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Designing supply chain network with discount policy and specific price-dependent demand function

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Abstract

In this study, supply chain network design is considered. Responsibility and profitability are the company's main features, so we proposed a model to maximize profit, reducing logistic costs, especially shortage costs, to increase responsibility. Adopting the right sales policy by a seller is an issue that needs to be addressed. In this research, we try to improve sales conditions based on each channel's capacity by considering the all unit discount policy for each channel's sale. We used a price-dependent demand function to bring the situation closer to the real world. We considered the factor of advertising and inflation on demand in this issue. A mixed-integer nonlinear programming model is introduced for the problem and formulated in GAMS software; then, sensitivity analysis examines some parameters.

Keywords: Supply chain network design, discount policy, price-dependent demand function, logistic cost, multi-channel, multi-product

1-Introduction & Literature review

Supply chain management is essential in the recent decade because the number of companies is increasing, and every company wants to maximize its market share and profit. They should employ some strategy or technique to attract more customers. In supply chain network design (SCND), being responsive to customer demand and reducing logistic cost is an important challenge in past studies. To increase customer satisfaction, researchers introduced multi-channel, multi-product models. SCND could include supplier, plant, distribution center, retailer, and customer zone. Designing a supply chain consists of making decisions according to which facility should be open and which set of downstream serves each upstream facility (Kheirabadi et al. 2019). All decisions should be optimized to be cost-effective and find the best flow between echelons. Suppliers should have the capacity to supply all the raw materials from the bill of material. Logistics cost includes holding costs, transporting costs, purchasing costs, etc. Quantity discount is an interesting subject in this field because each echelon proposes a particular policy (Mousavi et al. 2014). In the real world, each channel has a specific capacity. With the right ordering policy at the beginning of each period, it increases responsibility at its highest level and decreases total costs at the lowest level. Inventory maintenance is one of the cost factors of the system. Each channel can increase the customer's desire by providing the right policy. We design a supply chain network, multi-period, multi-product, multi-echelon. We investigated five-echelon, including supplier, plant, distribution center, retailer, and customer. Plants order raw material base on the bill of material for each product. Each echelon proposes a special discount policy.

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Customer demand is affected by different factors such as price, ads, and inflation rate. All facilities are capacitated. The shortage is allowed in the customer zone. In the following, a brief review of supply chain network design is provided. Location and allocation problems are some of the issues discussed in this area. In one article, considering the single product, they tried to find the best number of plants and the optimal amount of items which are transferred from plant to the distribution center and then respond demand with some assumptions such as limit capacity and fixed demand (Shankar et al. 2013). A sustainable supply chain network is proposed based on two scenarios; they use a robust optimization approach and weighted goal programming and LP metric method. The model seeks to minimize logistic costs and minimize environmental emissions; their model considers the “bill of material (BOM)” for each product. According to BOM, the raw material is ordered from the supplier (Isaloo and Paydar, 2020). Some researchers investigate inventory optimization as a base problem. Another researcher investigated multi-product, multi-period inventory control between two echelons under a budget constraint, and products are purchased under quantity discount with fuzzy discount rates (Mousavi et al. 2013a). Inventory control and selling policy issues between two channels are called buyer and vendor policy in literature. The buyer and vendor policy can usually be formed between each member of the supply chain. An article considered a two-stage supply chain network where a buyer purchases a product from the capacitated supplier, supplier proposed different prices; the buyer selects suppliers to minimize the total cost per time (Li et al. 2018). In supply chain issues, the main goal is to meet customer demand. Some studies have tried to estimate demand closer to the real-world as innovators. Researchers had examined uncertainty in the demand rate (Mousavi et al. 2013b). Customers are sensitive to the delivery time and selling price. They propose a demand function, in single product conditions and price, safety stock is also considered an influential factor on demand (Roy and Chaudhuri, 2007). Two types of demand functions are introduced for two sales modes; it examines the price-dependent demand function (Rabbani et al. 2018). In this situation, rebate and selling price factors are useful in the demand function, and inflationary conditions are considered to get closer to the real world. Using different strategies together can bring issues closer to the real world. There is a gap in this field; we suggest a model that considers different policies to fix this gap. The all unit discount policy can be implemented in many logistics companies, and it has excellent benefits in holding costs and transporting cost, Therefore, the balance between the amount of product in each breakpoint and demand is one of the issues that need to be addressed, in this research demand follows a specific function, so the amount of order is also affected by factors affecting demand and We try to optimize the network conditions with the price-dependent demand function, rebate, and inflation rate.

2- Model description and formulation

2-1- Model description

In this section, a new model is presented in the supply chain network system. This work is considered a multi-echelon, multi-period, and inventory control problem. The model includes different layers: supplier, plant, distribution center(DC), retailer, and customer. Plants buy raw material and use raw materials based on the bill of material(BOM). All members of the supply chain purchase different quantities base on the need to respond to downstream members. Upstream members sell their product under an “all unit discount” policy, and sellers propose different “breakpoints”; downstream members should choose the best amount of product in each period. As we know, products are transported through channels, with the discount policy. The number of products has been indifferent category, based on category price is different. In our case, increasing the number of products cause less price for each item. The proposed supply chain model is shown to be a mixed integer binary nonlinear programming type to maximize profit, the profitability of a supply chain depends on increasing revenue while reducing costs, total supply chain costs that includes the opening cost of facilities, transportation cost, purchasing cost, the penalty for unutilized opened capacities, and lost scale cost. In this research, by obtaining the amount of ordering and using the appropriate breaking points, Then the inventory of each facility for the next periods is determined; we try to respond better to the demand. Then we calculate demand with the demand function we introduced. The demand function we used in this study examined the factors influencing customer demand. In the beginning, a price is offered by the retailer, and then a rebate percentage is specified to attract the customer, then the final price for the product is obtained. Other factors affecting demand include advertising and inflation were considered. Of course, there are

customers whose decisions are not influenced by any factor, and they are always a buyer. We assume the model has five levels: supplier, plant, distribution center (DC), retailer, and customer, there is a fixed opening cost and holding cost for each unit. The plants can manufacture products with a limited capacity to hold them. Each member has limited capacity. The order is received at the beginning of the period.

2-2- Model formulation

Indices:

s	Supplier	k	Retailer	p	Product
i	Plant	c	Customer	r	Raw material
j	DC	t	Period	B	Price break

Parameters:

bom_p^r	amount of raw material r required to produce a unit of product p
$rcap_s^t$	The raw material capacity of a supplier s in period t
$pcap_i^t$	The production capacity of plant i in time period t
$pcap_j^t$	The capacity of DC j in period t
$pcap_k^t$	The capacity of retailer k in period t
a_s	Opening cost of supplier
b_i	Opening cost of plant
d_j	Opening cost of DC
e_k	Opening cost of the retailer
f_{si}	Transportation cost from the supplier to plant i
g_{ij}	Transportation cost from plant i to DC j
h_{jk}	Transportation cost from DC j to retailer k
l_{kc}	Transportation cost from retailer k to customer c
pp_{ip}^t	Production cost of product p in plant i in period t
Hi_{ip}^t	Holding cost of product p in plant i in period t
Hj_{jp}^t	Holding cost of product p in DC j in period t
Hk_{kp}^t	Holding cost of product p in retailer k in period t
$s\theta_s^t$	Un-utilized capacity penalty cost for supplier s in period t
$p\theta_i^t$	Un-utilized capacity penalty cost for plant i in period t
$dc\theta_j^t$	Un-utilized capacity penalty cost for DC j in period t
$r\theta_k^t$	Un-utilized capacity penalty cost for retailer k in period t
π_c^t	Penalty cost for unmet demand for customer c in period t
m_c^t	Coefficient of exponential purchasing demand function for customer c in period t
f^t	Annual inflation rate in period t
k^t	Inflation coefficient in sales advertising cost function
q_{cp}^t	Number of customers seeking to purchase who are not sensitive to demand variation
k_p^t	Rebate coefficient of exponential purchasing demand function
$MSRP_p^t$	retailer's suggested price for product p in period t
srr^{min}	The minimum allowable sales rebate rate
srr^{max}	The maximum allowable sales rebate rate
c_{ijp}^{tB}	Purchasing cost of product p from plant i to DC j in breakpoint B
c_{jkp}^{tB}	The purchasing cost of product p from DC j to retailer k in breakpoint B
u_{ijp}^{tB}	the B th discount breakpoint of p th product purchase by DC j from plant i in period t
u_{jkp}^{tB}	the B th discount breakpoint of p th product purchase by retailer k from DC j in period t

Variables:

x_{sir}^t	The flow raw material r from the supplier s to the plant i
v_s	If supplier s is selected 1,
h_{pi}^t	The amount of product p manufactured by the plant i in period t
x_{ijp}^t	The flow product p from the plant i to DC j

in_{jp}^t	The initial (remained) positive inventory of product p purchased by DC j from planet i in period t
v_i	If planet i is opened 1
x_{jkp}^t	The flow product p from the DC j to retailer k
in_{kp}^t	The initial (remained) positive inventory of product p purchased by plant i from supplier s in period t
v_j	If DC j is opened 1
x_{kcp}^t	The flow product p from the retailer k to customer c
v_k	If retailer k is opened 1
D_{cp}^t	The demand of customer c from production p in period t
p_{cp}^t	price of product p for customer c in period t
b_{cp}^t	Rebate value of product p on sales for customer c in period t
μ_{sir}^{tB}	A binary variable that is set to 1 if plant i purchases raw material r from the supplier s at price breakpoint B in period t , and set to 0 otherwise
μ_{ijp}^{tB}	A binary variable that is set to 1 if DC j purchases product p from the plant, i at a price breakpoint B in period t , and set to 0 otherwise
μ_{jkp}^{tB}	A binary variable that is set to 1 if retailer k purchases product p from DC j at price breakpoint B in period t , and set to 0 otherwise

$$\begin{aligned}
\max z = & \sum_t \sum_p \sum_c \sum_k x_{kcp}^t p_{cp}^t (1 + f_t) \\
& - \left(\sum_s a_s v_s + \sum_i b_i v_i + \sum_j d_j v_j + \sum_k e_k v_k + \sum_s \sum_i \sum_r \sum_t f_{sir} x_{sir}^t \right. \\
& + \sum_i \sum_j \sum_p \sum_t g_{ij} x_{ijp}^t + \sum_j \sum_k \sum_p \sum_t h_{jk} x_{jkp}^t + \sum_k \sum_c \sum_p \sum_t l_{kc} x_{kcp}^t \\
& + \sum_i \sum_j \sum_p \sum_t \sum_B x_{ijp}^t c_{ijp}^{tB} \mu_{ijp}^{tB} + \sum_j \sum_k \sum_p \sum_t \sum_B x_{kcp}^t c_{jkp}^{tB} \mu_{jkp}^{tB} \\
& + \sum_i \sum_j \sum_p \sum_t \sum_B x_{ijp}^t c_{ijp}^{tB} \mu_{ijp}^{tB} + \sum_j \sum_k \sum_p \sum_t \sum_B x_{kcp}^t c_{jkp}^{tB} \mu_{jkp}^{tB} \\
& + \sum_i \sum_t p \theta_i^t \left(v_i p cap_i^t - \sum_p in_{ip}^t \right) + \sum_j \sum_t dc \theta_j^t \left(v_j p cap_j^t - \sum_p in_{jp}^t \right) \\
& + \sum_k \sum_t r \theta_k^t \left(v_k p cap_k^t - \sum_p in_{kp}^t \right) \\
& + \sum_t \left(\sum_i \sum_p p p_{ip}^t h_{pi}^t + \sum_i \sum_p H i_{ip}^t in_{ip}^t + \sum_j \sum_p H j_{jp}^t in_{jp}^t \right. \\
& \left. + \sum_k \sum_p H k_{kp}^t in_{kp}^t \right) + \sum_c \sum_p \sum_t \pi_c^t \left(D_{cp}^t - \sum_k x_{kcp}^t \right)
\end{aligned} \tag{1}$$

The objective function (1) prefer to maximize profit. In this objective function, we consider the logistics cost for each channel.

$$\sum_r \sum_i x_{sir}^t \leq r cap_s^t v_s \quad \forall s, t \tag{2}$$

$$\sum_j x_{ijp}^t \leq p cap_i^t v_i \quad \forall p, t, i \tag{3}$$

$$\sum_k x_{jkp}^t \leq p cap_j^t v_j \quad \forall j, t, p \tag{4}$$

$$\sum_c x_{kcp}^t \leq p cap_k^t v_k \quad \forall p, k, t \tag{5}$$

Constraints (2), (3), (4), (5) show that the amount of product could not exceed capacity.

$$\sum_s x_{sir}^t = \sum_p h_{pi}^t bom_p^r \quad \forall i, r \tag{6}$$

Constraint (6) shows that raw material supplied from the supplier is equal to manufactured products.

$$in_{ip}^{t+1} = in_{ip}^t + h_{pi}^t - \sum_j x_{ijp}^t \quad \forall i. t. p \quad (7)$$

$$in_{jp}^{t+1} = in_{jp}^t + \sum_i x_{ijp}^t - \sum_k x_{jkp}^t \quad \forall j. p. t \quad (8)$$

$$in_{kp}^{t+1} = in_{kp}^t + \sum_j x_{jkp}^t - \sum_c x_{kcp}^t \quad \forall k. p. t \quad (9)$$

Constraints (7), (8), (9) show each facility's inventory.

$$\sum_k x_{kcp}^t \leq D_{cp}^t \quad \forall c. p. t \quad (10)$$

Constraint (10) shows that products that are transported are less than customer demand.

$$D_{cp}^t = m_c^{t ea} \sqrt{f^t k^t} \exp\left(-\frac{p_{cp}^t}{k_p^t B_{cp}^t} (f^t - a_{cp}^t)\right) + q_{cp}^t \quad \forall c. p. t \quad (11)$$

Constraint (11) shows demand function and related parameters.

$$p_{cp}^t = MSRP_p^t - B_{cp}^t \quad \forall c. p. t \quad (12)$$

Constraint (12) shows the price of selling products to the customer.

$$srr^{min} MSRP_p^t \leq B_{cp}^t \leq srr^{max} MSRP_p^t \quad \forall p. t \quad (13)$$

Constraint (13) shows upper and lower rebate bounds.

$$\sum_B \mu_{ijp}^{tB} = \begin{cases} 1 & x_{ijp}^t > 0 \\ 0 & \text{other wise} \end{cases} \quad \forall i. j. p \quad (14)$$

$$\sum_B \mu_{jkp}^{tB} = \begin{cases} 1 & x_{jkp}^t > 0 \\ 0 & \text{other wise} \end{cases} \quad \forall j. k. p \quad (15)$$

Constraints (14), (15) are about choosing breakpoints.

$$\left\{ \begin{array}{l} c_{ijp}^{t1} \quad u_{ijp}^{t1} \leq x_{ijp}^t \leq u_{ijp}^{t2} \\ c_{ijp}^{t2} \quad u_{ijp}^{t2} \leq x_{ijp}^t \leq u_{ijp}^{t3} \\ \dots \\ c_{ijp}^{tB} \quad u_{ijp}^{tB-1} \leq x_{ijp}^t \leq u_{ijp}^{tB} \end{array} \right. \quad \forall i. j. p \quad (16)$$

$$\left\{ \begin{array}{l} c_{jkp}^{t1} \quad u_{jkp}^{t1} \leq x_{jkp}^t \leq u_{jkp}^{t2} \\ c_{jkp}^{t2} \quad u_{jkp}^{t2} \leq x_{jkp}^t \leq u_{jkp}^{t3} \\ \dots \\ c_{jkp}^{tB} \quad u_{jkp}^{tB-1} \leq x_{jkp}^t \leq u_{jkp}^{tB} \end{array} \right. \quad \forall j. k. p \quad (17)$$

Also, constraints (16) and (17) show the purchasing cost for each amount of purchased product.

2-3- Numerical experiment

To investigate the proposed model, we use GAMS 25.13 software on a computer of Intel core i7 3.30 GHz and 8.00 GB of RAM to evaluate the proposed model's performance and applicability.

The following are our studies on the effects of parameters on decision variables and objective function. First, we try to evaluate our model; it is clear with an increase in shortage costs, the model will face a smaller amount of shortage to reduce logistics costs, so the shortage is inversely related to the amount of shortage cost, as shown in figure1. As you see in figure2, with increasing product advertising, demand for the product increases and as a result, profits change upwards. The numbers in the row figure3 are coefficients for the amount of product at each breakpoint. With the increase in the number of products, the system's profit has a downward trend, and one coefficient is more in line with customer demand. In the following analysis, we will examine the percentage of rebate for the customer as the rebate percentage increases also increase the product sent from the retailer. Figure4 shows a different range of rebates and their effect on selling products in each period.

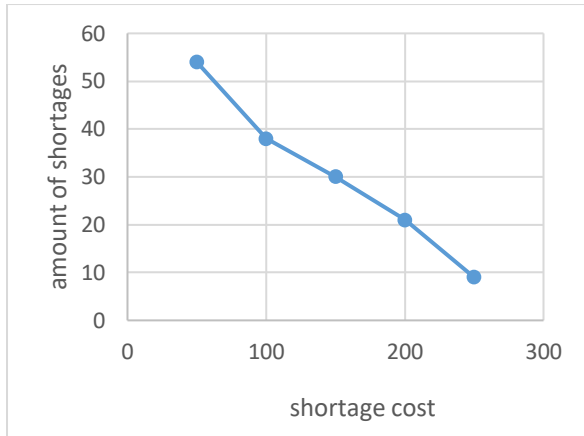


Fig 1. The impact of shortage cost

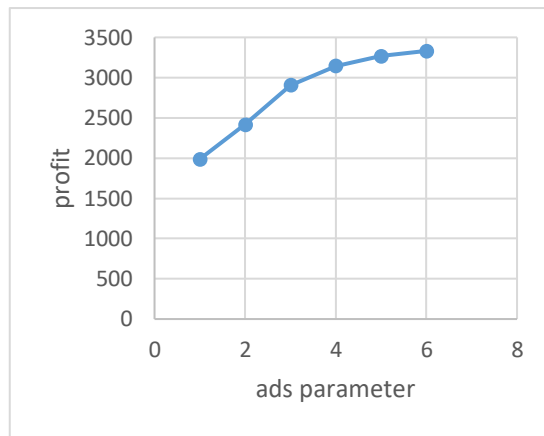


Fig 2. The impact of advertising

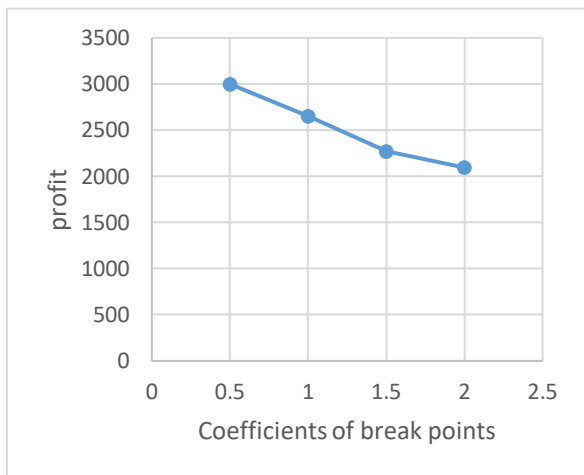


Fig 3. The impact of amount of product

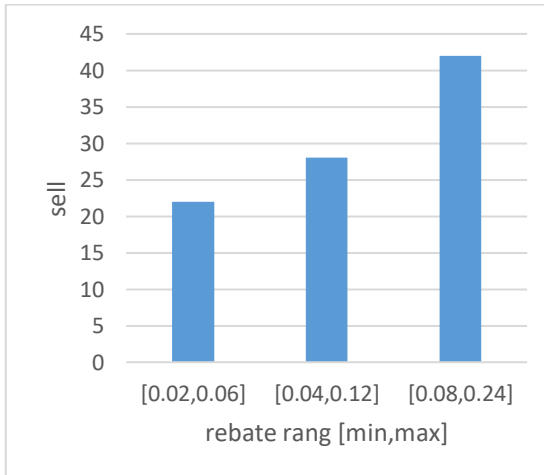


Fig 4. The impact of rebate rang on sell

3- Managerial insight and conclusion

The goal is to organize supply chain conditions in the best possible way, so profitability structure depends on more sales. One of the most critical factors in selling is advertising. In this way, the products offered to the customer, more advertising leads to the product's acceptance and visibility. In this section, the use of product marketing techniques can also be used. Studying customer behavior and the parameters that affect can significantly impact chain performance, so target markets that are sensitive to advertising can be profitable. The next point is to align the proposed breakpoint and the amount of demand. It can also reduce shortages and shortage costs. As we know, the product's price has a significant impact on the customer's behavior in each period when the system loses profitability, and the advertising agency does not have the necessary efficiency. There is another way to make the system more profitable by changing prices and increasing the rebate rate. The demand function is affected by these two factors, price, and rebate, which can benefit the system.

This study proposed a mathematical model that investigated SCND's aim to maximizing profits in the inflation situation. The model also considered some strategies to reduce logistic costs. We use the price-dependent demand function to be useful in the real world because most people decide by price. It is helpful to estimate demand in each period and produce optimally. Then we use all discount policies to let each channel buy number of products according to capacity and downstream demand. To buy more, we consider discounts to increasing buyer's desire, and on the other hand, the seller has more space for products in the next period. We also consider ads in supply chain management and its effect on the system. This strategy helps to make more money, also attracting more customers into the supply chain network. Calculating profit consider revenue, cost, and responsibility to the customer. For future research in this area, the purchase cost can be checked in case of uncertainty; it is even possible to generate a cost function for each breakpoint's purchase cost, which creates a more competitive

environment between sellers and solves the problem on a larger scale.

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