

## **An insight into the model structures applied in DEA-based bank branch efficiency measurements**

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### **Abstract**

In this paper, we focus on the Data Envelopment Analysis (DEA)-based model structures that have been used in assessing bank branch efficiency. Probing the methodologies of 75 published studies at the branch level since 1985 to early 2015, we found that these models can be divided into four categories: standard basic DEA models, single level and multi-level models, enriched (hybrid) models and special models. Also, summary statistics for DEA applications in bank branches from the perspectives of different measurement approaches adopted by researchers and the frequency of appearing the models of each category in the literature of discussion are derived and presented. The illustrated statistical comparisons show that the popularity of multi-level models than the single level models are on the rise. Furthermore, as a result, we can conclude that from the perspective of performance measurement approaches applied to bank branches, the production approach is more widely used than the others.

**Keywords:** Bank branch, data envelopment analysis, efficiency, model structures.

### **1- Introduction**

Data Envelopment Analysis (DEA) has become one of the most widely used instruments for measuring bank branch efficiency. As pointed in a citation-based DEA survey by Liu et al. (2013), it is expected that the literature will grow to at least double its current size. Fethi & Pasiouras (2010) identified DEA as the most widely used operational research technique in assessing bank performance. However, the majority of DEA banking studies have focused on banks at an institutional level, rather than at the branch level. This can be partially attributed to the difference of data availability. The majority of banks are publicly traded firms that are listed on major stock exchanges and thus, must provide their investors with quarterly and annual financial reports (LaPlante & Paradi, 2015). This makes the collection of data for institutional level analysis rather easy. On the contrary, branch level data is proprietary information and is not generally disclosed to the public. Instead, it is

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either aggregated into bank financial reports or not reported at all. Nonetheless, surveys have shown that there has been a steady increase in DEA branch studies, nearly doubling in the last five years alone (Paradi & Zhu, 2013).

To date, there are four survey papers that reviewed DEA applications in the banking industry. Three of them focused on the studies that analyzed efficiency at the bank level and one of them at the branch level. Berger & Humphrey (1997) were the first to review five major efficiency analysis techniques including DEA on the financial institutions level to make some useful comparisons between their average efficiency levels. Out of the total of 130 studies reviewed by them, there were 57 DEA based papers, 42 focusing at the bank level and 15 on the branch level. Berger (2007) reviewed over 100 applications of frontier techniques that compared bank efficiencies across nations. Fethi & Pasiouras (2010) reviewed 196 studies employing operational research and artificial intelligence techniques in the assessment of bank performance. Among the 196 studies, 151 of them used DEA-like techniques to measure bank efficiency and productivity growth, and only 30 studies focused on the branch level. Paradi & Zhu (2013) found 275 DEA applications in the banking sector between 1985 and 2011, among them 195 studies examined banking institutions as a whole, but only 80 on the branch level.

Despite these four surveys on the use of DEA at the bank and branch level, there is no research at the branch level (not at the bank level) concerning only the model structures and mathematical programming formulates applied in this context. Specifically, one can study on existing assessments carried out on bank branches from two perspectives: one is focusing on various input sets and output sets as indexes of assessment. The other is investigating multifarious model structures applied by users in these assessments. Ahn & Lee (2014) have provided an insight into the specification of inputs and outputs for DEA-based bank efficiency measurements. They aimed at examining whether the input-output specifications for banks in DEA applications are in consistence with the criteria upon which banks make decisions. Four bank behavior models which are most popularly employed to determine input and output factors in DEA studies - the intermediation approach, production approach, user cost approach and value added approach - were comprehensively discussed and reviewed by them. Our contribution in this paper is to focus specifically on investigating various DEA model structures and measurement approaches applied at the branch level in order to classify them into several categories. Cook & Seiford (2009) paid attention to various models of DEA, including basic DEA models and multi-level models, and other special considerations regarding the status of variables, different multiplier restrictions and data variations. But their study did not include bank branch-specific applications. However, we've found no study that specifically address and classify the different types of methodologies and models have been used in efficiency measurement of bank branches. With a growing number of studies using DEA in bank branch analysis, a survey of this field would be useful and timely.

Since the first published paper about DEA applications in a U.S. bank branch setting by Sherman & Gold (1985), our paper considers the main research purposes and model structures of 75 available DEA-based published studies on bank branch efficiency until 2015. In this study, we have tried to provide a summary of these models and approaches in order to achieve our judgments about them. Then we categorize these structures from different aspects. This paper is organized as follows. Section II statistically discusses several most common performance measurement approaches that have been applied in the banking industry at the branch level. Section III focuses on models from four points of view: basic DEA models, single level and multi-level models, hybrid models (combining DEA with other techniques and strategies), and special models. Other operational research and statistical techniques that are used for enriching DEA models are presented in this section. In section IV, we draw our conclusions.

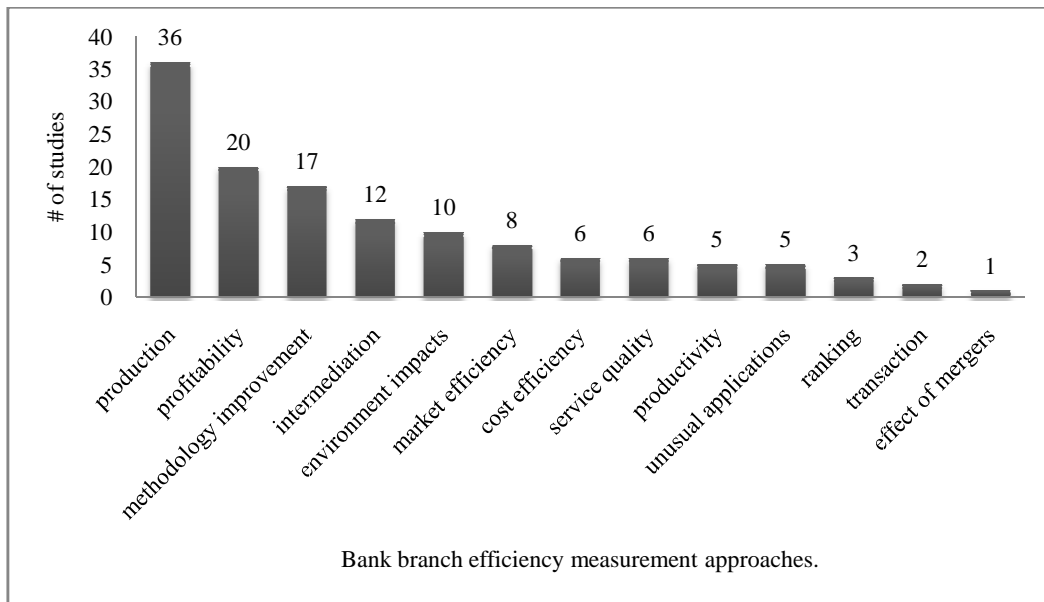
## **2- Statistical analysis of different measurement approaches in data envelopment analysis applied to assessing bank branches**

Different measurement approaches were proposed by Paradi & Zhu (2013). But we found other additional views in our survey that would be mentioned subsequently. These approaches that sometimes called as bank behavior models determine the purpose of an evaluation study. These approaches constitute an important part of model structure of a given study on bank branch efficiency. This purpose might be defined by either management of bank or by researcher. Some studies have

explicitly expressed their adopted approach, but most of them did not specify the main adopted views in the text of their paper. In these cases, we must distinguish the main research purpose of that respective study ourselves. Our diagnosis is based on the types of inputs and outputs were employed in the study. We should notice that the specification of input and output factors in DEA applications is in consistency with the criteria upon which banks make decisions. This section aims to provide an insight and statistical comparison into all existing approaches which are employed in DEA efficiency studies. They are:

1. Methodology improvement: Prior to 1995, the use of DEA in bank branch studies mainly focused on directly applying standard DEA models to assess branch efficiency. Since 1997, the DEA research has gradually shifted towards dealing with both the theoretical extensions and practical applications of DEA. The flexibility of DEA models and the complexity of bank branch operating characteristics offer researchers significant opportunities to develop new models, which are needed in different application situations and with specific purposes. Two lines of research have emerged around the DEA methodology improvement: extending the traditional DEA models and combining DEA models with other advanced operational research methodologies (Paradi & Zhu, 2013).
2. Branch production analysis: Production efficiency is one of most significant dimensions of bank branch performance. In bank branch analyses, the production model commonly views bank branches as producers of services using labor and other physical resources as inputs and providing services for taking deposits, making loans and others (number of transactions or document processing) as outputs.
3. Branch profitability analysis: Profitability is the measure of how well branches generate profits from their use of labor, assets and capital. It treats the branch as the producer of a product as opposed to the provider of a service.
4. Branch intermediation analysis: The branch's intermediary role is mainly studied to examine how efficient the branch is in collecting deposits and other funds from customers (inputs) and then lending the money in various forms of loans, mortgages, and other assets (investments).
5. Branch cost efficiency analysis: Cost efficiency evaluates the ability of a branch to produce current outputs at minimal cost.
6. Efficiency ranking.
7. Branch studies incorporating service quality: There are mainly two ways to incorporate service quality factors into branch performance analyses, either directly into the DEA model or conducting post-hoc analyses on the relationship between the DEA efficiency scores and the service quality reported.
8. Environmental impacts on branch performance: This approach accounts for the exogenous impacts, such as the impacts of locations, market power, regulations, organization, and new technologies in the evaluation of branches.
9. Effects of mergers and acquisition on branch performance.
10. Unusual banking applications of DEA.
11. Market efficiency: Market efficiency has an output maximization orientation and can be defined as the extent to which individual bank branches, given their capacity and resources available, utilize their market potential by maximizing sales (Athanasopoulos, 1998).
12. Transaction efficiency: Transactional efficiency is defined as the extent to which a bank branch moves general transactions away from a branch to alternative means of distribution (Portela & Thanassoulis, 2007).

Frequency of the use of the performance measurement approach mentioned above in the 75 studies we surveyed is illustrated in Figure (1). we can see that the production approach is the most widely used approach at the branch level. However, many studies have adopted more than one perspective.



**Fig 1.** Different measurement approaches used in DEA-based bank branch efficiency analysis.

The concept of transaction efficiency was first introduced by Giokas (2008). Later, Cook & Zhu (2010) used this approach to identify and build standards into the DEA analysis that led to introducing standard Decision Making Unit (DMU). Only one study by Sherman & Rupert (2006) has been found applying DEA to investigate the effects of mergers and acquisitions on branch performance. The production approach has been regarded from the first study at 1985 by Sherman & Gold (1985) until now.

### **3- Classification of DEA models have been used in bank branch efficiency literature**

#### **3-1- Standard basic DEA models**

Our discussion concentrates on four basic DEA models including the CCR model by Charnes et al. (1978), the BCC model by Banker et al. (1984), Additive model by Charnes et al. (1985) and the RAM (Range Adjusted Measure) model by Cooper et al. (1999). These models are most popularly employed in DEA-based bank efficiency studies and are here collectively referred to as the basic DEA models. At the advent of applications of DEA in assessing bank branch performance by Sherman & Gold (1985), most of the practitioners and modelers used the foregoing basic models sometimes along with weight restrictions and value judgments or without inputs/outputs. We found 15 studies that employed basic models with assurance region type I (ARI) and only study with absolute weight restrictions. The first study that made changes in basic models was Oral & Yolalan (1990). They added an equality constraint to the CCR model in order to force a special DMU called as 'global leader' 'to be in the reference set of all under evaluation DMUs. But almost from 1997, we found no study that uses a basic model in its original form. Today, models enriched with other operations research techniques and hybrid models (that will be explained in the next sections) are more favorable for practitioners. The conventional DEA models may fail to capture branch behaviors. In such cases, DEA results can hardly reflect the performance in its true sense, i.e. how bank branches perform against the goals that they decide to pursue. The findings suggest focusing on (DEA-based) performance measurement from a goal-oriented perspective, i.e. from the point of view of multi criteria decision making.

#### **3-2- Single level and multi-level models**

Some models generally pertain to single level situations in which we wish to evaluate the efficiency status of each member of a given set of DMUs at a given point in time. a number of efficiency

measurement situations can involve having to look at what might be regarded as multiple levels. We classify single level models as:

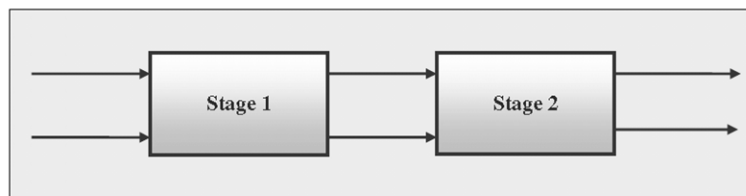
- The Constant Return to Scale (CRS) model
- The Variable Return to Scale (VRS) model
- The Additive model
- Slack-based measures (Green et al., 1997 and Tone, 2001)
- The Russell measure (Färe & Lovell, 1978)
- Other non-radial models (such as RAM)
- FDH- The Free Disposal Hull model
- Least distance projections

and multi-level models as:

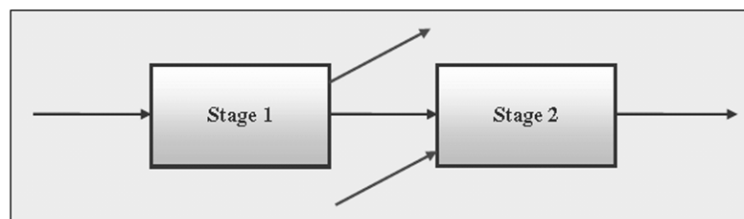
- multi-stage/serial models (such as network DEA and supply chains)
- multi-component/parallel models
- Hierarchical/nested models.

Although DEA can evaluate the relative efficiency of a set of DMUs, it cannot identify the sources of inefficiency in the DMUs because conventional DEA models view DMUs as black boxes that consume a set of inputs to produce a set of outputs (Avkiran, 2009). In such cases using single-stage DEA may result in inaccurate efficiency evaluation (Rho & An, 2007). In contrast, a two-stage DEA model allows one to further investigate the structure and processes inside the DMU, to identify the misallocation of inputs among sub-DMUs and generate insights about the sources of inefficiency within the DMU (Li et al., 2012).

The existing multi-stage DEA models in the literature can be classified into two categories as shown in Figure (2) and Figure (3): closed-system and open system models. In the closed-system DEA models, the intermediate outputs remain unchanged from one stage to another. In contrast, in the open-system DEA models, the intermediate outputs in one stage are partial inputs in a subsequent stage (Ebrahimnejad et al., 2014).



**Fig 2.** A closed two-stage DEA model



**Fig 3.** An open two-stage DEA model

In most applications of DEA presented in the literature, the models presented are designed to obtain a single measure of efficiency. In many instances however, the DMUs involved may perform several different and clearly identifiable functions, or can be separated into different components. There is a growing need to view performance in a more disaggregated sense, paying specific attention to different components of the operation. These components include different classes of products or sales activities, such as mutual funds and mortgages, and different elements of service. By measuring

a branch's performance on each of a set of such components, particular areas of strength and weakness can be identified and addressed, where necessary (Cook et al., 2000).

The idea of measuring efficiency relative to certain sub-processes or components of a DMU is not new. Färe and Grosskopf (1996), for example, look at a multi-stage process where in intermediate products or outputs at one stage, can be both final products and inputs to later stages of production. As an example of multi-component models, consider the following dual component model proposed by Cook et al. (2000). They supposed that all branch transactions are classified into two groups, sales and service, as shown in Figure (4). The branch performance is simultaneously assessed along with sales component and service component.

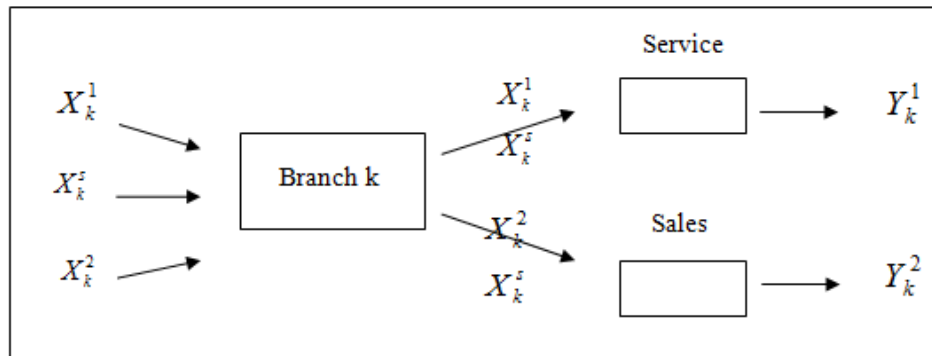


Fig 4. A dual component model in assessing bank branches

However, our studies show that the use of multi-level models is increasing. As Figure (5) shows, 47 from 75 studies used multi-level models.

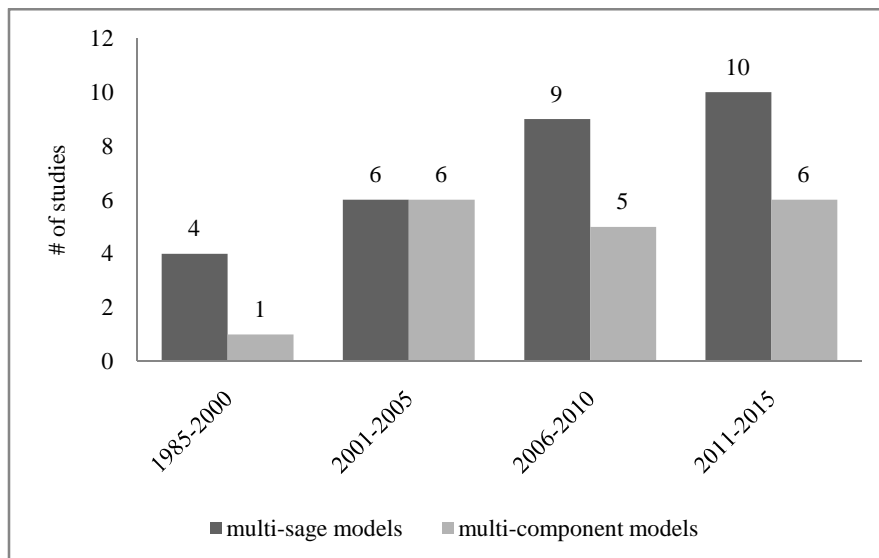
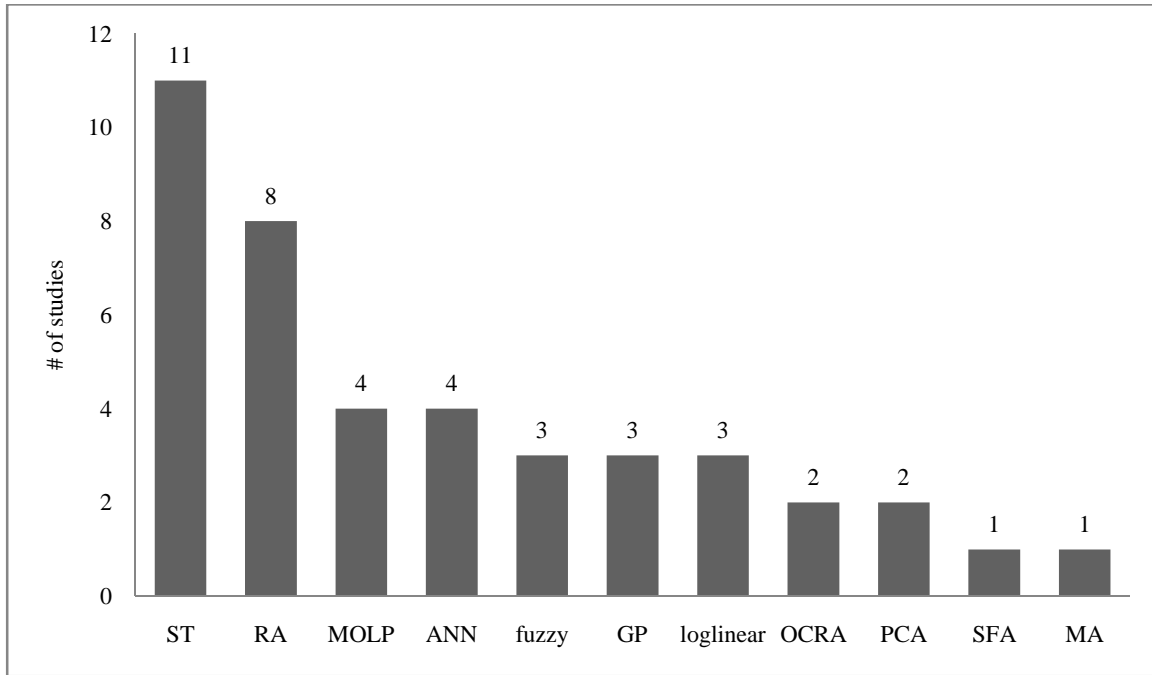


Fig 5. Multi-level models growth.

### 3-3- Hybrid models

Some studies combine DEA with other parametric and nonparametric performance evaluation methods, operational research techniques and etcetera. Figure (6) Shows all techniques that we found in 75 papers of the related literature and compares frequency of using them. These techniques are somehow included in DEA evaluation and construct hybrid models.



**Fig 6.** Techniques applied in hybrid DEA models and frequency of using them.

Different types of statistical tests (ST) and hypothesis tests could be seen in DEA-based bank branch studies such as Pearson correlation coefficient, Spearman's correlation coefficient, Kendall, Kruskal-Wallis' nonparametric test, Wilcoxon rank sum test, Hausman test, Kolmogorov-Smirnov test, and semi-parametrical statistical tests.

Different methods of regression analysis (RA) such as Tobit regression, logistic regression, etc. are employed for different purposes. For example, a combination approach of multiple regression analysis and DEA is used to estimate the service index of the branches under evaluation (Paradi et al., 2010). The field of multi-objective linear programming (MOLP) has attracted a lot of attention and many approaches were developed to address these problems. Lotfi et al. (2010) used an interactive decision making technique that encompasses both DEA and MOLP to incorporate preference information, without necessary prior judgment. They conducted a combined-oriented CCR model performance assessment using an MOLP method. Angiz et al. (2010) introduced a mathematical method for improving the discrimination power of DEA and to completely rank the efficient DMUs using fuzzy concept. The introduced model is a multi-objective linear model, endpoints of which are the highest and lowest of the weighted values. Yang et al. (2010) developed a hybrid approach to incorporate value judgments of both branch managers and head-office directors and to search for most preferred solution along the efficient frontier for each bank branch. Yang & Liu (2012) considered that the complementation of production and intermediation activities within a branch should be evaluated simultaneously and proposed a two-stage series model in the network framework to measure branch performance in Taiwan's banking system. In order to overcome the shortage of a traditional network DEA methodology about branches that cannot be assessed on the same base, they combined the MOLP and the fuzzy approach to propose the fuzzy multi-objective model to evaluate this network problem.

An artificial neural network (ANN) 'learns' relationship between input and output variables through being repeatedly shown example data and changing the internal structure of the network to represent the relationships more closely. Both DEA and neural networks are non-parametric methods in the sense that no assumptions are made concerning the functional form that links the inputs and outputs used to describe an operating process. In DEA, a set of weights is assessed for the inputs/outputs of each DMU in order to maximise its relative efficiency subject to the efficiency of the other DMUs in the study. Neural networks are also based on the estimation of sets of weights that link inputs with

outputs. In the latter case, however, the weights seek to derive the best possible fit through the observations of the assessed data set. DEA was being compared with ANN by Athanassopoulos & Curram, (1996). Wu et al. (2006a) applied DEA model as a data filter to create a sub-sample training data set used for neural networks (NN) to evaluate the branch efficiency. They introduced a two-stage DEA-NN model and claimed that this model produces a more robust frontier and identifies more efficient units since more good performance patterns are explored.

Fuzzy logic formulation could be introduced into DEA model to deal with environmental variables so that the branch performance from different regions could be assessed (Wu et al., 2006b).

The conventional DEA approach, as applied in bank related studies, has tended to concentrate on a single measure of performance for the DMU. Very often, however, there are multiple components or sub units within the DMU whose individual performance is required. By its very nature, the goal programming (GP) technique has the tendency to force the component measures toward each other (Cook & Hababou, 2001).

In many of the studies of economies of scale in banking, output is assumed to be produced according to a Cobb-Douglas production function. This function has the desirable property of being transformable into a logarithmic linear function that will allow the coefficients to be estimated by solving a linear programming model (Giokas, 1991). This loglinear function estimation has been compared with DEA in three studies of bank branch assessment literature.

Operational competitiveness rating analysis (OCRA) is a relative performance measurement approach based on a nonparametric model. With OCRA, one can obtain ratings for a set of production units (PUs) that gauge the performance of their operations in a relative sense. This technique is combined and compared with DEA in two studies as shown in Figure 6.

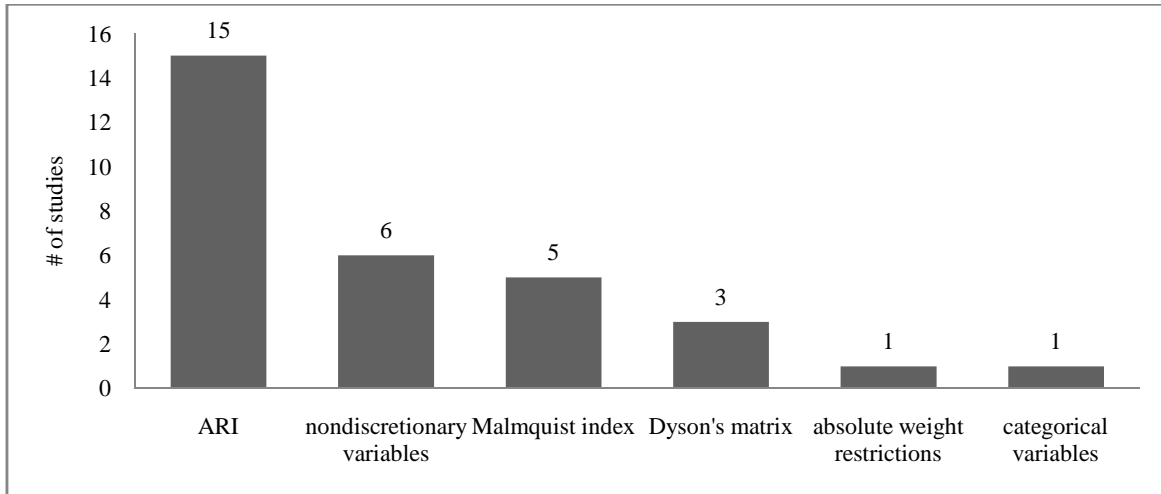
Principal component analysis (PCA) is used to evaluate the significance of a variable in a model by comparing the efficiency distributions obtained by running the model with and without the variable by means of a Wilcoxon rank sum test. The principle component analysis performed by LaPlante & Paradi (2015) provided insights into the relationships that exist between certain variables and indices and each type of branch efficiency. This method also used by Athanassopoulos (1998) identifies factors in clustering analysis.

Shyu & Chiang (2012) measured the true managerial efficiency of bank branches in Taiwan using a three-stage DEA model. They aimed to distinguish true managerial performance from that gained (lost) by favorable (unfavorable) environments or measurement errors. The method consists of a three-stage analysis that starts with the first stage DEA. The process continues with the stochastic frontier analysis (SFA) to explain the variations in organizational performance measured in the first stage, in terms of the operating environment, statistical noise, and managerial efficiency. It is noticeable that SFA is a parametric econometric frontier efficiency approach.

Multivariate analysis (MA) is a statistical analysis technique that has been combined with DEA in order to ensure the homogeneity of the branches under assessment.

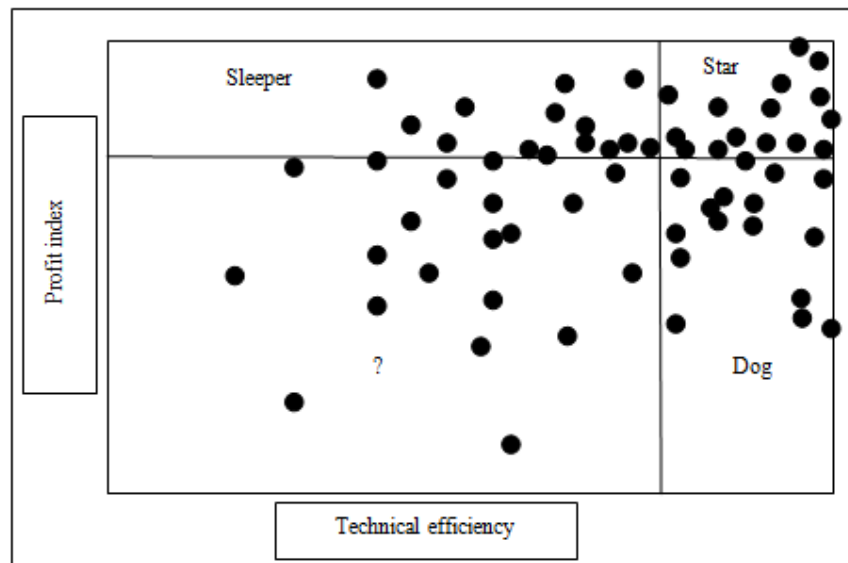
The frequency of using other special considerations in 75 related studies, regarding the status of variables, such as nondiscretionary inputs/outputs, data variations and time series data, such as Malmquist index (Färe et al. 1994) for total factor productivity growth, comparison of technical efficiency and profitability of DMUs by means of Dyson's matrix, and different types of weight restrictions applied by researchers, is illustrated in Figure (7).





**Fig 7.** Frequency of special considerations regarding the status of variables, data variation, and weight restrictions in DEA-based bank branch efficiency studies

In order to obtain an enhanced picture of branches' performance, the relation between the DEA efficiency measure and profitability of them is explored by means of Dyson's matrix proposed by Boussofiane et al., (1991). The joint use of the efficiency and profitability measures can highlight the potential performance improvements that management might be able to exploit, leading to higher profits. This analysis is based on the efficiency-profitability matrix' as illustrated in Figure (8).



**Fig 8.** Efficiency-profitability matrix

The efficiency-profitability matrix is divided in four quadrants, where different profiles of branches are likely to exist. The precise boundary positions between quadrants are subjective (Camanho & Dyson, 1999). Branches located in the 'star' quadrant, provide benchmarks for the network. Branches in 'dog' quadrant are operating efficiently but are relatively low on profitability. This may be due to an unfavorable environment, in which case their viability should be questioned, as the branches' profit may be critically affected by the presence of competition and low business potential in the catchment area. Branches located in the 'question mark' quadrant have the potential for both greater efficiency and profitability. The 'sleepers' are profitable, yet inefficient. Their profitability is likely to be a consequence of favorable environment rather than good management. They should be prime candidates for an efficiency improvement effort leading to greater profits.

### 3-4- Special models

The DEA models of this type cannot be incorporated in previous categories. Each special model have a special name that is selected by the author(s) of the paper. In Table I, we mentioned some of these special models appear in DEA-based bank branch efficiency studies and give a brief description about each of them.

In the 'radius of stability' model of Haag & Jaska (1995), A radius of stability technique has been demonstrated as an alternative mechanism for determining minimum simultaneous perturbations for technical efficiency reclassification. The radius of stability will facilitate the evaluation of the worth of new programs by identifying minimum targeted perturbations. The appropriate interpretation of inefficiency ratings in the additive model of DEA has been explored. The results demonstrate that efficiency scores can only be used to discriminate between efficient and inefficient DMUs. Additionally, the issue of data scaling has been addressed to create a units-invariant model and hence should be applied to all DEA models. The variable deletion test developed by Lovell & Pastor (1997), provides a test of hypothesis that a variable or a group of variables can be deleted from a linear programming assessment problem such as DEA models without statistically significant loss of information.

Multivariate analysis in order to ensure the homogeneity of the branches is the first step for introducing the nonparametric frontier models for assessing the market and cost efficiency of large-scale bank branch networks. Athanassopoulos (1998) used the nonparametric deterministic frontier analysis models for assessing site-specific and aggregate market and cost efficiency of bank branches.

As a branch's scale size can significantly affect its efficiency, the standard VRS model was modified in order to preclude from the peer set branches that are either too large or too small to be considered benchmarks for the assessed branch. In this study it was considered that the peers should not differ from the current size of the branch evaluated by more than two employees (Camanho & Dyson, 1999).

In contrast to BCC, the quasi-concave (QC) model does give consistent estimators. In addition, it suffers less from finite sample error than the FDH (Dekker & Post, 2001). This model assumes that the production possibility set is a quasi-concave set. Measure function (objective function) of the Normalized weighted additive VRS model is the weighted sum of the slacks and the weights are the inverse of standard deviation of the  $i$ -th input and  $j$ -th output variables. Ševčovič et al.(2001) introduced three measures of efficiency and compared the results.

Cascaded overall efficiency model of Manandhar & Tang (2002), is composed of three-stage, namely, internal service quality efficiency, operating efficiency, and profitability efficiency. Each stage uses the CCR model and the outputs of each stage are part of the inputs of the next stage. Eventually, the BCC model with three outputs and no inputs is used for aggregating these measures.

Two types of reference sets for benchmarking the DMUs were considered by Cook et al. (2004): benchmarking the electronic branches (e-branches) against the traditional branches, and benchmarking within e-branches.

The practical frontier obtained from P-DEA model of Sowlati & Paradi (2004), enhanced the discrimination power of standard basic DEA models by adding the unobserved DMUs to the set of observed DMUs.

Geometric Distance Function (GDF), that is the ratio of the geometric mean of input efficiency scores to the geometric mean of output efficiency scores, is used in calculating maximum profit targets and then overall profit efficiency is estimated in a four-stage process introduced by Portela & Thanassoulis (2005).

Quality Adjusted DEA model (QA-DEA) is an iterative algorithm developed by Sherman & Zhu (2006) in order to incorporating the high quality DMUs in the reference set of inefficient DMUs.

The methodology used by Yang & Paradi (2006) for incorporating environmental impacts in the assessment is a three-stage procedure within which a handicapping factor is used in the model of stage two for adjusting the cultural differences between branches of different regions.

In order to apply DEA to negative data, Conceição et al, (2007) developed a model called Range Directional Model (RDM) that is based on the directional distance model. The model used was output oriented with a nondiscretionary output. Input and output values to be used in the RDM model can be negative, since the RDM model is translation and unit's invariant. The efficiency measure reflecting the distance from observed points to its targets (with reference to the ideal point) is directly obtained from the optimal objective function of the model. The RDM provides efficiency scores similar in meaning to radial efficiency scores, which can be directly used to compare production units when some inputs and/or outputs are negative.

The culturally adjusted (CA) DEA model is designed to control the influences of corporate culture through two indices, Corporate Index (CI) and Service Index (SI), in order to assess the true managerial efficiency. The CI reveals the differences in corporate strategies, while the SI measures the effectiveness of the operating systems for delivering high quality customer service. The CI is included in the profitability analysis because the corporate strategies imposed upon a branch may limit its scope of business mix. The SI is incorporated into the operational model since the level of customer service quality affects the production abilities of the branches (Paradi et al., 2010).

Azizi & Ajirlu (2010) Evaluated branch performances from both optimistic and pessimistic perspectives. The optimistic efficient branches collectively delineate an efficiency frontier, while all pessimistic inefficient branches define an inefficiency frontier. The conventional form of DEA evaluates performances of DMUs only from the optimistic point of view. In other words, it chooses the most favorable weights for each DMU. There is another approach that measures efficiency of a DMU from the pessimistic point of view. This approach chooses the most unfavorable weights for evaluation of each DMU. Azizi & Ajirlu (2010) proposed to integrate both efficiencies in the form of an interval in order to measure the overall performance of a DMU. The proposed DEA models for evaluation of efficiencies are called bounded DEA model.

Incorporating decision maker's preference information into the process of DEA assessing efficiency using multi-objective linear programming (MOLP) is developed by Lotfi et al. (2010). A directional vector for showing the direction of moving an inefficient DMU toward the efficient frontier was introduced.

For computing our index and indicator of productivity change, Portela & Thanassoulis (2010) used a meta-frontier, which envelops the pooled data of a panel covering a number of time periods, to which we refer collectively as the meta-period. Using meta-frontiers under VRS makes it possible to compute the index for all units. It is recalled that some approaches to decomposing Malmquist indices of productivity change under VRS can encounter infeasible models for some units. In addition, meta-Malmquist indices have the advantage of being circular and computationally simpler because they do not resort to geometric averages. Luenberger indicators use directional distance functions to capture differences in productivity of a DMU between two time periods.

The HMRP-DEA approach is composed of three minimax models, including the super-ideal point model, the ideal point model and the shortest distance model, which share the same decision and objective spaces, are different from each other only in their reference points and weighting schema, and are proven to be equivalent to the output-oriented DEA dual models. The HMRP-DEA approach uses DEA as an ex-post-facto evaluation tool for past performance assessment and the minimax reference point approach as an ex-ante planning tool for future performance forecasting and target setting. Thus, the HMRP-DEA approach provides an alternative means for realistic target setting and better resource allocation (Yang et al., 2010).

**Table 1.** Some of the special models applied in DEA-based bank branch efficiency studies.

The name of the model	Creator	Objectives
Radius of stability	Haag & Jaska, 1995	Determining minimum simultaneous perturbations for technical efficiency reclassification in Additive model.
Deletion of variables technique	Lovell & Pastor, 1997	Examine the performance of target setting procedure and determine the optimal structure of targets
Site-specific/Aggregate market/cost efficiency	Athanassopoulos, 1998	Assessment of market efficiency and cost efficiency concerned with both central management (aggregate) and also local management (site-specific).
DEA model with restricted peer selection	Camanho & Dyson, 1999	The peers should not differ from the current size of the branch evaluated by more than two employees.
Dual component CRS model	Cook et al., 2000	Derive an aggregate measure of branch performance that involving the sales and service functions (multi-component efficiency measurements and shared inputs in DEA), split the shared inputs for optimizing the aggregate efficiency score.
QC-DEA model	Dekker & Post, 2001	Propose a quasi-concave DEA model to relax the standard DEA assumptions of concavity for the production frontier.
Normalized weighted Additive VRS model	Ševčovič et al., 2001	Compare and analysis DEA efficiencies obtained from different measures.
Cascaded overall efficiency model	Manandhar & Tang, 2002	Benchmarking performance of branches in different dimensions simultaneously: internal service quality, operating efficiency, and profitability in order to incorporating intangible aspects in assessment.
Variable-benchmark and fixed-benchmark models	Cook et al., 2004	A unit under benchmarking selects a portion of benchmark such that the performance is characterized in the most favorable light or is benchmarked against a fixed set of benchmarks.
P-DEA model	Sowlati & Paradi, 2004	Develop a DEA model that provides targets for empirically efficient units by defining a practical frontier.
Optimistic/pessimistic cost efficiency model	Camanho & Dyson, 2005	Estimation of upper and lower bounds for the cost efficiency measure in the situation of price uncertainty.
GDF	Portela & Thanassoulis, 2005	Calculate overall profit efficiency and decomposing it into allocative and technical profit efficiency.
QA-DEA model	Sherman & Zhu, 2006	Improve benchmarking ability of a DEA model by incorporating quality factor.
Handicapped DEA model	Yang & Paradi, 2006	Adjusting cultural differences due to corporate management's policies.
RDM	Conceição et al., 2007	Range directional model is developed in order to apply DEA to negative data.
Directional measurement of technical inefficiency	Deville, 2009	Determining one efficiency frontier for each type of environment.
CA-CCR model	Paradi et al., 2010	Propose a new strategy to benchmark business units that operate under different cultural (business) environments. Two cultural indices (corporate strategy index and service capacity index) are identified to represent a firm's unique operating environment.
Bounded DEA models	Azizi & Ajirlu, 2010	Measurement of interval efficiencies of DMUs based on optimistic efficiency of ADMU (Anti-ideal DMU) and pessimistic efficiency of IDMU (Ideal DMU).
Combined-oriented CCR model	Lotfi et al., 2010	Reflecting the DM's preferences in the process of assessing efficiency without necessary prior judgment using an interactive decision making technique that encompasses both DEA and MOLP.
Meta-Malmquist index and Meta-Luenberger indicator	Portela & Thanassoulis, 2010	Develop an index and an indicator of productivity change that can be used with negative data.
HMRP-DEA model	Yang et al., 2010	The HMRP-DEA approach provides an alternative means for realistic target setting and better resource allocation and incorporating of value judgments of both branch managers and head-office directors and to search for most preferred solution along the efficient frontier for each bank branch.

## 4- CONCLUSION

DEA became a mainstream technology in bank branch studies in recent years. Since its introduction in 1978, DEA has become one of the preeminent nonparametric methods for measuring efficiency and productivity of DMUs (Emrouznejad, 2014). However, there is a lack of a literature review in the field of model structures have been used by researchers and practitioners. Until now, there has been no classification and regularization of multifarious models applied in DEA-based bank branch efficiency measurements. Therefore, it is critical that the DEA community has an open mind on these issues, as DEA is being further developed and applied in various areas.

In this paper, we present the results of a survey of model structures of 75 DEA applications at the branch level published in journals since 1985, all known (to authors) studies in this area. Given the importance of bank branch modeling techniques and the focus on performance improvement, we believe that the basic DEA models as well as their many extensions would likely play a more important role in bank branch studies in future. By comparison of four different categories of models presented in this study, one can conclude that the inputs and outputs of the most DEA applications in bank branches are compatible with the production approach. On the other hand, the popularity of multi-level models is increasing among practitioners. Some of the special models, that could not be included in basic models and hybrid models, are referenced.

Although there have been many research thrusts in DEA techniques over the past four decades, there is still no reliable DEA model which can effectively handle the situations where some variables are mixed with both positive and negative entries. To point out a promising direction for future research, we suggest probing the model structure of each category introduced in this paper separately and more precisely and then offering a theoretical framework for each of the four classes presented here. Another interesting future research area is to find new ways to apply DEA in conjunction with other advanced methodologies in order to extend such methodologies and to complement each other's strengths while eliminating their weaknesses.

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