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Competitive vehicle routing problem projects with time windows and stochastic demands

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Abstract

The vehicle routing problem is one of the most important issues in transportation. Among these issues, competitive VRP is one of the new concepts that have attracted the attention of many researchers. In this study we introduced a new method for competitive VRP. In this method three time boundaries are given and the probability of arrival time between each time bound is assumed to be 1/3. Based on this information, demands of each customer will differ in each time window. Therefore, the revenue given in each time window is different. In this paper we consider a project with two companies in a city with eight customers and the best routing with maximum revenue is determined.

Keywords: Competitive, VRP, Time window, Stochastic demand, probabilistic

1- Introduction

Transportation systems play an important role in various areas, such as investment, economics and service systems. For this reason, investors, officials and manufacturers are interested in improving transport routes in order to eliminate the unnecessary journeys and to choose better routes. In addition, many problems, such as the travelling salesman problem (TSP), vehicle routing problem (VRP) and other similar problems, are researched and developed using these criteria. The vehicle routing problem (VRP) is a general term that is given to a category of problems including vehicles that require stations. The first VRP was expressed by Dantzig and Ramser (1959).

The most researched vehicle routing problem is probably the capacitated vehicle routing problem (CVRP). In the CVRP the number of customers, with specific demands, should be supported from a depot by homogeneous vehicles with known capacity. There are other kinds of VRP that can be seen in the table 1:

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Table1. Types of VRP

R	Type	Description
1	Vehicle Routing Problem with Clustered Backhauls (VRPCB)	This type is characterized by the requirement that the group or cluster of delivery customers has to be served before the first pick-up customer can be visited. Delivery customers are also denoted as linehaul customers, pick-up customers as backhaul customers. In the literature this problem class is denoted as Vehicle Routing Problem (VRP) with Backhauls.
2	VRP with Mixed linehauls and Backhauls (VRPMB)	Unlike VRPCB, mixed visiting sequences are explicitly allowed.
3	VRP with Divisible Delivery and Pick-up (VRPDDP)	The deliveries and pick-ups are divided. Rather, two visits, one for delivery and one for pick-up are possible. In this case, first a few customers are visited for delivery service only, in order to empty the vehicle partially. Then, customers are visited where goods are delivered and picked up. At the end, the pick-ups are performed for the customers initially visited for delivery. A customer can either be visited once for both pick-up and delivery or twice, first for delivery and then for pick-up.
4	VRP with Simultaneous Delivery and Pick-up (VRPSDP)	Every customer is associated with a linehaul (delivery) as well as a backhaul (pick-up) quantity. It is imposed that every customer can only be visited exactly once.
5	Pick-up and Delivery VRP (PDVRP)	Refers to problems where goods are transported from pick-up to delivery points. The delivery and pick-up locations are not paired.
6	Open Vehicle Routing Problem (OVRP)	The OVRP is very close to the CVRP. The difference between the two problems is that in the OVRP the vehicles do not have to return to the depot. Thus, an OVRP can be solved as an asymmetric CVRP by setting distances and travel times from every customer to the depot to zero.
7	Site-Dependent VRP (SDVRP)	SDVRP can be considered as a generalization of CVRP. In the SDVRP a customer may only be serviced by a given subset of the vehicles, typically because the access paths to the node do not allow given vehicles to pass, or because specific facilities are demanded in the vehicle (e.g. a freezing compartment). Furthermore, vehicles do not need to have the same capacity in the SDVRP.
8	Multi-Depot VRP (MDVRP)	The MDVRP extends the CVRP by allowing multiple depots. In the MDVRP each customer may be serviced by a vehicle originating at any of the available depots. It requires that each request is assigned to a specific depot. In general, this is a hard optimization problem of its own which needs to be handled together with the routing problem.
9	Multicriteria VRP (MVRP)	The Multicriteria vehicle routing problem (VRP) is a VRP which allows several relevant objectives to be achieved.
10	Periodic Vehicle Routing Problem (PVRP)	In the PVRP, customers specify a service frequency and sets of allowable combinations of visit days.
11	Dynamic Multi-Period Vehicle Routing Problem (DMVRP)	Dynamic Multi-Period Vehicle Routing Problem which deals with the distribution of orders from a depot to a set of customers over a multi-period time horizon. Customer orders and their feasible service periods are dynamically revealed overtime. The objectives are to minimize total travel costs and customer waiting, and to balance the daily work load over the planning horizon.
12	Split Delivery Vehicle Routing Problem (SDVRP)	In the split delivery vehicle routing problem (SDVRP) the restriction that each customer is visited once is removed. Moreover, the demand of each customer can be greater than the capacity of the vehicles.
13	Vehicle Routing Problem with Time Window (VRPTW)	The VRPTW is one of the most studied problems in the field of combinatorial optimization. It is a variant of the Vehicle Routing Problem (VRP). A solution to the VRPTW is a set of routes consisting of a sequence of visits to customers, where each route is assigned to a vehicle and all customers are visited within their time windows. The total volume assigned to each route must not exceed the capacity of the vehicle. The challenge is finding a solution that minimizes the total amount of vehicles used and distance traveled.
14	Vehicle Routing Problem with Soft Time Window (VRPSTW)	Can be violated but this causes additional costs, i.e., the respective pick-up or delivery-activity do not has to be executed within the time window
15	Vehicle Routing Problem with Hard Time Window (VRPSTW)	Do not allow any violations of the defined intervals, i.e. each pick-up or delivery activity has to be generated inside each time window
16	Vehicle Routing Problem with Stochastic Demands (VRPSD)	In the vehicle routing problem with stochastic demands a vehicle has to serve a set of customers whose exact demand is known only upon arrival at the customer’s location. The objective is to find a permutation of the customers (an a priori tour) that minimizes the expected distance traveled by the vehicle. Since the objective function is computationally demanding, effective approximations of it could improve the algorithms ‘performance.

Furthermore, the combination of the above methods can be seen as a kind of VRP. According to the subject of this paper, we explain the competitive vehicle routing problem with a time window and vehicle routing problem with stochastic demands separately.

2- Competitive Vehicle Routing Problem with Time Windows (VRPTW)

The vehicle routing problem with time windows (VRPTW) is an extension of the VRP where a transfer of goods to a customer must be done within a given set of intervals [t_1, t_2], so t_1 is the earliest and t_2 is the latest allowable time that the transfer should be done. The VRPTW is divided in two parts, called VRP with a soft time window (VRPSTW) and VRP with a hard time window (VRPHTW). The VRPSTW is free in the VRPHTW, so the delivery of goods can be transferred outside the time interval if a fine is paid. However, in the VRPHTW, non compliance from the time interval is strictly not authorized.

As a result of the significance of the service times presented by other companies in the real world, distribution companies design the routes of fleets with respect to the conditions of other competitors for obtaining the maximum sale.

The competitive vehicle routing problem follows a new approach for VRPs, in which the cost and distance of routes are minimized while the quantity of sales is simultaneously maximized. This approach needs to review some other parameters, such as competition between distributors, customer decision factors and service time visited by distributors for customers where the basic VRP cannot reach proper solutions for these types of assumptions.

The competitive vehicle routing problem is a different version of the vehicle routing problem with time windows that occurs in a competitive environment. In this situation, it is very important to obtain the service time required to visit customers as if the vehicle presents the service to customers later than its competitors, a part of its sale will be lost. Therefore, the distributor's delivery time to the customers influences the amount of sales.

Solomon (1986, 1987) and Solomon et al. (1988) studied time window constrained routing and scheduling problems in VRP. Golden et al. (1988) studied the methods used in VRP. Taillard et al. (1997) proposed a Tabu search heuristic for the vehicle routing problem with soft time windows. Cordeau et al. (2002) have a study on VRP with time windows. Geiger (2003) proposed a genetic crossover operator for multi-objective vehicle routing problem with soft time windows. Tavakkoli-Moghaddam (2005, 2011) studied a multi-criteria vehicle routing problem and proposed a new mathematical model for a competitive vehicle routing problem with time windows. Braysy et al. (2005) studied vehicle routing problem with time windows. Ombuki (2006) also proposed multi-objective genetic algorithm for vehicle routing problem with time windows. Qureshi et al. (2009) introduced an exact solution approach for vehicle routing and scheduling problems with soft time windows. Li (2010) studied models and algorithms of v**ehicle routing problems with time windows and stochastic travel and service times. Baldacci (2012) proposed an** exact algorithm for solving the vehicle routing problem under capacity and time window constraints. Errico et al. (2013) studied the vehicle routing problem with hard time windows and stochastic service times. Batista et al. (2014) proposed a bi-objective vehicle routing problem with time windows.

Table 2 describes the literature review of VRPTW:

Table 2. Literature review of VRPTW

Authors	Year	Title	Description
Solomon MM	1986	On the worst-case performance of some heuristics for the vehicle routing and scheduling problem with time window constraints	Variety of heuristics
Solomon MM.	1987	Algorithms for the vehicle routing and scheduling problem with time windows constraints.	Insertion-Type Heuristic
Golden BL and Assad AA	1988	Vehicle routing: methods and studies	Modeling and Implementation
Solomon MM, Desrosiers J	1988	Time window constrained routing and scheduling problems	Dial-a-Ride Problem
Taillard E et.al.	1997	A tabu search heuristic for the vehicle routing problem with soft time windows.	Tabu Search
Cordeau JF et.al.	2002	The VRP with time windows	Discrete Mathematics and Applications
Geiger MJ	2003	A computational study of genetic crossover operators for multi-objective vehicle routing problem with soft time windows.	Genetic Algorithm
Tavakkoli-Moghaddam R et.al.	2005	A multi-criteria vehicle routing problem with soft time windows by simulated annealing.	Simulated Annealing
Braysy O and Gendreau M	2005	Vehicle routing problem with time windows, part I: route construction and local search algorithms.	Local Search Algorithms
Ombuki B, Ross BJ andHanshar F.	2006	Multi-objective genetic algorithm for vehicle routing problem with time windows	Genetic Algorithm
Qureshi AG et.al.	2009	An exact solution approach for vehicle routing and scheduling problems with soft time windows	Dantzig-Wolf decomposition
Li X. Et.al	2010	Vehicle routing problems with time windows and stochastic travel and service times: Models and algorithm	Tabu Search
Tavakkoli-Moghaddam R et.al.	2011	A new mathematical model for a competitive vehicle routing problem with time windows solved by simulated annealing	Simulated Annealing
Baldacci R et.al.	2012	Recent exact algorithm for solving the vehicle routing problem under capacity and time window constraints	State-of-the-art exact Algorithm
Errico F. Et.al	2013	The Vehicle Routing Problem with Hard Time Windows and Stochastic Service Times	Branch-price-and-cut Algorithm
Batista BM et.al.	2014	A bi-objective vehicle routing problem with time windows: A real case in Tenerife	Mixed integer linear Model

3- Vehicle Routing Problem with Stochastic Demands (VRPSD)

The VRPSD is known as a NP-hard problem, in which a vehicle with limited capacity leaves the depot with a full load and has to serve a set of customers whose demands are known only when the vehicle arrives to them. A vehicle starts from the depot and visits each customer exactly once, and returns to the depot. This is called an a priori tour. An a priori tour is a map showing a vehicle's visits as a sequence of all visited customers. In a given instance, the customers should be visited based on the sequence of the a priori tour but the final route includes the depot, including returns to the depot when the vehicle needs reloading. The nodes that vehicle uses to return to the depot are stochastically expressed. Due to the difficulty of the VRPSD and the fact that it is an NP-hard problem, the problem with n customers (n is big) cannot be solved with an exact method in a reasonable time. For this reason, some approximation techniques were used for solving these kinds of problems. An initial solution of the VRPSD is a permutation of the customers in which the vehicle starts from the depot. To find the route the vehicle makes, an initial path which starts from the depot is considered. Based on the demand of the next customer in the path, the vehicle can return to the depot for reloading or it can continue with the next customer.

Sometimes the demand of a customer is less than the load of the vehicle, and the vehicle returns to the depot for reloading. This is called preventive reloading. The preventive reloading aims to avoid the risk of the vehicle to go to the next customer without having enough of a load to satisfy it. If this happens, the vehicle must go back to the depot and, then, return to the same customer.

Yang et al. (2000) studied on stochastic vehicle routing problem with restocking. Secomandi (2001) proposed a rollout policy for the vehicle routing problem with stochastic demands. Laporte (2002) proposed an integer L-Shaped algorithm for the capacitated vehicle routing problem with stochastic demands. Bent et al. (2003) studied dynamic vehicle routing with stochastic requests. Bianchi et al.(2004, 2006) proposed some metaheuristics for this problem. Chepuri et al. (2005) studied the problem by cross-entropy method. Haugland et al. (2006) studied on designing delivery districts for the vehicle routing problem with stochastic demands. Secomandi (2009) has a study on re-optimization Approaches for the vehicle routing problem with Stochastic Demands. Ismail (2008) solved the problem by Tabu search method. Mendoza (2010) proposed a memetic algorithm for the multi-compartment vehicle routing problem with stochastic demands. Gupta (2012) has a study on approximation algorithms for VRP with stochastic demands. Marinakis (2013) studied on Particle Swarm Optimization for the vehicle routing problem with stochastic demands.

Table 3 explains the literature review of VRPSD:

Table 3. Literature review of VRPSD

Authors	Year	Title	Description
Yang, W. H. Et.al	2000	Stochastic vehicle routing problem with restocking	heuristic algorithms
Secomandi N.	2001	A Rollout Policy for the Vehicle Routing Problem with Stochastic Demands	heuristic algorithms
Laporte G.	2002	An Integer L-Shaped Algorithm for the Capacitated Vehicle Routing Problem with Stochastic Demands	L-Shaped Algorithm
Bent R. and Van Hentenryck P.	2003	Dynamic Vehicle Routing with Stochastic Requests	Multiple Scenario Approach (MSA)
Bianchi L. et.al.	2004	Metaheuristics for the Vehicle Routing Problem with Stochastic Demands	Metaheuristics Algorithms
Chepuri K. and Homem-de-Mello T.	2005	Solving the Vehicle Routing Problem with Stochastic Demands using the Cross-Entropy Method	Cross-Entropy Method
Bianchi L. Et.al.	2006	Hybrid Metaheuristics for the Vehicle Routing Problem with Stochastic Demands	Hybrid Metaheuristics
Haugland D. Et.al	2007	Designing delivery districts for the vehicle routing problem with stochastic demands	Tabu Search
Ismail Z.	2008	Solving the Vehicle Routing Problem with Stochastic Demands via Hybrid Genetic Algorithm-Tabu Search	Hybrid Genetic Algorithm-Tabu Search
Secomandi N.	2009	Reoptimization Approaches for the Vehicle-Routing Problem with Stochastic Demands	Partial Reoptimization
Mendoza J.E. et.al	2010	A memetic algorithm for the multi-compartment vehicle routing problem with stochastic demands	MemeticAlgorithm
Mendoza J.E. et.al	2011	Constructive Heuristics for the Multicompartment Vehicle Routing Problem with Stochastic Demands	classical 2-Opt heuristic
Gupta A.	2012	Technical Note - Approximation Algorithms for VRP with Stochastic Demands	Cyclic Heuristic
Marinakis Y. Et.al.	2013	Particle Swarm Optimization for the Vehicle Routing Problem with Stochastic Demands	PSO Algorithm

In this paper we will use a very simple kind of stochastic demands problem. As the demands of each customer are not the same and vary over time.

4- Problem Definition

The problem is proposed in case there are two distributors in a city. Furthermore, there are a number of customers within the city that are scattered. One of the distributors, for example A, wants to get more revenue so it's necessary for it to choose the best path to service more customers than the other distributor, B. Thus, A obtains the range time of a rival that sells to every customer. Then designs a VRPTW problem in which it can sell its products before its rival. Therefore, the models are introduced as the following:

Assumptions:

1- All vehicles are homogenous with specific capacity.
2- Each customer is serviced only with one vehicle and only once in each period.
3- The demand of each customer must not exceed the capacity of each vehicle.
4- Each vehicle starts from the depot and visits a number of customers until its capacity is finished.
5- If a vehicle arrives at a customer before its rival, it can satisfy all demands of that customer. If that vehicle arrives at a customer with its rival at the same range time, it can satisfy half of the

demands of that customer. Finally, if that vehicle arrives at a customer after its rival and range time, it loses that customer. Furthermore, arriving at a customer in these three situations is probabilistic with a probability of 1/3.

Sets and indices:

I: set of customers ={1, 2, ..., n}

i, j: index of customers

K: set of vehicles ={1, 2, ..., nv}

k: index of vehicles

Parameters:

n: number of customers

t_{li}: lower bound of rival's arrival time to customer i

t_{ui}: upper bound of rival's arrival time to customer i

t_{di}: actual distributor's vehicle arrival time to customer i

D_i: demand of customer i

nv: number of vehicles

Cap_k: capacity of vehicle k

T_k: maximum travel time of vehicle k

t_{ik}: required time for serving customer i by vehicle k

t_{ijk}: required time for traveling from customer i to j by vehicle k

C_{ij}: travel cost between customer i and j

R: profit of selling each unit of product

Desicion variables:

$x_{ijk}=1$ if vehicle k travels between customer i and j
$x_{ijk}=0$ otherwise

$m_{ik}=1$ if vehicle k serves customer i in range time $(0, t_{li})$
$m_{ik}=0$ otherwise

$n_{ik}=1$ if vehicle k serves customer i in range time $[t_{li}, t_{ui}]$
$n_{ik}=0$ otherwise

$o_{ik}=1$ if vehicle k serves customer i in range time $(t_{ui}, t_{ui}+ \varepsilon]$, $\forall\ \varepsilon > 0$
$o_{ik}=0$ otherwise

t_{di} actual distributor's vehicle arrival time to customer i

Tables (4) and (5) describe the mathematical model.

Table 4. Probability of arrival and demand of customer i

	$(0, t_{li})$	$[t_{li}, t_{ui}]$	$(t_{ui}, t_{ui}+ \varepsilon]$
Probability of arrive	1/3	1/3	1/3
Demand of customer i	D_i	$D_i/2$	0

Model:

So the problem is formulated as follows:

$$\min \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{nv} C_{ij}x_{ijk} - \frac{R}{3}\sum_{i=1}^{n}\sum_{k=1}^{nv}\left(D_i m_{ik} + \frac{D_i}{2} n_{ik}\right) \qquad (1)$$

s.t. :

$$\sum_{i=1}^{n}\sum_{k=1}^{nv} x_{ijk} = 1, \quad \forall\, j \in I \tag{2}$$

$$\sum_{j=1}^{n}\sum_{k=1}^{nv} x_{ijk} = 1, \quad \forall\, i \in I \tag{3}$$

$$\sum_{i=1}^{n} x_{ipk} - \sum_{i=1}^{n} x_{pik} = 0, \quad \forall\, p \in I, \quad \forall\, k \in K \tag{4}$$

$$\sum_{i=1}^{n} t_{ik} \sum_{j=1}^{n} x_{ijk} + \sum_{i=1}^{n}\sum_{j=1}^{n} t_{ijk}\, x_{ijk} \le T_k, \quad \forall\, k \in K \tag{5}$$

$$t_{dj} = \sum_{i=1}^{n} t_{di} \sum_{k=1}^{nv} x_{ijk} + \sum_{i=1}^{n}\sum_{k=1}^{nv} (t_{ik} + t_{ijk}) x_{ijk}, \quad \forall\, j \in I \tag{6}$$

$$\sum_{i=2}^{n} \left(D_i m_{ik} + \frac{D_i}{2} n_{ik} \right) \sum_{j=1}^{n} x_{ijk} \le Cap_k, \quad \forall\, k \in K \tag{7}$$

$$m_{ik} + n_{ik} + o_{ik} = 1, \quad \forall\, i \in I \tag{8}$$

$$\sum_{i=2}^{n} x_{i1k} = 1, \quad \forall\, k \in K \tag{9}$$

$$x_{ijk}, m_{ik}, n_{ik}, o_{ik} \quad are\ binary \qquad t_{di} \ge 0 \tag{10}$$

Table 5. Description of constraints

Number of constraints	Description
(1)	Objective function: minimizes total cost and maximizes total profit
(2)& (3)	Ensure that each customer is serviced from only one vehicle.
(4)	States that if a vehicle arrives to a customer, it must leave it.
(5)	Shows that total time of servicing time to customer i and traveling time between customer i and j , must not exceed from maximum time that vehicle k can be used.
(6)	Calculate the time of arriving to j th customer
(7)	Ensure that demand of each customer do not exceed from capacity of each vehicle.
(8)	States that each vehicle arrives to customer only in one of three time range.
(9)	Shows that each vehicle returns to depot

5-Numerical Examples

We suppose that there are eight customers in a city and two vehicles in the depot, so with simulated data the routes for each vehicle are displayed on the above figure. Therefore, the vehicle 1 “- - - ►” has a tour (depot,4,3,8,depot) and vehicle 2 “——►” has a tour (depot,2,5,depot). Thus, for maximizing profit, the distributor does not have to service customers 1, 6 and 7. The time to service to each customer is demonstrated in table 6:

Table 6. Time of service to each customer

Customer	Vehicle	$(0,t_{li})$	$[t_{li},t_{ui}]$	$(t_{ui},t_{ui}+\varepsilon)$
1	-	0	0	1
2	2	1	0	0
3	1	0	1	0
4	1	0	1	0
5	2	1	0	0
6	-	0	0	1
7	-	0	0	1
8	1	1	0	0

The objective function value for this model is -1145.

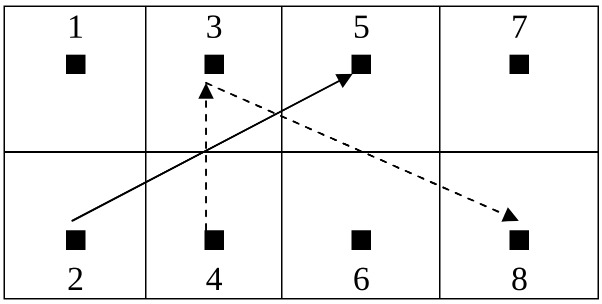

Fig 1. model results of numerical example

6-Conclusion

According to this paper, the idea of competitive VRP with stochastic demands is modelled. In this study we introduced a new method for competitive VRP. In this method three time boundaries are given and the probability of arrival time between each time bound is assumed to be 1/3. Based on this information, demands of each customer will differ in each time window. Therefore, the revenue given in each time window is different

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