

A multi-period fuzzy mathematical programming model for crude oil supply chain network design considering budget and equipment limitations

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Abstract

The major oil industry upstream activities include the exploration, drilling, extraction, pipelines installation, and production of crude oil. In this paper, we develop a mathematical model to plan for these operations as a crude oil supply chain network design problem. The proposed multi-period mixed integer linear programming model entails both strategic (e.g., facility location and allocation) and tactical (e.g., project and production planning) decisions. With the objective of maximizing total Net Present Value (NPV) at the end of planning horizon, the decisions to be made comprise the location of the facilities, the flow of commodities and the amount of investment. The uncertain natures of important input parameters such as capital and operational cost, demand and price of crude oil, are taken into account via fuzzy theory. Finally, the performance of the developed model is investigated using the real data of Iranian South Oilfields.

Keyword: Crude oil supply chain, oilfield development, multi-period programming, time horizon analysis, fuzzy optimization.

1- Introduction

Supply chain management (SCM) is the management of the flow of goods and services. It includes the movement and storage of raw materials, inventory, and finished goods from point of origin to point of consumption. Applying the concept of supply chain management can provide considerable competitive advantage for any companies from any sectors (Damghani, 2015). In this context, the petroleum/oil industry can be characterized as a supply chain which is of vital importance in the economy of the world oil-rich countries. The optimization of such a supply chain can significantly increase the profitability of oil companies and improve the national economic indices.

A typical oil industry supply chain is composed of an exploration operation at well heads, crude procurement and storage logistics, transportation to refineries, refinery operations, and distribution of products. These activities can be classified into two categories.

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The first category includes upstream activities such as exploration, well drilling, crude oil extraction and crude oil production. These operations are related to the crude oil supply chain. The second section includes downstream (Refinery) operations such as (I) crude-oil unloading, mixing and inventory control, (II) scheduling of production units, and (III) finish products blending and distribution. The problems related to the second category can be formulated as downstream oil supply chain problems. Also, all the activities can be considered as petroleum supply chain problems.

Upstream activities such as investment planning, facility location- allocation, and production planning, which also called oilfields development, can be modeled as crude oil supply chain (Sahebi et al., 2014) or upstream petroleum supply chain. Shah et al. (2011) explained crude oil supply chain based on the strategic and tactical levels of decision making as follows:

- Facility location and allocation: selection of optimal locations for drilling, installation of platforms and production units (PU), and allocation of facilities (Strategic level).
- Project planning: determination of the optimal time for drilling, installation of the equipment, and construction of facilities required for production (Tactical level).
- Production planning: determination of the optimal extraction of each well considering the reservoir behavior, and the amount of optimal production (Tactical and operational levels).

One of the earliest works in the oilfield development is contributed by Devine and Lesso (1972). They address to location and allocation of the well, and facility related to oilfield. Iyer et al. (1998) proposed a multi-period MILP model for optimal project planning as well as facility location and allocation of offshore oilfield development. They approximated reservoirs performance equations through piecewise linear approximations. This work was extended by van den Heever and Grossmann (2000) where they proposed a multi-period generalized disjunctive programming model for oil field infrastructure planning and a bi-level decomposition method to solve the developed model. Kosmidis et al. (2002) considered a mixed integer optimization strategy for integrated oil and gas production. They presented a dynamic optimization model and an efficient approximation solution strategy for this system. Kosmidis et al. (2004) also proposed a mixed integer nonlinear (MINLP) model for the daily well scheduling in oilfields, where nonlinearity behavior for reservoirs was considered. Carvalho and Pinto (2006a) extended the model developed by (Tsarboboulou, 2000), and proposed a bi-level decomposition algorithm for solving large scale problems where the master problem determines the allocation of platforms to wells and sub-problem is related to scheduling of the fixed facilities. This model was further extended by (Carvalho and Pinto, 2006b) where multiple reservoirs were considered within the model.

All the above-mentioned papers applied deterministic mathematical programming approaches; however, the assumption of certainty for some parameters of problem such as demand, oil price, costs, etc., and other important ones like productivity index, reservoirs behavior may lead to inefficient results. Aseeri et al. (2004) considered uncertainty in some parameters such as the oil prices and well productivity index (PI) into the model proposed by (Iyer et al., 1998) and used the approximation algorithm to solve the corresponding stochastic programming model. Tarhan et al. (2009) presented a multistage stochastic programming model for planning offshore oilfields infrastructure in stochastic environment. They considered uncertainty in initial maximum oil flow rate and recoverable oil volume.

In the recent papers of this context, Gupta and Grossmann (2012) proposed a novel mathematical model for optimal planning of offshore oil and gas field infrastructure. Then, this model was developed by a multistage stochastic programming to regard uncertainty of parameters (Gupta and Grossmann,

2014). Also, about the maritime transportation of crude oil, Hennig et al. (2012) applied a mathematical programming model to describe the distribution of crude oil from supply places to demand point. Shen et al. (2011) introduced a Lagrangian relaxation approach for an inventory-routing problem with transshipment in crude oil transportation.

In this paper, we present a multi-period mathematical programming model which takes all of the strategic, tactical and operational activity into account in a fuzzy environment integrally. According to this model, offshore oilfield development operations (e.g. drilling, extraction, pipelines installation, and production of crude oil) are expressed as a mixed integer linear programming (MILP) problem. Furthermore, the lack of consideration of budget, time, and equipment limitations constraints which may far previous study away from real world, is intended in this work. In other word, the aim of this paper is development of optimization model for upstream oil industry activities as crude oil supply chain, in which net present value (NPV) of crude oil sale is maximized, while all of the constraints about oilfield development such as extract-rig limitation, production capacity, transport-rig limitation as well as budget and investment limitation should be satisfied.

The rest of paper is organized as follows: Next section gives an explanation of problem and describes the crude oil supply chain structure precisely. The proposed fuzzy mathematical programming model is elaborated in the 'Modeling' Section. 'Model performance, result and discussion' section reports the numerical results, sensitivity analysis, validation of approach and a managerial discussion of the results. Finally, 'Conclusion' section concludes the paper as well as introducing a number of attractive future research directions.

2- Problem definition

Consider an oilfield which consists of a set of explored oil reservoirs and wells. To appropriately exploit oilfield reservoirs, it is necessary to provide the capital and equipment required for drilling, extraction, production, supply and transfer. The first step, after exploration of reservoirs and oilfields, is drilling wells and then extraction for which well platforms are installed. The extracted crude oil which is usually mixed with water is transferred through pipelines to the production unit (PU) where impurities are separated and then the crude oil is ready to be supplied to customers (local refineries and exportation) (see Figure1).

The crude oil is transferred to local customers mostly through pipelines. Although the same is possible for exportation, it is usually carried out through marine terminals from where the oil is transferred overseas by big tankers. The transportation part of this problem is the crude oil transfer from well to PUs as well as from PUs to local refineries and exportation terminals through pipelines.

Depending on the oil reservoirs behavior and the available equipment, exploitation usually takes about fifteen to twenty years (Gupta and Grossmann, 2012, Sahebi and Nickel, 2014). During the planning horizon, the revenue from crude oil selling, the costs rigs and facilities as well as operational cost should be considered simultaneously to make optimal decisions.

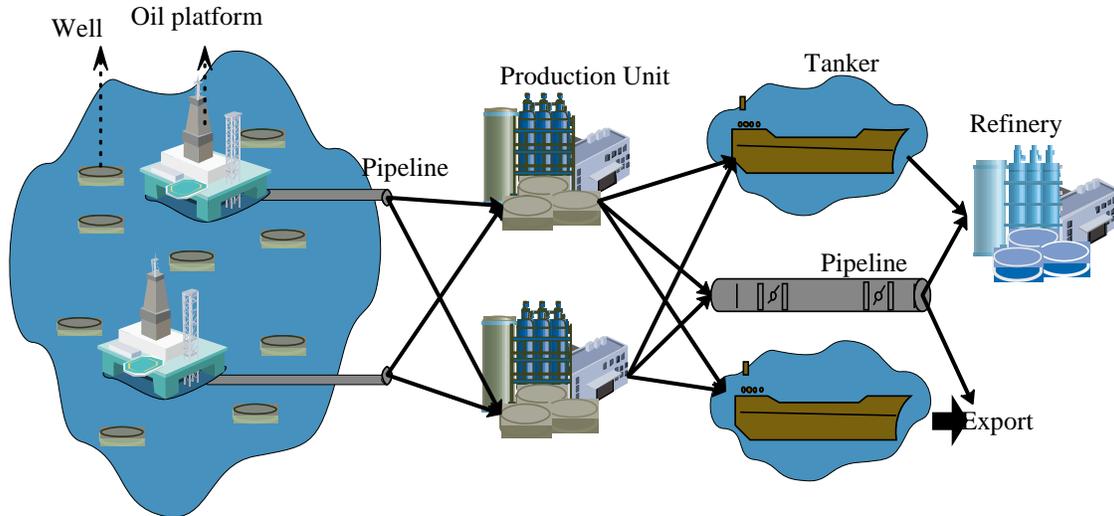


Figure 1.The network of crude oil supply chain

In order to provide a better description from the concerned problem, consider a proved oilfield consisting of several reservoirs where each of them contains a number of potential wells with specific productivity index (PI) and oil-water ratio (OWR) as well as some offshore platforms types and production rigs to recover oil from the wells and preparing it to supply. A set of pipes are available to pump crude oil from wells to PUs, and from PUs to customers, who have demand for produced crude oil. The optimal decision making about operations, transportation, and investment planning is main problem that divided to some sub-problems as follows:

- I. Which wells and in which periods should be drilled?
 - The answer of this problem is obtained by PI and OWR indices of wells.
- II. Which well/oil platforms (WPs) should be installed for drilling and extraction?
 - The answer of this problem is obtained by the capital and operational cost, and capacity of WPs.
- III. What type of pipes should be installed in different transportation lines?
 - The answer of this problem is obtained by the capital cost of pipes and their capacity.
- IV. Where PUs should be located?
 - The answer of this problem is obtained according to distances between wells to PUs, and PUs to customers.
- V. How much crude oil should/may be extracted in each period, and how much is production volume?
 - The answer of this problem is obtained according to PI and OWR indices of drilled wells, capacity of available PUs, and amount of customers demand.
- VI. How much capital should be invested in each period?
 - The answer of this problem is obtained according to facility and budget limitations.

In the concerned problem, an oil company (company that produce and supply crude oil) aims to make decisions optimally about above sub-problems. As mentioned above, the objective of this company is to maximize the NPV of crude oil sale. It is obvious that in order to achieve this objective, the company managers solve an optimization problem consisting facility selection (I, II, III), facility location and allocation (IV), production planning (V), and investment planning (VI).

Some parameters such as the capital and operational costs of facilities (WPs, PUs), transportation cost, wells indices (PI, OWR), and finally oil price and demand are to be specified during each period throughout the planning time horizon. Although estimation of the values of some of these parameters is possible based on the existing data and experts' opinions, they are undoubtedly imprecise and associated with uncertainties; therefore, in this research, these values have been considered as triangular fuzzy numbers to somewhat take into account the uncertainties governing the system. Thus, a fuzzy optimization problem is developed, which the solution of it can be useful for managers decision making.

3- Model formulation

3-1- Problem representation as a mathematical programming

In the following a mixed integer linear programming formulation is presented for the in hand problem. The nomenclature used in our model is as follows:

Index	Description
w	potential well, $w \in W = \{1, 2, \dots, W \}$
wp	potential well platform, $wp \in WP = \{1, 2, \dots, WP \}$
pu	location of Production Unit, $pu \in PU = \{1, 2, \dots, PU \}$
pl	pipeline types, $pl \in PL = \{1, 2, \dots, PL \}$
c_d	domestic customer, $c_d \in C_d = \{1, 2, \dots, C_d \}$
c_f	foreign customer, $c_f \in C_f = \{1, 2, \dots, C_f \}$
c	customer, $c \in C = \{1, 2, \dots, C_d + C_f \}$
t	Time horizon, $t \in T = \{1, 2, \dots, T \}$
Deterministic parameters	Description
U_t^{wp}	maximum extraction capacity of the wp th well platform in period t
U_t^{pu}	maximum production capacity of the pu th production unit in period t
U_t^{pl}	maximum transportation capacity of the pl th pipeline in period t
$Lng^{(wp, pu)}$	distance between wp th well platform (or well) and pu th production unit
$Lng^{(pu, c)}$	distance between pu th production unit and c th customer

Uncertain parameters**Description**

PI_t^w	Productivity Index of w th well in period t
pd_t^w	maximum Pressure Drop from w th well bore to well head in period t
OWR_t^w	maximum oil-to-water flow rate of the w th well in period t
Inv_t^W	maximum budget which can be invested in well drilling in period t
Inv_t^{WP}	maximum budget which can be invested in well platform installing in period t
Inv_t^{PU}	maximum budget which can be invested in production unit constructing in period t
L_t	maximum length of pipeline which can be installed during period t
UN_t^W	maximum number of the well which can be drilled in period t (Because of time and equipment limitation)
UN_t^{WP}	maximum number of the well platform which can be installed in period t
UN_t^{PU}	maximum number of the production unit which can be constructed in period t
D_t^c	demand volume of the c th customer in period t
α_t^{cf}	minimum percent of oil supply to c_f th foreign customer in period t
β_t^{cd}	minimum percent of oil supply to c_d th domestic customer in period t
d_t	price deflator index in period t
P_t^c	sale price of oil for c th customer in period t
B_t^*	Drilling ($*$ = w) or installing ($*$ = wp or pu or pl) cost of the f th facility in period t
FC_t^{wp}	fixed operation cost of wp th well platform in period t
FC_t^{pu}	fixed operation cost of pu th production unit in period t
Ex_t^{wp}	extraction cost per unit of fluid extracted by the wp th well platform in period t fixed operation cost of facility f in period t
PrC_t^{pu}	production cost per unit of crude oil produced by the pu th production unit in period t

Integer variables**Description**

Y_t^*	1 if the f th well ($*$ = w) is drilled, well platform ($*$ = wp) is installed, production unit ($*$ = pu) is built in period t ; else 0.
$Y_t^{(w,wp)}$	1 if the interconnection between w and wp is installed in period t ; else 0.

$Y_t^{pl,(wp,pu)}$	1 if the pl th type of pipes is installed in period t between wp and pp ; else 0.
$Y_t^{pl,(pu,c)}$	1 if the pl th type of pipes is installed in period t between pu and c ; else 0.
$open_t^*$	1 if the f th well ($* = w$) is open, well platform ($* = wp$) is open, production unit ($* = pu$) is open in period t ; else 0.
$open_t^{(w,wp)}$	1 if the interconnection between w and wp is open in period t ; else 0.
$open_t^{pl,(wp,pu)}$	1 if the pl th type of pipes is open in period t between wp and pp ; else 0.
$open_t^{pl,(pu,c)}$	1 if the pl th type of pipes is open in period t between pu and c ; else 0.
Continuous variables	Description
$oil_t^{w,wp}$	extracted oil volume from the w th well by the wp th well platform in period t
$water_t^{w,wp}$	extracted water volume from the w th well by the wp th well platform in period t
$oil_t^{wp,pu}$	transported oil volume from the wp th well platform to the pu th production unit in period t
$water_t^{wp,pu}$	transported water volume from the wp th well platform to the pu th production unit in period t
fld_t^{wp}	total extracted fluids at the wp th well platform in period t
$oil_t^{pu,c}$	total transported crude oil volume from the pu th production unit to the c th customer in period t
oil_t^{pu}	total oil produced by the pu th production unit in period t
S_t^{cf}	total oil supplied to the cd th domestic customer in period t
S_t^{cd}	total oil supplied to the cf th foreign customer in period t

The objective function (1) maximizes the NPV at the end of time horizon (1), which is obtained by summation of subtracting costs from crude oil sales (to domestic and foreign customers) revenue in each period. Costs can be classified into seven parts: the first three parts are associated with drilling well, installing WPs, and building PUs respectively. The fourth part is associated with installing pipes from WPs to PUs, and from PUs to customers. The last three parts are operational cost associated with extraction, production, and transportation, respectively.

$$\begin{aligned}
\text{Maximize } NPV = & \sum_t d_t \left(\sum_{c_d} p_t^{c_d} S_t^{c_d} + \sum_{c_f} p_t^{c_f} S_t^{c_f} - \sum_w B_t^w Y_t^w - \sum_{wp} B_t^{wp} Y_t^{wp} - \sum_{pu} B_t^{pu} Y_t^{pu} \right. \\
& - \left[\sum_{wp} \sum_{pu} \sum_{pl} Lng^{wp,pu} \cdot B_t^{pl} \cdot Y_t^{pl,(wp,pu)} + \sum_{pu} \sum_c \sum_{pl} Lng^{pu,c} \cdot B_t^{pl} \cdot Y_t^{pl,(pu,c)} \right] \\
& - \left[\sum_{wp} (\bar{F}C_t^{wp} \cdot open_t^{wp} + \bar{E}x_t^{wp} \cdot fld_t^{wp}) \right] - \left[\sum_{pu} (\bar{F}C_t^{pu} \cdot open_t^{pu} + \bar{P}rC_t^{pu} \cdot oil_t^{pu,c}) \right] \\
& - \left. \left[\sum_{wp} \sum_{pu} Lng^{wp,pu} \bar{T}r_t^{wp,pu} \cdot fld_t^{wp,pu} + \sum_{pu} \sum_c Lng^{pu,c} \bar{T}r_t^{pu,c} \cdot oil_t^{pu,c} \right] \right) \quad (1)
\end{aligned}$$

If the objective function is changed into a cost-minimizing objective, the impact of crude oil price in the model will be ignored. In this case, oilfield development is only affected by the crude oil demand which may not reflect the real situation within a strategic planning horizon.

- *Constraints*

$$\sum_w oil_t^{w,wp} = \sum_{pu} oil_t^{wp,pu} \quad \forall wp, t \quad (2)$$

$$\sum_w water_t^{w,wp} = \sum_{pu} water_t^{wp,pu} \quad \forall wp, t \quad (3)$$

$$\sum_{wp} oil_t^{wp,pu} = \sum_c oil_t^{pu,c} \quad \forall pu, t \quad (4)$$

$$fld_t^{wp} = \sum_w oil_t^{w,wp} + \sum_w water_t^{wp,pu} \quad \forall wp, t \quad (5)$$

$$S_t^{c_d} = \sum_{pu} oil_t^{pu,c_d} \quad \forall c_d, t \quad (6 \text{ I})$$

$$S_t^{c_f} = \sum_{pu} oil_t^{pu,c_f} \quad \forall c_f, t \quad (6 \text{ II})$$

$$oil_t^{pu} = \sum_c oil_t^{pu,c} \quad \forall pu, t \quad (7)$$

$$fld_t^{wp,pu} = oil_t^{wp,pu} + water_t^{wp,pu} \quad \forall wp, t \quad (8)$$

$$fld_t^{wp} \leq U_t^{wp} \cdot open_t^{wp} \quad \forall wp, t \quad (9)$$

$$\sum_{wp} fld_t^{wp,pu} \leq U_t^{pu} \cdot open_t^{pu} \quad \forall pu, t \quad (10)$$

$$(oil_t^{w,wp} + water_t^{w,wp}) \leq U_t^{wp} \cdot open_t^{w,wp} \quad \forall w, wp, t \quad (11)$$

$$fld_t^{wp,pu} \leq \sum_{pl} U_t^{pl} \cdot open_t^{pl,wp,pu} \quad \forall wp, pu, t \quad (12)$$

$$oil_t^{pu,c} \leq \sum_{pl} U_t^{pl} \cdot open_t^{pl,pu,c} \quad \forall pu, c, t \quad (13)$$

$$\sum_t Y_t^w \leq 1 \quad \forall w \quad (14)$$

$$\sum_t Y_t^{wp} \leq 1 \quad \forall wp \quad (15)$$

$$\sum_t Y_t^{pu} \leq 1 \quad \forall pu \quad (16)$$

$$\sum_T \sum_{wp} Y_t^{(w,wp)} \leq 1 \quad \forall w \quad (17)$$

$$\sum_{pl} \sum_{pu} \sum_t Y_t^{pl,(wp,pu)} \leq 1 \quad \forall wp \quad (18)$$

$$\sum_{pl} \sum_t Y_t^{pl,(pu,c)} \leq 1 \quad \forall pu, c \quad (19)$$

$$open_t^w = open_{t-1}^w + Y_{t-1}^w \quad \forall w, t \quad (20)$$

$$open_t^{wp} = open_{t-1}^{wp} + Y_{t-1}^{wp} \quad \forall wp, t \quad (21)$$

$$open_t^{pu} = open_{t-1}^{pu} + Y_{t-1}^{pu} \quad \forall pu, t \quad (22)$$

$$open_t^{(w,wp)} = open_{t-1}^{(w,wp)} + Y_{t-1}^{(w,wp)} \quad \forall w, wp, t \quad (23)$$

$$open_t^{pl,(wp,pu)} = open_{t-1}^{pl,(wp,pu)} + Y_{t-1}^{pl,(wp,pu)} \quad \forall pl, wp, pu, t \quad (24)$$

$$open_t^{pl,(pu,c)} = open_{t-1}^{pl,(pu,c)} + Y_{t-1}^{pl,(pu,c)} \quad \forall pl, pu, c, t \quad (25)$$

$$open_t^{w,wp} \leq open_t^w \cdot open_t^{wp} \quad \forall w, wp, t \quad (26)$$

$$open_t^{pl,w,wp} \leq open_t^{wp} \cdot open_t^{pu} \quad \forall wp, pu, pl, t \quad (27)$$

$$open_t^{pl,(pu,c)} \leq open_t^{pu} \quad \forall pu, pl, c, t \quad (28)$$

$$\sum_{w \in W} \beta_t^w Y_t^w \leq \#v_t^w \quad \forall t \quad (29)$$

$$\sum_{wp} \beta_t^{wp} Y_t^{wp} \leq \#v_t^{wp} \quad \forall t \quad (30)$$

$$\sum_{pp} \beta_t^{pu} Y_t^{pu} \leq \#v_t^{pu} \quad \forall t \quad (31)$$

$$\sum_w Y_t^w \leq \check{U}N_t^w \quad \forall t \quad (32)$$

$$\sum_{wp} Y_t^{wp} \leq \bar{U} N_t^{wp} \quad \forall t \quad (33)$$

$$\sum_{pu} Y_t^{pu} \leq \bar{U} N_t^{pu} \quad \forall t \quad (34)$$

$$\sum_{pl} \sum_{wp} \sum_{pu} Lng^{wp,pu} Y_t^{pl,wp,pu} + \sum_{pl} \sum_{pu} \sum_c Lng^{pu,c} Y_t^{pl,pu,c} \leq \bar{L}_t \quad \forall t \quad (35)$$

$$\sum_{wp} oil_t^{w,wp} + \sum_{wp} water_t^{w,wp} \leq \bar{A}_t^w \cdot \bar{F}_t^w \cdot open_t^w \quad \forall t, w \quad (36)$$

$$\sum_{wp} oil_t^{w,wp} = \bar{O}WR_t^w \sum_{wp} water_t^{w,wp} \quad \forall t, w \quad (37)$$

$$\alpha_t^{c_f} \bar{B}_t^{c_f} \leq S_t^{c_f} \leq D_t^{c_f} \quad \forall t, c_f \quad (38)$$

$$\beta_t^{c_d} \bar{B}_t^{c_d} \leq S_t^{c_d} \leq D_t^{c_d} \quad \forall t, c_d \quad (39)$$

Constraints (2)-(8) control the balance between input and output oil and water flow of wells, WPs, PUs, and customers at the end of each planning horizon. Constraints (9)-(10) are capacity constraints for WPs and PUs respectively. Constraint (11) states that fluid (Oil and water) can be extracted from a well during time period t if there is an available well platform in that time period. A pipeline should be open between WPs and PUs (12), as well as PUs and customers to transport crude oil (13). Not that, also, the capacity of pipeline is considered into two last constraints. Constraints (14-16) restrict the drilling wells, installation WPs and building PUs to take place at most once during time horizon, respectively. Constraints (17)-(19) state that the connection between well-WPs, WPs-PUs, and PUs-customers can be installed only once in each period during time horizon, respectively. Constraints (20)-(25) show the logic behind establishing network during time period. They mean that, in each period, a well, WP, PU and interconnection can be used if it existed in the previous period or started to come into being then. Note, it is assumed that if establishment of each part starts in period $t - 1$, it can be utilized in time period t . Constraint (26) shows that a well can be connected to a WP when both are available; Constraint (27) means that a pipeline between a PW and a PU is possible only when both exist, and Constraint (28) states that pipelines between customers and PU are possible only when the latter exist. Constraints (27)-(28) are nonlinear; they are substituted with the following four constraints to linearize the model:

$$open_t^{w,wp} \leq open_t^w \quad \forall w, wp, t \quad (26 \text{ I})$$

$$open_t^{w,wp} \leq open_t^{wp} \quad \forall w, wp, t \quad (26 \text{ II})$$

$$open_t^{pl,w,wp} \leq open_t^{wp} \quad \forall w, wp, t \quad (27 \text{ I})$$

$$open_t^{pl,w,wp} \leq open_t^{pu} \quad \forall wp, pp, pl, t \quad (27 \text{ II})$$

Constraints (29)-(31) express that the cost of well drilling, WP installing, and PU building cannot exceed the considered budget for each of them, respectively. On the other hand, in addition to budget constraints, development of crude oil supply network can still be limited because of the equipment and time limitation in each period. This fact is considered in the modeling by constraints (32)-(35).

The reservoir and well behavior are important factors in making decisions about selecting wells for drilling and the amounts of their production. A reservoir behavior is generally determined by Productivity Index (PI) and Oil-Water Ratio (OWR).

PI is the ratio of a well production (oil and water) to its pressure drop. Let us assume that in period t the well w is opened. Also, let us consider the related PI and pressure drop are equal to PI_t^w and Pd_t^w , respectively. Thus, according to PI definition, the amount of liquid (oil and water) extracted from this

well (Q_t^w) satisfies $PI_t^w \geq \frac{Q_t^w}{Pd_t^w}$. Since $Q_t^w = \sum_{wp} oil_t^{w,wp} + \sum_{wp} water_t^{w,wp}$, we have $PI_t^w \geq \frac{\sum_{wp} oil_t^{w,wp} + \sum_{wp} water_t^{w,wp}}{Pd_t^w}$.

The ratio of the net pure oil to the extracted mixture is calculated by OWR. Let us assume its OWR index for drilled well w in period t is OWR_t^w . According to OWR definition, the total oil and water extracted

from this well satisfies $OWR_t^w = \frac{\sum_{wp} oil_t^{w,wp}}{\sum_{wp} water_t^{w,wp}}$. Constraint (37) adjusts the amount of the pure oil produced

from a well with considering its OWR index.

Constraints regarding supplying customers demand are the last ones in the problem. During each period, it is necessary to supply a minimum specified amount of crude oil to respond to the customers demand. Demand satisfaction is considered in constraints (35)-(36) for domestic and foreign customers, respectively. Note, because of the sensitivity of foreign customers, usually α_t^{cf} is greater than β_t^{cd} .

3-2- Model indeterminacy and fuzzy approach

The input parameters of model are usually set based on the available data. Due to lack of available data and/or errors in some data, exact setting the parameters is impossible. Almost, in the strategic model, such as presented model, indeterminacy of parameters should be considered by an efficient approach. For modeling indeterminacy, there exist two mathematical systems; one is probability theory, and the other is uncertainty theory. Probability is interpreted as frequency, while uncertainty is interpreted as personal belief degree (Liu, 2010).

Here, we present a new fuzzy method to tackle model indeterminacy. Fuzzy mathematical programming is one of the most used approaches in the class of uncertainty theory. Methods proposed on this context are numerous; it was first introduced by (Tanaka† et al., 1973) based on (Bellman and Zadeh, 1970) fuzzy decision making model. Different methods have been proposed which are based on either fuzzy order or fuzzy measures (For more study, the reader is referred to a “survey on fuzzy linear programming”(Shams et al., 2012)).

In some FLP methods, a certain number replaces the fuzzy one in the model. Peidro et al. (2009) suggested the replacement of certain value $p_n + \frac{d_p - d'_p}{3}$ instead of triangular fuzzy number (TFN)

$\tilde{p}_n = (p_l, p_n, p_u)$ shown in Fig. 2. We first generalize this method and then, apply the proposed method to defuzzify.

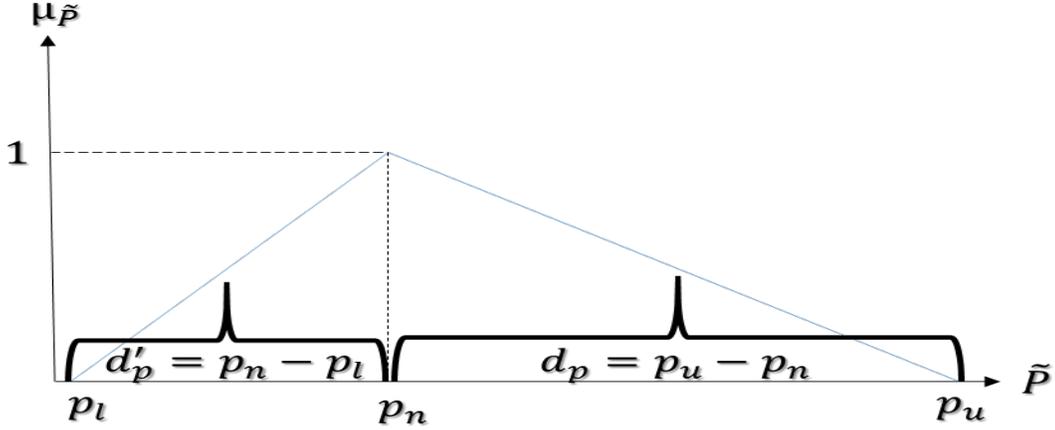


Figure 2. Triangular fuzzy number

Let the function $\varphi: TFN \rightarrow \mathbb{R}$ be defined in the following form:

$$\varphi(\tilde{P}) = \frac{p_l + p_n + p_u}{3} \quad (40)$$

$\tilde{P} = (p_l, p_n, p_u) \in TFN$

According to Eq. (41), function φ has been used in the method of (Peidro et al., 2009) for defuzzification. Therefore, they replace the mean of components instead of TFS.

$$p_n + \frac{d_p - d'_p}{3} = p_n + \frac{(p_u - p_n) - (p_n - p_l)}{3} = \frac{p_l + p_n + p_u}{3} \quad (41)$$

We define function φ^λ as follows:

$$\varphi^\lambda(\tilde{P} = (p_l, p_n, p_u)) = \frac{p_l + \lambda \cdot p_n + p_u}{3 + (\lambda - 1)} \quad (42)$$

It is clear that function φ^λ , contrary to function φ , takes into consideration the weight λ for the TFN nominal value p_n . In this new defuzzification method, we can analyze the effect of nominal values on the results. Note that if $\lambda = 1$, then we refer to the previous method.

Note that the problem is defined in a fixed network. Therefore, the model size (the number of variables and constraints) depends on the cardinality of the index sets. Before defuzzification, the number of variables and constraints is formulated, respectively, as follows:

$$\begin{aligned} & |T| (4|W| + 5|WP| + 3|PU| + (|C_f| + |C_d|) + 4|W| + |WP| \\ & + 3|WP| + |PU| + 3|PU| (|C_f| + |C_d|) \\ & + 3|PU| (|C_f| + |C_d|) + |PL| \\ & + 3|WP| + |PU| + |PL|) \end{aligned} \quad (43)$$

$$\begin{aligned}
& |T| (3|W| + 3|WP| + 2|PU| + (|C_f| + |C_d|) + 4|W| |WP| \\
& + 2|WP||PU| + 3|PU|(|C_f| + |C_d|) \\
& + 2|PU|(|C_f| + |C_d|)|PL| \\
& + 2|WP||PU||PL| + 7)
\end{aligned} \tag{44}$$

After the defuzzification, the size of the model is equal to the above formulas, since only TFN changes to a deterministic number in Eq. (42).

4- Model implementation

In this section, we investigate the application of the proposed model using the read data of Persian Gulf oilfields. The scales of the problem under investigation are as follows (see fig. 3):

- Wells: We assume 9 explored locations for drilling wells ($|W| = 9$);
- WPs: It is assumed there are 4 types of WPs for drilling wells and extracting crude oil ($|WP| = 4$);
- PUs: The number of locations for PUs is assumed to be 5 ($|PU| = 5$);
- Customers: We consider 2 refineries as local customers ($|C_d| = 2$);
- Pipes: We assume there are 3 types of pipes (26, 38 and 42 inch) for transporting crude oil from WPs to PUs, and from PUs to customers. ($|PL| = 3$).

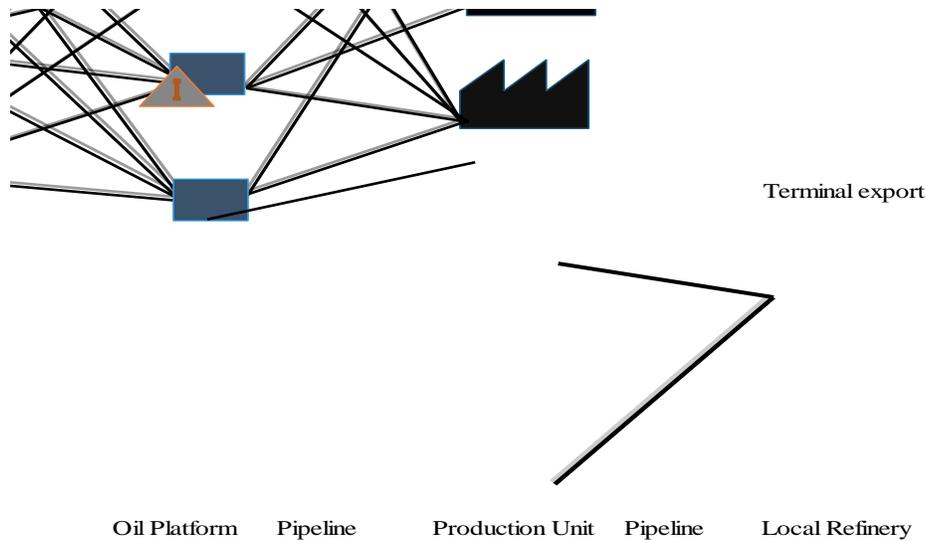


Figure 3. A schematic view of crude oil supply network in numerical study

4-1- Effects of time horizon on results

In this sub-section, the effects of the time horizon on the results are analyzed. The profitability of the activities are determined at the end of each period during 15-year time horizon, and at the end of each time horizon planning from 1-year to 15-year (Fig. 3).

Usually, in strategic/long-time planning horizon, in the early planning, the profitability is lesser than short-time planning horizon. To clarify the subject, consider 7-year and 15-year planning horizons in which we want to determine NPV, and show the difference between short and long time horizons. Fig. 4 states that although, from the beginning to the end of the 8th year, NPV earned by the 15-year time horizon is lesser than the value of the 7-year, conversely, from the 9th year to the end planning, NPV earned by the 7-year time horizon is less than the values of the 15-year, and the difference between them is enhanced. Thus, the length of the planning horizon problem has a significant impact on the decision-making.

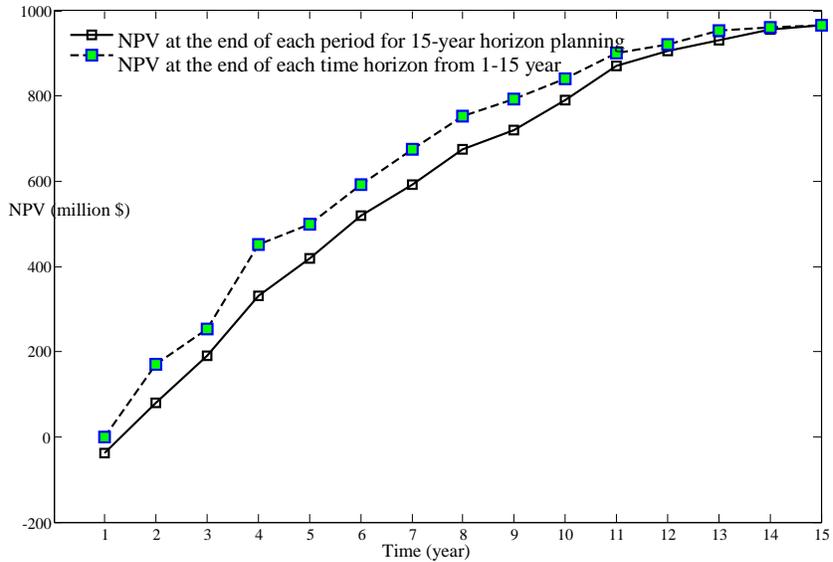


Figure 4. NPV during time horizon planning

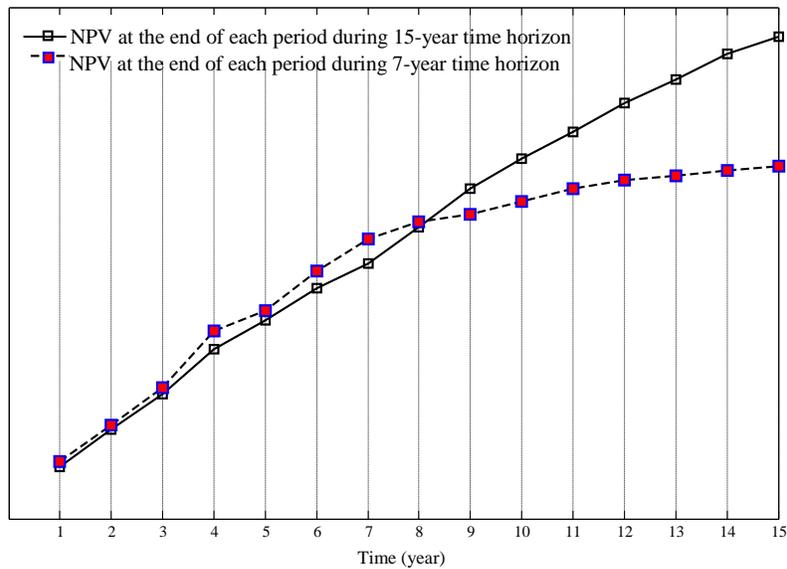


Figure 5. Trend of NPV during 7-year and 15-year time horizon planning

Table1. NPV at the 1-15 time horizon, and NPV at the different period during 7-year and 15-year horizon planning

Year	NPV (1×10^6 \$) End of each time horizon	NPV (1×10^6 \$) ; 7-year End of each time period	NPV (1×10^6 \$) ; 15-year End of each time period
1	0	-20.88	-37.06
2	170.92	92.29	80.87
3	253.43	209.31	190.55
4	452.18	338.42	330.62
5	500.90	452.50	420.58
6	593.97	578.08	520.20
7	674.27	674.27	593.74
8	752.11	730.80	675.47
9	794.25	753.02	761.23
10	841.40	793.62	790.84
11	900.59	834.73	870.19
12	922.26	860.48	905.22
13	954.60	873.57	930.17
14	962.71	891.23	955.22
15	965.22	903.74	965.22

4-2- Effect of “price and demand” on results

With regard to the economic point of view, price and demand are two important parameters in decision-making. We want to analyze the simultaneous effects of price and demand when they have their critical values (i.e. p_l and p_u). So, the results should be presented in the following four critical scenarios:

Scenario	1	2	3	4
Condition	$price = p_u$ $demand = p_u$	$price = p_u$ $demand = p_l$	$price = p_l$ $demand = p_u$	$price = p_l$ $demand = p_l$

The effect of the mentioned scenarios on oilfield development can be calculated by different criteria. Suppose the situation of an oilfield is specified based on two criteria: one shows the “*percent development (PD)*” and the other indicates the “*percent production (PP)*”. We define PD as the ratio of the available facilities (WPs, PUs, pipelines, etc.) to the minimum facilities required for full production while PP is defined as total production to the estimated capacity production from the beginning of development. Table 2 shows the results of the effects of price and demand on these two criteria at the end of the 15-year time horizon planning.

Table 2. The values of oilfield development criteria in critical scenarios

	Drilled wells	Installed WPs	Located PUs	PD (%)	PP (%)
Scenario 1	9	3	4	100	95.50
Scenario 2	8	2	4	81.33	70.00
Scenario 3	7	2	4	73.75	71.33
Scenario 4	5	1	2	43.66	40.13

4-3- Effects of uncertainty on results

As mentioned in the previous section, due to existent uncertainty in data, TFNs are considered for model parameters. Obtained fuzzy optimization problem is transformed to deterministic model by function ϕ^λ , and then solved by CPLEX solver.

For different values of λ different deterministic models and, hence, different optimal solutions (SOL_λ^*) are possible. Suppose the real deterministic model is known; substituting every SOL_λ^* in it, some constraints may be unsatisfied and the objective function value may deviate from its optimum. Therefore, “percent of unsatisfied constraint” and “percent of deviation from the optimal objective function” are introduced as two criteria for the validation of the SOL_λ^* . But, since the deterministic model does not exist, several such models are simulated through considering membership functions of the fuzzy parameters, and the mentioned criteria are calculated for every one of them. It is obvious that the solution with average values less than those of the two criteria, is more reliable and its decision risks are less.

We first simulate 20 certain models using membership function of TFNs, and then calculate the average of mentioned criteria. Computational results are tabulated in Table 3. Also, the value of criteria for $\lambda = 1$ and $\lambda = 1.2$ is shown in Fig. 5 for every simulation.

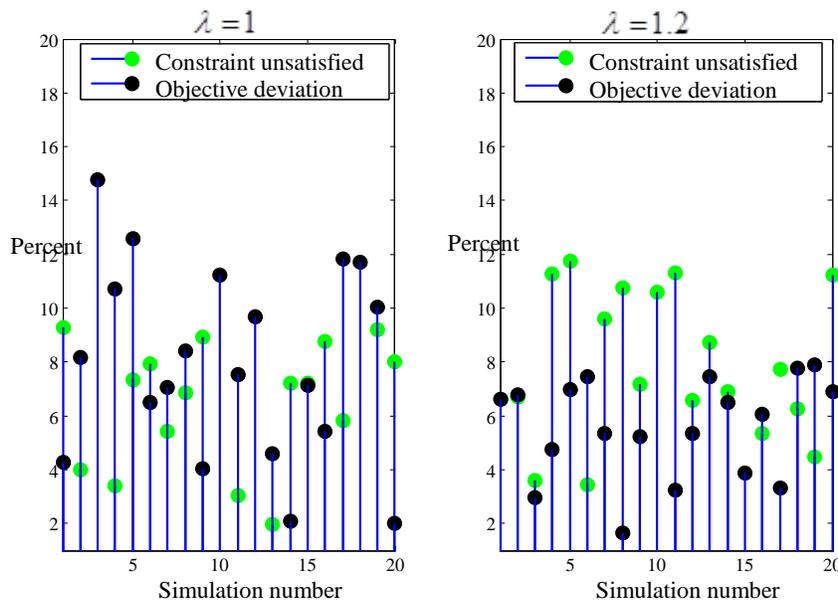


Figure 6. The value of Constraint unsatisfied and objective deviation criteria for $\lambda = 1$ and $\lambda = 1.2$

Table 3. Sensitivity analysis of λ

λ	Average of unsatisfied constraints (%)	Average objective deviation (%)	Maximum unsatisfied constraints (%)	Maximum objective deviation (%)
0.7	7.94	14.23	11.06	20.54
0.8	8.33	10.83	10.54	18.33
0.9	7.43	11.93	10.12	19.06
1	7.23	10.48	9.70	15.09
1.1	7.80	8.44	12.27	11.12
1.2	7.95	5.23	12.00	8.65
1.3	8.11	6.73	13.46	8.44
1.4	9.08	7.64	13.94	9.31
1.5	9.13	8.41	18.12	8.86
1.6	9.75	7.19	15.67	9.13
1.7	9.53	5.41	19.07	8.07
1.8	9.74	7.29	17.14	9.00
1.9	10.03	6.53	15.44	8.73
2	9.38	7.88	15.90	7.94

In order to validate the proposed fuzzy method, again 20 certain models are simulated using membership function of TFNs. We compare $SOL_{\lambda=1.2}^*$ with $SOL_{Nominal}^*$, in which $SOL_{Nominal}^*$ is optimum solution which is obtained by replacement nominal values instead of TFNs. As is clear from Fig. 6, although both solutions have approximately equal value in average, the deviations and risk of $SOL_{Nominal}^*$ is very greater than $SOL_{\lambda=1.2}^*$. So, computational tests using randomly generated data are presented showing that the fuzzy approach is worth considering in these types of problems.

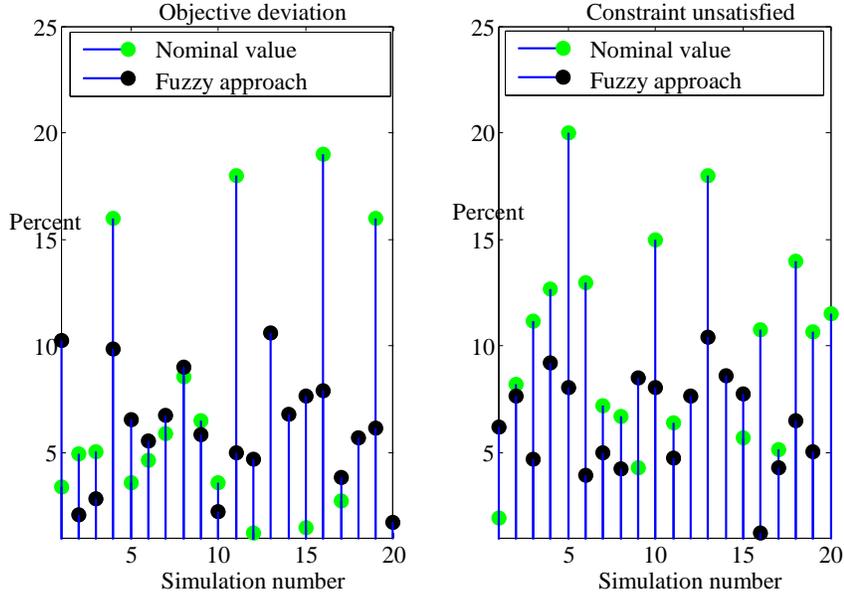


Figure 7. The compression of fuzzy approach ($SOL_{\lambda=1.2}^*$) and nominal value approach ($SOL_{Nominal}^*$)

5- Conclusions

In this work, we presented a mathematical programming model for the design of crude oil supply chain and planning of offshore oilfields development. The proposed model is a multi-period mixed-integer linear program which accounts for the optimization of the economic performance. The developed model considers the strategic decisions such as facility location (i.e., locations of wells and production units (PU)), facility allocation (i.e., assigning the wells to the PUs, and the PUs to the customers), and technology selection with respect to the capacity and cost. The tactical decisions include the optimal time for drilling, installation of the equipment facilities required for the production and transportation, optimal extraction of each well, and the amount of optimal production.

The application of the proposed model was examined through a case problem in which the real data of Gulf Oilfields and Iranian South Oilfields was utilized. In order to validate the performance of the fuzzy optimization model, some random problems were simulated using membership functions. We showed the proposed fuzzy formulation was more effective than the deterministic model in handling the real situations, where certain information is not available for the oilfield development planning. Furthermore, the fuzzy model did not result in an excessive computational time. Additionally, we analyzed the effects of the time horizon on the results. Our computational results implied the length of the planning horizon has a significant impact on the decision-making. The obtained results, in addition, revealed that a long time horizon could provide a larger NPV at the end of the oilfield Life-cycle.

For the future work, it would be interesting to investigate the nonlinearity behaviors of the oil reservoirs, the pressure drops of the oil wells, and the impacts of the activities on the environment and society.

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