

Integrated production planning and warehouse layout problem under uncertainty: A robust possibilistic approach

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Abstract

This study investigates the joint production planning and warehouse layout under uncertainty. Today's competitive business world needs to be investigated by models which are capable of considering uncertain nature of the problems, especially when the historical data is not available or the level of uncertainty is high. Joint production planning and warehouse layout problems is almost a novel and new area in both academics and practice. For warehousing problem, the eventually of rental warehouses and new allocations is enabled in each planning horizon period. A bi-objective MILP model is proposed and fuzzy distributed parameters and chance constraints are taken into considerations. One of the objective functions deals with the cost associated parameters and variables while the second one minimizes the fluctuations of the work labor in each planning period. A simple test problem along with a case study is investigated by the proposed model. The obtained results prove the applicability of the proposed model in real-world scale problems.

Keywords: Warehouse layout, production planning, robust possibilistic programming, fuzzy programming

1-Introduction and literature review

Production planning and warehouse layout decisions have been studied thoroughly by many researchers. Warehouse space is one of the key issues in both optimizing the warehouse management and smooth production planning. This importance leads us toward an integrated decision-making process which will tackle with capacitated lot-sizing problem along with warehouse layout problem in order to coordinate both decisions effectively. There are some significant studies reviewing the models and problems of lot-sizing. For instance see (Buschkühl et al., 2010, Jans and Degraeve, 2007). Existing studies indicate that the lot-sizing problems are generally hard to solve. This complexity suggests the use of heuristics or relaxations and decomposition methods in order to reduce the complexity of the problem. On the other hand, production planning is one of the very first problems introduced in business optimization environment. But the joint of these two important decisions haven't been studied well enough.

Recently, an interesting study is done by Zhang et al. (2017). They have introduced a MILP model to integrate the warehouse layout problem with production planning in food industry but they didn't consider the highly uncertain nature of the real-world class problems to avoid complexity.

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A large body of research focuses on the modeling and design of various components of production-distribution problems. Benefits and challenges of integration are summarized along with emphasizing on the need for practical analytic models and efficient solution methods.

One of the major challenges for manufacturers is providing the retailer by retaining stockpiles. However, retaining actions for large inventory require increased operational costs. In addition, a stock-out may not only result in order cancellations, but may also affect the probability of future customer demand. In order to evade such a difficulties and notoriousness, integrated decision making can assure the decision makers and supply managers of availability of the stock in the right place within a right cost.

The production-distribution problems become more challenging when information on parameters of the model like demand and availability of raw materials is imprecise. To address the uncertainty in our model we apply fuzzy parameters on the proposed model. Fuzzy set theory offers strong analytical support for capturing uncertainty. Fuzzy logic has found numerous applications due to its simplicity on implementation, flexibility and tolerant nature for handling imprecise data. Uncertainty is a lethal factor if it is ignored, especially in real-world problems where the historical data is not available or the nature of the problem needs fuzzy applications in decision making (Moradi et al., 2019, Babazadeh and Sabbaghnia, 2018).

Storage location assignment and warehouse layout problem can be used interchangeably. As mentioned earlier there have been lots of studies investigating the details of this problem. Details such as different operations and amendment in storage process. Storage location assignment and warehouse layout problem is about determining the location of goods and items while considering the storage requirements like structure and capacity. Some all-inclusive and extensive reviews are done on warehousing and storage operations, for instance see Gu et al. (2010). Researchers divide the layout policies according to the different rules applied to locate the goods in a warehouse, in general there are five main policies introduced by De Koster et al. (2007). Besides, lots of studies have investigated the effect of each policy and examined the utilization can be achieved and one of the most applied policies is the class-based storage policy (Sabbaghnia and Taleizadeh, 2020). Regarding this policy there is wide range of studies that can be mentioned for instance interested readers can refer to Pan et al. (2014).

As mentioned, capacitated lot-sizing problem (CLSP) is one the most studied problems in production planning. This problem is proved to belong to the NP hard classification (Florian and Klein, 1971). To escape the computational complexity many exact and heuristic algorithms have been developed. The review of the literature indicates that the integration decision making of warehouse layout and production planning has been neglected through past decades and developing a mathematical model which will able the decision makers to determine the optimal coordinated and integrated decisions while considering the uncertain nature of the problem is quite necessary. CLSP still is an open area and studies are developing regarding production planning problem (Jiang et al., 2017, Menezes et al., 2017, Song et al., 2017, Vogel et al., 2017).

The conclusion of the above-mentioned insights could be presented as follows; production planning has an undeniable role in performance of a production system. The attributed decisions play a critical role in survival of the company in today's competitive business. On the other hand, short term decisions like storage layout also have a strong effect on performance of the production system and costs of the system. Fortunately, the storage location problem and warehouse layout problem have been investigated thoroughly and comprehensively by many researchers and practitioners. But the joint problem of production planning and warehouse layout has been neglected and there are a few studies regarding this integration and cooperative decision making.

This study aims to cope with the integration of production planning as a medium-term decision and warehouse layout as a short-term decision with uncertainty considerations. The main contribution of this work is the insertion of uncertainty considerations in the model, specifically the parameters of the proposed model. As mentioned, fuzzy theory has the potential to aid the decision makers especially when there is no historical data or the type of the problem needs subjective data rather than objective. Joint production planning and warehouse layout problems is almost a novel and new area in both academics and practice. For warehouses, the eventually of rental warehouses and new allocations in each period of planning horizon is considered. A bi-objective MILP model is proposed and the fuzzy distributed parameters and chance constraints are taken into considerations. One of the objective functions deals with the cost associated

parameters and variables while the second one minimizes the fluctuations of the work labor in each planning period. A simple test problem along with a case study is investigated by the proposed model.

As for the warehouses, we consider them in two main categories, first are the facilities that the firm owns them and second are facilities that can be rent at any time if a contract has already been signed for them or they are just available upon request with no need of pre-reservation. Another decision about warehouses is the location of them, whether they are close to production sites or consumer area. Numerical examples are developed to study the performance of the proposed approach and analyze the sensitivity of the obtained solutions. A case study is applied to investigate the solutions of the model in real case industries.

2-Model description

Here, a warehousing system in auto-industry is analyzed and studied. Production quantities and inventory decisions are considered along with the warehousing decisions. In each period, storage assignments, production rate and inventoried amount of the items are determined with an interactive and integrated mathematical model. At the beginning of each period the possibility of rental warehouses is taken into account. The first objective of this model is to minimize the total cost of the system and the second one is to minimize the fluctuations of the hiring and firing the work force of the production area. In today's business environment, crisp and deterministic models can't deal with the complexity arising in this area. Most of the parameters, e.g., demand data, production costs, traveling time etc., are tainted with a high degree of uncertainty in real-life situations. Lack of knowledge about the exact values of parameters leads us toward the stochastic programming models. One of the well-known procedures is the applications of the fuzzy-sets theory in managerial problems. Here, a fuzzy mathematical programming (FMP) approach is used to deal with the uncertain nature of the parameters and insufficient available objective data. The following are the notations used in modeling procedure.

Table 1. Parameters and Decision variables

Indices	
i	Index of products
w	Index of warehouses
j	Index of the warehousing and storage location in storage area
l	Index of work force skill level and experience
s	Index of work hours types (regular, overtime and subcontracted)
k	Index of demand area
t	Index of the periods of the planning horizon
Parameters	
\tilde{c}_{is}^t	Variable production cost per unit of item i in working shift s in period t
\tilde{f}_{is}^t	Production setup cost of item i in working shift s in period t
cs_{ls}^t	Salary cost of work force with skill level l in working shift s in period t
ch_{ls}^t	Hiring cost of work force with skill level l in working shift s in period t
cl_{ls}^t	Laying-off cost of work force with skill level l in working shift s in period t
\tilde{b}_{il}^t	Unit process time of item of item i by l -skilled work force in period t
b_{il}^{tt}	1, if item i requires a l -skilled operator in period t ; 0, otherwise
$\tilde{Cap}T_s^t$	Time capacity in working shift s in period t
h_{iw}^t	Holding cost per unit of item i in warehouse w period t
π_i^t	Shortage cost per unit of item i in period t
\tilde{d}_{jw}^1	Distance from production area to the j^{th} location in warehouse w
\tilde{d}_{jw}^2	Distance from the j^{th} location in warehouse w to the output point

Parameters

\tilde{d}_{wk}^t	Distance from the warehouse w^{th} to the demand point k in period t
p_{ijw}^t	Unit travel cost for item i form production area to the j^{th} location in warehouse w in period t
q_{ijw}^t	Unit travel cost for item i form j^{th} location in w^{th} warehouse to the output point in period t
r_{iwk}^t	Unit travel cost for item i from w^{th} warehouse to the demand point k in period t
$\tilde{d}e_{ik}^t$	Demand of item i in demand point k in period t
\tilde{a}_w^t	Rental fee of warehouse w in period t
$\tilde{C}ap_{is}^t$	Production capacity of item i in working shift s in period t
$\tilde{C}apW_{iw}^t$	Warehouse w capacity for item i in period t
<hr/>	
Decision Variables	
y_{is}^t	Production amount of item i in working shift s in period t
x_{iwk}^t	Amount of item i delivered to k^{th} demand point from warehouse w in period t
WF_{ls}^t	Number of work force with skill level l in working shift s in period t
HWF_{ls}^t	Number of hired work force with skill level l in working shift s in period t
LWF_{ls}^t	Number of laid-off work force with skill level l in working shift s in period t
I_{iw}^t	Inventory level of item i at warehouse w in end of the period t
z_{ik}^t	Amount of shortage of item i in demand point k in period t
y_{is}^t	1, if item i is produced in working shift s in period t ; 0, otherwise
u_{ijw}^t	1, if item i is assigned to j^{th} location in warehouse w in period t ; 0, otherwise
u_{ijw}^t	1, if item i is moved to j^{th} location in warehouse w in period t ; 0, otherwise
v_{ijw}^t	1, if item i is retrieved from j^{th} location in warehouse w in period t ; 0, otherwise
n_{ijw}^t	1, if item i is inventoried in j^{th} location in warehouse w in period t ; 0, otherwise
R_w^t	1, if warehouse w is rented in period t ; 0, otherwise

The proposed model is formulated as following:

$$\begin{aligned}
 Min Z_1 = & \sum_{i,s,t} (\tilde{f}_{is}^t y_{is}^t + \tilde{c}_{is}^t y_{is}^t) + \sum_{i,j,w,t} (\tilde{d}_{jw}^1 p_{ijw}^t u_{ijw}^t + \tilde{d}_{jw}^2 q_{ijw}^t v_{ijw}^t) + \sum_{i,k,w,t} \tilde{d}_{wk}^t r_{iwk}^t x_{iwk}^t + \sum_{w,t} \tilde{a}_w^t R_w^t \\
 & + \sum_{i,w,t} h_{iw}^t I_{iw}^t + \sum_{i,k,t} \pi_i^t z_{ik}^t + \sum_{l,s,t} (cS_{ls}^t WF_{ls}^t + ch_{ls}^t HWF_{ls}^t + cl_{ls}^t LWF_{ls}^t)
 \end{aligned} \tag{1}$$

$$Min Z_2 = \sum_{l,s,t} (HWF_{ls}^t + LWF_{ls}^t) \tag{2}$$

First objective function is consisted of seven terms, in first term variable and fixed cost of production in each period is calculated. The second is the travel cost of the items from production area to the storage

locations and from the storage locations to the output point. The third term is the transportation delivery cost to the demand points in each period. The next is the cost of the renting warehouses. The fifth is the holding cost of inventoried items in each warehouse in each planning period, the sixth term is the cost associated with the amount of shortage happened in each period and finally the last one is the labor cost. For the second objective function denoted by(2), this objective is to minimize the fluctuations of the work force level during the planning horizon. It is obvious that the two objective functions are contradictory. The workforce level in each period would not be necessarily aligned with the minimum costs of the system. As for the second objective for the managers a smooth workforce plan has a crucial role in the productivity of the system.

The constraints of the model are presented as follows;

$$\sum_i u''_{ijw} \leq 1, \quad \forall j, w, t \quad (3)$$

$$\sum_i u^t_{ijw} \leq 1, \quad \forall j, w, t \quad (4)$$

$$\sum_i v^t_{ijw} \leq 1, \quad \forall j, w, t \quad (5)$$

$$\sum_i n^t_{ijw} \leq 1, \quad \forall j, w, t \quad (6)$$

$$\sum_{j,w} v^t_{ijw} \leq \sum_k (\tilde{d}e^t_{ik} + z^{t-1}_{ik}) \quad \forall i, t \quad (7)$$

$$\sum_{j,w} u''_{ijw} = \sum_s y''_{is} \quad \forall i, t \quad (8)$$

$$\sum_{j,w} n^t_{ijw} = \sum_w I^t_{iw} \quad \forall i, t \quad (9)$$

$$v^t_{ijw} \leq u^t_{ijw} + n^{t-1}_{ijw} \quad \forall i, j, w, t \quad (10)$$

$$u^t_{ijw} + n^{t-1}_{ijw} \leq u''_{ijw} \quad \forall i, j, w, t \quad (11)$$

$$n^{t-1}_{ijw} \leq u''_{ijw} \quad \forall i, j, w, t \quad (12)$$

$$u^t_{ijw} \leq u''_{ijw} \quad \forall i, j, w, t \quad (13)$$

$$v^t_{ijw} \leq u''_{ijw} \quad \forall i, j, w, t \quad (14)$$

$$n_{ijw}^t \leq u_{ijw}'' \quad \forall i, j, w, t \quad (15)$$

$$n_{ijw}^t = u_{ijw}^t - v_{ijw}^t + n_{ijw}^{t-1} \quad \forall i, j, w, t \quad (16)$$

$$WF_{ls}^t = WF_{ls}^{t-1} + HWF_{ls}^t - LWF_{ls}^t \quad \forall l, s, t \quad (17)$$

$$LWF_{ls}^t \leq WF_{ls}^{t-1} + HWF_{ls}^t \quad \forall l, s, t \quad (18)$$

$$\sum_w I_{iw}^t - \sum_k z_{ik}^t = \left(\sum_{j,w} v_{ijw}^t + \sum_w I_{iw}^{t-1} \right) - \sum_k \left(de_{ik}^t + z_{ik}^{t-1} \right) \quad \forall i, t \quad (19)$$

$$\sum_s y_{is}'' + \sum_w I_{iw}^{t-1} - \sum_k z_{ik}^{t-1} = \sum_{w,k} x_{iwk}^t + \sum_w I_{iw}^t - \sum_k z_{ik}^t \quad \forall i, t \quad (20)$$

$$\sum_i y_{is}'' \left(\sum_l \tilde{b}_{il}' b_{il}'' \right) \leq \tilde{Cap} T_s^t \quad \forall s, t \quad (21)$$

$$\sum_l WF_{ls}^t \tilde{Cap} T_s^t \geq \sum_i y_{is}'' \left(\sum_l \tilde{b}_{il}' b_{il}'' \right) \quad \forall s, t \quad (22)$$

$$y_{is}'' \leq \tilde{Cap}_{is}^t \quad \forall i, s, t \quad (23)$$

$$y_{is}'' \leq M \cdot y_{is}^t \quad \forall i, s, t \quad (24)$$

$$I_{iw}^t \leq \tilde{Cap} W_{iw}^t R_w^t \quad \forall i, w, t \quad (25)$$

$$y_{is}'', I_{iw}^t, z_{ik}^t \geq 0 \quad \& \quad y_{is}^t, u_{ijw}'', u_{ijw}^t, v_{ijw}^t, n_{ijw}^t, R_w^t \in \{0, 1\} \quad \forall i, j, w, k, t \quad (26)$$

Constraints (3) ensure that each storage location only can be assigned to one item at each period. In other words, two items can't be assigned to the same storage location at the same time. Constraints (4) ensure that at most one item can travel to one specified location in each warehouse in each time period. Constraints (5) ensure that at most one item can be demanded from each location of each warehouse in each time period. Constraints (6) ensure that at each time period, at most one item can be inventoried in each location of each warehouse. Constraints (7) state that number of retrieved items in each period and for each item is less than the summation of demand of that item and backordered demand for that item from the previous period. Constraints (8) state that the total number of products assigned to storage locations are equal to the amount of the production in each period. Constraints (9) ensure that the summation of the inventoried items in all storage locations are equal to the inventory level of that item in each period. Constraints (10) ensure that an item can be demanded from a storage location if it has been moved to that location already or has been

inventoried in that location in previous period. Constraints (11) state that an item could be moved to a location only if it is already assigned to that location and there is no inventoried item from the previous period. Constraints (12) ensure that if an item is inventoried in the previous period the binary assignment variable for that location is equal to one. Constraint (13) state that an item can be moved to a specified storage location only if it has been already assigned to that location. Constraints (14) indicate that an item can be demanded from a specific storage location only if it has been already assigned to that location. Constraints (15) ensure that an item can be inventoried in a specific storage location only if it has been already assigned to that location. Constraints (16) state the balance of material flow in the system, it starts from production area to the output point in each period. Constraints (17) state the balance of work force considering hired and laid-off employees in each period. Constraints (18) ensure that the number of laid-off work labors do not exceed the number of available work force in each period. Constraints (19) denote the balance of material flow with respect to the shortage and inventory of the items at the end of each period. Constraints (21) ensure that the required production time do not exceed the total available time in each working shifts. Constraints (22) indicate that the required production process time should be less than or equal to the available work labor in each working shift in each period of planning horizon. Constraints (23) ensure that the production quantity of each item do not exceed the production capacity of that item in each period. Constraints (24)-(25) indicate that items can be produced in a specific period if only the fixed-fee for set up is considered. In other words, in each period production process can be engaged only if the set-up cost is taken into account. Constraints(26), are the non-negativity and binary restrictions of variables.

Proposed model can be simplified by some manipulations on the constraints and equations. By implanting the equation (16) into constraints (10) and some calculation leads to $n'_{ijw} \geq 0$ which is satisfied due to the non-negativity restrictions of variables and therefor can be eliminated. Recall that $u''_{ijw}, u'_{ijw}, v'_{ijw}, n'_{ijw}, R'_w \in \{0,1\}$ and this can be deduced that constraints (13) and (15) are satisfied by constraints (11) thus they can be eliminated. From both (10) and (11) one can drive out the(14). Furthermore, with(3), (10) and (11) the constraints denoted by (5) can be simply deduced.

3-Accounting for data uncertainty

As already discussed in previous sections, in today's competitive business environment, deterministic models can't deal with the complexity arising in this area. Along with that, most of the parameters of the real-life problems are tainted with a high degree of uncertainty. In situations like this, the robustness degree of the solution is very vital to the managers. In this study, we consider a mid-term decision policy (warehouse layout decisions) along with a short-term decision policy (production planning) for the system and robustness of the determined decisions for the mid-term decisions are of particular importance. Fuzzy mathematical programming (FMP) is believed to be a strong and effective modeling approach especially in where, there is no historical data or the parameters values are vague and ambiguous. Following the FMP and robust programming are briefly introduced. Then based on the very most recent fuzzy measures in the literature the proposed model is defuzzied.

3-1-Robust programming approach

In mid-term and long-term level decision making problems, feasibility and optimality robustness of the final solutions is important. Pishvae et al. (2012a) describe the feasibility robustness and optimality robustness as follows; feasibility robustness means that obtained solution must continue to be feasible for (almost) all possible realization of the uncertain parameters whereas optimality robustness means that the deviations from the optimal objective function value should be negligible for (almost) all possible realization of the uncertain parameters. According to Pishvae et al. (2012a), robust programming approaches can be classified into three subcategories; I) the hard worst-case robust programming, II) the soft worst-case robust programming, and III) the realistic robust programming. Interested readers can refer to Ben-Tal et al. (2009), Pishvae et al. (2012a), Zahiri et al. (2014) and Mousazadeh et al. (2017).

3-2-Fuzzy mathematical programming approach

FMP is known to be a proper programming approach in addressing both epistemic uncertainty of the data and flexibility and elasticity in goals and constraints of the models. The two main classifications on FMP are known as possibilistic programming (Inuiguchi and Ramik, 2000) and flexible programming (Mula et al., 2006). Possibilistic programming is used when DMs deal with parameters where there is no historical data or the values are vague and ambiguous. Nevertheless, flexible programming is used when flexibility in objective function value or elasticity in some constraints are needed. One of the most well-known possibilistic programming approaches is the possibilistic chance-constraint programming (PCCP) approach. This approach is one of the wide spread methods in the literature because of its ability on controlling the confidence level of constraints and its compatibility with different types of fuzzy numbers (Pishvae et al., 2012a). Necessity (N) and Possibility (π) measures are representing the extreme attitude of the parameters of a chance constraint programming (CCP) model and they do not have self-duality property (Pishvae et al., 2012b).

Given a trapezoidal fuzzy variable $\xi = (a_1, a_2, a_3, a_4)$ where $a_1 < a_2 < a_3 < a_4$ and its membership function is given by (27). Following the Liu and Iwamura (1998) and Inuiguchi and Ramik (2000) the possibility and necessity measures of given trapezoidal variable can easily calculated for any confidence level. Equations (28)-(31) are representing the calculated values for all confidence levels equal or greater than 0.5. Similarly, the credibility (Cr) measure could be calculated.

$$\mu_{(r)} = \begin{cases} \frac{r-a_1}{a_2-a_1} & a_1 \leq r \leq a_2 \\ 1 & a_2 \leq r \leq a_3 \\ \frac{a_4-r}{a_4-a_3} & a_3 \leq r \leq a_4 \\ 0 & O.W \end{cases} \quad (27)$$

$$\pi(\tilde{\xi} \leq r) \geq \alpha \Leftrightarrow \frac{r-a_1}{a_2-a_1} \geq \alpha \Leftrightarrow r \geq (1-\alpha)a_1 + \alpha a_2 \quad (28)$$

$$\pi(\tilde{\xi} \geq r) \geq \alpha \Leftrightarrow \frac{a_4-r}{a_4-a_3} \geq \alpha \Leftrightarrow r \leq \alpha a_3 + (1-\alpha)a_4 \quad (29)$$

$$N(\tilde{\xi} \leq r) \geq \alpha \Leftrightarrow \frac{r-a_3}{a_4-a_3} \geq \alpha \Leftrightarrow r \geq (1-\alpha)a_3 + \alpha a_4 \quad (30)$$

$$N(\tilde{\xi} \geq r) \geq \alpha \Leftrightarrow \frac{a_2-r}{a_2-a_1} \geq \alpha \Leftrightarrow r \leq \alpha a_1 + (1-\alpha)a_2 \quad (31)$$

Me measure introduced by Xu and Zhou (2013), is a strong tool in fuzzy problems environment. In this approach a spectrum of decisions is provided between necessity and possibility extreme points as follows:

$$Me(\tilde{\xi}) = \lambda.\pi(\tilde{\xi}) + (1-\lambda).N(\tilde{\xi}) \quad (32)$$

Where $0 \leq \lambda \leq 1$, is the tuning parameter which states the optimistic or pessimistic attitude of the DM. It is clear that *Possibility*, *Necessity* and *Credibility* measures are special cases of *Me* for special values of λ . Similar to possibility and necessity measures the *Me* measure can be calculated. The crisp counterparts of both $Me(\tilde{\xi} \geq r) \geq \alpha$ and $Me(\tilde{\xi} \leq r) \geq \alpha$ would be piecewise functions and would not normally fit into one equation. We have:

$$Me(\tilde{\xi} \leq r) \geq \alpha \Leftrightarrow \begin{cases} \text{if } \lambda < 0.5 \Rightarrow \lambda + (1-\lambda) \frac{r-a_3}{a_4-a_3} \geq \alpha \Rightarrow r \geq \frac{(\alpha-\lambda)a_4 + (1-\alpha)a_3}{1-\lambda} \\ \\ \text{if } \lambda \geq 0.5 \Rightarrow \begin{cases} \lambda \frac{r-a_1}{a_2-a_1} \geq \alpha \Leftrightarrow r \geq \frac{(\lambda-\alpha)a_1 + \alpha a_2}{\lambda} & a_1 \leq r \leq a_2 \\ \lambda & a_2 \leq r \leq a_3 \\ \lambda + (1-\lambda) \frac{r-a_3}{a_4-a_3} \geq \alpha \Leftrightarrow r \geq \frac{(\alpha-\lambda)a_4 + (1-\alpha)a_3}{1-\lambda} & a_3 \leq r \leq a_4 \end{cases} \end{cases} \quad (33)$$

$$Me(\tilde{\xi} \geq r) \geq \alpha \Leftrightarrow \begin{cases} \text{if } \lambda < 0.5 \Rightarrow \lambda + (1-\lambda) \frac{a_2-r}{a_2-a_1} \geq \alpha \Rightarrow r \leq \frac{(\alpha-\lambda)a_1 + (1-\alpha)a_2}{1-\lambda} \\ \\ \text{if } \lambda \geq 0.5 \Rightarrow \begin{cases} \lambda + (1-\lambda) \frac{a_2-r}{a_2-a_1} \geq \alpha \Leftrightarrow r \leq \frac{(\alpha-\lambda)a_1 + (1-\alpha)a_2}{1-\lambda} & a_1 \leq r \leq a_2 \\ \lambda & a_2 \leq r \leq a_3 \\ \lambda \frac{a_4-r}{a_4-a_3} \geq \alpha \Leftrightarrow r \leq \frac{\alpha a_3 + (\lambda-\alpha)a_4}{\lambda} & a_3 \leq r \leq a_4 \end{cases} \end{cases} \quad (34)$$

Please recall that Necessity (*N*) and Possibility (π) measures have extreme attitude toward the problem in CCP models. And that's the main reason of adopting a more moderate approach namely *Me* measures to cope with the uncertainty of the parameters. To work more convenient, we develop the compact form of the proposed model as follows:

$$\begin{aligned} \text{Min } Z_1 &= FY + VX \\ \text{Min } Z_2 &= U \\ \text{s.t.} \\ AY &\leq 1 \\ BY &\leq CY \\ EY &\geq GY \\ HX &\leq JY \\ NX &\leq OU \\ PX &\geq QX \\ RX &\geq D \\ SX &\leq K \\ Y &\in \{0,1\} \quad X, U \geq 0 \text{ \& integer} \end{aligned} \quad (35)$$

Now without losing any generality, assume that vectors F , V , D and K representing the fixed setup costs, variable costs, demand for each item and capacity of the production sites are the imprecise and fuzzy parameters. In order to convert the possibilistic objective functions to their crisp equivalent the expected value operator is used, and to cope with constraints including imprecise parameters the Me measure is adopted. The parameters are assumed to follow a trapezoidal possibility distribution, $\tilde{\xi} = (\xi_1, \xi_2, \xi_3, \xi_4)$ and note that when $\xi_2 = \xi_3$ it is simply altered into a triangular fuzzy number. With these descriptions the (35) model can be reformulated as follows:

$$\begin{aligned}
Min E[Z_1] &= E[\tilde{F}]Y + E[\tilde{V}]X \\
Min Z_2 &= U \\
s.t. \\
Me\{RX \geq \tilde{D}\} &\geq \alpha \\
Me\{SX \leq \tilde{K}\} &\geq \beta \\
AY &\leq 1 \\
BY &\leq CY \\
EY &\geq GY \\
HX &\leq JY \\
NX &\leq OU \\
PX &\geq QX \\
Y \in \{0,1\} \quad X, U &\geq 0 \text{ \& integer}
\end{aligned} \tag{36}$$

It's clear that the first objective function along with first two sets of constraints include fuzzy parameters denoted by trapezoidal possibility distributions mainly based on the expert subjective ideas and partially based on some historical data. Using transformations discussed over equations, the crisp equivalent of the proposed model based on the expected value operator and Me measure is as follows:

$$\begin{aligned}
Min E[Z_1] &= \left[\frac{1-\lambda}{2}(F_1 + F_2) + \frac{\lambda}{2}(F_3 + F_4) \right] Y + \left[\frac{1-\lambda}{2}(V_1 + V_2) + \frac{\lambda}{2}(V_3 + V_4) \right] X \\
Min Z_2 &= U \\
s.t. \\
RX &\geq \frac{(\alpha - \lambda)D_4 + (1 - \alpha)D_3}{1 - \lambda} \\
SX &\leq \frac{(\beta - \lambda)K_1 + (1 - \beta)K_2}{1 - \lambda} \\
AY &\leq 1 \\
BY &\leq CY \\
EY &\geq GY \\
HX &\leq JY \\
NX &\leq OU \\
PX &\geq QX \\
Y \in \{0,1\} \quad X, U &\geq 0 \text{ \& integer}
\end{aligned} \tag{37}$$

Although the (37) model can effectively tackle with imprecise parameters, it fails to react to the deviations of objective function values from their expected value. Fluctuations from expected value of objective functions can cost a lot especially in today's extreme competitive business. Furthermore, in this programming approach the minimum confidence level of constraints is ascertained by the experts or based on DM's preferences which cannot assure the optimality of attained solutions. To cope with these deficiencies, as investigated in the literature review, a combination of robust and fuzzy approaches are introduced and widely applied Zahiri et al. (2014).

A solution is called robust if has both feasibility robustness and optimality robustness (Pishvae et al., 2012a). If a solution remains feasible for all possible values of uncertain parameters it has feasibility robustness and if the objective function value remains optimal or near optimal for all possible value of imprecise parameters it meets the optimality robustness. Classical robust possibilistic programming has used *Necessity* to tackle with the uncertainty but as discussed earlier, *Me* fuzzy measure has a more realistic point of view to the imprecise data and parameters. We provide the robust possibilistic programming model based on *Me* measures as follows:

$$\begin{aligned}
\text{Min } Z_1^{\text{Crisp}} &= E[Z_1] + \gamma(Z_1^{\text{Max}} - E[Z_1]) + \sigma \left(D_4 - \frac{(\alpha - \lambda)D_4 + (1 - \alpha)D_3}{1 - \lambda} \right) + \delta \left(\frac{(\beta - \lambda)K_1 + (1 - \beta)K_2}{1 - \lambda} - K_1 \right) \\
\text{Min } Z_2^{\text{Crisp}} &= E[Z_2] + \sigma' \left(D_4 - \frac{(\alpha - \lambda)D_4 + (1 - \alpha)D_3}{1 - \lambda} \right) + \delta' \left(\frac{(\beta - \lambda)K_1 + (1 - \beta)K_2}{1 - \lambda} - K_1 \right) \\
&\quad \text{s.t.} \\
RX &\geq \frac{(\alpha - \lambda)D_4 + (1 - \alpha)D_3}{1 - \lambda} \\
SX &\leq \frac{(\beta - \lambda)K_1 + (1 - \beta)K_2}{1 - \lambda} \\
AY &\leq 1 \\
BY &\leq CY \\
EY &\geq GY \\
HX &\leq JY \\
NX &\leq OU \\
PX &\geq QX \\
Y &\in \{0,1\} \quad X, U \geq 0 \text{ \& integer, } 0.5 \leq \beta, \alpha \leq 1
\end{aligned} \tag{38}$$

In first objective function of model, the expected value of Z (first term), improves the expected performance value of the objective function. The second term controls the optimality robustness of the final solution through minimizing the deviations of upper bound of the objective function from the expected value of it. As it's clear, the second objective function has no fuzzy parameter; therefore, there is no need to add any term to control optimality robustness into it. Furthermore, the parameter γ is the preference weight of the optimality robustness over the feasibility robustness. The upper bound of the first objective function can easily be calculated as follows:

$$Z_1^{\text{max}} = F_4 Y + V_4 X \tag{39}$$

The other two terms of first objective function of model, control the feasibility robustness of the solutions by minimizing the violations of RHS of chance constraints from their worst-case value of the uncertain parameters by penalty values $\delta, \sigma, \delta', \sigma'$. This is very important to note that these penalty values can be defined and determined due to the problem context. For example, shortage penalty costs and costs off idle capacity of the system can be calculated and adjusted to these penalty costs. Bear in mind that in former

model the confidence level (i.e., α and β) of the chance constraints are variables and their final values are optimized in solving procedure. This is worth mentioning that the cost-benefit aspect of this problem allows us to develop a realistic robust programming rather than a hard/soft worst case robust programming model.

4-Coping with objective functions

There are three main classes of approaches to deal with multi objective functions in mathematical programming literature, priori, interactive and posteriori classes. Interactive approaches try to accumulate the favorable features of the other two approaches while averting the inefficiencies of them. Despite of the priori approaches, interactive methods aim to look into the preferences of the decision makers attentively and generating different Pareto-optimal solutions. Here, we will apply the interactive method proposed by Torabi and Hassini (2008), (TH) to our model to deal with objective functions of the model.

The steps of the TH approach are as follows:

- ✓ Determine the positive ideal solution (PIS) and negative ideal solution (NIS) for each objective function
- ✓ Calculate a linear fuzzy membership function for each objective function as follows:

$$\mu_{Z_j}(x) = \begin{cases} 1 & Z_j < Z_j^{PIS} \\ \frac{Z_j^{NIS} - Z_j}{Z_j^{NIS} - Z_j^{PIS}} & Z_j^{PIS} \leq Z_j \leq Z_j^{NIS} \\ 0 & Z_j > Z_j^{NIS} \end{cases} \quad (40)$$

Where μ_{Z_j} denotes the satisfaction degree of the j th objective function.

- ✓ Convert the corresponding crisp bi-objective model into a single-objective model applying TH aggregation function as follows:

$$\begin{aligned} \text{Max } & \phi\tau_0 + (1-\phi) \sum_k w_k \mu_k(x) \\ \text{s.t. } & \\ & \mu_k(x) \geq \tau_0; \quad \forall k \\ & x \in F_x \text{ and } \tau_0 \in [0,1] \end{aligned} \quad (41)$$

Where τ_0 indicate the minimum satisfaction degree of objective functions, ϕ and w_k stand for objective functions compromise coefficient and importance of the j th objective function (denote that $\sum_k w_k = 1, w_k \geq 0$).

- ✓ Determine the values of importance weight of the objective functions and coefficient of compromise between objective functions and solve the single-objective model.

5-Implementation and evaluation

Here, the proposed model's performance is investigated through a test problem. The proposed model is coded in GAMS 24.7.4 optimization software using CPLEX solver and all the executions are implemented on a Corei7 2.40 GHz laptop with 8 GB of RAM. The trapezoidal fuzzy numbers of given uncertain

parameters are considered as $(0.9\varphi, 0.95\varphi, 1.05\varphi, 1.1\varphi)$. The test problem size is $10*5*500*3*2*5*5$ (Table) and input parameters follow a uniform distribution as given in Table .

Table 2. Size of the problem indices

Indices	
i	/1*10/
w	/1*5/
j	/1*500/
l	/1*3/ (novice, intermediate, expert)
s	/1*2/ (regular time and overtime)
t	/1*5/

Table 3. Input parameters of the test problem

Parameter	Range	Parameter	Range
\tilde{c}_{is}^t	$\sim uniform(10, 30)$	\tilde{d}_{wk}^t	$\sim uniform(25, 40)$
\tilde{f}_{is}^t	$\sim uniform(500, 2500)$	\tilde{r}_{iwk}^t	$\sim uniform(5, 12)$
$\tilde{c}s_{ls}^t$	$\sim uniform(30, 300)$	p_{ijw}^t	$\sim uniform(8, 15)$
$\tilde{c}h_{ls}^t$	$\sim uniform(40, 70)$	q_{ijw}^t	$\sim uniform(8, 15)$
$\tilde{c}l_{ls}^t$	$\sim uniform(80, 120)$	$\tilde{Cap}W_{iw}^t$	$\sim uniform(450, 500)$
b_{il}^t	$\sim uniform(1, 10)$	$\tilde{Cap}T_s^t$	$\sim uniform(450, 510)$
$\tilde{d}e_{ik}^t$	$\sim uniform(70, 250)$	Cap_{is}^t	$\sim uniform(150, 550)$
\tilde{h}_{iw}^t	$\sim uniform(3, 8)$	\tilde{a}_w^t	$\sim uniform(150, 250)$
$\tilde{\pi}_i^t$	$\sim uniform(15, 20)$		

The optimal results of the test problem are presented on Table. The τ_0 is the minimum satisfaction degree of the objective functions as already presented in model (41). The controlling parameters are considered to be equal to 0.6, namely, $\delta, \sigma, \delta', \sigma', \lambda, \gamma, \phi$. Here, weight coefficients run the gamut from 0 to 1 by step size 0.2 for both objective functions. As expected, objective function values increase (decreases), as the weight coefficient of that objective decreases (increases). This trend is same for objective functions satisfaction degree. More analysis on controlling parameters is conducted, but it seems with a moderate set of controlling parameters an acceptable lower bound for TH objective function (model(41)) is achievable.

Table 4. Optimal solution of the test problem with controlling parameters all equal 0.6

Weight coefficients	TH Objective Function	τ_0	μ	Z1	Z2	CPU time (min)
w= (0.0,1.0)	0.857	0.857	(0.857, 0.907)	4.382299E+9	11658	62
w= (0.2,0.8)	0.855	0.864	(0.864, 0.877)	4.306002E+9	12458	81
w= (0.4,0.6)	0.854	0.873	(0.873, 0.851)	4.127302E+9	13158	108
w= (0.6,0.4)	0.852	0.877	(0.884, 0.847)	4.086544E+9	14059	142
w= (0.8,0.2)	0.858	0.850	(0.889, 0.830)	3.971884E+9	14659	67
w= (1.0,0.0)	0.857	0.812	(0.899, 0.812)	3.518649E+9	15464	88

To shed light on the obtained results of the test problem, an illustration is presented in Fig , to demonstrate the conflicts of the objective functions and their behavior through the weight coefficients spectrum. For Transparency considerations, the objective function values are normalized. CPU time is considerably high when the weight coefficients values are approximately near each other, e.g., $w=(0.4, 0.6)$ and $w=(0.6, 0.4)$. This higher CPU time occurs because of the trade-off between objectives and higher computational effort for that purpose. (Note that the notation Z in all the tables and figures is representing the crisp Z).

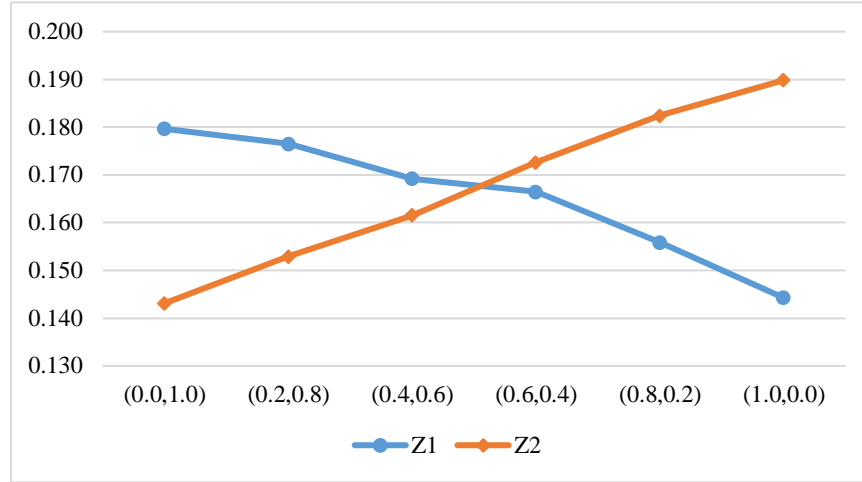


Fig 1. Normalized Pareto solutions based on optimal values of objective functions

The parameter ϕ in model(41) plays a balancing role between objective functions minimum satisfaction degrees and summation of objective function satisfaction degrees. For small amounts of ϕ the importance of high weighted objective function is highlighted while for large values of ϕ , the minimum satisfaction degree is given more importance. Table runs the gamut from 0.1 to 0.9 for the parameter ϕ . The weight factor for first objective function (cost) is high as it is the first priority in any production planning program. From model(41) we learn that the objective function(s) with higher weight coefficient is drastically sensitive to the values of the ϕ . under controlling parameters equal to 0.7 and weight coefficients (0.7, 0.3)

Table 5. Optimal solution values for the spectrum of ϕ

ϕ	TH Objective Function	τ_0	μ	Z1	Z2	CPU time (min)
0.9	0.848	0.848	(0.848, 0.852)	4.595500E+9	13857	150
0.8	0.847	0.844	(0.864, 0.844)	4.465049E+9	13918	97
0.7	0.852	0.848	(0.867, 0.848)	4.405344E+9	13979	30
0.6	0.851	0.848	(0.856, 0.848)	4.495372E+9	14359	103
0.5	0.854	0.811	(0.935, 0.811)	3.865798E+9	14464	22
0.4	0.865	0.803	(0.950, 0.803)	3.746038E+9	14595	16
0.3	0.875	0.773	(0.981, 0.773)	3.499516E+9	14669	16
0.2	0.893	0.788	(0.975, 0.788)	3.549613E+9	14767	13
0.1	0.899	0.868	(0.973, 0.773)	3.562581E+9	14969	77

Table is analyzing the behavior of TH objective function and satisfaction degree of the proposed model's objectives under different values of γ . These executions are taken under (0.7, 0.3) weight coefficients for the objectives and all the other controlling parameters are considered to be equal to 0.6. The parameter γ is penalizing the objective because of its possible violation from the average optimal amounts, see the crisp model and equation(39). With no surprise, as the value of γ raises the optimal value of first

objective function increases. To point out the impact of this parameter, one should analyze the minimum satisfaction degree of objectives along with TH objective function value and satisfaction degree for each different rows of the presented table. Minimum satisfaction degree is slightly increasing as the penalty coefficient raising, the same behavior is traceable and noticeable on TH objective function values. Second objective function is independent from values of γ , but it is under influence of minimum satisfaction degree, so as the τ_0 increases, we expect an improvement on objective values.

Table 6. Optimal solutions under different values of γ

γ	TH Objective Function	τ_0	μ	Z1	Z2	CPU time (min)
0	0.851	0.848	(0.882, 0.868)	3.523589E+9	16746	75
1	0.857	0.856	(0.862, 0.856)	4.428870E+9	16740	43
5	0.857	0.855	(0.863, 0.855)	7.586376E+9	16727	160
10	0.857	0.856	(0.861, 0.856)	1.15216E+10	16715	44
20	0.863	0.860	(0.870, 0.860)	1.89787E+10	16703	81
50	0.864	0.863	(0.871, 0.863)	4.22695E+10	16688	72

Controlling parameters of the proposed model have a crucial role in finding the optimal solution and execution time. The computational power used in this paper are a Corei7 2.40 GHz laptop with 8 GB of RAM. As discussed earlier, to escape the complexity of the non-linear models, α and β are treated as parameters. Here, on Table, a sensitivity analyses are conducted based on different values of controlling parameter, ranging from 0.5 to 0.9 by step size 0.1. As controlling parameters increases, the feasibility robustness and optimality robustness are tightened and in result the optimal values are growing and thus the newly calculated optimal values don't outperform the older ones.

Table 7. Optimal solutions under different values of controlling parameters

Controlling Parameters	TH Objective Function	τ_0	μ	Z1	Z2	CPU time (min)
0.5	0.846	0.798	(0.941, 0.798)	3.404970E+9	11978	62
0.6	0.847	0.844	(0.856, 0.844)	4.149143E+9	14358	86
0.7	0.851	0.860	(0.864, 0.860)	4.169530E+9	16745	90
0.8	0.854	0.840	(0.856, 0.840)	4.406998E+9	19129	74
0.9	0.861	0.850	(0.865, 0.850)	4.518971E+9	21515	94

To do more investigations on α and β parameters, the model is conducted under following setting as well, all controlling parameters are considered to be equal to 0.6 and weight coefficients are (0.7, 0.3). The obtained results are given in Table. As the values of α and β increase the objective function values for both objectives of the proposed model are outperforming the objective function values with lower α and β values. However, modest values for these parameters lead into conservative results and a smooth degradation in second objective function's optimal values.

Table 8. Optimal solutions under different values of α, β

α, β	TH Objective Function	τ_0	μ	Z1	Z2	CPU time (min)
0.5	0.860	0.860	(0.861, 0.860)	4.221304E+9	17933	217
0.6	0.853	0.878	(0.884, 0.847)	4.083374E+9	14279	192
0.7	0.892	0.877	(0.897, 0.870)	3.987201E+9	14028	113
0.8	Infeasible					
0.9	Infeasible					

6-Conclusions

In this study the joint production planning and warehouse layout problem is investigated under uncertainty. Joint production planning and warehouse layout problems is almost a novel and new area in both academics and practice. For warehouses, the eventually of rental warehouses and new allocations in each period of planning horizon is considered. A bi-objective MILP model is proposed and the fuzzy distributed parameters and chance constraints are taken into considerations. A simple test problem along with a case study is investigated by the proposed model. The results indicate the efficiency of the proposed model in optimizing the behavior of the under-study problem. With the high enough cost sensitivity coefficients, the minimization of the total cost is achievable while the work labor lay-offs are not that much. The same results could be obtained if the decision-makers attend the work labor fluctuations more than the total cost of the production system.

Although the proposed model considers the real-world conditions, there is further room for further developments, complex warehousing policies needs to be addressed in an integrated environment with operational and tactical level decisions of a production system. The joint strategic and tactical decisions of a supply chain, considering warehousing in an uncertain environment could be another research avenue. If a more realistic setting is about to be investigated, the internal operations in warehouses could introduce some major managerial insights in this field of research. On computational approaches, along with the novel TH approach, evolutionary algorithms have proven their applicability in dealing with complex structured problems.

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