particle swarm optimization approach was proposed to solve the model. A corresponding framework for value-based performance and risk optimization in a single-stage supply chain problem was developed by Hahn and Kuhn (2012). They applied Economic Value Added as a prevalent metric of value-based performance to mid-term sales and operations planning. Due to the uncertainty of future events in a scenario based problem, they also used robust optimization methods to deal with operational risks in physical and financial supply chain management. Nickel S. et al (2012) provided a multi-period supply chain network design problem. In this problem, uncertainty was assumed for demand and interest rates, which was described by a set of scenarios. Accordingly, the problem was formulated as a multi-stage stochastic mixed-integer linear programming problem. Recently, Hosseininezhad et al. (2013) proposed a continuous covering location model with risk consideration in which the objective function consists of installation and risk costs and introduced a risk analysis method based on Response Surface Methodology (RSM) to consider risk management in the location models. Also, Hosseininezhad et al. (2014) presented a continuous capacitated location-allocation model with fixed cost as a risk management model. In the presented model, the fixed cost consists of production andinstallation costs. The model considered risk as percent of unsatisfied demands.

In the rest of this section, aforementioned articles are classified based on location model, risk type, space and uncertainty as shown in Table 1 in order to help the reader appreciate the symmetry associated with the facility location problems.

Table1. Comparison between the works

Author(s)	*Location model*	*Risk type*	*Space*	*Uncertainty*
Guillen et al (2005)	*Multi objective supply chain*	*Scenario based*	*Discrete*	*Stochastic*
Snyder et al (2007)	*Location with risk pooling*	*Scenario Based*	*Discrete*	*Stochastic*
Ozsen et al.(2008)	*Warehouse location*	*Uncertain demand*	*Discrete*	*Stochastic*
Azaron et al.(2008)	*Multi-objective stochastic*	*Scenario Based*	*Discrete*	*Stochastic*
Wagner et al.(2009)	*Uncapacitated p-median*	*Value-at-Risk*	*Discrete*	*Stochastic*
You et al. (2009)	*Multi-product supply chain*	*Uncertain demand*	*Discrete*	*Stochastic*
Mete and Zabinsky (2010)	*Location with vehicle routing*	*Disaster*	*Discrete*	*Stochastic*
Cui et al.(2010)	*Reliable facility location*	*Risk of disruption*	*Discrete*	*Stochastic*
Liu et al. (2010)	*Two-echelon inventory/logistics*	*Stochastic demand*	*Discrete*	*Stochastic*
Peng et al. (2011)	*Reliable logistics network design*	*Disruption*	*Discrete*	*Stochastic*
Chen et al. (2011)	*Location selection*	*Disaster*	*Discrete*	*judgmental*
Wang and Watada (2012)	*Fuzzy facility location*	*Value-at-Risk*	*Discrete*	*Fuzzy*
Hahn and Kuhn (2012)	*Single-stage supply chain*	*Scenario Based*	*Discrete*	*Stochastic*
Nickel S. et al (2012)	*Multi-stage supply chain*	*Scenario Based*	*Discrete*	*Stochastic*
Hosseininezhad et al.(2013)	*Continues covering location*	*Uncertain covering radius*	*Continuous*	*Fuzzy*
Hosseininezhad et al.(2014)	*Continues location allocation*	*Uncertain demand*	*Continuous*	*Fuzzy*
This research	*Continues covering location*	*Uncertain coverage*	*Continuous*	*Fuzzy*

In the next section, the continuous covering location model with risk consideration is introduced. Because of uncertain covering radius, the problem is formulated as a risk management model. Therefore, the main differences of our research compared to the mentioned works are as follows.

1. Providing a continuous model for the covering location problem
2. Investigating fuzzy coverage and possibility of covering in the covering location model.
3. Investigating risk in continuous space as a multiobjective model

3- Continuous Covering Location problem (CCLP)

In this section, a continuous covering location model is introduced which is the extension of the model P_1in continuous space. For this model, the space is divided into *n* zones.We are interested in finding the location of *k* facilities in continuous space. Assuming x_j , y_j are the coordinates of the facility and a_i , b_i are coordinates of the center of zone *I* (or the customer *i*) and *D* is the maximum distance that a facility could be located from the center of a zone for assigning to the zone. Two new variables z_{ji} and u_{ji} are also introduced in the model.z_{ji}, namely, the selected zone variable shows whether or not the facility *j* is located in the zone *i*; it is assumed that the distance between each customer and each facility is euclidean. Then constraints (3) and (4) are as follows.

$$\sum_{i=1}^{n} z_{ji}\left(\sqrt{\left(x_j - a_i\right)^2 + \left(y_j - b_i\right)^2}\right) \leq D, \qquad \forall j = 1,2,\dots,k \tag{3}$$

$$\sum_{i=1}^{n} z_{ji} = 1, \qquad \forall j = 1,2,\dots,k \tag{4}$$

Constraint (3) guarantees that if the distance between the facility j and the zone i is greater than $D, z_{ji} = 0$, so the facility j won't be located in the zone i and the facility j will be located in the zone i, if the distance between the facility j and the zone i is smaller than D, constraint (4) Guarantees that the facility j is installed only in one zone. Also, we set a constraint to locate facilities in each zone as follows.

$$\sum_{j=1}^{k} z_{ji} \leq 1, \qquad \forall i = 1,2,\ldots,n \tag{5}$$

Constraint (5) guarantees that at most one facility could be located in the zone i. Another new variable u_{ji}, namely, the covering variableshows whether or not the customer i is covered by the facility j. If R is the covering radius and L is a large value, then

$$\sqrt{\left(x_j - a_i\right)^2 + \left(y_j - b_i\right)^2} \leq R + L(1 - u_{ji}), \qquad \forall i = 1,2,\ldots,n, \quad \forall j = 1,2,\ldots,k \tag{6}$$

$$\sqrt{\left(x_j - a_i\right)^2 + \left(y_j - b_i\right)^2} \geq R - Lu_{ji}, \qquad \forall i = 1,2,\ldots,n, \quad \forall j = 1,2,\ldots,k \tag{7}$$

Constraints (6) and (7) are the covering constraints and guarantee that each customer can be covered by a facility if the distance between them is smaller than$R$; u_{ji}'s ($\forall i = 1,2,\ldots,n \; and \; \forall j = 1,2,\ldots,k$) constitute the covering matrix. If the distance between the customer i and the facility j is greater than R then $u_{ji} = 0$ and $u_{ji} = 1$ otherwise; since L is a large value constraint sets (6), (7) will be satisfied, simultaneously. Assuming that if the customer i is not covered by any of facilities, then $q_i = 1$,we set a constraint similar to constraint (2) as follows.

$$\sum_{j=1}^{k} u_{ji} + q_i \geq 1, \qquad i = 1,2,\ldots,n \tag{8}$$

Constraint (8) indicates the demand constraint which guarantees that q_i is *1* if u_{ji} is zero, it means that the customer i is not covered. If C_i is the importance of customer i, the objective function of the model constitutes of the risk cost which is the cost of the uncovered customers based on the importance of each customer. Accordingly, by integration (3)-(8), the continuous covering location model as the *0-1* nonlinear programming model $P_2$is as shown in (9):

$$P_2 : min \sum_{i=1}^{n} C_i \, q_i \tag{9}$$

st:

$$\sum_{i=1}^{n} z_{ji} \left(\sqrt{\left(x_j - a_i\right)^2 + \left(y_j - b_i\right)^2} \right) \leq D, \qquad \forall j = 1,2,\ldots,k$$

$$\sum_{i=1}^{n} z_{ji} = 1, \qquad \forall j = 1,2,\ldots,k$$

$$\sum_{j=1}^{k} z_{ji} \leq 1, \qquad \forall i = 1,2,\ldots,n$$

$$\sqrt{\left(x_j - a_i\right)^2 + \left(y_j - b_i\right)^2} \leq R + L(1 - u_{ji}), \qquad \forall i = 1,2,\ldots,n, \quad \forall j = 1,2,\ldots,k$$

$$\sqrt{\left(x_j - a_i\right)^2 + \left(y_j - b_i\right)^2} \geq R - Lu_{ji}, \qquad \forall i = 1,2,\ldots,n, \quad \forall j = 1,2,\ldots,k$$

$$\sum_{j=1}^{k} u_{ji} + q_i \geq 1, \qquad i = 1,2,\ldots,n$$

$$z_{ji}, u_{ji}, q_i \in \{0,1\} x_j, y_j \in \mathbb{R} \forall i = 1,2,\ldots,n, \qquad \forall j = 1,2,\ldots,k$$

In next section, by applying the possibility of covering,a multiobjective model is presented.

4- Multi objective continuous covering location model

In this section, at first the multiobjective model is presented and then a fuzzy programming is applied to convert the model to a single objective one. Since we investigate the problem in uncertain conditions, assuming the covering radius is a triangle fuzzy number$\tilde{R} = (R_1, R_1, R_2)$ as shown in Figure1, in the model P_2 we replace (6) and (7) by (10) and (11), respectively as follows.

$$\sqrt{(x_j - a_i)^2 + (y_j - b_i)^2} \le \tilde{R} + L(1 - u_{ji}), \qquad \forall i = 1,2,\dots,n, \quad \forall j = 1,2,\dots,k \tag{10}$$

$$\sqrt{(x_j - a_i)^2 + (y_j - b_i)^2} \ge \tilde{R} - Lu_{ji}, \qquad \forall i = 1,2,\dots,n, \quad \forall j = 1,2,\dots,k \tag{11}$$

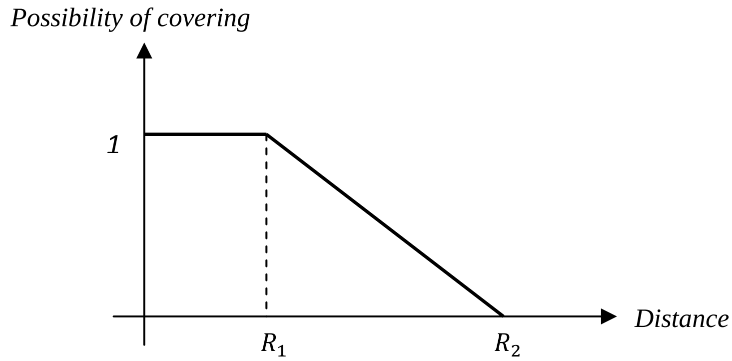

Figure 1. fuzzy covering radius $\tilde{R}$

In this paper, we introduce the possibility of covering concept as follows. If the distance between a customer and a facility is smaller than R_1 , the possibility of covering is *1*, if the distance is greater than R_2, the possibility of covering is *0* and if the distance is between R_1 and R_2, the possibility of covering is between *0* and *1*.So,

$$S_{ji} = \begin{cases} 1 & \sqrt{(x_j - a_i)^2 + (y_j - b_i)^2} \le R_1 \\ \left(R_2 - \sqrt{(x_j - a_i)^2 + (y_j - b_i)^2}\right)/(R_2 - R_1) & R_1 \le \sqrt{(x_j - a_i)^2 + (y_j - b_i)^2} \le R_2 \\ 0 & R_2 \le \sqrt{(x_j - a_i)^2 + (y_j - b_i)^2} \end{cases} \quad ,\forall i\, ,j \tag{12}$$

According to constraint (12), S_{ji} is the possibility of covering the customer *i* by the facility *j* and constraints (13) and (14) are provided as follows.

$$\sqrt{(x_j - a_i)^2 + (y_j - b_i)^2} - R_2 + S_{ji}(R_2 - R_1) \le L(1 - u_{ji}), \qquad \forall i = 1,2,\dots,n, \quad \forall j = 1,2,\dots,k \tag{13}$$

$$S_{ji} \le Lu_{ji}, \qquad \forall i = 1,2,\dots,n, \quad \forall j = 1,2,\dots,k \tag{14}$$

Constraints (13) and (14) are the possibility of covering constraints; since $0 \le S_{ji} \le 1$ and L is a large value, if the customer I can be covered by the facility j then $u_{ji} = 1$ and the constraint (13) is activated. If $u_{ji} = 0$ the constraint (14) is activated and $S_{ji} = 0$. Constraints (13) and (14) guarantee feasibility of the model. Also for providing a crisp model, $\tilde{R}$ is replaced by R_2 which guarantees that if the distance between the customer *i* and the facility *j* is greater than R_2,then $u_{ji} = 0$.In the model P_2, the risk cost, namelyF_1, is minimized. But because of uncertainty, it is desired to maximize the possibility of covering by facilities. So we add the possibility of covering concept by each facility, namely F_2, to the model, then the multiobjective continuous location model $P_3$is as provided in (15).

$$P_3: \left\{\begin{array}{l} F_1: max \sum_{j=1}^{k}\sum_{i=1}^{n}\left(C_i \cdot S_{ji}\right)\Big/ n \\ F_2: min \sum_{i=1}^{n} C_i\, q_i \end{array}\right\} \tag{15}$$

S.t.

$$\sqrt{\left(x_j - a_i\right)^2 + \left(y_j - b_i\right)^2} \le R_2 + L(1-u_{ji}), \qquad \forall i = 1,2,\dots,n, \qquad \forall j = 1,2,\dots,k$$

$$\sqrt{\left(x_j - a_i\right)^2 + \left(y_j - b_i\right)^2} \ge R_2 - Lu_{ji}, \qquad \forall i = 1,2,\dots,n, \qquad \forall j = 1,2,\dots,k$$

$$\sum_{j=1}^{k} u_{ji} + q_i \ge 1, \qquad i = 1,2,\dots,n$$

$$\sum_{i=1}^{n} z_{ji}\left(\sqrt{\left(x_j - a_i\right)^2 + \left(y_j - b_i\right)^2}\right) \le D, \qquad \forall j = 1,2,\dots,k$$

$$\sum_{i=1}^{n} z_{ji} = 1, \qquad \forall j = 1,2,\dots,k$$

$$\sum_{j=1}^{k} z_{ji} \le 1, \qquad \forall i = 1,2,\dots,n$$

$$\sqrt{\left(x_j - a_i\right)^2 + \left(y_j - b_i\right)^2} - R_2 + S_{ji}(R_2 - R_1) \le L(1-u_{ji}), \qquad \forall i = 1,2,\dots,n, \qquad \forall j = 1,2,\dots,k$$

$$S_{ji} \le Lu_{ji}, \qquad \forall i = 1,2,\dots,n, \qquad \forall j = 1,2,\dots,k$$

$$z_{ji}, u_{ji}, q_i \in \{0,1\} \qquad ,0 \le S_{ji} \le 1, \qquad x_j, y_j \in \mathbb{R} \forall i = 1,2,\dots,n, \qquad \forall j = 1,2,\dots,k$$

For converting the multiobjectivemodel P_3 to a single objective one a fuzzy programming is applied.At first, we solve two single models with the objective F_1 and F_2 , separately. If X^1 and X^2bethe obtained solution of $F_1$and F_2, respectively, we replace X^1 and X^2 in F_2 and F_1, respectively, to obtain lower and upper bound for each objective, so a pay-off matrix is obtained as shown in Table 2.

Table 2. Pay-off matrix for the multi objective model

	$F_1(X)$	$F_2(X)$
X^1	$F_1(X^1) = U_1$	$F_2(X^1) = U_2$
X^2	$F_1(X^2) = L_1$	$F_2(X^2) = L_2$

Two variables $T_1(X)$ and $T_2(X)$ are introduced which are satisfaction degree of the objective values $F_1$and F_2, respectively, then constraints (16) and (17) are as follows.

$$T_1(X) = \begin{cases} 0 & F_1(X) \le L_1 \\ \dfrac{F_1(X) - L_1}{U_1 - L_1} & L_1 \le F_1(X) \le U_1 \\ 1 & U_1 \le F_1(X) \end{cases} \tag{16}$$

$$T_2(X) = \begin{cases} 1 & F_2(X) \le L_2 \\ \dfrac{U_2 - F_2(X)}{U_2 - L_2} & L_2 \le F_2(X) \le U_2 \\ 0 & U_2 \le F_2(X) \end{cases} \tag{17}$$

The satisfaction degree concept of the objective values is shown in Figure2 and Figure3.

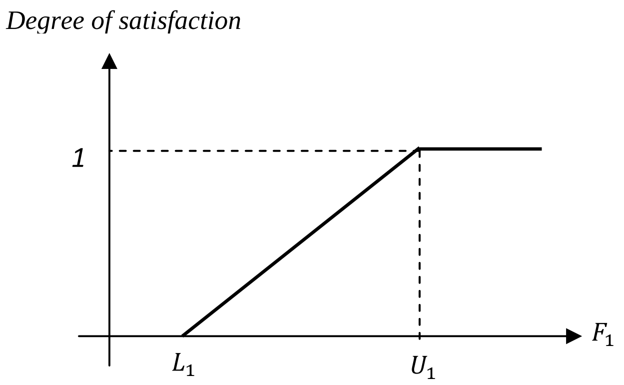

Figure 2. Satisfaction degree of F_1 values

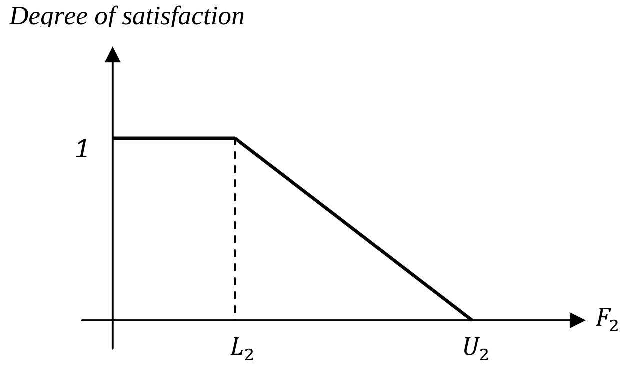

Figure 3. Satisfaction degree of F_2 values

Finally, we use Zimmermann Max-min operator as shown in constraint (18),

$$\max\min\{T_1(X), T_2(X)\} \tag{18}$$

If $\lambda = \min\{T_1(X), T_2(X)\}$ and $0 \le \lambda \le 1$, Since $\lambda \le T_1(X)$then, $\lambda \le (F_1(X) - L_1)/(U_1 - L_1)$, then constraint (19) is as follows,

$$L_1 - \left(\sum_{j=1}^{k}\sum_{i=1}^{n}(C_i \cdot S_{ji})\Big/n\right) + \lambda(U_1 - L_1) \le 0, \tag{19}$$

We carry out similar calculations for $T_2(X)$which lead toconstraint set (20) as follows,

$$\sum_{i=1}^{n} C_i\, q_i - U_2 + \lambda(U_2 - L_2) \le 0 \tag{20}$$

Then, the final single objective model $P_4$is as provided in (21)

$$P_4\colon \max \lambda \tag{21}$$

st:

$$\sqrt{(x_j - a_i)^2 + (y_j - b_i)^2} \le R_2 + L(1 - u_{ji}), \qquad \forall i = 1,2,\dots,n, \quad \forall j = 1,2,\dots,k$$

$$\sqrt{(x_j - a_i)^2 + (y_j - b_i)^2} \ge R_2 - Lu_{ji}, \qquad \forall i = 1,2,\dots,n, \quad \forall j = 1,2,\dots,k$$

$$\sum_{j=1}^{k} u_{ji} + q_i \ge 1, \qquad i = 1,2,\dots,n$$

$$\sum_{i=1}^{n} z_{ji}\left(\sqrt{(x_j - a_i)^2 + (y_j - b_i)^2}\right) \le D, \qquad \forall j = 1,2,\dots,k$$

$$\sum_{i=1}^{n} z_{ji} = 1, \qquad \forall j = 1,2,\dots,k$$

$$\sum_{j=1}^{k} z_{ji} \le 1, \qquad \forall i = 1,2,\dots,n$$

$$\sqrt{(x_j - a_i)^2 + (y_j - b_i)^2} - R_2 + S_{ji}(R_2 - R_1) \le L(1 - u_{ji}), \qquad \forall i = 1,2,\dots,n, \quad \forall j = 1,2,\dots,k$$

$$S_{ji} \le Lu_{ji}, \qquad \forall i = 1,2,\dots,n, \quad \forall j = 1,2,\dots,k$$

$$L_1 - \left(\sum_{j=1}^{k} \sum_{i=1}^{n} (C_i \cdot S_{ji}) \Big/ n \right) + \lambda(U_1 - L_1) \leq 0,$$

$$\sum_{i=1}^{n} C_i\, q_i - U_2 + \lambda(U_2 - L_2) \leq 0,$$

$$z_{ji}, u_{ji}, q_i \in \{0,1\} \quad ,0 \leq S_{ji} \leq 1, \qquad 0 \leq \lambda \leq 1,\ \ x_j, y_j \in \mathbb{R} \forall i = 1,2,\dots,n, \qquad \forall j = 1,2,\dots,k$$

Finally, λ is the objective value, (x_j, y_j) provide the best location of the facility *j* and $u_{ji}'s$ for *i* , *j* provide the covering matrix.

5- Numerical example

In this section, a numerical example is expressed to illustrate the introduced model**.** Suppose that we want to locate *3* new facilities in a region including *16* zones (customers). Specifications of customers are shown in Table 3. As shown in Figure4, Fuzzy covering radius is$\tilde{R} = (0.70, 0.70, 1.10)$ and*D=0.60*.

Table 3. specifications of customers

Customer i	*Coordinate* (a_i, b_i)	*Importance* c_i
1	*(2,1)*	*1*
2	*(3,1)*	*1.5*
3	*(1,2)*	*1.1*
4	*(2,2)*	*1.2*
5	*(3,2)*	*1.0*
6	*(1,3)*	*1.3*
7	*(2,3)*	*1.4*
8	*(3,3)*	*1.6*
9	*(4,3)*	*1.0*
10	*(1,4)*	*1.0*
11	*(2,4)*	*1.1*
12	*(3,4)*	*2.0*
13	*(4,4)*	*1.0*
14	*(2,5)*	*1.0*
15	*(3,5)*	*1.5*
16	*(4,5)*	*1.3*

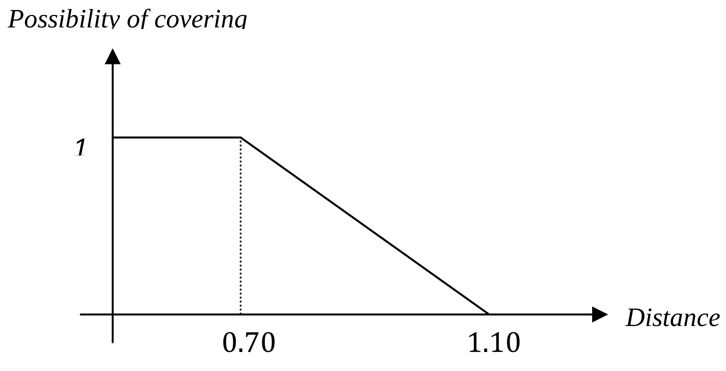

Figure 4. fuzzy covering radius$\tilde{R}$ for the numerical example

At first, we solve the model P_3 for the objective F_1andF_2, separately, the example was solved by optimization software which uses the *branch and reduce* algorithm. The Pay-off matrix for the numerical example is as shown in Table 4.

Table 4. Pay-off matrix for the numerical example

	$F_1(X)$	$F_2(X)$
X^1	$U_1 = 0.273$	$U_2 = 0.570$
X^2	$L_1 = 0.153$	$L_2 = 0.165$

Finally, the model $P_4$is solved and coordinates of new facilities are shown in Table 5.

Table 5. Coordinate of new facilities

Facility	Coordinate x	Coordinate y
1	*3.399*	*4.576*
2	*1.567*	*2.586*
3	*2.592*	*1.569*

The covering variable and the selecting zone variables are
$u_{1,12} = u_{1,13} = u_{1,15} = u_{1,16} = u_{2,3} = u_{2,4} = u_{2,6} = u_{2,7} = u_{3,1} = u_{3,2} = u_{3,4} = u_{3,5} = 1.00$
$z_{1,15} = z_{2,7} = z_{3,5} = 1$
As shown, the facilities are located in zones 5, 7 and 15. Also, the possibilities of covering values are

$S_{1,12} = 1.00, S_{1,13} = 0.67, S_{1,15} = 1.00, S_{1,16} = 0.91$
$S_{2,3} = 0.71, S_{2,4} = 0.93, S_{2,6} = 1.00, S_{2,7} = 1.00$
$S_{3,1} = 0.70, S_{3,2} = 1.00, S_{3,4} = 0.92, S_{3,5} = 1.00$
and,
$q_8 = q_9 = q_{10} = q_{11} = q_{14} = 1.00$

That means customers $\{8,9,10,11,14\}$ are not covered. Finally, the objective value is$\lambda = 0.702$. The final solution is shown in Figure 5.

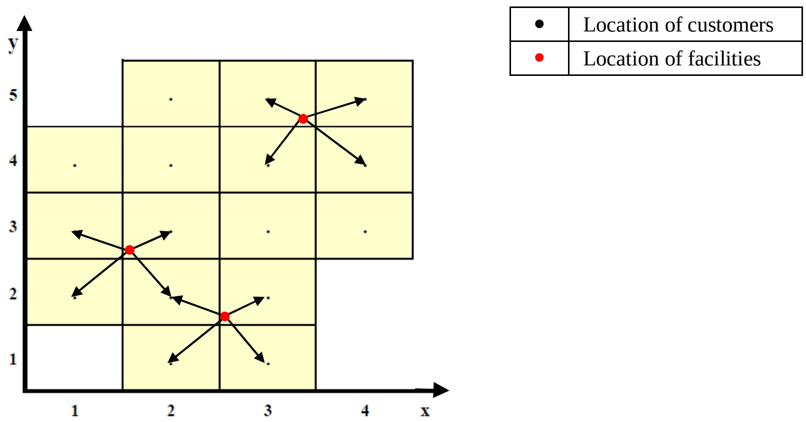

Figure 5. Results of the numerical example

6- Sensitivity Analysis

In this section, we analyze the presented models based on the numerical example. At first, a sensitivity analysis is carried out for the *CCLP* model by changing parameter *R* which is shown in Figure6. As can be seen, by increasing the covering radius, the risk is decreased.This shows usability of the presented continuous covering location model P_2based on different covering radius.

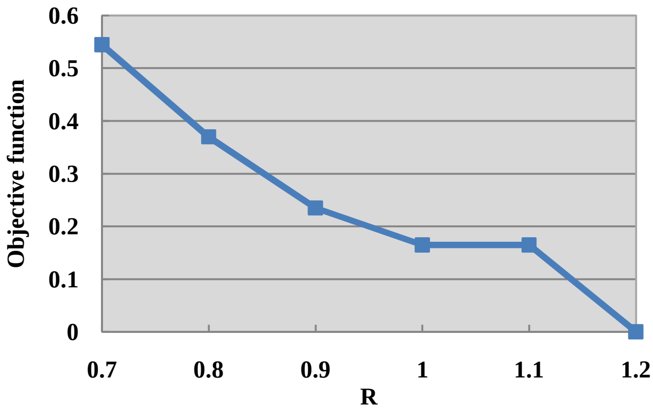

Figure 6. Sensitivity analysis of the CCLP model

The final analysis deals with the importance of considering two objective functions simultaneously as introduced namely the multi objective continuous location model. Assuming F_{11} and F_{12} are the value of objective F_1 in the model P_3 with merely the objective F_1 and F_2, respectively, and F_{21} andF_{22} are the value of objective F_2 in the model P_3 with merely the objectiveF_1 and F_2, respectively, we solve the model P_3 by changing parameter $R_2$with fixing $R_1 = 0.70$, as shown in Figure 7 and Figure 8. Also, if F_1^* and $F_2^*$are the values of the objective F_1 and F_2 obtained by solving the presented model with different R_2 values, respectively. The best results for $F_1$are obtained viaF_{11}but in this case the worst results for F_2areobtained here. On the other hand, the best results for $F_2$are obtained via F_{22}but in this case the worst results for F_1 are obtained here. Obviously considering merely one objective may sacrifice the other. Comparison of results shows that the presented model makes a tradeoff between these two objective functions.

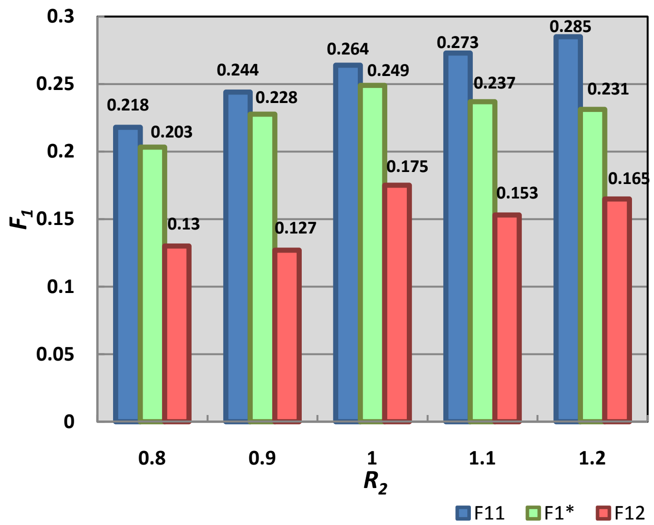

Figure 7. Sensitivity analysis of the presented model with merely the objectiveF_1

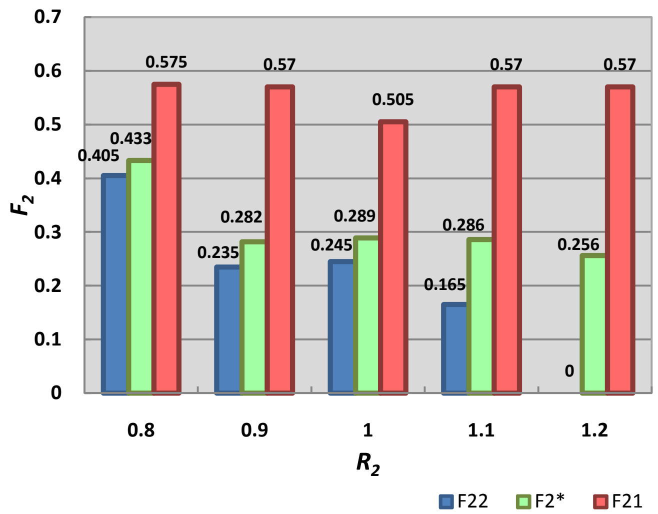

Figure 8. Sensitivity analysis of the presented model with merely the objective F_2

7- Conclusion

This paper presents a multi objective continuous covering location problem in fuzzy environment as the risk management model. Because of uncertain covering radius, the possibility of covering concept was introduced. The presented model's advantage over the traditional covering location ones was the consideration of continuous space for the covering problems. Two variables were introduced for extending the discrete model to the continuous one; the selected zone and the covering variables. Providing the continuous risk management location model is another usability of the presented model. Also, the paper introduces the possibility of covering based on the distance between the customers and the facilities. The two-objective modelwas constituted ofthe maximum possibility of covering by each facility and the minimum risk cost of uncovered objectives. Then, the fuzzy programming was applied for converting the model to the single objective one. Finally, sensitivity analysis was carried out to show the usability's of the continuous covering location problem and the presented two-objective model. Extension of the model as a continuous covering location allocation model with uncertain supply and demand and considering uncertain budget could be investigated in future researches. Providing a heuristic method for large scale instances is another research issue which we think may need future investigations.

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