Journal of Industrial and Systems Engineering
Vol. 9, special issue on location allocation and hub modeling, pp 23-39
Winter (February) 2016

Uncapacitated phub center problem under uncertainty

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Abstract

Hubs are facilities to collect arrange and distribute commodities in telecommunication networks, cargo delivery systems, etc. In this paper, an uncapacitated phub center problem in case of single allocation and also multiple allocations is introduced in which travel times or transportation costs are considered as fuzzy parameters. Then, by proposing new methods, the presented problem is converted to deterministic mixed integer programming (MIP) problems where these methods are obtained through the implementation of the possibility theory and fuzzy chance-constrained programming. Both possibility and necessity measures are considered separately in the proposed new methods. Finally, the proposed methods are applied on the popular CAB data set. The computational results of our study show that these methods can be implemented for the phub center problem with uncertain frameworks.

Keywords: phub center, fuzzy chance-constrained programming, uncertainty, hub loation

1-Introduction

Hub location is one of the most attractive fields in facility location problems. Hub location problems (HLPs) are classical optimization problems that have many practical applications in telecommunication networks, cargo delivery systems, railroad transports systems, airlines, postal networks and other delivery networks that have multiple send and receive nodes. In hub location problem, commodities (such as cargo, passengers, mails, express packages etc.) are consolidated and distributed by hub nodes to the none-hub nodes (whom are also called spokes).The goal of the HLPs is to optimize the objective function by locating hub nodes and allocating spokes to the hubs. Minimization of transportation costs in hub location problems is achieved by the economy of scale, which happens due to existence of discount factor (α) in inter-hub connections. Hub location problems are classified by their objective function (Mini-max or Mini-sum), solution space (continuous, discrete or network), determination of the number of hubs to locate, capacity of hubs or links, fixed or variable cost for establishing hubs and allocating spokes and other classification factors. In most of the classical hub location problems, demand of the nodes or in other words, the flow between any origin-destination (O-D) nodes and also transportation cost (or travel time) is considered as deterministic parameters.

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ISSN: 1735-8272,

However, because of many environmental aspects such as traffic intensity or climate changes, it is required to assume these deterministic parameters as uncertain parameters. One of the suggested approaches to confront uncertain parameters in linear models is fuzzy linear programming. In this research, we study and develop one of the popular hub location problems under fuzzy framework by fuzzy parameters. We considered the uncapacitated phub center problem as the primary model for proposing fuzzy counterpart of this problem. The major properties of this problem are:

- The problem is uncapacitated and there is no limitation in capacity of hubs.
- The objective function is mini-max which means that the maximum flow from any pair of origin - destination will be minimized.
- The number of hubs to be located is exogenous and must be equal to p.
- No cost has been defined for locating hub nodes.
- Both single and multiple allocations are considered: in single allocation each spoke must be allocated only to one hub but in multiple allocations, none of hub nodes' could be allocated to more than one hub.

This paper proposes the phub center problem with uncertain travel time (or transportation cost) in which the transportation times are considered as fuzzy variables.

The reminder of this paper is organized as follows: Section 2 reviews some related researches to this work. In section 3, the fuzzy uncapacitated phub center mathematical model for both single and multiple allocations are proposed (in possibility and necessity condition),. In section 4 numerical experiments on the problems are presented and finally conclusion and future research are presented in section 5.

2- Literature Review

In the last two decades, hub location problems have gained more attention from researchers and practitioners; however, hub location under uncertain environment is newly discussed and it is state-of-the-art field. In this section at first, we review the researches about classical and original hub location problems briefly. Then some related works to this paper, specifically those considering uncertainty are reviewed in two sub sections containing mathematical modeling and solution methods respectively.

O'Kelly (1987) introduced the first mathematical model in HLP. He presented a quadratic integer programming whose objective is to minimize the total delivery cost between nodes and locating a pre-specified number of hubs. The later hub location literatures focused on different kinds of problems such as criterion (objective function), number of hubs to locate (fixed or variable), hub capacity (capacitated or uncapacitated), and kinds of allocation (single or multiple) and so on. The interested reader could review the papers by Campbell and O'Kelly (2012) and Farahani et al(2013) to read full survey of hub location problems and its sub categories.

Campbell (1994) proposed two new hub location problems, which are hub covering and phub center problems. In phub center problem a given number of hubs (p) is located while the maximum flow or travel time is minimized. The specified flow in these problems is considered between all origin-destination nodes. The none-hub nodes in some literatures are called spokes, so the networks containing hubs and none-hubs are called hub and spoke networks. Kara and Tansel (2000) and Ernst et al. (2009) represented different formulations for the phub center problem. In the phub center problem the main issue is time, which is mostly considered in cargo delivering systems.

2-1- Hub location problems with uncertainty

In real world problems, there might be vagueness or ambiguity in the parameters of the model. For example, the flow of commodities from one city to another could be uncertain for the decision makers. This is why optimization under uncertainty is discussed. In the literature of HLPs, there is less attention to the uncertainty of problem and most of the models have been formulated in deterministic environment. Mahdi and Mirzaei (2008) introduced a fuzzy capacitated hub center location problem that locates hub facilities based on qualitative variables. They proposed a hybrid formulation that performs both location and allocation phases with qualitative and quantitative criteria simultaneously. Makui et al. (2002) presented a robust optimization model for multi-objective operation of capacitated phub location problems under uncertainty. They used scenario based robust approach to encounter with uncertainties (Mulvey and Ruszczynsk, 1995). Alumur et al. (2012) proposed a comprehensive model considering all sources of uncertainty and used direct approach for solution. Ghodratnama et al. (2013) proposed a novel fuzzy bi-objective model for a hub covering location–allocation problem, whom its first objective minimizes total cost and its second objective is to minimize the summation of shipping times ‘of commodities by transporters from the origin node to the destination node via hubs. A fuzzy goal programming approach is proposed to obtain solution. A sustainable hub location under mixed uncertainty is formulated by Mohammadi et al. (2014). Niakan et al. (2014) studied on a multi-objective hub location under uncertainty with an inexact rough-interval fuzzy approach. Recently, Yang et al. (2014) developed fuzzy phub center problem with generalized value-at-risk. Also, Qin and Gao(2014) discussed phub location with uncertain flows.Mohammadi and Tavakkoli-Moghaddam (2015) designed a novel bi-objective reliable p-hub center problem. They considered arrival time of shipments as a fuzzy M/M/1 queuing system. As well as fuzzy programming, some researchers interested in robust optimization for confronting uncertainties and proposing robust hub location formulations (Boukani et al. 2014; Shahabi and Unnikrishnan 2014; Ghaffari-Nasab et al. 2015).

2-2- Solution approaches to HLPs under uncertainty

One of the most efficient metaheuristic algorithmswhich is used by many researchers, is genetic algorithm (Kratica and Stanimirović, 2006).The other metaheuristic method is particle swarm optimization (PSO) algorithm. For example, Kai Yang et al. (2012) proposed a hybrid particle swarm optimization algorithm for fuzzy phub center. They combined PSO with genetic operators and local search (LS) to improve solutions of the problem. Other papers that have been focused on solution approaches for HLPs under uncertainty are as follows: Bashiri et al. (2013) presented a genetic based heuristic to solve the capacitated p-hub center problem. They tested their solution on an example obtained by the fuzzy VIKOR method and the AP (Australian Post) data set to explain the effectiveness of the heuristic. Kai Yang et al. (2012) proposed a new fuzzy phub center with value-at-risk criterion in the objective and presented a genetic algorithm incorporating with local search for solution approach. After that Zade et al. (2014) presented a multi-objective hub maximal covering. They assumed uncertain shipments in the context of the problem and a modified NSGA-II metaheuristic was proposed for the solution of the multi-objective problem. Furthermore, Ghaderi and Rahmaniani (2015) presented metaheuristic approaches for robust hub location problem.

In most of the articles related to phub center problems under uncertainty, the uncertainty approach that has been applied is fuzzy programming and robust optimization. Especially, those who observed fuzzy programming, proposed diverse solution methods for it or presented mathematical modeling with fuzzy parameters, and confronted them by different techniques. According to our literature review, we could not find any papers that encounter uncertain parameters of phub center problem by offering possibility and necessity measures. The most important aim of this paper is to introduce new approaches for the uncapacitated phub center problem in both single and multiple allocation states under fuzzy framework based on the possibility theory (Dubois & Prade, 2001). Therefore, the theorems are obtained to convert the original problem to the deterministic mixed integer programming (MIP) problem for optimistic and pessimistic decision makers separately.

3- Mathematical models

Let G= (N, E) be an undirected complete graph with node set N= {1, 2, … , n} and arc set E. Each arc (i,j) has a cost (time, flow, distance, etc.) c_{ij} where $c_{ij} = c_{ji}$ and satisfies triangular inequity ($c_{ij} \leq c_{im} + c_{mj}\ \forall\ i, j, m$). Each origin-destination pair i-j should be connected through hub nodes and it is assumed that there is a pre-defined reduction factor (α such that$0 \leq \alpha \leq 1$) between hub nodes so the cost between pairs is reduced, compared to direct connection. Also a given integer number of p hubs should be located.

We will discuss fuzzy uncapacitated phub center problem (FUpHCP) in single (FUSApHCP) and multiple (FUMApHCP) states. Mathematical model of phub center originally is proposed by Campel (1994). Then, Ernst et al. (2009) presented linear formulation for phub center. In this research, linear model of Ernst et al. (2009) is used for fuzzy programming.

3-1 The FUSApHCP

The original objective function of phub center model is the following equation:

$\min \max_{i,j,k,m \in N}\{C_{ij}^{km} X_{ik} X_{jm}\}$,

Which has a quadratic objective function and C_{ij}^{km} represents the cost (time, money, etc.) between node i and node j that flows through hub k and hub m. In other words, the route from node i to node j is the following scheme:

$$\underset{none-hub}{i} \rightarrow \underset{hub}{k} \rightarrow \underset{hub}{m} \rightarrow \underset{none-hub}{j}$$

To include the discount factor in the model, the cost coefficient is transformed as$C_{ij}^{km} = C_{ik} + +\alpha C_{km} + C_{mj}$ where α is the discount factor of cost between hub k and m. X_{ik} is a binary variable such that $X_{ik} = 1$ if and only if node i is allocated to node k .The objective function of the linearized USApHCP model and its constraints, proposed by Ernst et al (2009), are as follows:

Indices are:

- i , j : none-hub node index
- k , m : hub node index

$\min z$

$$s.t. \quad z \geq \sum_{k=1}^{N} (C_{ik} + \alpha C_{km}) X_{ik} + C_{mj} X_{jm} \quad i,j,m = 1, \dots, n \tag{1}$$

$$\sum_{k=1}^{N} X_{ik} = 1 \; i = 1, \dots, n \tag{2}$$

$$X_{ik} \leq X_{kk} i,k = 1, \dots, n \tag{3}$$

$$\sum_{k=1}^{N} X_{kk} = p \tag{4}$$

$$X_{ik} \epsilon \{0,1\} \qquad i,k = 1, \dots, n \tag{5}$$

In the above model, objective function minimizes z, where z is the maximum flow or cost between all origin-destination nodes, which is obtained in the first constraint. The second constraint assures that each none-hub node i is allocated to only one hub node k. The third constraint means that node k must be a hub, if a node like$i$ is allocated to it, and the last constraint shows that precisely p hubs should be located.

In our hub location problem, there are two parameters that could be assumed as uncertain parameters: flow (or monetary cost or travel time) between any O-D pair and the cost of establishing hubs in any node. As noted in the model assumptions, there is no establishing cost for hubs, so the only uncertain parameter is the flow between nodes. For proposing our fuzzy models we use the method which is discussed in details by Nematian (2015).

A LR fuzzy number $\tilde{B} = (B^0, B^-, B^+)_{LR}$ is represented by the following membership function:

$$\tilde{B}(x) = \begin{cases} L\left(\frac{B^0 - x}{B^-}\right) B^0 - B^- \le x \le B^0 \\ R\left(\frac{x - B^0}{B^+}\right) B^0 < x \le B^0 + B^+, \end{cases} \quad (6)$$

WhereB^0 defines the center, B^+ defines the right spread and B^- is the left spread. $L, R: [0,1] \to [0,1]$ with $R(0) = L(0) = 1$ and $L(1) = R(1) = 0$. R and L are decreasing continuous functions.

By the following problem, the USApHCP is developed to a model with fuzzy variables (FUSApHCP):

Problem1:

$Min\ z$

$$s.t. \quad z \ge \left[\sum_{k=1}^{N} \left(\tilde{C}_{ik} + \alpha \tilde{C}_{km}\right) X_{ik} + \tilde{C}_{mj} X_{jm}\right] i, j, m = 1, \dots, n \quad (7)$$

Constraint (2) – (5),

where $\tilde{C}_{ik} = (C_{ik}^0, \beta_{ik}, \gamma_{ik})_{LR}$, $\tilde{C}_{km} = (C_{km}^0, \beta_{km}, \gamma_{km})_{LR}$ and $\tilde{C}_{mj} = (C_{mj}^0, \beta_{mj}, \gamma_{mj})_{LR}$. In the above model, each variable with "tilde sign (~)" over it , shows a fuzzy variable or uncertain parameter.

In order to solve the FUSApHLP, the fuzzy model should be transformed into a deterministic model by using possibility and necessity measures in each constraint with fuzzy variables and applying fuzzy chance-constrained programming (FCCP). Now, problem 1 is converted into the following problem by applying the FCCP:

Problem 2:

$Min\ z$

$$s.t. \quad Pos\left(\left[\sum_{k=1}^{N} \left(\tilde{C}_{ik} + \alpha \tilde{C}_{km}\right) X_{ik} + \tilde{C}_{mj} X_{jm}\right] \le z\right) \ge \eta i, j, m = 1, \dots, n \quad (8)$$

Constraint (2) – (5),

where η is a predetermined possibility level and $Pos\left(\left[\sum_{k=1}^{N} \left(\tilde{C}_{ik} + \alpha \tilde{C}_{km}\right) X_{ik} + \tilde{C}_{mj} X_{jm}\right] \le z\right)$ is defined as follwes:

$$Pos\left(\tilde{z}_{ijm} \le z\right) = \sup_{y_1, y_2} \left\{min\left[\mu_{\tilde{z}_{ijm}}(y_1), \mu_z(y_2)\right] | y_1 \le y_2\right\}, \quad (9)$$

where $\tilde{z}_{ijm} = \sum_{k=1}^{N} \left(\tilde{C}_{ik} + \alpha \tilde{C}_{km}\right) X_{ik} + \tilde{C}_{mj} X_{jm}$

Now, we obtain the following theorem to convert problem 2 to deterministic programming.

Theorem 1:

$$Pos\left(\tilde{z}_{ijm} \le z\right) \ge \eta \iff \sum_{k=1}^{N} (C_{ik}^0 + \alpha C_{km}^0) X_{ik} + C_{mj}^0 X_{jm} - L^*(\eta) \left[\sum_{k=1}^{N} (\beta_{ik} + \alpha \beta_{km}) X_{ik} + \beta_{mj} X_{jm}\right] \le z \quad (10)$$

Where $L^*(\eta)$ is pseudo inverse function and is defined as $L^*(\lambda) = \sup\{t|L(t) \geq \lambda\}$. η indicates the level of possibility, for example if $\eta = 1$ then the model output would be the same as non-fuzzy mode.

So the complete possibility model is represented by the following problem:

Problem 3:

$Min\ z$

$$s.t. \quad \sum_{k=1}^{N}(C_{ik}^0 + \alpha C_{km}^0)X_{ik} + C_{mj}^0 X_{jm} - L^*(\eta)\left[\sum_{k=1}^{N}(\beta_{ik} + \alpha\beta_{km})X_{ik} + \beta_{mj}X_{jm}\right] \leq z \qquad i,j,m = 1,\dots,n$$

Constraint (2) – (5).

(11)

Furthermore, for pessimistic decision makers, we apply the necessity measures in the FCCP approach like the previous model as follows:

Problem 4:

$Min\ z$

$$s.t. \quad Nec\left(\left[\sum_{k=1}^{N}\left(\tilde{C}_{ik} + \alpha\tilde{C}_{km}\right)X_{ik} + \tilde{C}_{mj}X_{jm}\right] \leq z\right) \geq \eta i,j,m = 1,\dots,n \qquad (12)$$

Constraint (2) – (5),

where $Nec\left(\left[\sum_{k=1}^{N}\left(\tilde{C}_{ik} + \alpha\tilde{C}_{km}\right)X_{ik} + \tilde{C}_{mj}X_{jm}\right] \leq z\right)$ is defined as

$$Nec\left(\tilde{z}_{ijm} \leq z\right) = \inf_{y_1,y_2}\left\{max\left[1 - \mu_{\tilde{z}_{ijm}}(y_1), 1 - \mu_z(y_2)\right]|y_1 \leq y_2\right\} \qquad (13)$$

Like the possibility model, we obtain the following theorem to transform problem 4 to a deterministic problem.

Theorem 2:

$$Nec\left(\tilde{z}_{ijm} \leq z\right) \geq \eta \Leftrightarrow \sum_{k=1}^{N}(C_{ik}^0 + \alpha C_{km}^0)X_{ik} + C_{mj}^0 X_{jm} - L^*(1-\eta)\left[\sum_{k=1}^{N}(\beta_{ik} + \alpha\beta_{km})X_{ik} + \beta_{mj}X_{jm}\right] \leq z$$

(14)

Problem 5:

$Min\ z$

$$s.t. \quad \sum_{k=1}^{N}(C_{ik}^{0}+\alpha C_{km}^{0})X_{ik}+C_{mj}^{0}X_{jm}-L^{*}(1-\eta)\left[\sum_{k=1}^{N}(\beta_{ik}+\alpha\beta_{km})X_{ik}+\beta_{mj}X_{jm}\right]\leq z \qquad i,j,m=1,\dots,n$$

Constraint (2) – (5).

(15)

3-2- FUMApHCP

In multiple allocations each none-hub node can be allocated to more than one hub node. The mathematical model for multiple allocation of pHub center proposed by Ernst et al. (2009) is as follows:

$Min\ z$

$$s.t. \quad z\geq\sum_{k=1}^{N}\sum_{m=1}^{N}y_{ijkm}\left(C_{ik}+\alpha C_{km}+C_{mj}\right) \qquad i,j=1,\dots,n$$

$$\sum_{k=1}^{N}\sum_{m=1}^{N}y_{ijkm}=1\,i,j=1,\dots,n \tag{16}$$

$$\sum_{k=1}^{N}y_{ijkm}\leq Z_{m}i,j,m=1,\dots,n \tag{17}$$

$$\sum_{m=1}^{N}y_{ijkm}\leq Z_{k}i,j,k=1,\dots,n \tag{18}$$

$$\sum_{k=1}^{N}Z_{k}=p \tag{19}$$

$$Z_{k},y_{ijkm}\epsilon\{0,1\} \qquad i,j,k,m=1,\dots,n \tag{20}$$

The variable y_{ijkm} represents the allocation of node i to hub k and node j to hub m, so the origin-destination path is $i-k-m-j$. The variable $Z_k$indicates the index of the hubs that will be established. The process of developing UMApHCP to FUMApHCP is the same as previous section that mentioned above:

Problem 6:

$Min\ z$

$$s.t. \quad z\geq\sum_{k=1}^{N}\sum_{m=1}^{N}y_{ijkm}\left(\tilde{C}_{ik}+\alpha\tilde{C}_{km}+\tilde{C}_{mj}\right)\ i,j=1,\dots,n \tag{21}$$

Constraint (16) – (20).

By applying FCCP approach with possibility measures for the above problem, we have:

Problem 7:

$Min\ z$

$$s.t.\quad Pos(z \geq \sum_{k=1}^{N}\sum_{m=1}^{N} y_{ijkm}\big(\tilde{C}_{ik} + \alpha\tilde{C}_{km} + \tilde{C}_{mj}\big)) \geq \eta\ i,j = 1,\dots,n \tag{22}$$

Constraint (16) – (20).

Like previous section, we achieve the following proposition:

Proposition 1:

$Pos\big(\tilde{z}_{ij} \leq z\big) \geq \eta\ \Leftrightarrow$

$$\sum_{k=1}^{N}\sum_{m=1}^{N} y_{ijkm}\big(C_{ik}^{0} + \alpha C_{km}^{0} + C_{mj}^{0}\big) - L^{*}(\eta)\left[\sum_{k=1}^{N}\sum_{m=1}^{N} y_{ijkm}\big(\beta_{ik} + \alpha\beta_{km} + \beta_{mj}\big)\right] \leq z \tag{23}$$

where $\tilde{z}_{ij} = \sum_{k=1}^{N}\sum_{m=1}^{N} y_{ijkm}\big(\tilde{C}_{ik} + \alpha\tilde{C}_{km} + \tilde{C}_{mj}\big)$.

Then, the possibility model of FUMApHCP is represented as

Problem 8:

$Min\ z$

$$s.t.\quad \sum_{k=1}^{N}\sum_{m=1}^{N}\big(C_{ik}^{0} + \alpha C_{km}^{0} + C_{mj}^{0}\big)y_{ijkm} - L^{*}(\eta)\sum_{k=1}^{N}\sum_{m=1}^{N}\big(\beta_{ik} + \alpha\beta_{km} + \beta_{mj}\big)y_{ijkm} \leq z \tag{24}$$

$$i,j = 1,\dots,n$$

Constraint (16) – (20).

Furthermore, based on the necessity measures, we have the following problem:

Problem 9:

$Min\ z$

$$s.t.\quad Nec(z \geq \sum_{k=1}^{N}\sum_{m=1}^{N} y_{ijkm}\big(\tilde{C}_{ik} + \alpha\tilde{C}_{km} + \tilde{C}_{mj}\big)) \geq \eta\ i,j = 1,\dots,n \tag{25}$$

Constraint (16) – (20).

Proposition 2:

$Nec\big(\tilde{z}_{ij} \leq z\big) \geq \eta\ \Leftrightarrow$

$$\sum_{k=1}^{N}\sum_{m=1}^{N} y_{ijkm}\big(C_{ik}^{0} + \alpha C_{km}^{0} + C_{mj}^{0}\big) - L^{*}(1-\eta)\left[\sum_{k=1}^{N}\sum_{m=1}^{N} y_{ijkm}\big(\beta_{ik} + \alpha\beta_{km} + \beta_{mj}\big)\right] \leq z \tag{26}$$

Finally, the necessity model of FUMApHCP is represented as follows:

Problem 10:

$Min\ z$

$$s.t.\quad \sum_{k=1}^{N}\sum_{m=1}^{N} y_{ijkm}\left(C_{ik}^{0}+\alpha C_{km}^{0}+C_{mj}^{0}\right)-L^{*}(1-\eta)\left[\sum_{k=1}^{N}\sum_{m=1}^{N} y_{ijkm}\left(\beta_{ik}+\alpha\beta_{km}+\beta_{mj}\right)\right] \leq z \qquad i,j=1,\dots,n \tag{27}$$

Constraint (16) – (20).

All obtained deterministic problems are easily solved by one of the MIP solvers.

4-Numerical Experiments

In this paper, we used popular CAB data set for numerical tests. The CAB data set was represented by O'Kelly (1987) for hub location problems. The CAB data set is based on Civil Aeronautics Board in 1970, which is generated from the flow of airline passengers in 25 cities in United States. Numbers in CAB data set are symmetric and satisfies triangular inequity. We used GAMS v24.1.2 to solve fuzzy phub center problem. A PC with Core i5 processor and 8GB RAM was used for performing experiments.
For solving our fuzzy models we need to use input data in form of (x_1, x_2, x_3) where x_2 is the crisp number, and x_1, x_3 are left and right values. The crisp and middle number (x_2) is assumed to be the original number in CAB data set. Assume that φ is the original number in data set, to generate right and left values the following relation is used: $(1 \pm r)\varphi$, Where $0 \leq r \leq 1$. The value of r depends on level of uncertainty and the decision maker can change it according to his opinion. We assumed $r = 0.2$ so the right and left values are obtained respectively by 1.2φ and 0.8φ.

We divided numerical tests for solving these two problems (both single and multiple allocations for fuzzy phub center) into several sub problems. These sub problems are generated by using different values for model features. The features conclude the following cases:

- Size of problem: different problem sizes are found by taking the top 10, 15, 20 and 25 nodes from CAB data set.
- Discount factor: various varies of $\alpha = \{0.2, 0.4, 0.6, 0.8\}$
- Number of hubs to locate: different values of $p = \{2, 3, 4, 5\}$
- Possibility or necessity
- Pseudo inverse functions: the functions $R^*(h)$ and $L^*(\eta)$ in models represent probability and possibility levels, where $R^*(h) = L^*(h) = 1 - h$. So different probability
  Levels are obtained by using h, η as 0.1 , 0.3 , 0.5 , 0.7 , 0.9 .

Sub problems are shown as $x.y.z$, where x is the size of problem, y is the discount factor value and z is the value of p. For example sub problem 25.2.3 represents 25 nodes with $\alpha = 0.2$ and $p = 3$.

Results for the fuzzy single allocation phub center model are shown in Table I and for the fuzzy multiple allocation phub center model the results are shown in Table II. According to the results of Table I and Table II, in the same problem with the lowest possibility level for possibility-based model and the highest possibility level for necessity-based model, the optimal solutions for both possibility and necessity-based models are same.

Fig.1 and Fig.2 represent the optimal solution for different problems in the case that h changes. For the possibility cases of minimizing objective functions like phub center, with the increase of h the objective function increases too and in necessity cases the optimal value decreases.

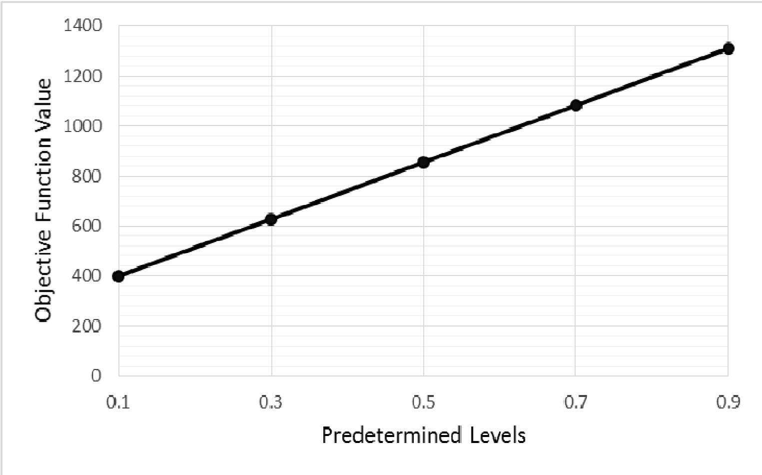

Fig. 1. Problem: 10.2.2 of single allocation phub center for possibility

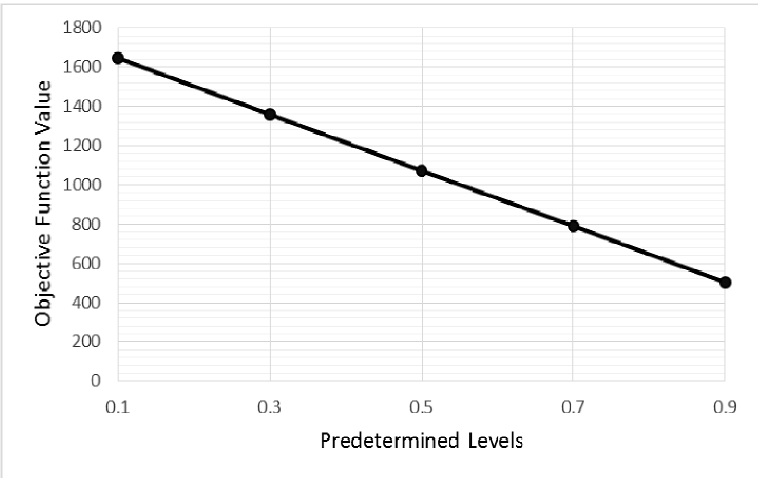

Fig. 2. Problem: 20.4.3 of Single allocation phub center for necessity

Any decision maker can consider other levels based on his/her circumstances or any other constraints. Therefore, the decision maker's opinion can be classified as follows:

1. *Best optimal solution:* this vision of the decision maker does not deal with any levels of possibility or necessity. The decision maker chooses the best allocation of spokes and the optimal hubs to locate and there is no restriction for selecting the possibility/necessity levels.

2. *The lowest or highest level:* in this point of view, the decision maker chooses only lowest or highest levels of possibility/necessity for locating hubs and allocating none-hub nodes. We considered $h = 0.1$ as the lowest level and $h = 0.9$ as the highest level.

3. *The middle levels:* in this perspective, the decision maker wants to have middle levels. This view happens when the DM does not have absolute information about the levels and decides to have middle levels.

In this section, we treated the possibility/necessity levels of $\{0.1, 0.3, 0.5, 0.7, 0.9\}$onlyas sample levels to obtain optimal solutions, so there is no limitation for the decision maker to choose only them to find the optimal solutions. One can choose any other levels between$(0,1]$ to find his/her optimal solutions.

The map of the geographical locations of the cities in the CAB data set and the optimal solution for some of the problems is shown on Fig.3 and Fig.4. The optimal solution among all possibility or necessity level is chosen for the problem which is illustrated. Both figures considered only single allocation of the problem.

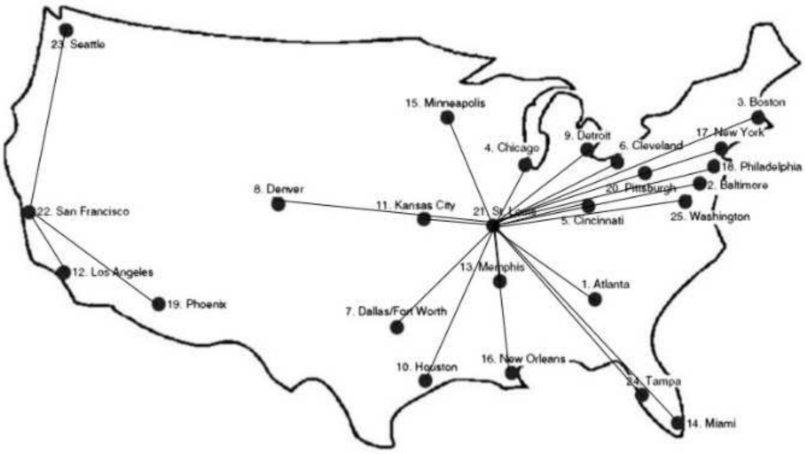

Fig. 3. Optimal solution of CAB data set with 25 nodes and two numbers of hubs to located with $\alpha = 0.2$

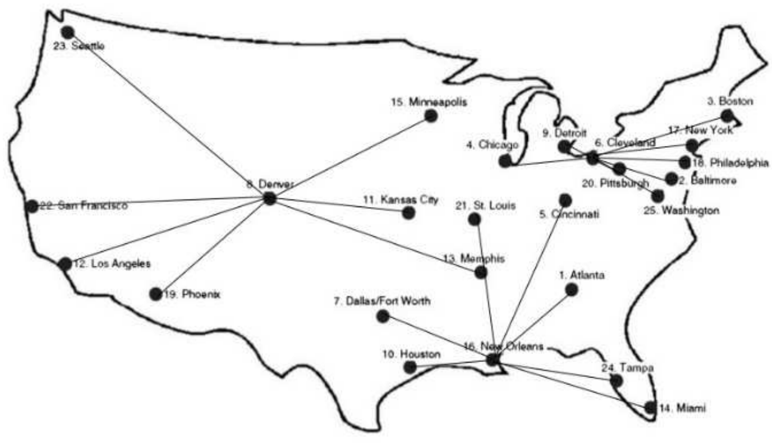

Fig. 4. Optimal solution of CAB data set with 25 nodes and three numbers of hubs to located with $\alpha = 0.8$

Table I. Numerical Results of FUSApHCP for CAB dataset

prob	Possibility					Necessity				
	h=0.1	h=0.3	h=0.5	h=0.7	h=0.9	h=0.1	h=0.3	h=0.5	h=0.7	h=0.9
10.2.2	399.163	627.256	855.349	1083.442	1311.535	1311.535	1083.442	855.349	627.256	399.163
10.2.3	313.470	492.595	671.721	850.846	1029.972	1029.972	850.846	671.721	492.595	313.47
10.2.4	235.017	369.313	503.608	637.904	772.199	772.199	637.904	503.608	369.313	235.017
10.2.5	206.148	323.947	441.746	559.545	677.343	677.343	559.545	441.746	323.947	206.148
10.4.2	470.026	738.613	1007.199	1275.786	1544.372	1544.372	1275.786	1007.199	738.613	470.026
10.4.3	331.818	521.429	711.039	900.649	1090.260	1090.26	900.649	711.039	521.429	331.818
10.4.4	271.096	426.008	580.919	735.831	890.743	890.743	735.831	580.919	426.008	271.096
10.4.5	241.152	378.953	516.754	654.556	792.357	792.357	654.556	516.754	378.953	241.152
10.6.2	492.556	774.017	1055.478	1336.938	1618.399	1618.399	1336.938	1055.478	774.017	492.556
10.6.3	401.071	630.254	859.437	1088.620	1317.804	1317.804	1088.62	859.437	630.254	401.071
10.6.4	320.932	504.322	687.711	871.101	1054.491	1054.491	871.101	687.711	504.322	320.932
10.6.5	301.653	474.026	646.399	818.773	991.146	991.146	818.773	646.399	474.026	301.653
10.8.2	492.556	774.017	1055.478	1336.938	1618.399	1618.399	1336.938	1055.478	774.017	492.556
10.8.3	466.158	732.533	998.909	1265.285	1531.661	1531.661	1265.285	998.909	732.533	466.158
10.8.4	410.636	645.285	879.935	1114.584	1349.233	1349.233	1114.584	879.935	645.285	410.636
10.8.5	395.313	621.206	847.100	1072.993	1298.886	1298.886	1072.993	847.1	621.206	395.313
15.2.2	568.340	893.105	1217.871	1542.637	1867.402	1867.402	1542.637	1217.871	893.105	568.34
15.2.3	492.841	774.464	1056.087	1337.711	1619.334	1619.334	1337.711	1056.087	774.464	492.841
15.2.4	381.197	599.024	816.851	1034.678	1252.505	1252.505	1034.678	816.851	599.024	381.197
15.2.5	321.887	505.822	689.758	873.693	1057.629	1057.629	873.693	689.758	505.822	321.887
15.4.2	605.009	950.729	1296.449	1642.168	1987.888	1987.888	1642.168	1296.449	950.729	605.009
15.4.3	492.841	774.464	1056.087	1337.711	1619.334	1619.334	1337.711	1056.087	774.464	492.841
15.4.4	401.626	631.127	860.628	1090.129	1319.630	1319.63	1090.129	860.628	631.127	401.626
15.4.5	359.682	565.215	770.748	976.280	1181.813	1181.813	976.28	770.748	565.215	359.682
15.6.2	619.945	974.200	1328.454	1682.709	2036.964	2036.964	1682.709	1328.454	974.2	619.945
15.6.3	516.578	811.766	1106.953	1402.141	1697.328	1697.328	1402.141	1106.953	811.766	516.578
15.6.4	491.263	771.985	1052.707	1333.428	1614.150	1614.15	1333.428	1052.707	771.985	491.263
15.6.5	436.813	686.421	936.028	1185.636	1435.243	1435.243	1185.636	936.028	686.421	436.813
15.8.2	700.539	1100.847	1501.154	1901.462	2301.770	2301.77	1901.462	1501.154	1100.847	700.539
15.8.3	606.631	953.277	1299.923	1646.569	1993.215	1993.215	1646.569	1299.923	953.277	606.631
15.8.4	582.417	915.227	1248.037	1580.847	1913.657	1913.657	1580.847	1248.037	915.227	582.417
15.8.5	598.148	939.946	1281.745	1623.543	1965.342	1965.342	1623.543	1281.745	939.946	598.148
20.2.2	530.037	832.916	1135.794	1438.673	1741.552	1741.552	1438.673	1135.794	832.916	530.037
20.2.3	434.350	682.550	930.750	1178.950	1427.150	1427.15	1178.95	930.75	682.55	434.35
20.2.4	379.516	596.382	813.248	1030.114	1246.980	1246.98	1030.114	813.248	596.382	379.516
20.2.5	340.358	534.848	729.339	923.829	1118.319	1118.319	923.829	729.339	534.848	340.358
20.4.2	605.009	950.729	1296.449	1642.168	1987.888	1987.888	1642.168	1296.449	950.729	605.009
20.4.3	501.370	787.867	1074.364	1360.861	1647.358	1647.358	1360.861	1074.364	787.867	501.37
20.4.4	412.358	647.991	883.624	1119.257	1354.891	1354.891	1119.257	883.624	647.991	412.358
20.4.5	389.153	611.525	833.898	1056.271	1278.644	1278.644	1056.271	833.898	611.525	389.153
20.6.2	636.908	1000.855	1364.802	1728.749	2092.696	2092.696	1728.749	1364.802	1000.855	636.908
20.6.3	559.382	879.029	1198.676	1518.323	1837.969	1837.969	1518.323	1198.676	879.029	559.382
20.6.4	513.753	807.326	1100.899	1394.471	1688.044	1688.044	1394.471	1100.899	807.326	513.753
20.6.5	479.468	753.450	1027.432	1301.414	1575.396	1575.396	1301.414	1027.432	753.45	479.468
20.8.2	702.287	1103.595	1504.902	1906.209	2307.516	2307.516	1906.209	1504.902	1103.595	702.287
20.8.3	651.407	1023.640	1395.872	1768.105	2140.337	2140.337	1768.105	1395.872	1023.64	651.407
20.8.4	633.884	996.104	1358.324	1720.543	2082.763	2082.763	1720.543	1358.324	996.104	633.884
20.8.5	622.443	978.124	1333.806	1689.487	2045.169	2045.169	1689.487	1333.806	978.124	622.443
25.2.2	596.735	937.727	1278.719	1619.710	1960.702	1960.702	1619.71	1278.719	937.727	596.735
25.2.3	538.473	846.172	1153.871	1461.570	1769.269	1769.269	1461.57	1153.871	846.172	538.473
25.2.4	467.798	735.110	1002.423	1269.736	1537.049	1537.049	1269.736	1002.423	735.11	467.798
25.2.5	381.197	599.024	816.851	1034.678	1252.505	1252.505	1034.678	816.851	599.024	381.197
25.4.2	672.713	1057.120	1441.527	1825.935	2210.34	2210.34	1825.935	1441.527	1057.12	672.713
25.4.3	596.735	937.727	1278.719	1619.710	1960.702	1960.702	1619.71	1278.719	937.727	596.735
25.4.4	527.756	829.332	1130.907	1432.482	1734.057	1734.057	1432.482	1130.907	829.332	527.756
25.4.5	447.927	703.885	959.844	1215.802	1471.760	1471.76	1215.802	959.844	703.885	447.927
25.6.2	733.631	1152.848	1572.066	1991.283	2410.500	2410.5	1991.283	1572.066	1152.848	733.631
25.6.3	655.271	1029.712	1404.153	1778.593	2153.034	2153.034	1778.593	1404.153	1029.712	655.271
25.6.4	617.809	970.843	1323.876	1676.910	2029.944	2029.944	1676.91	1323.876	970.843	617.809
25.6.5	573.751	901.609	1229.467	1557.324	1885.182	1885.182	1557.324	1229.467	901.609	573.751
25.8.2	760.179	1194.567	1628.956	2063.344	2497.732	2497.732	2063.344	1628.956	1194.567	760.179
25.8.3	721.911	1134.431	1546.952	1959.472	2371.993	2371.993	1959.472	1546.952	1134.431	721.911
25.8.4	719.517	1130.669	1541.822	1952.974	2364.127	2364.127	1952.974	1541.822	1130.669	719.517
25.8.5	695.903	1093.561	1491.220	1888.879	2286.538	2286.538	1888.879	1491.22	1093.561	695.903

Table II. Numerical Results of FUMApHCP for CAB dataset

prob	Possibility					Necessity				
	h=0.1	h=0.3	h=0.5	h=0.7	h=0.9	h=0.1	h=0.3	h=0.5	h=0.7	h=0.9
10.2.2	398.128	625.629	853.131	1080.632	1308.134	1308.134	1080.632	853.131	625.629	398.128
10.2.3	313.470	492.595	671.721	850.846	1029.972	1029.972	850.846	671.721	492.595	313.47
10.2.4	226.622	356.120	485.619	615.117	744.615	744.615	615.117	485.619	356.12	226.622
10.2.5	206.148	323.947	441.746	559.545	677.343	677.343	559.545	441.746	323.947	206.148
10.4.2	438.486	689.050	939.613	1190.176	1440.740	1440.74	1190.176	939.613	689.05	438.486
10.4.3	330.783	519.802	708.821	897.840	1086.859	1086.859	897.84	708.821	519.802	330.783
10.4.4	271.096	426.008	580.919	735.831	890.743	890.743	735.831	580.919	426.008	271.096
10.4.5	241.152	378.953	516.754	654.556	792.357	792.357	654.556	516.754	378.953	241.152
10.6.2	508.206	798.610	1089.014	1379.418	1669.821	1669.821	1379.418	1089.014	798.61	508.206
10.6.3	371.771	584.211	796.651	1009.092	1221.532	1221.532	1009.092	796.651	584.211	371.771
10.6.4	343.802	540.260	736.718	933.176	1129.634	1129.634	933.176	736.718	540.26	343.802
10.6.5	302.505	475.365	648.224	821.084	993.944	993.944	821.084	648.224	475.365	302.505
10.8.2	489.730	769.576	1049.422	1329.268	1609.113	1609.113	1329.268	1049.422	769.576	489.73
10.8.3	420.718	661.129	901.540	1141.950	1382.361	1382.361	1141.95	901.54	661.129	420.718
10.8.4	395.313	621.206	847.100	1072.993	1298.886	1298.886	1072.993	847.1	621.206	395.313
10.8.5	395.313	621.206	847.100	1072.993	1298.886	1298.886	1072.993	847.1	621.206	395.313
15.2.2	561.405	882.208	1203.011	1523.813	1844.616	1844.616	1523.813	1203.011	882.208	561.405
15.2.3	480.520	755.103	1029.685	1304.268	1578.851	1578.851	1304.268	1029.685	755.103	480.52
15.2.4	360.577	566.621	772.666	978.710	1184.754	1184.754	978.71	772.666	566.621	360.577
15.2.5	321.887	505.822	689.758	873.693	1057.629	1057.629	873.693	689.758	505.822	321.887
15.4.2	603.351	948.122	1292.894	1637.666	1982.438	1982.438	1637.666	1292.894	948.122	603.351
15.4.3	486.731	764.862	1042.994	1321.126	1599.258	1599.258	1321.126	1042.994	764.862	486.731
15.4.4	413.349	649.548	885.748	1121.947	1358.146	1358.146	1121.947	885.748	649.548	413.349
15.4.5	370.417	582.084	793.751	1005.418	1217.085	1217.085	1005.418	793.751	582.084	370.417
15.6.2	615.954	967.928	1319.902	1671.876	2023.850	2023.85	1671.876	1319.902	967.928	615.954
15.6.3	529.068	831.392	1133.717	1436.041	1738.366	1738.366	1436.041	1133.717	831.392	529.068
15.6.4	492.906	774.566	1056.227	1337.888	1619.548	1619.548	1337.888	1056.227	774.566	492.906
15.6.5	436.813	686.421	936.028	1185.636	1435.243	1435.243	1185.636	936.028	686.421	436.813
15.8.2	678.664	1066.472	1454.280	1842.088	2229.897	2229.897	1842.088	1454.28	1066.472	678.664
15.8.3	614.255	965.258	1316.261	1667.264	2018.267	2018.267	1667.264	1316.261	965.258	614.255
15.8.4	582.417	915.227	1248.037	1580.847	1913.657	1913.657	1580.847	1248.037	915.227	582.417
15.8.5	582.417	915.227	1248.037	1580.847	1913.657	1913.657	1580.847	1248.037	915.227	582.417
20.2.2	530.499	833.641	1136.783	1439.925	1743.067	1743.067	1439.925	1136.783	833.641	530.499
20.2.3	464.007	729.154	994.301	1259.448	1524.595	1524.595	1259.448	994.301	729.154	464.007
20.2.4	374.712	588.834	802.955	1017.076	1231.198	1231.198	1017.076	802.955	588.834	374.712
20.2.5	328.511	516.232	703.953	891.674	1079.394	1079.394	891.674	703.953	516.232	328.511
20.4.2	567.754	892.185	1216.616	1541.047	1865.477	1865.477	1541.047	1216.616	892.185	567.754
20.4.3	486.731	764.862	1042.994	1321.126	1599.258	1599.258	1321.126	1042.994	764.862	486.731
20.4.4	429.520	674.961	920.401	1165.841	1411.281	1411.281	1165.841	920.401	674.961	429.52
20.4.5	387.412	608.790	830.168	1051.546	1272.924	1272.924	1051.546	830.168	608.79	387.412
20.6.2	629.476	989.177	1348.878	1708.579	2068.280	2068.28	1708.579	1348.878	989.177	629.476
20.6.3	555.828	873.445	1191.061	1508.677	1826.293	1826.293	1508.677	1191.061	873.445	555.828
20.6.4	519.347	816.117	1112.887	1409.657	1706.427	1706.427	1409.657	1112.887	816.117	519.347
20.6.5	465.452	731.424	997.396	1263.368	1529.341	1529.341	1263.368	997.396	731.424	465.452
20.8.2	719.890	1131.255	1542.621	1953.986	2365.352	2365.352	1953.986	1542.621	1131.255	719.89
20.8.3	664.318	1043.928	1423.538	1803.148	2182.758	2182.758	1803.148	1423.538	1043.928	664.318
20.8.4	602.839	947.319	1291.798	1636.278	1980.758	1980.758	1636.278	1291.798	947.319	602.839
20.8.5	582.417	915.227	1248.037	1580.847	1913.657	1913.657	1580.847	1248.037	915.227	582.417
25.2.2	1081.979	1700.252	2318.526	2936.800	3555.073	3555.073	2936.8	2318.526	1700.252	1081.979
25.2.3	535.249	841.106	1146.962	1452.819	1758.676	1758.676	1452.819	1146.962	841.106	535.249
25.2.4	467.798	735.110	1002.423	1269.736	1537.049	1537.049	1269.736	1002.423	735.11	467.798
25.2.5	364.370	572.581	780.792	989.003	1197.214	1197.214	989.003	780.792	572.581	364.37
25.4.2	1200.170	1885.982	2571.793	3257.605	3943.416	3943.416	3257.605	2571.793	1885.982	1200.17
25.4.3	578.107	908.454	1238.801	1569.148	1899.495	1899.495	1569.148	1238.801	908.454	578.107
25.4.4	521.646	819.730	1117.814	1415.897	1713.981	1713.981	1415.897	1117.814	819.73	521.646
25.4.5	472.240	742.091	1011.942	1281.794	1551.645	1551.645	1281.794	1011.942	742.091	472.24
25.6.2	766.981	1205.255	1643.530	2081.805	2520.080	2520.08	2081.805	1643.53	1205.255	766.981
25.6.3	638.964	1004.087	1369.210	1734.332	2099.455	2099.455	1734.332	1369.21	1004.087	638.964
25.6.4	611.070	960.253	1309.436	1658.620	2007.803	2007.803	1658.62	1309.436	960.253	611.07
25.6.5	556.976	875.248	1193.520	1511.792	1830.064	1830.064	1511.792	1193.52	875.248	556.976
25.8.2	795.967	1250.805	1705.643	2160.481	2615.319	2615.319	2160.481	1705.643	1250.805	795.967
25.8.3	760.713	1195.406	1630.099	2064.792	2499.485	2499.485	2064.792	1630.099	1195.406	760.713
25.8.4	716.226	1125.498	1534.770	1944.042	2353.314	2353.314	1944.042	1534.77	1125.498	716.226
25.8.5	670.440	1053.548	1436.657	1819.765	2202.874	2202.874	1819.765	1436.657	1053.548	670.44

5- Conclusion

In this paper, we studied uncapacitated phub center problem under uncertainty in the travel time or transportation costs. We presented generic models capturing these different sources of uncertainty for the single and the multiple allocation cases of the problem. Also we proposed new methods to solve the problem for optimistic and pessimistic decision makers separately. Our new approach uses differentpossibility and necessity measures to obtain the optimal solution of the phub center problem. The presented problem is converted to deterministic mixed integer programming problems for convenience of solving with MIP solvers. Finally, for the numerical experiments we performed extensive computational analysis with more than 250 sub problems on the CAB data set.

As one of the future research activities, the proposed approach in confronting uncertain parameters could be implemented on other hub location problems such as hub covering problems, multi objective hub location problems, other capacitated hub location problems and also some new hub location problems like the hub line location problem (Martins et. al. 2015). Another future research suggestion is providing solution methods to efficiently solve more realistic, large-scale instances for this class of fuzzy phub center problem. This includes solving the formulation with met-heuristic algorithms.

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