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Developing a joint sustainable pricing, EOQ and EPQ model for a two-echelon supply chain considering economic, environmental and social issues

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Abstract

In this paper, we studied a joint sustainable lot-sizing/ pricing problem in a two echelon supply chain consists of a retailer and a supplier. For each member of the supply chain, Mathematical profit function containing revenue function and different cost functions based on different factors of sustainability such as economic, environmental and social parameters is formulated and for each member of the chain, optimal lot-sizing (Sustainable EOQ or Sustainable EPQ) and pricing decisions are made. Also, a new procedure for problem solving is presented. The contribution of this paper is integrating sustainable pricing and lot-sizing decisions of a supply chain in one model considering all main pillars of sustainability. We conducted a numerical example based on the real data of an Iranian petrochemical two-echelon supply chain and for better analyzing of numerical example results we performed a sensitivity analysis on production capacity of the supplier and profit margin of the retailer. The results show that in this case the decision variables values are not sensitive to production capacity, but they are so sensitive to profit margin of the retailer.

Key words: Sustainable EOQ, sustainable EPQ, pricing, two-echelon supply chain, sustainable supply chain management

1- Introduction

Sustainable Supply Chain Management (SSCM) as a new paradigm in supply chain and operations management body of knowledge is increasingly noticed by the researchers and the leading organizations because of the importance of the sustainable development matter. Also, after Harris (1913) and Taft (1915) papers, determining optimal economic order quantity (EOQ) and economic production quantity (EPQ) as an important area of operations management is attended by researchers in more than a century up to now. Because of global concerns on sustainability issues, many papers have been published in the field of solving

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sustainable economic order quantity (EOQ) and sustainable economic production quantity (EPQ) problems considering different sustainability issues such as economic, environmental and social aspects, in the recent decade. Many of previous works on the sustainable EOQ/ EPQ problem are focused on economic and environmental factors and a few studies has been done considering all of them in the supply chain context. In this paper we proposed a new model for determining optimum sustainable EOQ for the retailer and sustainable EPQ for the supplier as two members of the two-echelon supply chain and in addition we determine optimum whole sale and retail price for the retailer and the supplier, respectively.

This work consists of seven sections. After introduction (section 1), we reviewed on previous researches and studies in section 2 (Literature Review). In section 3 (Problem Description), the main dimensions of the problem and relationships between supply chain members is defined properly. In section 4 (Mathematical formulation of the problem) we defined notations and then the mathematical formulation of the profit function of the retailer and the supplier is done and the problem is parametrically solved. Also, a procedure of problem solving is proposed. In section 5 a numerical example based on real data of an Iranian petrochemical supply chain is presented. In section 6 (Results and discussions), we interpret the results of the numerical example and conduct a sensitivity analysis on two parameters of the problem. Finally, we have presented the managerial insights and the conclusions of this research in sections 7 and 8.

2- Literature review

Many researchers have been studied on sustainable EOQ/ EPQ problems. Turkay (2008), Bouchery et al. (2010), Bonney and Jaber (2011), Csutora et al. (2012), Glock et al. (2012), Ozlu (2013), Chen et al. (2013), Digiesi et al. (2013), Battini et al. (2014), Jawad et al. (2014), Digiesi et al. (2015), Hovelaque and Bironneau (2015), Soleymanfar et al. (2015a), Soleymanfar et al. (2015b), Kazemi et al. (2018), Taleizadeh et al. (2017a) and Taleizadeh et al. (2018) developed various sustainable EOQ/EPQ models in the recent decade. Also, Heuvel et al. (2012) and Bouchery et al. (2012) proposed multi-objective EOQ model to minimize both inventory costs and emissions. Abdallah et al. (2012), Gurtu et al. (2015) and Hammami et al. (2015) modeled sustainable lot-sizing problem for different members of the supply chain. Jana et al. (2013) modeled a Fuzzy multi-item EPQ model considering shortage and process reliability and solve that problem with geometric programming technique. Nobil and Taleizadeh (2016) formulated a single machine multi-item EPQ problem in a defective manufacturing system considering auction and reworking. Rossi et al. (2017) proposed a new approach for modeling single-machine multi-product EOQ problem considering capacity limitation in a particular inventory/ production system. Mishra et al. (2020) studied a sustainable EPQ problem under carbon tax and cap regulation considering controllable carbon emissions and inventory shortage. Mishra et al. (2021) proposed a sustainable inventory model with price-dependent demand and controllable carbon emissions considering inventory deterioration and backordering. Rabta (2020) proposed an EOQ model in a circular economy considering linear or nonlinear relationships between demand, price and cost of the product. Also, Hasan et al. (2021) developed new models for joint technology investment and determining inventory level problem considering carbon emissions with cap and trade and carbon tax approaches.

Also, lot sizing problem in supply chains has been studied by many researchers. Kasthuri and Seshaiah (2013) proposed a new multi- product EOQ model with price-dependent demand considering investing and warehousing constraints. Shafieezadeh and Sadegheih (2014) developed a new approach in their paper to resolve inventory management problem in all echelons of a supply chain by presenting an integrated multi-product model. Behnamian et al. (2017) studied a multi-period, multi-product lot-sizing problem in a multi-level production system with uncertainty of scrap, inspection and rework in different levels, using Markovian approach. Shahidul Islam and Abdul Hoque (2017) developed a joint lot-sizing model for a three-echelon agricultural supply chain, considering collaboration between the chain members to make more profit. Jaber et al. (2019) formulated an EOQ problem with price-dependent demand considering buyer's investment in vendor's process improvement.

Some reviews on sustainable systems have been published. Andriolo et al. (2013) discussed on a "Sustainable Inventory Management Framework" that identifies associated sub-problems, decision variables, and the sources of sustainable achievement. They also explained that material transportation and waste have a major role in environmental sustainability. Survey of Andriolo et al. (2014) indicates that sustainable inventory and production models are interested increasingly. Dubey et al. (2016) predicted a mail role for inventory systems area in further reasarch on sustainable supply chain management. Lukman et al. (2016) reviewed on principles, approaches and sub-systems of sustainable consumption and production (SCP) area with real examples of sustainability-oriented companies.

Some works are accomplished on operations management decision making problems considering social sustainability factors. Castellini et al. (2012) used a MCDA method for sustainability assessment of a production system, considering safety of the workers as main criteria of social sustainability. Bouchery et al. (2012) taking into account injuries of inventory ordering and holding for formulation of sustainable inventory models. Jaber et al. (2017) consider worker's stress as a dimension of social sustainability to develop an economic manufacturing quantity model. Sierra et al. (2017) proposed a 5-stages procedure for project selection regarding social sustainability issues. They considered some social indices such as employment, health, safe environment and etc. for the short term and long term analysis.

Some papers are published related to pricing in supply chains. González-Ramírez et al. (2011) proposed a heuristic method to solve a joint pricing and lot-sizing problem in a multi-item multi-period inventory system. Shafiee-Gol et al. (2016) formulated a new model to make both pricing and production decisions in a multi-item production system with rework and discrete delivery. Bajwa et al. (2016) developed a multi-item model to determine optimal lot sizes and prices of products with production capacity constraint and dependent demand. Teksan and Geunes (2016) developed a new model for joint pricing and EOQ problem where both demand and supply of a product are depended on price. Xu and Wang (2017) investigated a two-echelon supply chain with dominant manufacturer taking into account emission reduction to find optimal price when demand is price-dependent. Franchetti et al. (2017) proposed that the green or sustainable supply chain members can jointly determine optimal pricing decisions with a game theoretical approach. The retailer and supplier, as the main members of the chain, can determine the retail price and the wholesale price based on their revenue functions, respectively. Taleizadeh et al. (2017b) developed a new model for joint pricing and alliance strategy selection in a two-echelon supply chain with dominant retailer considering product return policy and game theory approach. Feng and Chan (2018) used learning curves to set optimal price and production levels for new products of a company in order to maximizing profit. Rogetzer et al. (2019) proposed a new framework for sustainable procurement of recycled materials with capacity reserve considering uncertain demand and price. Taleizadeh et al. (2019) proposed a joint pricing and discount decision model to determine price of products and discounts for returned used products in a sustainable closed loop supply chain. Mishra et al. (2019) developed a new model for re-manufactured product to find optimal inventory solution, price and NPV of cash flow under deterioration. Wang et al. (2019) developed pricing models for different members of a two-echelon sustainable closed-loop supply chain considering production capacity constraint using different game models. Wang et al. (2020) designed a supply chain network which both chains compete on retail price and carbon emission. Their study shows that in competition between two chains, emission is minimized, capacity utilization is optimized and total costs are minimized. Dye (2020) modeled a joint inventory control, pricing and advertising problem for a single firm with perishable products. Assarzadegan and Rasti-Barzoki (2020) solved a pricing problem in a Closed Loop Supply Chain with a manufacturer and two retailers considering Money full/partial refund guarantee policy for returned defective/non-defective products. Also, Ventura et al. (2021) proposed a framework to determine optimum supplier selection, order quantity and price considering production constraint in a supply chain with a buyer and multiple suppliers.

3- Problem description

Most of previous papers, did not assume social issues in formulation of sustainable lot-sizing models. Also, sustainable pricing is not attended by previous works, especially in combination with joint sustainable EOQ/ EPQ problem in the supply chain context. The contribution of this paper is integrating sustainable pricing and sustainable lot-sizing (EOQ and EPQ) decisions of a two-echelon supply chain in a model considering all dimensions of sustainability by applying 5 economic (ordering, production setup, purchasing, production and inventory holding), 3 environmental (production emissions, obsolescence emissions and inventory holding emissions) and 5 social factors (social costs of ordering, inventory holding, setup, transportation and work stress) with direct accounting approach.

The supply chain which is considered in this research has been composed with two echelons. In the top tier, the retailer sales *D* units of the product to the customers in a year and customer pays *s* dollars per unit to the retailer. Based on yearly demand of the customers, retailer determines Sustainable Economic Order Quantity (*Q*) to order in each time to the supplier and the supplier earns *w* dollars per unit for satisfying retailer's needs. In the bottom tier, the supplier determines Sustainable Economic Production Quantity (*q*) to order to his/ her production system that each product unit costs c_p dollars for him/her. Figure 1 demonstrates the relationships between supply chain members.

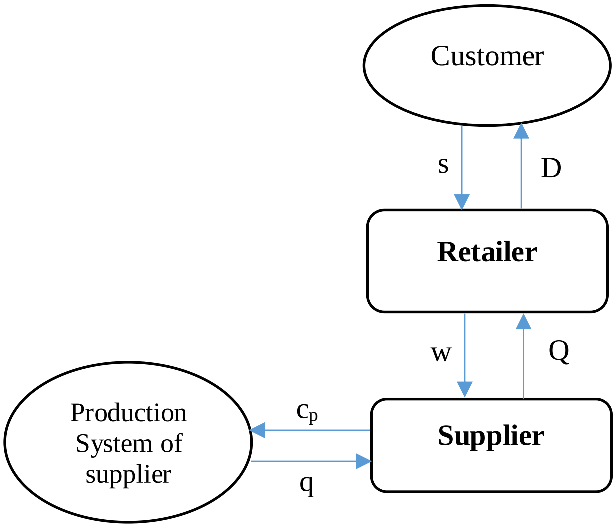

Fig 1. Relations between supply chain members

In the next section, we formulated the mathematical model of each member of the supply chain to determine optimum values of sustainable EOQ, sustainable EPQ, whole sale price and retail price as decision variables of the problem.

4- Mathematical formulation of the problem

The mathematical model of the problem is formulated in this section. At the first step, the notation of the problem is defined.

4-1- Notations

The notations that we used to develop the mathematical model are as below:

<u>*Parameters*</u>

d: Demand of product (quantity per year)
p: Production capacity (quantity per year)
s': Scrap value of product for the retailer ($/unit)
w': Scrap value of product for the supplier ($/unit)
c_p: Production cost of product ($/unit)
c_o: Ordering cost of product i for retailer ($/order)
c_s: Cost of production setup for product i ($/setup)
c_h: Cost of Inventory holding of product i for the retailer ($/unit)

c'_h: Cost of Inventory holding of product i for the supplier ($/unit)
c_{st}: Social cost of one-hour transportation ($/ hour)
a: Weight of an obsolete inventory (tons/ unit)
b: Occupied space of a unit of product in the warehouse (cubic meters/ unit)
α: Rate of inventory obsolescence of product i (percent)
c_{eh}: Average emission cost of inventory holding ($/cubic meters)
c_{eo}: Average emission cost of inventory obsolescence ($/ton)
c_{ep}: Average emission cost of production for a product unit ($/unit)
c_{sh}: Inventory holding social cost of a product ($/ unit)
c_{so}: Ordering social cost of an order ($/ order)
c_{ss}: Social cost of production setup ($/ setup)
t_1: Average duration to transport products from the supplier to the retailer (hours)
t_2: Average duration to transport products from the retailer to the customers (hours)
ω: The coefficient of social cost of work stress (percent)
u: Profit margin of one unit of product for the retailer
F: Average capacity of transportation vehicles of retailer (tons/ vehicle)
f: Average capacity of transportation vehicles of supplier (tons/ vehicle)

Decision Variables
Q_i: Sustainable economic order quantity (SEOQ) that could be ordered by the retailer (unit)
q_i: Sustainable economic production quantity (SEPQ) that could be produced by the supplier (unit)
s: Retail price of product ($/unit)
w: Wholesale price of product ($/unit)

Dependent Variables and functions
SF_r: Sales function of retailer ($/year) = sd
SF_s: Sales function of supplier ($/year) = wd
CF_{rp}: Purchasing cost function of retailer ($/year) = wd
CF_{sp}: Production cost function of supplier ($/year) = $c_p d$
CF_{ro}: Ordering cost function of retailer ($/year) = $\frac{c_o d}{Q}$
CF_{si}: Setup cost function of supplier ($/year) = $\frac{c_s d}{q}$
CF_{rh}: Cost function of inventory holding of retailer ($/year) = $\frac{c_h Q}{2}$
CF_{shi}: Cost function of inventory holding of supplier ($/year) = $c'_h \frac{q}{2}(1-\frac{d}{p})$
CF_{rob}: Inventory obsolescence cost function of retailer ($/year) = $\alpha(s-s')\frac{Q}{2}$
CF_{sob}: Inventory obsolescence cost function of supplier ($/year) = $\alpha(w-w')\frac{q}{2}(1-\frac{d}{p})$
CF_{sep}: Cost function of supplier production emission ($/year) = $c_{ep} d$

CF_{reh}: Cost function of inventory holding emission of retailer (\$/year) = $\frac{c_{eh}bQ}{2}$

CF_{seh}: Cost function of inventory holding emission of supplier (\$/year) = $c_{eh}\frac{bq}{2}(1-\frac{d}{p})$

CF_{reo}: Emission of Inventory obsolescence cost function of retailer (\$/year) = $\alpha ac_{eo}\frac{Q}{2}$

CF_{seo}: Emission of Inventory obsolescence cost function of supplier (\$/year) = $\alpha ac_{eo}\frac{q}{2}(1-\frac{d}{p})$

CF_{rso}: Social cost function of ordering for retailer (\$/year) = $\frac{c_{so}d}{Q}$

CF_{sss}: Social cost function of supplier production setup (\$/year) = $\frac{c_{ss}d}{q}$

CF_{rsh}: Social cost function of inventory holding of retailer (\$/year) = $\frac{c_{sh}Q}{2}$

CF_{ssh}: Social cost function of inventory holding of supplier (\$/year) = $c_{sh}\frac{q}{2}(1-\frac{d}{p})$

CF_{sws}: Social cost function of work stress for product i (\$/year) = $\omega\frac{c_p}{q}$

CF_{rst}: Social cost function of transportation for the retailer (\$/year) = $2c_{st}t_2\frac{d}{F}$

CF_{sst}: Social cost function of transportation for the supplier (\$/year) = $2c_{st}t_1\frac{d}{f}$

TP_r: Total profit function of the retailer (\$/ year)
TP_s: Total profit function of the supplier (\$/ year)

4-2- Model development for the retailer

The demand of product can be defined as below:

$$d = 1 - s \tag{1}$$

Total profit function of retailer (TP_R) can be written as follows:

$$TP_r = SF_r - CF_{rp} - CF_{ro} - CF_{rh} - CF_{rob} - CF_{reh} - CF_{reo} - CF_{rso} - CF_{rsh} - CF_{rst} \tag{2}$$

Then by substituting cost function formulas from notation section to equation (2) we have

$$TP_r = (1-s)s - w(1-s) - \frac{c_o(1-s)}{Q} - \frac{c_hQ}{2} - \frac{bc_{eh}Q}{2} - \alpha(s-s')\frac{Q}{2} - \alpha ac_{eo}\frac{Q}{2} - c_{sh}\frac{Q}{2} - \frac{c_{so}(1-s)}{Q} \\ -2c_{st}t_2\frac{(1-s)}{F} \tag{3}$$

Because TP$_r$ is concave on Q and s (see *Appendix*), by taking first derivative of TP$_R$ respect to Q and set it equal to zero we get

$$\frac{\partial TP_r}{\partial Q}=\frac{c_o(1-s)}{Q^2}-\frac{c_h}{2}-\frac{bc_{eh}}{2}-\frac{\alpha(s-s')}{2}-\frac{\alpha ac_{eo}}{2}-\frac{c_{sh}}{2}+\frac{c_{so}(1-s)}{Q^2}=0 \tag{4}$$

Also by taking first derivative of TP$_r$ respect to s and set it equal to zero we get

$$\frac{\partial TP_r}{\partial s}=1-2s+w+\frac{c_o+c_{so}}{Q}-\alpha\frac{Q}{2}+\frac{2c_{st}t_2}{F}=0 \tag{5}$$

Then

$$s=\frac{1}{2}+\frac{w}{2}+\frac{c_o}{2Q}-\alpha\frac{Q}{4}+\frac{c_{so}}{2Q}+\frac{c_{st}t_2}{F} \tag{6}$$

By substituting equation (6) into equation (4) we get

$$(\frac{c_o+c_{so}}{Q^2})(\frac{1}{2}-\frac{w}{2}-\frac{c_o}{2Q}+\frac{\alpha Q}{4}-\frac{c_{so}}{2Q}-\frac{c_{st}t_2}{F})-\frac{\alpha}{2}(\frac{1}{2}+\frac{w}{2}+\frac{c_o}{2Q}-\frac{\alpha Q}{4}+\frac{c_{so}}{2Q}+\frac{c_{st}t_2}{F})=\lambda \tag{7}$$

That we consider

$$\lambda=c_h+bc_{eh}+\alpha ac_{eo}+c_{sh}-\alpha s' \tag{8}$$

Then from (7) we have

$$\begin{aligned}&2(c_o+c_{so})(2QF-2QFw-2Fc_o+\alpha FQ^2-2Fc_{so}-4Qc_{st}t_2)-\alpha(2FQ^3+2FQ^3w+2FQ^2c_o\\&-\alpha FQ^4+2Fc_{so}Q^2+4Q^3c_{st}t_2)-4F\lambda Q^3=0\end{aligned} \tag{9}$$

Finally, we can find a quadratic equation as follows

$$AQ^4+BQ^3+CQ+D=0 \tag{10}$$

Which

$$A=\alpha^2F \tag{11}$$

$$B=-2\alpha Fw-2\alpha F-4\alpha c_{st}t_2-4F\lambda \tag{12}$$

$$C=(c_o+c_{so})(4F-4Fw-8c_{st}t_2) \tag{13}$$

$$D=-4F(c_o+c_{so})^2 \tag{14}$$

We can find four roots of the Q by solving the quadratic equation (10) which has acceptable or unacceptable values.

4-3-Model development for the supplier

The total profit function of the supplier (TP_S) can be defined as bellow:

$$TP_s = SF_s - CF_{sp} - CF_{si} - CF_{sob} - CF_{sep} - CF_{seh} - CF_{shi} - CF_{seo} - CF_{sss} - CF_{ssh} - CF_{sws} - CF_{sst} \tag{15}$$

Then we have

$$\begin{aligned} TP_s = {} & (1-s)(w-c_p-c_{ep}) - \omega\frac{c_p}{q} - \frac{c_s(1-s)}{q} - \frac{c_h q}{2}(1-\frac{1-s}{p}) - \frac{bc_{eh}q}{2}(1-\frac{1-s}{p}) - \alpha(w-w')\frac{q}{2}(1-\frac{1-s}{p}) \\ & -\alpha a c_{eo}\frac{q}{2}(1-\frac{1-s}{p}) - c_{sh}\frac{q}{2}(1-\frac{1-s}{p}) - \frac{c_{ss}(1-s)}{q} - 2c_{st}t_1\frac{(1-s)}{f} \end{aligned} \tag{16}$$

We know that

$$s = w + u \tag{17}$$

If we substitute equation (17) into equation (16) we get

$$\begin{aligned} TP_s = {} & (1-w-u)(w-c_p-c_{ep}) - \omega\frac{c_p}{q} - \frac{c_s(1-w-u)}{q} - \frac{c'_h q}{2}(1-\frac{1-w-u}{p}) - \frac{bc_{eh}q}{2}(1-\frac{1-w-u}{p}) \\ & -\alpha(w-w')\frac{q}{2}(1-\frac{1-w-u}{p}) - \alpha a c_{eo}\frac{q}{2}(1-\frac{1-w-u}{p}) - c_{sh}\frac{q}{2}(1-\frac{1-w-u}{p}) - \frac{c_{ss}(1-w-u)}{q} - 2c_{st}t_1\frac{(1-w-u)}{f} \end{aligned} \tag{18}$$

Considering

$$\lambda' = c'_h + \alpha a c_{eo} + bc_{eh} + c_{sh} \tag{19}$$

We can rewrite equation (18) as below:

$$\begin{aligned} TP_s = {} & (1-w-u)(w-c_p-c_{ep}) - \omega\frac{c_p}{q} - \frac{(c_s+c_{ss})(1-w-u)}{q} - \frac{\lambda' q}{2}(\frac{p+w+u-1}{p}) - \\ & \alpha(w-w')\frac{q}{2}(\frac{p+w+u-1}{p}) - 2c_{st}t_1\frac{(1-w-u)}{f} \end{aligned} \tag{20}$$

Because TP_s is concave on q and w (see *Appendix*), by taking partial derivative of TP_S into q we have:

$$\frac{\partial TP_s}{\partial q} = \frac{1}{q^2}\left[\omega c_p + (c_s + c_{ss})(1-w-u)\right] - \alpha(w-w')(\frac{p+w+u-1}{2p}) - \lambda'(\frac{p+w+u-1}{2p}) \tag{21}$$

Setting equation (21) equal to zero we can find q value as below:

$$q = \sqrt{\frac{2p\left[\omega c_p + (c_s+c_{ss})(1-w-u)\right]}{(p+w+u-1)\left[\lambda' + \alpha(w-w')\right]}} \tag{22}$$

By taking partial derivative of TP_S into w and setting it equal to zero, we have:

$$\frac{\partial TP_s}{\partial w} = c_p + c_{ep} + 1 - 2w - u + \frac{c_s}{q} - \frac{\alpha q}{2p}(p + w + u - 1) - \alpha(w - w')\frac{q}{2p} - \frac{q\lambda'}{2p} + \frac{c_{ss}}{q} + \frac{2c_{st}t_1}{f} = 0 \tag{23}$$

$$w = \frac{c_p + c_{ep} + 1 - u - \dfrac{\alpha q(p + u + w' - 1)}{2p} - \dfrac{q\lambda'}{2p} + \dfrac{c_s + c_{ss}}{q} + \dfrac{2c_{st}t_1}{f}}{2 + \dfrac{\alpha q}{p}} \tag{24}$$

Substituting equation (24) into equation (22) we have

$$\begin{aligned} &q^2(A' + B'q + C'p + D'p/q - pqE - \lambda'q/2)\begin{pmatrix} (\lambda' - \alpha w')(2p + \alpha q) + \alpha pC' + \alpha p(c_s + c_{ss})/q \\ -\alpha^2 q(p + u - w' - 1)/2 - \alpha\lambda'q/2 \end{pmatrix} \\ &= (2p + \alpha q)^2 G - (2p + \alpha q)H(C' + D'/q - qE) \end{aligned} \tag{25}$$

Then

$$\begin{aligned} &(A'q^2 + B'q^3 + c_p q^2 - PEq^3 - q^3\lambda'/2 + D'Pq)(N + Kq + L/q - qM) \\ &= 4P^2G + \alpha^2 q^2 G + 4P\alpha qG - 2PHC' - 2PHD' + 2PHEq - \alpha HC'q - HD'\alpha + HE\alpha q^2 \end{aligned} \tag{26}$$

After some algebra we get

$$\begin{aligned} &\left[N(B' - PE - \lambda'/2) + (K - M)(A' + C'P)\right]q^4 + \left[N(A' + C'P) + D'P(K - M) + L(B' - PE - \lambda'/2) - \alpha^2 G - HE\alpha\right] \\ &q^3 + \left[ND'P + L(A' + C'P) - 4P\alpha G - 2PHE + \alpha HC'\right]q^2 + (LD'P - 4P^2G + 2PHC' + HD'\alpha)q + 2PHD' = 0 \end{aligned} \tag{27}$$

And then we can rewrite this equation as follows

$$A''q^4 + B''q^3 + C''q^2 + D''q + E' = 0 \tag{28}$$

That is a quadratic equation where:

$$A'' = N(B' - PE - \lambda'/2) + (K - M)(A' + C'P) \tag{29}$$

$$B'' = N(A' + C'P) + D'P(K - M) + L(B' - PE - \lambda'/2) - \alpha^2 G - HE\alpha \tag{30}$$

$$C'' = ND'P + L(A' + C'P) - 4P\alpha G - 2PHE + \alpha HC' \tag{31}$$

$$D'' = LD'P - 4P^2G + 2PHC' + HD'\alpha \tag{32}$$

$$E' = 2PHD' \tag{33}$$

$$A' = 2P(P + u - 1) \tag{34}$$

$$B' = \alpha(P + u - 1) \tag{35}$$

$$C' = 1 - u + c_p + c_{ep} + 2c_{st}t_1 / f \tag{36}$$

$$D' = c_s + c_{ss} \tag{37}$$

$$E = \frac{\alpha(P + u - w' - 1) - \lambda'}{2P} \tag{38}$$

$$G = 2P\omega c_p + 2P(c_s + c_{ss})(1 - u) \tag{39}$$

$$H = 2P^2(c_s + c_{ss}) \tag{40}$$

$$K = \alpha(\lambda' - \alpha w') \tag{41}$$

$$L = \alpha P(c_s + c_{ss}) \tag{42}$$

$$M = \frac{\alpha^2(p + u - w' - 1) + \alpha\lambda'}{2} \tag{43}$$

$$N = 2P(\lambda' - \alpha w') + \alpha CP \tag{44}$$

4-4- Procedure of problem solving

Steps of the procedure are proposed as follows:

Step 1. Calculate q value from solving equation(28). Four different roots may be get by solving that quadratic equation.

Step 2. Insert acceptable results of q value which calculated in step 1 in equation (24) and find the optimal value of w. value of w must be greater than zero and greater than c_p value.

Step 3. Calculate Q value from solving equation(10). Four different roots may be get by solving that quadratic equation.

Step 4. Calculate s value from equation(17).

Step 5. Calculate TP_r and TP_s from equation (3) and equation(20), respectively.

5- Numerical example

In this section, a numerical example is conducted, based on the real data of an Iranian Petrochemical two-echelon supply chain as our case study. Most of the parameters of this numerical example are obtained from our case study. For determining a few parameters of the problem which is not determined by the supply chain members of our case study, we used the values of parameters in previous related researches (Bouchery et al., 2012; Battini et al., 2014; Jaber et al., 2016). Parameters values of the problem are presented in table 1.

Table 1. Parameters values of the problem

Parameter	Value	Unit	Parameter	Value	Unit
p	100000	Tons per year	C_{ss}	0.17	Thousands $ per setup
d	30000	Tons per year	C_{st}	0.001	Thousands $ per hour
s'	0.05	Thousands $ per ton	a	1	tons
C_o	0.2	Thousands $ per order	b	0.7	cubic meters/ ton
C_h	0.003	Thousands $ per ton	α	1	percent
C_h'	0.002	Thousands $ per ton	t_1	1.25	hours
C_{eh}	0.0006	Thousands $ per cubic meters	t_2	3	hours
C_{eo}	0.06	Thousands $ per ton	F	4	Tons per vehicle
C_{sh}	0.0003	Thousands $ per ton	f	4	Tons per vehicle
C_{so}	0.12	Thousands $ per order	ω	10	percent
C_p	0.20	Thousands $ per ton	w'	0.05	Thousands $ per ton
C_{ep}	0.0168	Thousands $ per ton	u	0.1	Thousands $ per ton
C_s	0.75	Thousands $ per setup			

This supply chain is composed of two members: a supplier/producer (a petrochemical company) and a retailer (a commercial company), who buys only a determined type of product from the supplier and sell it to the various types of the customers. Based on the procedure of problem solving that presented in 4-4 section, we solved the numerical example step by step as follows.

Step 1. Equation (28) can be formed as $A''q^4 + B''q^3 + C''q^2 + D''q + E' = 0$ which this equation coefficients and other dependent variable values are calculated from equation (19) and equations (29) to (44), that these values are presented in table 2.

Table 2. Values of dependent variables of supplier EPQ-Pricing problem

Dependent Variable	Value
A''	$-1*10^{11}$
B''	$3.36*10^{13}$
C''	$5.17*10^{8}$
D''	$3.95*10^{15}$
E'	$3.39*10^{15}$
A'	$2*10^{10}$
B'	999.99
C'	1.12
D'	0.92
E	0.005
G	4001.66
H	$1.84*10^{10}$
K	0.00003
L	920.00
M	5.00
N	1681.43
λ'	0.003

By solving equation (28) that can rewrite as below:

$$-1*10^{11}q^4+3.36*10^{13}q^3+5.17*10^{8}q^2+3.95*10^{15}q+3.39*10^{15}=0 \tag{45}$$

We get sustainable EPQ value or q=0.251.

Step 2. Substituting q=0.251 in equation (24) we get optimal whole sale price or w=2.392.

Step 3. Equation (10) can be formed as $AQ^4+BQ^3+CQ+D=0$ which this equation's coefficients are calculated and shown in table 3. By solving this equation we get sustainable EOQ value or Q=0.102.

Table 3. Values of dependent variables of retailer EOQ-Pricing problem

Dependent Variable	Value
A	0.0004
B	-0.33237
C	-7.12
D	-1.6384

Step 4. From equation (17) we get optimum retail price or s=2.492 for the retailer.

Step 5. Total profit amount of the retailer and the supplier (TP_r and TP_s) can be calculated from equation (3) and equation (20) and their optimum values are TP_r=4.546 and TP_s=2.147.

6-Results and discussions

The final results of numerical example are demonstrated in figure 2. As can be seen in figure 2, total profit value of retailer (TP_r=4.546) is 112% more than total profit value of supplier (TP_s=2.147). Optimum retail price (s=2.492) is 4% more than optimal whole sale price (w=2.392). Also, the supplier sustainable EPQ value (q=0.251) is 146% more than the retailer sustainable EOQ value (Q=0.102).

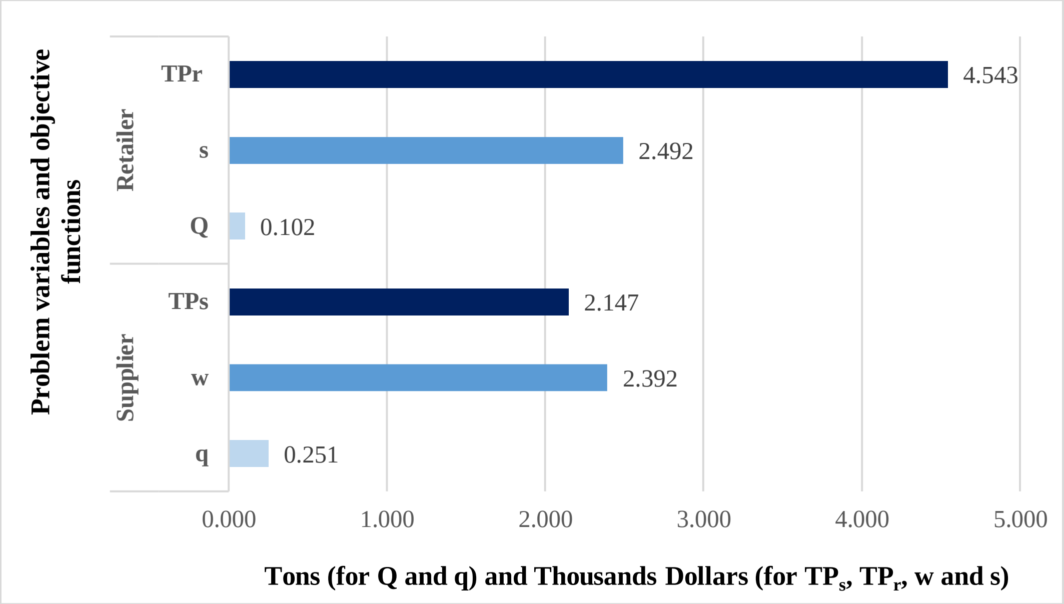

Fig 2. Results of numerical example for each member of supply chain

For better analyzing of numerical example results we performed a sensitivity analysis on two key parameters of the problem: (1) production capacity of the supplier (p) and (2) profit margin of the retailer (u). Results of sensitivity analysis are presented in table 4.

Table 4. Results of sensitivity analysis on p and u parameters (Tons for Q and q and Thousands Dollars for TP_s, TP_r, w and s)

	Parameter	Variation Percent	q	w	TP_s	Q	s	TP_r
Values	p	+50	0.251	2.392	2.147	0.102	2.492	4.546
		+25	0.251	2.392	2.147	0.102	2.492	4.546
		-25	0.251	2.392	2.147	0.102	2.492	4.546
		-50	0.251	2.392	2.147	0.102	2.492	4.546
	u	+50	0.269	2.243	1.867	0.116	2.393	3.64
		+25	0.26	2.317	2.003	0.108	2.442	4.076
		-25	0.242	2.468	2.299	0.096	2.543	5.05
		-50	0.234	2.545	2.46	0.09	2.595	5.592
Percent	p	+50	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		+25	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		-25	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		-50	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	u	+50	7.17%	-6.23%	-13.04%	13.73%	-3.97%	-19.93%
		+25	3.59%	-3.14%	-6.71%	5.88%	-2.01%	-10.34%
		-25	-3.59%	3.18%	7.08%	-5.88%	2.05%	11.09%
		-50	-6.77%	6.40%	14.58%	-11.76%	4.13%	23.01%

As shown in table 4, no variables of the problem (q, Q, w and s) are sensitive to supplier production capacity (p). Also, total profit amount of both retailer and supplier (TP_r and TP_s) are not sensitive to supplier production capacity (p) variation. But, variables of the problem are sensitive to variation of profit margin of retailer. Economic Production Quantity (q), changes from -6.77% to +7.17% by variation of profit margin of retailer (u) from -50% to +50%, respectively. Also, Economic Order quantity (Q) value changes from -11.76% to +13.73% by variation of profit margin of retailer (u) from -50% to +50%, respectively. Whole sale price of supplier (w) changes from +6.40% to -6.23% by variation of profit margin of retailer (u) from -50% to +50%, respectively. Retail price (s), changes from +4.13% to -3.97% by variation of profit margin of retailer (u) from -50% to +50%, respectively. Total profit value of supplier (TP_s) changes from -13.04% to +14.58% by 50% variation of profit margin of retailer (u). Total profit value of retailer (TP_r) changes from -19.93% to +23.01% by 50% variation of profit margin of retailer (u). The results show that Q variable is more sensitive to u parameter variation than other variables of the problem.

7- Managerial insight

In this section we presented some managerial insights of this research. Our proposed model is developed based on real conditions of modern industry that they are listed as follows:

1) In this work, we linked a *sustainable pricing* problem to a *sustainable EOQ/ EPQ* problem and formulated this model for a two-echelon supply chain (consist of a retailer and a supplier) to find optimal values of sustainable retail price and sustainable EOQ for the retailer and sustainable whole sale price and sustainable EPQ considering economic, environmental and social issues. This helps operations and supply chain managers in each tier of the chain to find the best answers to these key questions: *"How much should we order?"* and *"At what price should we sell?"* considering different economic, environmental and social factors of sustainability.

2) Our new joint sustainable pricing/ EOQ/ EPQ model is developed based on different functions that covered all three dimensions of sustainability (five economic, three environmental and five social functions

that are related to different sustainability aspects). Therefore, this model is s really sustainable model (and not only a green model) that can be used and customized by other researchers and operations managers as a basic sustainable model to solve a joint EOQ/ EPQ/ pricing problem.
3) In this research we proposed new sustainable joint pricing/ EOQ/ EPQ models to be applied in a two-echelon supply chain. If sustainability issue considered in all tiers of the supply chain as a chain management policy, all of members (the retailer and the supplier) can use these models, but if only a member wants to assume the sustainability issue, the retailer or the supplier can use our proposed model, independently.
4) Our developed models are applied in a petrochemical supply chain, but these models can be applied to many two-echelon supply chains in other businesses and industries by setting the parameters of the problem, accurately. Also, these models are easy to use for supply chain managers because of applying *direct accounting approach* in the process of mathematical formulation and proposing a careful *problem solving procedure*.

8- Conclusions

In this work, two different mathematical models for each member of a two-echelon supply chain is formulated and sustainable EOQ/EPQ and optimum whole sale / retail prices are calculated. The two-echelon supply chain consists of a retailer and a supplier with his production system. For each member of the supply chain, Mathematical profit function containing revenue function and different cost functions considering different aspects of sustainability such as economic, environmental and social parameters is formulated and for each member of the chain, optimal lot-sizing (Sustainable EOQ or Sustainable EPQ) and pricing (Retail price or Whole sale price) decisions are made. Also, a new integrated procedure for problem solving and determining the optimal solution is developed and proposed.

The contribution of this paper is integrating sustainable pricing and sustainable lot-sizing (EOQ and EPQ) decisions of a two-echelon supply chain in a model considering all dimensions of sustainability by applying 5 economic (ordering, production setup, purchasing, production and inventory holding), 3 environmental (production emissions, obsolescence emissions and inventory holding emissions) and 5 social factors (social costs of ordering, inventory holding, setup, transportation and work stress) with direct accounting approach.

We conducted a numerical example based on the real data of an Iranian petrochemical two-echelon supply chain. We found optimum sustainable retail price and EOQ for a commercial company (we called him “retailer” in this model) and optimal whole sale price and EPQ for a petrochemical unit (we called him “supplier” in this model) considering environmental and social responsibilities of the supply chain in addition to economic performance. Finally, for better analyzing of numerical example results we performed a sensitivity analysis on capacity constraint of the supplier (producer) and profit margin of the retailer. The results show that in this case, the problem is not sensitive to production capacity variation, but the results are so sensitive to profit margin variation of the retailer. We can propose some research titles for further research: (1) This problem can be modeled the uncertainty of the parameters (specially, sustainability parameters) using fuzzy numbers and determine optimal results using fuzzy logic. (2) The supply chain members can be more than two members. In this case, For example two different suppliers can be compete with each other or ally with the retailer to compete against another one that the optimum solution and alliance strategy can be determined using game theory. (3) Products of the supply chain can be more than one and the problem can be redesign with multi product approach.

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Appendix

1) Concavity proof of total profit function of retailer (*TP*$_r$):

$$\begin{pmatrix} \frac{\partial^2 TP_r}{\partial Q^2} & \frac{\partial^2 TP_r}{\partial s \partial Q} \\ \frac{\partial^2 TP_r}{\partial Q \partial s} & \frac{\partial^2 TP_r}{\partial s^2} \end{pmatrix} \begin{pmatrix} Q \\ \\ s \end{pmatrix} = \begin{pmatrix} \frac{-2(1-s)(c_o+c_{so})}{Q^3} & -\frac{c_o+c_{so}}{Q^2}-\frac{\alpha}{2} \\ -\frac{c_o+c_{so}}{Q^2}-\frac{\alpha}{2} & -2 \end{pmatrix} \begin{pmatrix} Q \\ \\ s \end{pmatrix}$$

$$= \frac{4(1-s)(c_o+c_{so})}{Q^3} - \left(\frac{c_o+c_{so}}{Q^2} + \frac{\alpha}{2} \right)^2 \ge 0 \tag{1-A}$$

To meet concavity condition of TP$_r$, the relation (1-A) must be met (established). Also, other concavity conditions are always met as follows:

$$\frac{\partial^2 TP_r}{\partial Q^2} = \frac{-2(1-s)(c_o+c_{so})}{Q^3} \le 0 \tag{2-A}$$

$$\frac{\partial^2 TP_r}{\partial s \partial Q} = \frac{\partial^2 TP_r}{\partial Q \partial s} = -\frac{c_o+c_{so}}{Q^2} - \frac{\alpha}{2} \le 0 \tag{3-A}$$

$$\frac{\partial^2 TP_r}{\partial s^2} = -2 \le 0 \tag{4-A}$$

2) Concavity proof of total profit function of supplier (*TP*$_S$):

$$\begin{pmatrix} \frac{\partial^2 TP_S}{\partial q^2} & \frac{\partial^2 TP_S}{\partial w \partial q} \\ \frac{\partial^2 TP_S}{\partial q \partial w} & \frac{\partial^2 TP_S}{\partial w^2} \end{pmatrix} \begin{pmatrix} q \\ \\ w \end{pmatrix} =$$

$$\begin{pmatrix} \frac{-2\left[\omega c_p + (1-w-u)(c_s+c_{ss})\right]}{q^3} & -\frac{c_s+c_{ss}}{q^2} - \frac{\alpha}{2p}(p+2w+u-1-w') - \frac{\lambda'}{2p} \\ -\frac{c_s+c_{ss}}{q^2} - \frac{\alpha}{2p}(p+2w+u-1-w') - \frac{\lambda'}{2p} & -2-\frac{\alpha q}{p} \end{pmatrix} \begin{pmatrix} q \\ \\ w \end{pmatrix} \tag{5-A}$$

$$= \frac{2\left[\omega c_p + (1-w-u)(c_s+c_{ss})\right]}{q^3}\left(2+\frac{\alpha q}{p}\right) - \left(\frac{c_s+c_{ss}}{q^2} + \frac{\alpha}{2p}(p+2w+u-1-w') + \frac{\lambda'}{2p} \right)^2 \ge 0$$

Three other relations that must be fulfilled (met/ observed) are as follows:

$$\frac{\partial^2 TP_s}{\partial q^2} = \frac{-2\left[\omega c_p + (1-w-u)(c_s + c_{ss})\right]}{q^3} \leq 0 \tag{6-A}$$

$$\frac{\partial^2 TP_s}{\partial q \partial w} = \frac{\partial^2 TP_s}{\partial w \partial q} = -\frac{c_s + c_{ss}}{q^2} - \frac{\alpha}{2p}(p + 2w + u - 1 - w') - \frac{\lambda'}{2p} \leq 0 \tag{7-A}$$

$$\frac{\partial^2 TP_s}{\partial w^2} = -2 - \frac{\alpha q}{p} \leq 0 \tag{8-A}$$