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A location-inventory problem considering restrictions on storing different perishable products with uncertain demands and lead times

Mehdi Seifbarghy[1], Tina Behroo[1]

[1]*Department of Industrial Engineering, Faculty of Engineering, Alzahra University, Tehran, Iran*

m.seifbarghy@alzahra.ac.ir, tinabehroo7394@gmail.com

Abstract

This paper studies a location-inventory problem with uncertain demands and lead times in a three-level supply chain including a producer, multiple distribution centres (DCs) and multiple retailers. A number of perishable products such as food and medicine goods are considered with a specific shelf life; unlike the previous studies in the literature, the restrictions of storing different perishable products in identical DC is considered. The objective is to determine the number and location of DCs, the allocation of retailers to DCs, the reorder point and demand rate at each DC. Due to the uncertainty on demands and lead times, a queuing approach is utilized to model the problem. The problem is an integer nonlinear programming model and solved using the Genetic and the Imperialist Competitive algorithms.

Keywords: Location-inventory, perishable products, uncertain demands and lead times, Genetic Algorithm, Imperialist Competitive Algorithm

1- Introduction

Supply chain design is one of the most critical issues for effective supply chain management because it reduces costs and increases service quality. One of the essential steps in the supply chain design is determining the number and location of distribution centres (DCs). In the most of location-inventory problems with multi-products, restrictions on storing different kinds of products is not studied. Unlike prior researches in the literature, we consider restrictions on storing different perishable products with uncertain demands and lead-times. The mentioned restrictions will be presented using a matrix. Each Matrix element indicates the possibility of storing a specific type of product in a particular DC.

In this paper, we study a location-inventory problem for a three-level supply chain considering restrictions on storing different perishable products with uncertain demands and lead times. The supply chain is composed of a producer, multiple DCs, and arbitrary number of retailers. Considering restrictions on storing different perishable products with uncertain demands and lead times makes the model closer to the real world applications. On the restrictions on storing different perishable products, holding perishable products in DCs is not as easy as other normal products. Perishable products like meat and fruit need special holding systems such as refrigerators, shelving equipment and so on.

*Corresponding author
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These systems may be involved with high establishment and operational costs. This is why, it is recommended that each DC to be designed for holding and distributing a particular family of product. On the other hand, uncertainty of demands and lead times may be originated from some outside factors such as sudden increase in demand, seasonal trends and disruptions in suppliers. These factors make uncertainty on demand and lead time. The objective is to determine the number and location of DCs, the allocation of retailers, and the reorder point and demand rate at each DC. Due to the uncertainty on demands and lead times, a queuing approach is utilized to solve the model. Two heuristics based on the genetic algorithm (GA) and imperialist competitive algorithm (ICA) are proposed to solve the problem for large sized instances. The literature related to location-inventory problems and inventory management problems with storage restrictions and the solution approaches are presented in the next sections.

1-1- Location-inventory problem

The research by Baumol and Wolfe, (1958) is one of the first studies that consider inventory costs into location models. Firstly, they investigated the number and location of limited capacity warehouses; then, they considered transportation and inventory costs in the model, which resulted in a local optimum solution. Erlebacher and Meller, (2000) developed an analytical model that keeps acceptable service while minimizing system costs. Since the proposed model was NP-Hard, they developed a heuristics solution, and evaluated its performance. Daskin, (2002) proposed a distribution centre location model that consists of working inventory, safety stock inventory, and transportation costs. The model was formulated as a non-linear integer-programming problem. A Lagrangian relaxation solution algorithm was proposed to solve the problem. After the termination of Lagrangian relaxation, they applied a variant of heuristics for finding reasonable solutions. Shen et al., (2003) considered a joint location-inventory problem in which each retailer came across a varying demand. They considered safety stock to achieve a suitable service level. The objective was to determine which retailer should serve as distribution centre and how to assign other retailers to the addressed distribution centres. The authors formulated the problem as a non-linear integer programming. Snyder et al., (2007) presented the stochastic version of the location model with risk pooling to optimize location, inventory, and allocation decisions under random parameters described by discrete scenarios. The objective was to minimize the system costs across all scenarios. Furthermore, they proposed a Lagrangian relaxation algorithm to solve the problem. Miranda and Garrido, (2008) proposed an approach in order to incorporate inventory control decisions into location problem with stochastic demand and safety stock. They formulated the problem as a non-linear mixed integer model. They presented a solution based on the Lagrangian relaxation approach and the sub-gradient method to solve the problem. Park et al., (2010) considered a three-level supply chain, including suppliers, DCs, and retailers with risk pooling strategy and lead-time; the objective is to determine the number and locations of suppliers and DCs, the assignment of each DC to a supplier, and each retailer to DCs. The model was formulated as a non-linear integer programing problem. The problem was solved using a Lagrangian relaxation based heuristic algorithm. Chen et al., (2011) studied reliable join location-inventory problem with stochastic facility disruptions due to natural or human-made hazards. When a facility fails, its customers are reassigned to other facilities to avoid the penalty cost related to losing service. They formulated the problem as an integer programing model and proposed a Lagrangian relaxation based heuristic in order to solve the problem. Tancrez et al., (2012) studied location-inventory problem integrating three decisions: location of the DCs, allocation of customers to DCs, and the size of the shipments. The model was formulated as a nonlinear programming model with cost minimization objective function. They developed an iterative heuristic to solve the problem. Shahabi et al., (2014) developed location- inventory problem with correlated demand. The authors presented a mixed integer conic quadratic program formulation of the model. The solution was based on an outer approximation strategy proposed to solve the model. Ahmadi-Javid and Hoseinpour, (2015) Presented a location-inventory-pricing model with price sensitive demands and inventory-capacity constraints. A Lagrangian relaxation algorithm was proposed to solve the problem. The proposed approach can be applied to other supply chain design problems with sensitive demands.

Escalona et al.(2015) studied location-inventory problems with fast-moving items. They considered critical-level policies to provide different service levels for products with two types of demand. The model was formulated as a mixed integer non-linear programming (MINLP) and a heuristic approach was proposed to solve the problem. Sadjadi et al., (2015) studied a three-level supply chain network with uncertain demands and lead times, which include a supplier, multiple DCs, and retailers. The queuing approach is used to obtain the amount of annual ordering, purchase, shortage, and the average inventory in the steady-state condition. They formulated the model as MINLP. Moreover, the expected average inventory was calculated by two different methods and the result were compared. Diabat et al., (2017) developed location-inventory problem proposed by Sadjadi et al., (2015). The objective is to determine the number and location of DCs, the allocation of retailers to DCs, and the size and timing of orders for each DC. Due to uncertain demands and lead times, they utilized a queuing approach for calculating expected average inventory in each DC. To solve the presented problem, they proposed simulated annealing and the direct search method.

1-2- Inventory problems with uncertain demands

Berman and Kim, (2004) presented an optimal inventory control problem with uncertain demands and lead times with an outside supplier. The authors formulated the model as a Markov decision problem with continuous review policy, which maximized the facility's benefit subject to the system's costs and analytical examination of how the changes in system parameters affect the optimal profit. Mak and Shen, (2009) studied the problem of designing two-level spare parts inventory system. The model consists of a central plant and some service centres with stochastic demand. The manufacturing process is modelled as a queuing system in the central plant. The model was formulated as a MINLP with system costs minimization as the objective function. Since the problem is NP-hard, a Lagrangian heuristic approach was proposed. Atamtürk et al., (2012) studied facility location and inventory management problems with stochastic demand. They considered incapacitated facilities, capacitated facilities, correlated demand, stochastic lead times and multi commodities in their modeling. Saffari et al., (2013) considered the queueing approach to model the inventory problem with continuous review policy, lost sales, stochastic demands and lead times, which followed the Poisson process and exponential distribution.

Rashid et al., (2015) presented an inventory control system with stochastic demand and supplier's service time. The queuing theory was used to tackle the stochastic nature of the model for single product; then, the proposed model was extended to multi-item inventory model. According to the complexity of this problem, a new heuristic algorithm was developed. Wang (2018) proposed an inventory control model for an incapacitated warehouse. They considered two-stage and three-stage supply chains with uncertain demand and lead time. The objective was to minimize total system costs. They proposed an exact algorithm to solve the problem. They compared the result from the given model with three decision-making strategies: optimistic, moderate, and pessimistic.

1-3- Inventory problems with perishable product

Lee et al. (2014) studied a perishable inventory system. The objective was to maximize the profit of the system under a linearly decreasing price structure assumption. They showed that last in, first-out (LIFO) policy is optimal with a linearly decreasing price structure and it has a better performance than first-in, first-out policy (FIFO). White and Censlive, (2015) considered the effects of shelf life on a three-level supply chain's performance with sudden demand sales rate. It assumed that each product with a shelf life follows the FIFO policy. They showed a significant reduction in the number of discards leading to reduce carbon footprint and wasted resources. Kim et al., (2015) presented a multi-period newsvendor model to optimize the total logistic cost for perishable products. The model was formulated as multi-stage stochastic programming with integer decisions-the progressive hedging method used to solve the model efficiency. Hiassat et al., (2017) studied a location-inventory-routing problem for perishable products in such a way that location decision was added to the inventory-routing problem to make it more realistic. Since the problem is NP-hard, they proposed a GA based heuristic to solve it. Azadeh et al., (2017) presented an inventory-routing problem with a single perishable product. Since the problem is NP-hard, a GA based heuristic is used to solve the problem. Rafie-Majd et al., (2017) proposed an inventory-location-routing model with multi-perishable products in a three-level supply chain. They considered heterogeneous vehicle fleet and multiple DCs in the supply chain. A Lagrangian Relaxation Method was used to solve the problem.

Considering the given review on the literature of location-inventory problems shows that previous researchers do not study the assumption of restrictions on storing different types of perishable products in the identical DC while it is an important issue when designing and allocating a DC to a family of products. Furthermore, uncertainty of demand and lead time, makes the model closer to the real life applications. On the solution method side, application of GA and ICA based heuristics and comparing them can be for solving the large-sized instances can be another feature of the current paper.

The remainder of this paper is organized as follows: In section 2, we define the problem and then formulate it. In section 3, the solution methods are proposed. In section 4, numerical results are presented and the two proposed heuristics are compared. Finally, conclusions and recommendations for future research are presented in section 5.

2- Problem definition and formulation

2-1- Problem definition

This paper studies a three-level supply chain including a Producer, multiple potential DCs, and A retailers. The locations of the producer and retailers are known. In the first level, the producer manufactures multiple perishable products with independent demands; the producer gives the products to a set of retailers in different areas via some DCs. Since the establishing and operating cost of each DC is high, have considered restrictions on storing different perishable products in each DC. DCs receive orders from the retailers and place orders at the producer. The structure of the understudy supply chain is shown in figure 1.

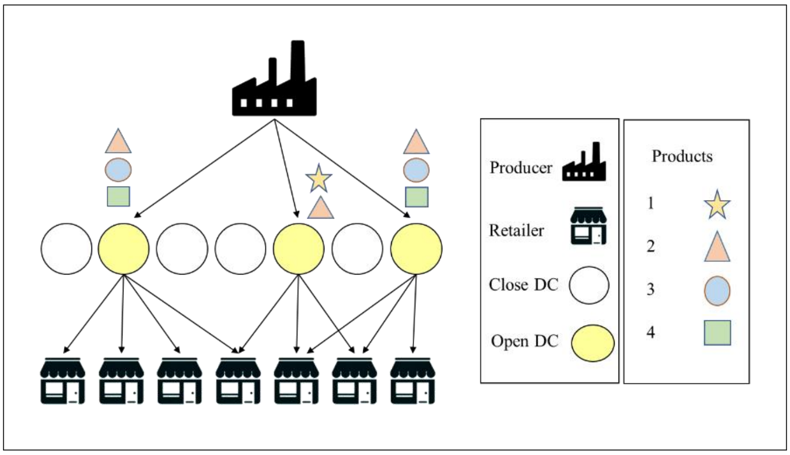

Fig 1. Structure of the understudy supply chain

The major assumptions of the model are:

- Our three-level supply chain includes a producer, multiple distribution centres and multiple retailers
- A number of perishable products such as food and medicine goods are considered with a specific shelf life

- In the first level, the producer manufactures multiple perishable products with independent demands; the producer gives the products to a set of retailers in different areas via some DCs. There exist restrictions of storing different perishable products in distribution centres
- Demand and lead time are considered to be uncertain
- The locations of DCs should be determined, while the locations of the producer and retailers are known

2-2- Notation

The following notations are used in order to model the problem:

Sets:

K Set of potential DCs,

S Set of products,

I Set of retailers.

Parameters:

C_{ks} Unit purchase cost of product *s* from the producer by DC *k*

π_{ks} Unit shortage cost of product *s* at DC *k*

F_k Fixed cost of locating a DC at location *k*

A_{ks} Unit ordering cost of product *s* at DC *k*

h_{ks} Unit holding cost of product *s* at DC *k*

U_{ks} Storage capacity of product *s* at DC *k*

shf_s Shelf-life time of product *s*

λ'_{is} Demand rate of product *s* at retailer *i*

μ_s Exponential distribution parameter of lead time of product *s*

M A big number

T_{kis} Unit transportation cost from DC *k* to retailer *i* for product *s*

β Weight factor associated with transportation cost

θ Weight factor associated with inventory cost

V_s Minimum service level of product *s*

p_s Maximum number of DCs that can hold product *s*

X_{ks} Is equal to 1 if it is possible to hold product *s* in DC *k*; 0, otherwise

The major decisions of the given model are as follows:

a- Number and location of each DC.
b- Best allocation of retailers to opened DC(s) based on restrictions on storing different perishable products.
c- Reorder point and demand rate of different products at each DC.

Based on the given descriptions, the following decision variables are given:

Z_k Is equal to 1 if DC k is opened; 0, otherwise.

Y_{kis} Is equal to 1 if retailer i is assigned to DC k for replenishing product s; 0, otherwise.

S_{ks} Reorder point at DC k for product s

Q_{ks} Demand rate of DC k for product s

2-3- Problem formulation

X_{ks} represents a Matrix with k rows and s columns. Its arrays are set to one or zero depending on what kind of product can be stored in each DC. For example, as figure 2, the first product can be stored in the first and second potential locations for the DCs, while, the second products can be stored in the second and third potential locations for the DCs.

$$
\begin{array}{cc}
 & \begin{array}{cc} S=1 & S=2 \end{array} \\
X_{ks} = & \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{array}{l} \Leftarrow \text{DC=1} \\ \Leftarrow \text{DC=2} \\ \Leftarrow \text{DC=3} \end{array}
\end{array}
$$

Fig 2. Representation of restrictions on storing products in each DC for each product by Matrix of X_{ks}

We utilized a continuous-time Markov chain for modeling the on-hand inventory levels at each DC for each product. Markov chain is a powerful analyzing tool for modeling uncertain inventory problems (Markov and Models, 2006; Diabat, Dehghani and Jabbarzadeh, 2017). Initially, we formulate on-hand inventory level for each product at each DC based on a Markov chain. Each DC places order at the producer according to the continuous review policy which is represented by ($S_{ks}.Q_{ks}$), where S_{ks} is the reorder point, and Q_{ks} is the reorder quantity of product s. When on-hand inventory for each product becomes equal or less than S_{ks}, the DC places an order at the producer with size of Q_{ks}. We assume that ($S_{ks}>Q_{ks}$) in order to prevent permanent shortage. In each DC, the on-hand inventory level can be changed between 0 and ($S_{ks}+Q_{ks}$). Figure 3 shows the states of on-hand inventory level for product s at the DC k.

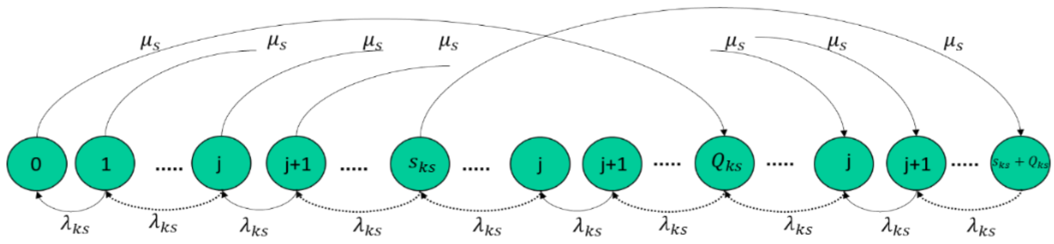

Fig 3. The on-hand inventory states at DC k for product s with demand rate of λ_{ks} and service rate of μ_s

Let $I_{ks}(t)$represent the on-hand inventory level at time t in DC k for product s. Thus $\{I_{ks}(t); t > 0\}$ with state space $E_{ks} = \{Q_{ks} + S_{ks}, Q_{ks} + S_{ks} - 1, \dots, 1, 0\}$ is a Markov process. Therefore, the probabilities of transitioning from one state to another state in a time unit can be obtained as in equations (1)-(2)

$$P_{ks}(i, j, t) = Pr\ [I_{ks}(t) = i |\ I_{ks}(0) = i] \qquad i, j \in E_{ks} \tag{1}$$

$$P_{ks}(j) = \lim_{t\to\infty} P_{ks}(i, j, t) \tag{2}$$

By equating the input and output rates of each state in Figure 3, the following equilibrium equations are obtained as in equations (3)-(7).

$$\mu_s p_{ks}(0) = \lambda_{ks} p_{ks} \tag{3}$$

$$(\lambda_{ks} + \mu_s) p_{ks}(j) = \lambda_{ks} p_{ks}(j+1) \qquad 1 \leq j \leq S_{ks} \tag{4}$$

$$\lambda_{ks} p_{ks}(j) = \lambda_{ks} p_{ks}(j+1) \qquad S_{ks}+1 \leq j \leq Q_{ks}-1 \tag{5}$$

$$\lambda_{ks} p_{ks}(j) = \lambda_{ks} p_{ks}(j+1) + \mu_{ks} p_{ks}(j - Q_{ks}) \qquad Q_{ks} \leq j \leq S_{ks} + Q_{ks} - 1 \tag{6}$$

$$\lambda_{ks} p_{ks}(S_{ks} + Q_{ks}) = \mu_s p_{ks}(S_{ks}) \tag{7}$$

Where $p_{ks}(j)$denotes the steady-state probability that the inventory level of product s will be equal to level j at DC k. Equations (3)-(7) based on the Markov process properties, ensure that the input and output rates for each state are equal. By solving the equations (3) - (7), we achieve to equations (8) - (11).

$$p_{ks}(j) = \left(1 + \frac{\lambda_{ks}}{\mu_s}\right)^{j-1} \frac{\mu_s}{\lambda_{ks}} p_{ks}(0) \qquad 1 \leq j \leq S_{ks} \tag{8}$$

$$p_{ks}(j) = \left(1 + \frac{\mu_s}{\lambda_{ks}}\right)^{S_{ks}} \frac{\mu_s}{\lambda_{ks}} p_{ks}(0) \qquad S_{ks}+1 \leq j \leq Q_{ks} \tag{9}$$

$$p_{ks}(j) = \left[\left(1 + \frac{\mu_s}{\lambda_{ks}}\right)^{S_{ks}} - \left(1 + \frac{\mu_s}{\lambda_{ks}}\right)^{j - Q_{ks} - 1}\right] \left(\frac{\mu_s}{\lambda_{ks}}\right) p_{ks}(0) \qquad Q_{ks}+1 \leq j \leq Q_{ks} + S_{ks} \tag{10}$$

$$p_{ks}(0)=\frac{\lambda_{ks}}{\lambda_{ks}+Q_{ks}\mu_s\left(1+\frac{\mu_s}{\lambda_{ks}}\right)^{S_{ks}}} \tag{11}$$

Where p_{ks} represents the probability of steady state where the inventory level of product s will be equal to 0 at DC k. The other performances measures computed as follows:
R_{ks} represents the number of reorders of product s at opened DC k; it is calculated by equation (12). Each DC places order at the producer when the inventory level of product s is equal or less than$(S_{ks}+1)$.

$$R_{ks}=\lambda_{ks}p(S_{ks}+1)=\mu_S\left(1+\frac{\mu_s}{\lambda_{ks}}\right)^{S_{ks}}p_{ks}(0) \tag{12}$$

Γ_{ks}represents he number of lost sales of product s at opened DC k; it is calculated by equation (13). When the inventory level for of product s in DC k is equal to 0, receiving demands are lost.

$$\Gamma_{ks}=\lambda_{ks}p_{ks}(0) \tag{13}$$

MI_{ks}represents the expected amount of average inventory level of product s in the steady-state and is calculated by equation (14).

$$MI_{ks}=\sum_{j=0}^{Q_{ks}+S_{ks}}(jp_{ks}(j)) \tag{14}$$

By replacing $p_{ks}(j)$ in equation (14) with its equivalent in the aforementioned equations, we can obtain Equation (15).

$$MI_{ks}=\frac{P_{ks}(0)(m_2\left(\frac{S_{ks}}{2}+S_{ks}Q_{ks}+\frac{S_{ks}^2}{2}\right)-\frac{\lambda_{ks}(\lambda_{ks}+\mu_s)^{Q_{ks}}\lambda_{ks}^{S_{ks}+1}-m_3+Q_{ks}\mu_s(\lambda_{ks}+\mu_s)^{S_{ks}}+m_1-Q_{ks}\lambda_{ks}^{S_{ks}}\mu_s}{\lambda_{ks}^{Q_{ks}}\lambda_{ks}^{S_{ks}}\mu_s^2\left(\frac{\lambda_{ks}+\mu_s}{\lambda_{ks}}\right)^{Q_{ks}}}}{\lambda_{ks}}$$

$$+\frac{P_{ks}(0)\left(\lambda_{ks}^{S_{ks}+1}-m_3+m_1\right)}{\lambda_{ks}^{s_{ks}}\mu_s}+\frac{P_{ks}(0)\mu_s\left(\frac{Q_{ks}(Q_{ks}+1)}{2}-\frac{S_{ks}(S_{ks}+1)}{2}\right)m_2}{\lambda_{ks}} \tag{15}$$

Where $m_1.m_2$ and m_3 are obtained from equations (16)-(18).

$$m_1=S_{ks}\mu_s(\lambda_{ks}+\mu_s)^{S_{ks}} \tag{16}$$

$$m_2=\left(1+\frac{\mu_S}{\lambda_{ks}}\right)^{S_{ks}} \tag{17}$$

$$m_3=(\lambda_{ks})(\lambda_{ks}+\mu_s)^{S_{ks}} \tag{18}$$

λ_{ks}represents demand rate of product s at DC k; it is calculated by equation (19).

$$\lambda_{ks}=\sum_i y_{kis}\lambda_{is}' \tag{19}$$

The objective function of the model consists of the following components as in equations (20) - (22):

The total cost of locating DCs is obtained from equation (20).

$$\sum_{k} F_k Z_k \tag{20}$$

The expected total inventory costs is obtained from equation (21). It consist of the holding, ordering, shortage, and purchase costs for all products in all DCs.

$$\sum_{s}\sum_{k} X_{ks}(h_{ks}MI_{ks}+A_{ks}R_{ks}+\Pi_{ks}\Gamma_{ks}+C_{ks}R_{ks}Q_{ks}) \tag{21}$$

Total Transportation cost can be calculated by equation (22).

$$\sum_{k\in K}\sum_{i\in I}\sum_{s\in S} X_{ks}T_{kis}y_{kis}\lambda'_{is}\left(1-p_{ks}(0)\right) \tag{22}$$

The optimization model is formulated as in equations (23)-(34).

$$\begin{aligned} Min\ TC:\ & \sum_{k} F_k Z_k + \theta \sum_{s}\sum_{k} X_{ks}(h_{ks}MI_{ks}+A_{ks}R_{ks}+\Pi_{ks}\Gamma_{ks}+C_{ks}R_{ks}Q_{ks}) \\ & +\beta(\sum_{k\in K}\sum_{i\in I}\sum_{s\in S} X_{ks}T_{kis}y_{kis}\lambda'_{is}\left(1-p_{ks}(0)\right) \end{aligned} \tag{23}$$

S.t:

$$\sum_{k\in K} y_{kis} = 1 \qquad \forall i\in I.\ \forall s\in S \tag{24}$$

$$\sum_{i\in I} y_{kis} \leq M\times Z_k \qquad \forall k\in K,\ \forall s\in S \tag{25}$$

$$\sum_{i\in I} \lambda'_{is} y_{kis} = \lambda_{ks} \qquad \forall k\in K,\ \forall s\in S \tag{26}$$

$$X_{ks}\left(Q_{ks}+S_{ks}\right)\leq U_{ks} \qquad \forall k\in K,\ \forall s\in S \tag{27}$$

$$\left(1-p_{ks}(0)\right)\geq V_s \qquad \forall k\in K,\ \forall s\in S \tag{28}$$

$$\frac{\left(Q_{ks}+S_{ks}\right)}{\lambda_{ks}}\leq shf_s \qquad \forall k\in K,\ \forall s\in S \tag{29}$$

$$1\leq \sum_{k\in K} Z_k X_{ks} \leq p_s \qquad \forall s\in S \tag{30}$$

$$S_{ks}+1\leq Q_{ks} \qquad \forall k\in K,\ \forall s\in S \tag{31}$$

$y_{kis} \in \{0.1\}$ $\quad \forall i \in I,\ \forall k \in K,\ \ \forall s \in S$ (32)

$Z_k \in \{0.1\}$ $\quad \forall k \in K$ (33)

$S_{ks}.Q_{ks} \geq 0$ $\quad S_{ks}$ *and* Q_{ks} *are Integer* $\quad \forall k \in K.\ \ \forall s \in S$ (34)

Equations *(8)-(19)*

The objective function (23) minimizes the total costs with the terms described earlier. Constraint (24) states that each retailer must be assigned to only one DC for replenishing each product. Constraint (25) guarantees that each retailer is assigned to merely the opened DCs. Constraint (26) states that each DC's demand rate for each product is equal to the sum of the demand rates of its assigned retailers. Constraint (27) guarantees the capacity limitations according to the constraint of facilities for holding products in each DC for each product. Constraint (28) states that the minimum service level of each product at each *DC*. Constraint (29) is for the shelf-life time of each product at each DC. Constraint (30) gives the maximum number of DCs which are assigned for each product noting that at least one DC should be assigned to each product. Constraint (31) makes sure that the order quantity of each product at each DC is at least one unit higher than the reorder point. Constraints (32), (33), and (34) give the status of the decision variables of the model. Equations (8)-(19) are also considered in the model.

3-Solution algorithms

Since the problem is NP-hard, two heuristics based on GA and ICA are proposed to solve the problem.

3-1- Genetic Algorithm (GA)

GA starts with an initial population; each individual in the population is called a chromosome and has three operators including mutation, crossover, and selection.

3-1-1- Chromosome representation in GA

The most important part of the GA is the chromosome structure. A chromosome is a tool for defining a proposed solution for the problem that the GA is trying to solve. Matrix notations are used for demonstrating the chromosomes. In this paper, due to the impossibility of displaying all the variables in one matrix, each chromosome consists of three matrices. Figure 5 is an example of a chromosome, in which the number of potential locations for DCs is three, the number of retailers is six, and the number of products is two.

The decision variable Y_{kis} gives the best assignment of retailers to the opened DCs based on the limitation of facilities in holding products. For the decision variable Y_{kis}, a matrix with s columns and $k \times i$ rows is generated. In this matrix, the third DC is open. As shown in figure 5, the matrix Y_{kis} indicates that the retailers of zone 1, 2, 3, 4, 5, and 6 are assigned to the third DC for both products. For the decision variable S_{ks} the matrix $s \times k$ is generated. This matrix shows the reorder point for the products based on the matrix X_{ks}. The matrix S_{ks} in Fig. 5 indicates that the reorder points for the first product in the second DC and for the second product in the first potential DC are zero because there are holding restrictions for them. Furthermore, the matrix Q_{ks} gives the order quantity for the products based on the matrixX_{ks} given in figure 4. For example, the order quantity for the first product in the second DC is eleven.

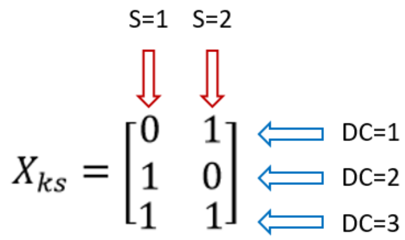

Fig 4. Representation of restrictions on storing products in each DC for each product by the matrix X_{ks}

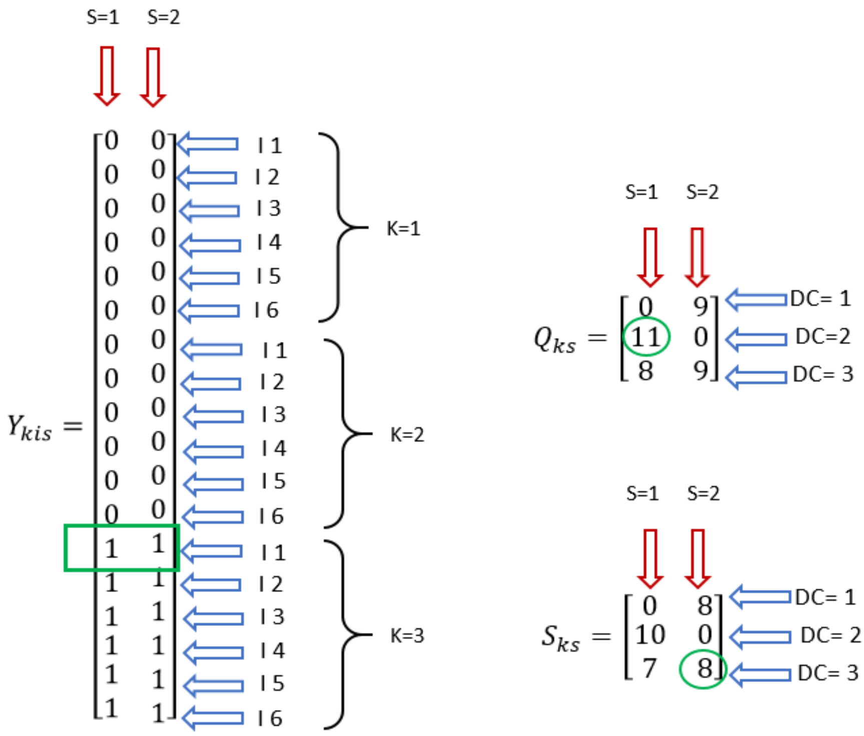

Fig 5. The representation of the chromosome with two products, six retailers and three DCs

3-1-2- GA operators:

In this section, we describe the applied operators of GA based heuristic.

Selection: the roulette wheel mechanism is usually used for selection in GA to select the potential higher quality solutions. Suppose that the fitness value of chromosome i, p_i in each population is given as

$$\left(p_i = \frac{e^{-\beta \frac{c_i}{c_{worst}}}}{\sum_i e^{-\beta \frac{c_i}{c_{worst}}}}\right)$$

where β is a constant parameter and represents the selection pressure, c_{worst} and c_i represent the maximum cost of a chromosome in the population (i.e. the chromosome with worst fitness value) and the cost of chromosome i, respectively. Considering F as the sum of the addressed fitness values as $F=\sum_{i=1}^{npop} p_i$ ($npop$ represents the number of population), then $A_i=p_i/F$ gives the probability of selecting chromosome i.

Crossover: Crossover is used to combine parental genetic information to produce new offspring. In this step, parents are selected with a roulette wheel selection strategy. The crossover rate is shown by $P_{crossover}$. Here, the two-point crossover is used. The genes between these two points are swapped between the parents and the off springs are obtained as shown in figure 6. Since each chromosome consists of three matrices, the crossover operator is randomly applied to one of them with equal probabilities.

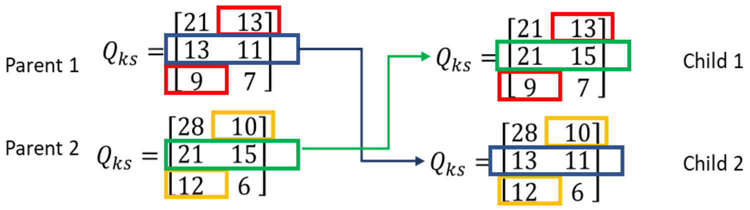

Fig 6. Crossover application on order quantity matrix

Mutation: Mutations occurs randomly with equal probabilities in one of the three matrices of each chromosome with the probability $P_{mutation}$. The genes selected for the mutation change within the allowable range of each variable. According to figure 7, mutation occurred randomly in the matrix Q_{ks}.

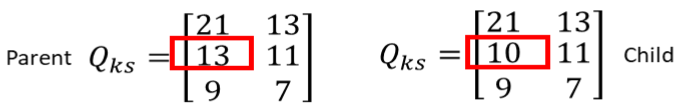

Fig 7. Mutation application on order quantity matrix

3-2- Colonial competition algorithm

ICA was presented by Atashpaz-Gargari and Lucas (2007). This algorithm starts with an initial population; each individual in the population is called a country. Countries in the ICA are the counterpart of chromosomes in GA. The main operators of this algorithm are the assimilation and revolution processes. The steps of this algorithm are given in the following:

3-2-1- Empire formation

ICA starts with an initial population. Several of the best individuals of this population, called countries, are picked up as the imperialist states based on the profit function; the rest of the population are called the colonies of these imperialists. Based on the roulette wheel selection method, the colonies of the initial population are divided among them. Any empire that does not improve in the imperialist competition will be diminished.

3-2-2- ICA operators

The operators of ICA are as follows:

***Assimilation process*:** Assimilation makes the colonies of each empire get closer to the imperialist state in the optimization search space. This is performed by moving the colonies toward their imperialists as shown in figure 8 for the matrix Q_{ks}. Each colony moves toward the imperialist by N_a which is a random element in each row of the matrix as in equation (35) in which assimilation is applied.

$$N_a \sim rand\big(1{:}b.\ ceil(b\times\beta)\big) \tag{35}$$

In equation (35) parameter β is the assimilation coefficient that is a positive number less than one; furthermore, a and b are the number of rows and columns in the matrix, respectively.

$$\underset{\text{imperialist}}{Q_{ks}=\begin{bmatrix} 0 & \boxed{9} \\ \boxed{10} & 12 \\ 11 & \boxed{13} \end{bmatrix}} \Longrightarrow \underset{\text{Colony}}{Q_{ks}=\begin{bmatrix} 0 & \boxed{12} \\ \boxed{14} & 15 \\ 17 & \boxed{25} \end{bmatrix}} \Longrightarrow Q_{ks}=\begin{bmatrix} 0 & 9 \\ 10 & 15 \\ 17 & 13 \end{bmatrix}$$

Fig 8. Assimilation process for a colony toward the imperialist

***Revolution process*:** This step helps the solution not to get trapped in the local optimum as shown in figure 9. The revolution rate for each colony, $P_{revolution}$ is calculated, then a random number in a uniform range [0, 1] is generated; if the generated random number is lower than $P_{revolution}$, the revolution is performed. Parameter *Nmu* calculates the revolutions for imperialists for each matrix of decision variables as in Equation (36); it is the total number of output vectors in which a revolution is performed randomly. Parameter *mu* is a positive number less than 1. If the imperialist's fitness value becomes better than the earlier value, the revolution is performed.

$$Nmu=Rand\ up(mu*output\ vector\ size) \tag{36}$$

$$Q_{ks}=\begin{bmatrix} 0 & 9 \\ 10 & \boxed{12} \\ 11 & 13 \end{bmatrix} \xRightarrow{\text{Revolution}} Q_{ks}=\begin{bmatrix} 0 & 9 \\ 10 & \boxed{17} \\ 11 & 13 \end{bmatrix}$$

Fig 9. The revolution process of the colony on second part of the output vector

3.2.3. Total power of an empire

The total power of each empire is defined as in equation (37):

$$TC_n=Cost\ (Imperialist)+\ \xi(mean\ \big(Cost(colonies\ of\ empire)\big) \tag{37}$$

Where ξ is a positive factor less than one.

4- Numerical results

4-1- Parameter tuning

Since the performance of metaheuristic algorithms depends on their parameters' value, in this paper, the Taguchi method is used to tune the parameters of GA and ICA based heuristics. According to research that conducted by Ahmadzadeh and Vahdani, (2017), we consider different levels for the parameters in both algorithms; then, based on the Taguchi method, which uses the concept of "signal to noise ratio" (S/N), the best level for each parameter is obtained. Table 1 and 2 show the levels for the GA and ICA based heuristics.

Table 1. The parameters levels for the GA based heuristic

Parameter	Index of Level	Level
N_{pop}	1	100
	2	150
	3	200
$p_{mutation}$	1	0.1
	2	0.2
	3	0.3
$p_{Crossover}$	1	0.6
	2	0.7
	3	0.8
α	1	1
	2	2
	3	3
Max it	1	100
	2	200
	3	300

Table 2. The parameters levels for the ICA based heuristic

Parameter	Index of Level	Level
N_{pop}	1	100
	2	150
	3	200
α	1	1
	2	2
	3	3
β	1	0.1
	2	0.2
	3	0.3
$p_{revolution}$	1	0.1
	2	0.2
	3	0.3
mu	1	0.03
	2	0.04
	3	0.05
ξ	1	0.1
	2	0.2
	3	0.3
N_{imp}	1	10
	2	20
	3	30
Max it	1	100
	2	200
	3	300

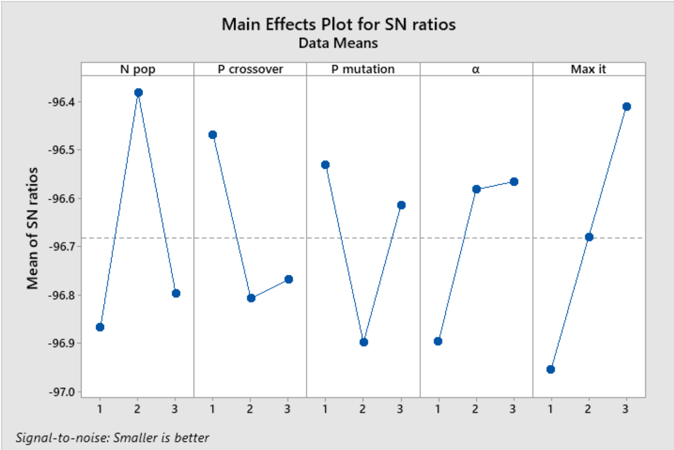

Fig 10. The mean of S/N ratio at each level for the GA parameters.

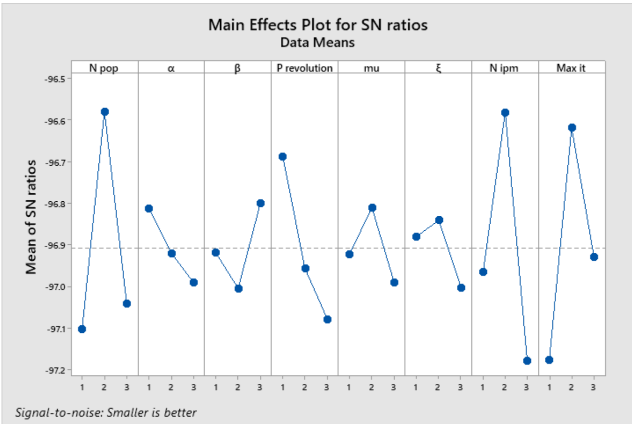

Fig 11. The mean of S/N ratio at each level for the ICA parameters.

The means of S/N ratio at each level of the GA and ICA parameters are illustrated in figures 10 and 11, respectively.

The optimal levels for the parameters of GA turn out to be:
$N_{pop}=150$,$P_{crossover}=0.6$,$P_{mutation}=0.2$.$\alpha= 1$, *Max it=300.*

The optimal levels for the parameters of ICA turn out to be:

$N_{pop}=200$, $\alpha= 1$, $\beta=0.3$, $p_{revolution}=0.1$, $mu=0.04$, $\xi=0.2$, $N_{imp}=20$, *Max it=200.*

4-2-Numerical examples

We design a number of test problems to study the performance of the model and given heuristics. We generate the values of parameters as in table 3, where U [a, b] means the parameter's values is randomly generated in the uniform range of a and b. The unit time for the shelf-life time is considered equal to one day for each product. The other unit times are considered equal to one hour.

Table 3. Ranges of parameters' values for the test problems

Parameter	Probability distribution function	Description
C_{ks}	U [15, 25]	Unit purchase cost of the product *s* from the producer at DC *k*
π_{ks}	U [65, 85]	Unit shortage cost of the product *s* at DC *k*
F_k	U [4500, 6500]	Fixed cost of locating a DC *k*
A_{ks}	U [5, 15]	Unit ordering cost of the product *s* at DC *k*
h_{ks}	U [25, 35]	Unit holding cost of the product *s* at DC *k*
U_{ks}	U [15, 25]	Storage capacity of the product *s* at DC *k*
shf_s	365	Lifetime period for product *s*
λ'_{is}	U [80, 110]	Demand rate of the product *s* for retailer *i*
μ_s	U [150, 350]	Exponential parameter of lead time for the product *s*
M	100	A big number
T_{kis}	U [4, 10]	Unit transportation cost from DC *k* to retailer *i* for the product *s*
β	1	Weight factor associated with transportation costs
θ	1	Weight factor associated with inventory costs
V_s	0.5	Minimum service level of product *s*
P_s	2	Maximum number of DCs that hold product *s*

We have also solved a few test problems utilizing optimization package of GAMS 25.1.2 in a reasonable time. The GA and ICA based heuristics are used to solve large sized instances of the problem. The objective function values and the required time spent for solving are given as in tables 4 and 5.

4-3- Assessing the performance of the heuristics

In this section, we evaluate the efficiency of both heuristics. The stopping condition for both heuristics is considered the number of iterations. Based on the results from table 5, the GA based heuristics outperforms than the ICA one in most of the cases.

Table 4. Objective function values for different sizes of the problem by GAMS, GA and ICA.

Test problem	I	K	S	GAMS	GA	ICA
1	5	2	2	28378	26134	24497
2	6	2	2	34040	28075	29253
3	7	2	2	40068	31211	33970
4	6	2	3	43570	43354	39679
5	8	3	2	-	34139	34881
6	15	3	2	-	60922	67034
7	65	23	2	-	421605	469381
8	75	30	2	-	512895	539140
9	12	4	3	-	70734	81417
10	20	4	3	-	129232	131431
11	30	4	3	-	230166	236050
12	50	4	3	-	404964	496713
13	80	4	3	-	831294	910304
14	30	5	3	-	226549	216152
15	40	5	3	-	263713	341180
16	80	5	3	-	904380	883722
17	40	10	3	-	382950	348315
18	50	12	3	-	500790	418555
19	45	14	3	-	392136	375122
20	55	17	3	-	568786	568596
21	65	20	3	-	704317	675247
22	25	3	4	-	318842	302983
23	35	3	4	-	535792	436752
24	25	4	4	-	246126	228282
25	25	5	4	-	234851	249019
26	35	5	4	-	335330	403470
27	45	5	4	-	596931	527987
28	65	5	4	-	930269	944980
29	50	10	4	-	673327	594130
30	35	6	5	-	457855	482867
31	45	6	5	-	589326	676323
32	45	8	5	-	678917	782148
33	30	2	6	-	586165	542279
34	40	3	6	-	626077	605137
35	30	2	7	-	675179	731728
36	40	3	7	-	742590	709677
37	40	4	7	-	784304	797047

Table 5. Time spent for different sizes of the problem by GA and ICA

Test problem	I	K	S	GA		ICA	
				Best Cost	Time (S)	Best Cost	Time (S)
1	5	2	2	26134	12.4	24497	18.8
2	6	2	2	28075	13.3	29253	18.9
3	7	2	2	31211	12.7	33970	18.6
4	6	2	3	43354	16.8	39679	25.1
5	8	3	2	34139	13	34881	19.5
6	15	3	2	60922	13.7	67034	21.1
7	65	23	2	421605	56.3	469381	79
8	75	30	2	512895	78.8	539140	103.8
9	12	4	3	70734	18.8	81417	25.7
10	20	4	3	129232	23.3	131431	27.7
11	30	4	3	230166	20.2	236050	30.1
12	50	4	3	404964	23.5	496713	35.9
13	80	4	3	831294	29.9	910304	43.4
14	30	5	3	226549	21.6	216152	32.3
15	40	5	3	263713	24.1	341180	35.3
16	80	5	3	904380	32.4	883722	50.4
17	40	10	3	382950	32.5	348315	48.5
18	50	12	3	500790	41.6	418555	57.1
19	45	14	3	392136	42.3	375122	61
20	55	17	3	568786	55.4	568596	75.4
21	65	20	3	704317	71.76	675247	102.9
22	25	3	4	318842	24.8	302983	38.4
23	35	3	4	535792	27.1	436752	41.2
24	25	4	4	246126	26.1	228282	44.2
25	25	5	4	234851	30.8	249019	43.3
26	35	5	4	335330	31.4	403470	45.1
27	45	5	4	596931	41.69	527987	51.6
28	65	5	4	930269	43.3	944980	61.5
29	50	10	4	673327	55.4	594130	78.7
30	35	6	5	457855	38	482867	62.1
31	45	6	5	589326	47.5	676323	65.2
32	45	8	5	678917	53.4	782148	92.2
33	30	2	6	586165	30.3	542279	49.9
34	40	3	6	626077	34	605137	54.3
35	30	2	7	675179	34.4	731728	55.5
36	40	3	7	742590	42.2	709677	65.8
37	40	4	7	784304	47.7	797047	73.3

Table 6. Objective function values for different sizes by GA, and ICA with identical running times

Test problem	I	K	S	Time (S)	GA	ICA
1	15	3	2	180	73862	70671
2	65	23	2	180	516114.5	401669
3	75	30	2	180	584175	491065
4	80	5	3	180	854478	845025
5	40	10	3	180	316884	316766
6	50	12	3	180	478626	462278
7	45	14	3	180	465822	420298
8	55	17	3	180	587682	563757
9	65	20	3	180	716859	683605
10	25	4	4	180	300682	286381
11	65	5	4	180	103783	972540
12	50	10	4	180	672044	633830
13	35	6	5	180	466945	444537
14	45	8	5	180	732349	660483
15	40	3	6	180	655059	624508
16	40	4	7	180	799433	764085

As given in table 6, for identical running times of both heuristics, the ICA based heuristic outperforms than GA based heuristic. Figures 12 and 13 show the variations of best solution for test problem with K=5, S=4 and I=65 by ICA and GA for running time equal to 180.

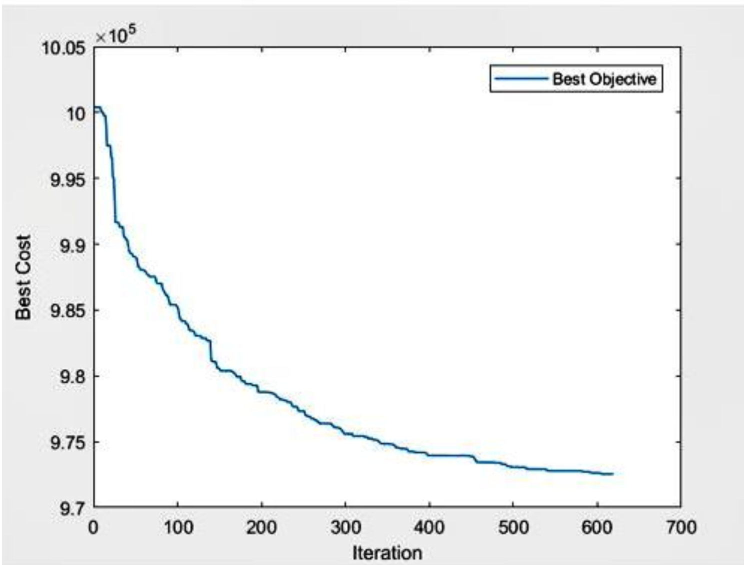

Fig 12. Variations of best solution for test problem with K=5, S=4 and I=65 by ICA

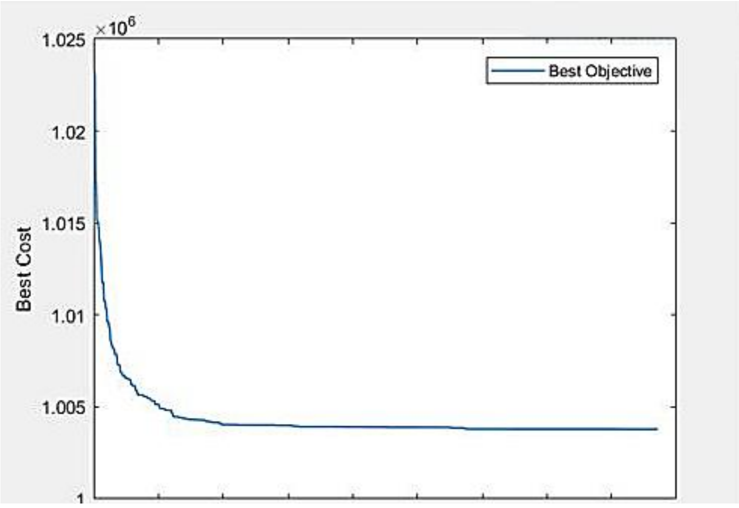

Fig 13. Variations of best solution for test problem with K=5, S=4 and I=65 by GA

5- Conclusions and further researches

We studied a location-inventory problem considering restrictions on storing different perishable products with uncertain demands and lead-times. By considering the restrictions on storing products in each DC, inventory-holding costs are expected to decrease because only products with similar storage conditions are stored in each DC. Furthermore, DC establishment costs are decreased since different storage equipment like shelving systems are not required in each DC. Considering shelf-life time for products was another major assumption in the given model. The relevant literature reviews did not show a model with such assumptions to be studied. We have also considered other constraints such as capacity limitation, and service level to make the model closer to the real life conditions. A model with these assumptions is usable for food and medicine industries with perishable items and specialized storage areas because of different storage requirements for different products. For example in the food industry, we may have some products, which require sub-zero storage temperatures. In medicine industry, we can mention different temperature conditions for different types of vaccines. A clear example can be COVID-19 vaccines, which require different temperature conditions in their logistic systems.

We also developed two heuristics based on GA and ICA in order to solve the problem especially large sizes. To evaluate the performance of the given heuristics, a number of numerical examples were generated. The results showed that the proposed ICA based heuristic could produce more efficient solutions than the GA based heuristic when two heuristics are run in equal processing times.

Further ideas for extension of this research can be considering multiple producers in the higher echelon of the supply chain, considering other assumptions like using multi-modal transportation systems, considering other objectives except for the cost such as environmental issues, or reliability of some facilities and application of some novel heuristics to solve the problem specially, for large sizes.

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