Uncapacitated phub center problem under uncertainty

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Abstract

Hubs are facilities to collect arrange and distribute commodities in telecommunication networks, cargo delivery systems, etc. In this paper, an uncapacitated phub center problem in case of single allocation and also multiple allocations is introduced in which travel times or transportation costs are considered as fuzzy parameters. Then, by proposing new methods, the presented problem is converted to deterministic mixed integer programming (MIP) problems where these methods are obtained through the implementation of the possibility theory and fuzzy chance-constrained programming. Both possibility and necessity measures are considered separately in the proposed new methods. Finally, the proposed methods are applied on the popular CAB data set. The computational results of our study show that these methods can be implemented for the phub center problem with uncertain frameworks.

Keywords: phub center, fuzzy chance-constrained programming, uncertainty, hub location

1-Introduction

Hub location is one of the most attractive fields in facility location problems. Hub location problems (HLPs) are classical optimization problems that have many practical applications in telecommunication networks, cargo delivery systems, railroad transports systems, airlines, postal networks and other delivery networks that have multiple send and receive nodes. In hub location problem, commodities (such as cargo, passengers, mails, express packages etc.) are consolidated and distributed by hub nodes to the none-hub nodes (whom are also called spokes). The goal of the HLPs is to optimize the objective function by locating hub nodes and allocating spokes to the hubs. Minimization of transportation costs in hub location problems is achieved by the economy of scale, which happens due to existence of discount factor (α) in inter-hub connections. Hub location problems are classified by their objective function (Mini-max or Mini-sum), solution space (continuous, discrete or network), determination of the number of hubs to locate, capacity of hubs or links, fixed or variable cost for establishing hubs and allocating spokes and other classification factors. In most of the classical Hub location problems, demand of the nodes or in other words, the flow between any origin-destination (O-D) nodes and also transportation cost (or travel time) is considered as deterministic parameters.

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However, because of many environmental aspects such as traffic intensity or climate changes, it is required to assume these deterministic parameters as uncertain parameters. One of the suggested approaches to confront uncertain parameters in linear models is fuzzy linear programming. In this research, we study and develop one of the popular hub location problems under fuzzy framework by fuzzy parameters. We considered the uncapacitated phub center problem as the primary model for proposing fuzzy counterpart of this problem. The major properties of this problem are:

- The problem is uncapacitated and there is no limitation in capacity of hubs.
- The objective function is mini-max which means that the maximum flow from any pair of origin-destination will be minimized.
- The number of hubs to be located is exogenous and must be equal to \( p \).
- No cost has been defined for locating hub nodes.
- Both single and multiple allocations are considered: in single allocation each spoke must be allocated only to one hub but in multiple allocations, none of hub nodes' could be allocated to more than one hub.

This paper proposes the phub center problem with uncertain travel time (or transportation cost) in which the transportation times are considered as fuzzy variables.

The reminder of this paper is organized as follows: Section 2 reviews some related researches to this work. In section 3, the fuzzy uncapacitated phub center mathematical model for both single and multiple allocations are proposed (in possibility and necessity condition). In section 4 numerical experiments on the problems are presented and finally conclusion and future research are presented in section 5.

2- Literature Review

In the last two decades, hub location problems have gained more attention from researchers and practitioners; however, hub location under uncertain environment is newly discussed and it is state-of-the-art field. In this section at first, we review the researches about classical and original hub location problems briefly. Then some related works to this paper, specifically those considering uncertainty are reviewed in two sub sections containing mathematical modeling and solution methods respectively.

O’Kelly (1987) introduced the first mathematical model in HLP. He presented a quadratic integer programming whose objective is to minimize the total delivery cost between nodes and locating a pre-specified number of hubs. The later hub location literatures focused on different kinds of problems such as criterion (objective function), number of hubs to locate (fixed or variable), hub capacity (capacitated or uncapacitated), and kinds of allocation (single or multiple) and so on. The interested reader could review the papers by Campbell and O’Kelly (2012) and Farahani et al.(2013) to read full survey of hub location problems and its sub categories.

Campbell (1994) proposed two new hub location problems, which are hub covering and phub center problems. In phub center problem a given number of hubs (\( p \)) is located while the maximum flow or travel time is minimized. The specified flow in these problems is considered between all origin-destination nodes. The none-hub nodes in some literatures are called spokes, so the networks containing hubs and none-hubs are called hub and spoke networks. Kara and Tansel (2000) and Ernst et al. (2009) represented different formulations for the phub center problem. In the phub center problem the main issue is time, which is mostly considered in cargo delivering systems.

2-1- Hub location problems with uncertainty

In real world problems, there might be vagueness or ambiguity in the parameters of the model. For example, the flow of commodities from one city to another could be uncertain for the decision makers. This is why optimization under uncertainty is discussed. In the literature of HLPs, there is less attention to the uncertainty of problem and most of the models have been formulated in deterministic environment. Mahdi and Mirzaei (2008) introduced a fuzzy capacitated hub center location problem that locates hub facilities based on qualitative variables. They proposed a hybrid formulation that performs both location
and allocation phases with qualitative and quantitative criteria simultaneously. Makui et al. (2002) presented a robust optimization model for multi-objective operation of capacitated phub location problems under uncertainty. They used scenario based robust approach to encounter with uncertainties (Mulvey and Ruszczynsk, 1995). Alumur et al. (2012) proposed a comprehensive model considering all sources of uncertainty and used direct approach for solution. Ghodratnama et al. (2013) proposed a novel fuzzy bi-objective model for a hub covering location–allocation problem, whom its first objective minimizes total cost and its second objective is to minimize the summation of shipping times ‘of commodities by transporters from the origin node to the destination node via hubs. A fuzzy goal programming approach is proposed to obtain solution. A sustainable hub location under mixed uncertainty is formulated by Mohammadi et al. (2014). Niakan et al. (2014) studied on a multi-objective hub location under uncertainty with an inexact rough-interval fuzzy approach. Recently, Yang et al. (2014) developed fuzzy phub center problem with generalized value-at-risk. Also, Qin and Gao(2014) discussed phub location with uncertain flows.Mohammadi and Tavakkoli-Moghaddam (2015) designed a novel bi-objective reliable p-hub center problem. They considered arrival time of shipments as a fuzzy M/M/1 queuing system. As well as fuzzy programming, some researchers interested in robust optimization for confronting uncertainties and proposing robust hub location formulations (Boukani et al. 2014; Shahabi and Unnikrishnan 2014; Ghaffari-Nasab et al. 2015).

2-2- Solution approaches to HLPs under uncertainty

One of the most efficient metaheuristic algorithm which is used by many researchers, is genetic algorithm (Kratica and Stanimirović, 2006). The other metaheuristic method is particle swarm optimization (PSO) algorithm. For example, Kai Yang et al. (2012) proposed a hybrid particle swarm optimization algorithm for fuzzy phub center. They combined PSO with genetic operators and local search (LS) to improve solutions of the problem. Other papers that have been focused on solution approaches for HLPs under uncertainty are as follows: Bashiri et al. (2013) presented a genetic based heuristic to solve the capacitated p-hub center problem. They tested their solution on an example obtained by the fuzzy VIKOR method and the AP (Australian Post) data set to explain the effectiveness of the heuristic. Kai Yang et al. (2012) proposed a new fuzzy phub center with value-at-risk criterion in the objective and presented a genetic algorithm incorporating with local search for solution approach. After that Zade et al. (2014) presented a multi-objective hub maximal covering. They assumed uncertain shipments in the context of the problem and a modified NSGA-II metaheuristic was proposed for the solution of the multi-objective problem. Furthermore, Ghaderi and Rahmaniani (2015) presented metaheuristic approaches for robust hub location problem.

In most of the articles related to phub center problems under uncertainty, the uncertainty approach that has been applied is fuzzy programming and robust optimization. Especially, those who observed fuzzy programming, proposed diverse solution methods for it or presented mathematical modeling with fuzzy parameters, and confronted them by different techniques. According to our literature review, we could not find any papers that encounter uncertain parameters of phub center problem by offering possibility and necessity measures. The most important aim of this paper is to introduce new approaches for the uncapacitated phub center problem in both single and multiple allocation states under fuzzy framework based on the possibility theory (Dubois & Prade, 2001). Therefore, the theorems are obtained to convert the original problem to the deterministic mixed integer programming (MIP) problem for optimistic and pessimistic decision makers separately.

3- Mathematical models

Let G = (N, E) be an undirected complete graph with node set N = {1, 2, … , n} and arc set E. Each arc (i,j) has a cost (time, flow, distance, etc.) $c_{ij}$ where $c_{ij} = c_{ji}$ and satisfies triangular inequity ($c_{ij} \leq c_{im} + c_{mj}$ $\forall i,j,m$). Each origin-destination pair i-j should be connected through hub nodes and it is assumed that there is a pre-defined reduction factor ($\alpha$ such that $0 \leq \alpha \leq 1$) between hub nodes so the cost between pairs is reduced, compared to direct connection. Also a given integer number of $p$ hubs should be located.
We will discuss fuzzy uncapacitated phub center problem (FUpHCP) in single (FUSApHCP) and multiple (FUMApHCP) states. Mathematical model of phub center originally is proposed by Campel (1994). Then, Ernst et al. (2009) presented linear formulation for phub center. In this research, linear model of Ernst et al. (2009) is used for fuzzy programming.

3-1 The FUSApHCP

The original objective function of phub center model is the following equation:

$$\min \max_{i,j,k,m \in N} (C^k m_{ij} X_{ik} X_{jm}),$$

Which has a quadratic objective function and $C^k m_{ij}$ represents the cost (time, money, etc.) between node $i$ and node $j$ that flows through hub $k$ and hub $m$. In other words, the route from node $i$ to node $j$ is the following scheme:

$$\text{none} \rightarrow k \rightarrow m \rightarrow j \rightarrow \text{none}$$

To include the discount factor in the model, the cost coefficient is transformed as $C^k m_{ij} = C_{ik} + \alpha C_{km} + C_{mj}$ where $\alpha$ is the discount factor of cost between hub $k$ and $m$. $X_{ik}$ is a binary variable such that $X_{ik} = 1$ if and only if node $i$ is allocated to node $k$. The objective function of the linearized USApHCP model and its constraints, proposed by Ernst et al (2009), are as follows:

Indices are:

- $i, j$: none-hub node index
- $k, m$: hub node index

$$\begin{align*}
\min z \\
\text{s.t.} \quad z & \geq \sum_{k=1}^{N} (C_{ik} + \alpha C_{km}) X_{ik} X_{jm} \quad i,j,m = 1,...,n \\
\sum_{k=1}^{N} X_{ik} & = 1 \quad i = 1,...,n \\
X_{ik} & \leq X_{kkl} \quad k = 1,...,n \\
\sum_{k=1}^{N} X_{kk} & = p \\
X_{ik} & \in \{0,1\} \quad i,k = 1,...,n
\end{align*}$$

In the above model, objective function minimizes $z$, where $z$ is the maximum flow or cost between all origin-destination nodes, which is obtained in the first constraint. The second constraint assures that each none-hub node $i$ is allocated to only one hub node $k$. The third constraint means that node $k$ must be a hub, if a node like $i$ is allocated to it, and the last constraint shows that precisely $p$ hubs should be located.

In our hub location problem, there are two parameters that could be assumed as uncertain parameters: flow (or monetary cost or travel time) between any O-D pair and the cost of establishing hubs in any node. As noted in the model assumptions, there is no establishing cost for hubs, so the only uncertain parameter is the flow between nodes. For proposing our fuzzy models we use the method which is discussed in details by Nematian (2015).

A LR fuzzy number $\tilde{B} = (B^0, B^-, B^+)$, is represented by the following membership function:
\[ B(x) = \begin{cases} L \left( \frac{B^0 - x}{B^-} \right) B^0 - B^- \leq x \leq B^0 \\ R \left( \frac{x - B^0}{B^+} \right) B^0 < x \leq B^0 + B^+ \end{cases} \]  

(6)

Where \( B^0 \) defines the center, \( B^+ \) defines the right spread and \( B^- \) is the left spread. \( L, R : [0,1] \rightarrow [0,1] \) with \( R(0) = L(0) = 1 \) and \( L(1) = R(1) = 0 \). \( R \) and \( L \) are decreasing continuous functions.

By the following problem, the USApHCP is developed to a model with fuzzy variables (FUSApHCP):

**Problem 1:**

\[ \text{Min } z \]

\[ s.t. \quad z \geq \sum_{i=1}^{N} \left( \tilde{c}_{ik} + \alpha \tilde{c}_{km} \right) x_{ik} + \tilde{c}_{mj} x_{jm} \]

(7)

Constraint (2) – (5),

where \( \tilde{c}_{ik} = (C_{ik}, \beta_{ik}, \eta_{ik})_{LR} \), \( \tilde{c}_{km} = (C_{km}, \beta_{km}, \eta_{km})_{LR} \) and \( \tilde{c}_{mj} = (C_{mj}, \beta_{mj}, \eta_{mj})_{LR} \). In the above model, each variable with "tilde sign (-)" over it, shows a fuzzy variable or uncertain parameter.

In order to solve the FUSApHLP, the fuzzy model should be transformed into a deterministic model by using possibility and necessity measures in each constraint with fuzzy variables and applying fuzzy chance-constrained programming (FCCP). Now, problem 1 is converted into the following problem by applying the FCCP:

**Problem 2:**

\[ \text{Min } z \]

\[ s.t. \quad \text{Pos} \left( \sum_{i=1}^{N} \left( \tilde{c}_{ik} + \alpha \tilde{c}_{km} \right) x_{ik} + \tilde{c}_{mj} x_{jm} \right) \leq z \]

(8)

Constraint (2) – (5),

where \( \eta \) is a predetermined possibility level and \( \text{Pos} \left( \sum_{i=1}^{N} \left( \tilde{c}_{ik} + \alpha \tilde{c}_{km} \right) x_{ik} + \tilde{c}_{mj} x_{jm} \right) \leq z \) is defined as follows:

\[ \text{Pos} \left( \bar{z}_{ijm} \leq z \right) = \sup_{y_1, y_2} \left\{ \min_{\eta_{ijm}} \left[ \mu_{2_{ijm}}(y_1), \mu_{2}(y_2) \right] \right\} \]

(9)

where \( \bar{z}_{ijm} = \sum_{i=1}^{N} \left( \tilde{c}_{ik} + \alpha \tilde{c}_{km} \right) x_{ik} + \tilde{c}_{mj} x_{jm} \)

Now, we obtain the following theorem to convert problem 2 to deterministic programming.

**Theorem 1:**

\[ \text{Pos} \left( \bar{z}_{ijm} \leq z \right) \geq \eta \iff \sum_{i=1}^{N} \left( C_{ik} + \alpha C_{km} \right) x_{ik} + C_{mj} x_{jm} - L'(\eta) \left[ \sum_{i=1}^{N} \left( \beta_{ik} + \alpha \beta_{km} \right) x_{ik} + \beta_{mj} x_{jm} \right] \leq z \]

(10)
Where $L^*(\eta)$ is pseudo inverse function and is defined as $L^*(\lambda) = \sup \{t | L(t) \geq \lambda\}$. $\eta$ indicates the level of possibility, for example if $\eta = 1$ then the model output would be the same as non-fuzzy mode.

So the complete possibility model is represented by the following problem:

**Problem 3:**

\[
\min z \\
\text{s.t. } \sum_{k=1}^{N} (c^0_{ik} + \alpha c^0_{km})x_{ik} + c^0_{mj}x_{jm} - L^*(\eta) \left[ \sum_{k=1}^{N} (\beta_{ik} + \alpha \beta_{km})x_{ik} + \beta_{mj}x_{jm} \right] \leq z \quad i, j, m = 1, \ldots, n
\]

Constraint (2) – (5).

(11)

Furthermore, for pessimistic decision makers, we apply the necessity measures in the FCCP approach like the previous model as follows:

**Problem 4:**

\[
\min z \\
\text{s.t. } \text{Nec} \left( \sum_{k=1}^{N} (\tilde{c}_{ik} + \alpha \tilde{c}_{km})x_{ik} + \tilde{c}_{mj}x_{jm} \right) \leq z \quad \eta, i, j, m = 1, \ldots, n
\]

(12)

Constraint (2) – (5),

where $\text{Nec} \left( \sum_{k=1}^{N} (\tilde{c}_{ik} + \alpha \tilde{c}_{km})x_{ik} + \tilde{c}_{mj}x_{jm} \right) \leq z$ is defined as

\[
\text{Nec}(\tilde{z}_{ijm} \leq z) = \inf_{y_1, y_2} \left\{ \max \left[ 1 - \mu_{z_{ijm}}(y_1), 1 - \mu_z(y_2) \right] | y_1 \leq y_2 \right\}
\]

(13)

Like the possibility model, we obtain the following theorem to transform problem 4 to a deterministic problem.

**Theorem 2:**

\[
\text{Nec}(\tilde{z}_{ijm} \leq z) \geq \eta \iff \sum_{k=1}^{N} (c^0_{ik} + \alpha c^0_{km})x_{ik} + c^0_{mj}x_{jm} - L^*(1 - \eta) \left[ \sum_{k=1}^{N} (\beta_{ik} + \alpha \beta_{km})x_{ik} + \beta_{mj}x_{jm} \right] \leq z
\]

(14)
Problem 5:

\[ \text{Min } z \]

\[ \text{s.t. } \sum_{k=1}^{N} (c_{ik}^0 + \alpha c_{km}^0) x_{ik} + c_{mj}^0 x_{jm} - L^* (1 - \eta) \left( \sum_{k=1}^{N} (\beta_{ik} + \alpha \beta_{km}) x_{ik} + \beta_{mj} x_{jm} \right) \leq z \quad i, j, m = 1, \ldots, n \]

Constraint (2) – (5).

3-2- FUMApHCP

In multiple allocations each none-hub node can be allocated to more than one hub node. The mathematical model for multiple allocation of pHub center proposed by Ernst et al. (2009) is as follows:

\[ \text{Min } z \]

\[ \text{s.t. } z \geq \sum_{k=1}^{N} \sum_{m=1}^{N} y_{ijkm} (c_{ik} + \alpha c_{km} + c_{mj}) \quad i, j, m = 1, \ldots, n \]

\[ \sum_{k=1}^{N} \sum_{m=1}^{N} y_{ijkm} = 1 \quad i, j = 1, \ldots, n \]

\[ \sum_{k=1}^{N} y_{ijkm} \leq z_{m} i, j, m = 1, \ldots, n \]

\[ \sum_{m=1}^{N} y_{ijkm} \leq z_{k} i, j, k = 1, \ldots, n \]

\[ \sum_{k=1}^{N} z_{k} = p \]

\[ z_{k}, y_{ijkm} \in \{0,1\} \quad i, j, k, m = 1, \ldots, n \]

The variable \( y_{ijkm} \) represents the allocation of node \( i \) to hub \( k \) and node \( j \) to hub \( m \), so the origin-destination path is \( i - k - m - j \). The variable \( z_{k} \) indicates the index of the hubs that will be established.

The process of developing UMApHCP to FUMApHCP is the same as previous section that mentioned above:

Problem 6:

\[ \text{Min } z \]

\[ \text{s.t. } z \geq \sum_{k=1}^{N} \sum_{m=1}^{N} y_{ijkm} (\tilde{c}_{ik} + \alpha \tilde{c}_{km} + \tilde{c}_{mj}) \quad i, j, m = 1, \ldots, n \]

Constraint (16) – (20).

By applying FCCP approach with possibility measures for the above problem, we have:
Problem 7:

\[ \text{Min } z \]

\[ \begin{align*}
    \text{s.t. } & \quad \text{Pos}(z \geq \sum_{k=1}^{N} \sum_{m=1}^{N} y_{ijkm}(c_{ik} + \alpha c_{km} + \bar{c}_{jm}) \geq \eta, i, j = 1, \ldots, n} \\
\end{align*} \]  

(22)

Constraint (16) – (20).

Like previous section, we achieve the following proposition:

Proposition 1:

\[ \text{Pos}(\tilde{z}_{ij} \leq z) \geq \eta \iff \]

\[ \sum_{k=1}^{N} \sum_{m=1}^{N} y_{ijkm}(c_{ik}^{0} + \alpha c_{km}^{0} + \bar{c}_{jm}^{0}) - L^{*}(\eta) \left[ \sum_{k=1}^{N} \sum_{m=1}^{N} \beta_{ik} + \alpha \beta_{km} + \beta_{mj} \right] \leq z \]

(23)

where \( \tilde{z}_{ij} = \sum_{k=1}^{N} \sum_{m=1}^{N} y_{ijkm}(\tilde{c}_{ik} + \alpha \tilde{c}_{km} + \tilde{c}_{jm}) \).

Then, the possibility model of FUMApHCP is represented as

Problem 8:

\[ \text{Min } z \]

\[ \begin{align*}
    \text{s.t. } & \quad \sum_{k=1}^{N} \sum_{m=1}^{N} (c_{ik}^{0} + \alpha c_{km}^{0} + \bar{c}_{jm}^{0})y_{ijkm} - L^{*}(\eta) \left[ \sum_{k=1}^{N} \sum_{m=1}^{N} \beta_{ik} + \alpha \beta_{km} + \beta_{mj} \right] \leq z, \quad i, j = 1, \ldots, n \\
\end{align*} \]  

(24)

Constraint (16) – (20).

Furthermore, based on the necessity measures, we have the following problem:

Problem 9:

\[ \text{Min } z \]

\[ \begin{align*}
    \text{s.t. } & \quad \text{Nec}(z \geq \sum_{k=1}^{N} \sum_{m=1}^{N} y_{ijkm}(c_{ik}^{0} + \alpha c_{km}^{0} + \bar{c}_{jm}^{0}) \geq \eta, i, j = 1, \ldots, n} \\
\end{align*} \]  

(25)

Constraint (16) – (20).

Proposition 2:

\[ \text{Nec}(\tilde{z}_{ij} \leq z) \geq \eta \iff \]

\[ \sum_{k=1}^{N} \sum_{m=1}^{N} y_{ijkm}(c_{ik}^{0} + \alpha c_{km}^{0} + \bar{c}_{jm}^{0}) - L^{*}(1 - \eta) \left[ \sum_{k=1}^{N} \sum_{m=1}^{N} \beta_{ik} + \alpha \beta_{km} + \beta_{mj} \right] \leq z \]

(26)
Finally, the necessity model of FUMApHCP is represented as follows:

**Problem 10:**

\[
\begin{align*}
\text{Min } & z \\
\text{s.t. } & \sum_{k=1}^{N} \sum_{m=1}^{N} \gamma_{ijk} \left( C_{ik}^0 + \alpha C_{km}^0 + C_{mj}^0 \right) - L^*(1 - \eta) \left[ \sum_{k=1}^{N} \sum_{m=1}^{N} \gamma_{ijk} \left( \beta_{ik} + \alpha \beta_{km} + \beta_{mj} \right) \right] \\
& \leq z \quad i, j = 1, \ldots, n
\end{align*}
\]

Constraint (16) – (20).

All obtained deterministic problems are easily solved by one of the MIP solvers.

### 4-Numerical Experiments

In this paper, we used popular CAB data set for numerical tests. The CAB data set was represented by O’Kelly (1987) for hub location problems. The CAB data set is based on Civil Aeronautics Board in 1970, which is generated from the flow of airline passengers in 25 cities in United States. Numbers in CAB data set are symmetric and satisfies triangular inequity. We used GAMS v24.1.2 to solve fuzzy phub center problem. A PC with Core i5 processor and 8GB RAM was used for performing experiments.

For solving our fuzzy models we need to input data in form of \((l, l, \{\})\) where \(l\) is the crisp number, and \(l, l\) are left and right values. The crisp and middle number \((l)\) is assumed to be the original number in CAB data set. Assume that \(l\) is the original number in data set, to generate right and left values the following relation is used: \(1.2l\) and \(0.8l\).

We divided numerical tests for solving these two problems (both single and multiple allocations for fuzzy phub center) into several sub problems. These sub problems are generated by using different values for model features. The features conclude the following cases:

- **Size of problem:** different problem sizes are found by taking the top 10, 15, 20 and 25 nodes from CAB data set.
- **Discount factor:** various varies of \(\alpha = \{0.2, 0.4, 0.6, 0.8\}\)
- **Number of hubs to locate:** different values of \(p = \{2, 3, 4, 5\}\)
- **Possibility or necessity**
- **Pseudo inverse functions:** the functions \(R^*(h)\) and \(L^*(\eta)\) in models represent probability and possibility levels, where \(R^*(h) = L^*(\eta) = 1 - h\). So different probability Levels are obtained by using \(h, \eta\) as \(0.1, 0.3, 0.5, 0.7, 0.9\).

Sub problems are shown as \(x, y, z\), where \(x\) is the size of problem, \(y\) is the discount factor value and \(z\) is the value of \(p\). For example sub problem 25.2.3 represents 25 nodes with \(\alpha = 0.2\) and \(p = 3\).

Results for the fuzzy single allocation phub center model are shown in Table I and for the fuzzy multiple allocation phub center model the results are shown in Table II. According to the results of Table I and Table II, in the same problem with the lowest possibility level for possibility-based model and the highest possibility level for necessity-based model, the optimal solutions for both possibility and necessity-based models are same.
Fig. 1 and Fig. 2 represent the optimal solution for different problems in the case that $h$ changes. For the possibility cases of minimizing objective functions like phub center, with the increase of $h$ the objective function increases too and in necessity cases the optimal value decreases.

Any decision maker can consider other levels based on his/her circumstances or any other constraints. Therefore, the decision maker’s opinion can be classified as follows:

1. **Best optimal solution:** this vision of the decision maker does not deal with any levels of possibility or necessity. The decision maker chooses the best allocation of spokes and the optimal hubs to locate and there is no restriction for selecting the possibility/necessity levels.
2. The lowest or highest level: in this point of view, the decision maker chooses only lowest or highest levels of possibility/necessity for locating hubs and allocating none-hub nodes. We considered $h = 0.1$ as the lowest level and $h = 0.9$ as the highest level.

3. The middle levels: in this perspective, the decision maker wants to have middle levels. This view happens when the DM does not have absolute information about the levels and decides to have middle levels.

In this section, we treated the possibility/necessity levels of $\{0.1, 0.3, 0.5, 0.7, 0.9\}$ only as sample levels to obtain optimal solutions, so there is no limitation for the decision maker to choose only them to find the optimal solutions. One can choose any other levels between $(0,1]$ to find his/her optimal solutions.

The map of the geographical locations of the cities in the CAB data set and the optimal solution for some of the problems is shown on Fig.3 and Fig.4. The optimal solution among all possibility or necessity level is chosen for the problem which is illustrated. Both figures considered only single allocation of the problem.
Fig. 3. Optimal solution of CAB data set with 25 nodes and two numbers of hubs to located with $\alpha = 0.2$

Fig. 4. Optimal solution of CAB data set with 25 nodes and three numbers of hubs to located with $\alpha = 0.8$
<table>
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<th>Necessity</th>
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<td>Table 1. Numerical Results of FUSAPHC for CAB dataset</td>
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<tr>
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Table II. Numerical Results of FUMaCHp for CAB dataset
5- Conclusion

In this paper, we studied uncapacitated phub center problem under uncertainty in the travel time or transportation costs. We presented generic models capturing these different sources of uncertainty for the single and the multiple allocation cases of the problem. Also we proposed new methods to solve the problem for optimistic and pessimistic decision makers separately. Our new approach uses different possibility and necessity measures to obtain the optimal solution of the phub center problem. The presented problem is converted to deterministic mixed integer programming problems for convenience of solving with MIP solvers. Finally, for the numerical experiments we performed extensive computational analysis with more than 250 sub problems on the CAB data set.

As one of the future research activities, the proposed approach in confronting uncertain parameters could be implemented on other hub location problems such as hub covering problems, multi objective hub location problems, other capacitated hub location problems and also some new hub location problems like the hub line location problem (Martins et. al. 2015). Another future research suggestion is providing solution methods to efficiently solve more realistic, large-scale instances for this class of fuzzy phub center problem. This includes solving the formulation with met-heuristic algorithms.

References


