

Coordinating Pricing and Ordering Decisions in a Multi-Echelon Pharmacological Supply Chain under Different Market Power using Game Theory

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Abstract

The importance of supply chains in pharmacological industry is remarkable so that nowadays many pharmacological supply chains have an effective and critical role for supplying and distributing drugs in health area. So, this article studies a three-echelon pharmacological supply chain containing multi-distributor of raw materials, a pharmaceutical factory, and multi-drug distributors companies. The distributors of raw material order raw materials of some drugs to own suppliers and sell them to the pharmaceutical factory. The factory transmutes raw materials to the several finished products and sells them to some drug distributors companies. There are several types of raw materials and finished products. Here, it is supposed that the market powers of partners are different. So, the Stackelberg game among the members of the chain is deemed to analyze the coordination behavior of the members of the proposed chain. The aim of the research is to maximize the total profit of supply chain by employing the optimal pricing and ordering decision policies where the order quantities of the distributors and the selling prices of pharmaceutical factory (manufacturer) and the distributors are the decision variables. Besides, the closed form solutions of the decision variables are presented. At the end, numerical example and some sensitivity analysis are presented.

Keywords: Pricing; Inventory; Production; Health Care; Game theory; Supply Chain.

1- Introduction and literature review

Many industries focus on pricing as a marketing tool to improve their profits. In addition, for production industries, coordination of price decisions with other aspects of the supply chain such as manufacturing and inventory, are useful and essential. The coordination of these decisions needs an integrated model to optimize the integrated system rather than individual components of the supply chain. This integration between manufacturing, pricing, and inventory decisions is still in its early stages in many firms. But recently, many researchers have focused on this topic and even each component of mentioned integrated system, separately. For instance, Lee and Rosenblatt (1986) introduced the problem of offering price discount and pricing, at the same time. Boyaci and Gallego (2002) studied coordination cases under deterministic demand which is price sensitive in a two echelons supply chain containing one wholesaler and one or more retailers and they considered joint lot sizing and pricing decisions for optimizing their model.

Abad (2003) analyzed pricing and lot sizing policies for a perishable item when partial backordering shortage is allowed. Dai et al. (2005) discussed the pricing policy for multi-competing firm supporting the same service for the customers where each firm's demand is price sensitive.

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Rosenthal (2008) evaluated the problem of setting prices in an integrated supply chain. Szmerkovsky and Zhang (2009) investigated two tiers advertising levels and pricing decisions between one manufacturer and one retailer. In their model end user's demand depends on the retail price and manufacturer and retailer advertisement costs.

Xiao et al. (2010) utilized wholesale pricing, ordering and lead time decisions in a three echelons supply chain where the manufacturer produces deteriorating items. Cai et al. (2009) surveyed pricing and ordering models with partial lost sales in a supply chain with one buyer and one seller. Soon (2011) reviewed multiple products pricing models where pricing is combined with other decision policies like distribution of resources or production. Seyed-Esfahani et al. (2011) studied vertical cooperative advertising with pricing decision policy in a supply chain involving one manufacturer and one retailer. Hua et al. (2012) analyzed the optimal pricing and ordering policies where the supplier suggests free shipping. Mutlu and Cetinkaya (2013) designed a carrier-retailer channel under both centralized and decentralized structures with price-sensitive demand to examine and also compare the profitability of chain under both structures. Giri and Sharma (2014) employed a pricing strategy in a two-echelon supply chain involving a manufacturer and two competing retailers where demand depends on advertising cost. Taleizadeh and Noori-daryan (2015a) proposed an economic production quantity (EPQ) model in a three-echelon supply chain composed of a supplier, a manufacturer and a group of retailers to study the reaction of the chain members. Their proposed model determines the optimal pricing and inventory policies to promote the profit of the chain. Ekici et al. (2015) considered a supply chain with two rival suppliers and one retailer under asymmetric information to evaluate the optimal pricing decisions where order consolidation strategy and order quantity constraint are considered. Another aim of each supply chain is to control its inventory level; also the members of the chain employ the optimal inventory decisions to satisfy the customers' demand. The authors considered different decision policies in their researches. For example, Whiting (1955) considered an inventory and pricing policies to optimize the objective function of an inventory system. Hill (1999) studied a problem in which the vendor supplies a product to a buyer by employing the production and shipment policies. Jorgensen and Kort (2002) proposed inventory replenishment and pricing policies in a system with serial inventories under decentralized and centralized decision making. Khouja (2003) considered a three-echelon supply chain by using three inventory decision policies. Ben-Daya et al. (2008) presented a comprehensive review of the joint economic lot sizing problem. Yu and Huang (2010) investigated the optimal inventory policies in a VMI supply chain containing a manufacturer and multi-retailers where a manufacturer receives raw material from multiple suppliers to produce a family of product in order to sell to the retailers. Ru and Wang (2010) studied a supply chain with a single period where a supplier contracts with a retailer. The demand is assumed to be uncertain and price sensitive. In their article, they compared two consignment arrangements to manage the inventory. Guan and Zhao (2011) developed an inventory-pricing model with multiple competing retailers in which each retailer faces to stochastic demand in an infinite horizon. Arkan and Hejazi (2012) offered a coordination mechanism in a two echelons supply chain containing one supplier and one buyer under uncertain demand and positive lead time. Kovacs et al. (2013) studied the principal challenge of inventory control, logistics and production under information asymmetry in supply chains. Cardenas-Barron and Sana (2014) investigated the coordination problem for a single-manufacturer-single-retailer supply chain where demand is sensitive to promotional sales teams' initiative. Two researches subsequently analyzed EPQ models in multi-echelon chains for deteriorating and reworkable items, respectively to determine optimal pricing and inventory strategies of partners of the chains performed by Taleizadeh et al. (2015a, 2015b).

Game theory approaches have been employed to model the relation among the chains member's in many researches (Giri, B.C., Sharma, S., 2014), (Taleizadeh and Noori-daryan, 2015a), (Cardenas-Barron, and Sana, 2014), (Taleizadeh, Noori-daryan and Cárdenas-Barrón, 2015a) and (Taleizadeh, Noori-daryan and Tavakkoli-Moghaddam, 2015b). But, recently, due to different market power of the partners of the chain, powerful partners do not agree to join a supply chain and collaborate with other ones as a whole supply chain with equal market share and prefer to work separately in a supply chain under a decentralized decision process. So, in these chains in which the members have different market power, non-cooperative game theoretic approaches as Nash and Stackelberg game have been established to analyze the decisions of the chains partners. To name a few, Cachon and Netessine (2004) studied a supply chain in which game theory had been used. Yu et al. (2006) employed a Stackelberg game in a VMI system where the manufacturer is leader and the multiple retailers are the followers. Nagarajan and Susic (2008) presented applications of cooperative game theory to manage supply chain. Cai et al. (2011) analyzed the influence of pricing schemes and price discount contracts on the dual-channel supply chain competition from supplier and

retailer. Zhao and Atkins (2009) evaluated a transshipment game and a substitution game between competing retailers. Huang et al. (2011) studied the coordination of enterprise decisions such as pricing and inventory in a multi-echelon supply chain where multi-supplier, one manufacturer and multiple retailers are the members of the chain and non-cooperative game between the members of the chain is used. Chen et al. (2012) presented a review of the related issues to the manufacturer's pricing policies in a two-echelon supply chain including one manufacturer and two competing retailers. They used the Stackelberg game approach such that manufacturer who is leader, offers wholesale prices to two competing retailers. Pal et al. (2012) developed a multi-product three echelons supply chain involving multi-supplier, one manufacturer and multi-retailer where each supplier supplies one kind of required raw material of the manufacturer and the manufacturer combines the fixed percentage of raw material to produce the products which should be sold to the retailers. Zhao and Wang (2015) applied three different games study pricing and retail service decisions in a single-manufacturer two-retailer supply chain under uncertainty, as manufacturer-leader Stackelberg, retailer-leader Stackelberg, and vertical Nash. Qi et al. (2015) studied game theoretic models to evaluate the impact of market search reaction of customer on the decisions of supply chain members where one manufacturer and two retailers are considered as the chain partners with considering lost sale shortage at the retailers.

In this paper, an EPQ model is developed in a multi-echelon supply chain for pharmacological raw material and products by applying optimal pricing and ordering policies. According to our review of literature, the proposed model in this article has not been considered by any of the recent researches. The novelty of the proposed model is that several raw materials distributors', a pharmaceutical factory, and some drug distributors are the members of the chain. It is assumed that demand is price-sensitive and shortage is not allowed. Also thanks to different market power of the chain members, the Stackelberg game approach is considered among the chain partners to survey and model the total profit of the chain. Moreover, the closed form solutions of the optimal values of decision variables are presented so that the concavity of the total profit functions is proven. The order quantities of the distributors and the selling prices of the pharmaceutical factory and the drug distributors are the decision variables of the presented model. The rest of this paper is arranged as follows. The problem is defined in section 2. The mathematical model is presented in section 3. Section 4 shows the solution method. The numerical example and sensitivity analysis is provided in section 5 and section 6 presents conclusion.

2- Problem Definition: A Real Case Study

Consider a three-echelon supply chain of pharmacological industry including several non-competing distributors of raw materials (distributors) of drugs such as Talk, Sodium Benzoate, Gelatin Capsule, Guar Gum, Dibasic Calcium Phosphate, Flavor, Crystal Sugar, Color, and a pharmaceutical factory (manufacturer) and some non-competing drug distributors companies (retailers). A configuration of this supply chain is shown in Figure (1).

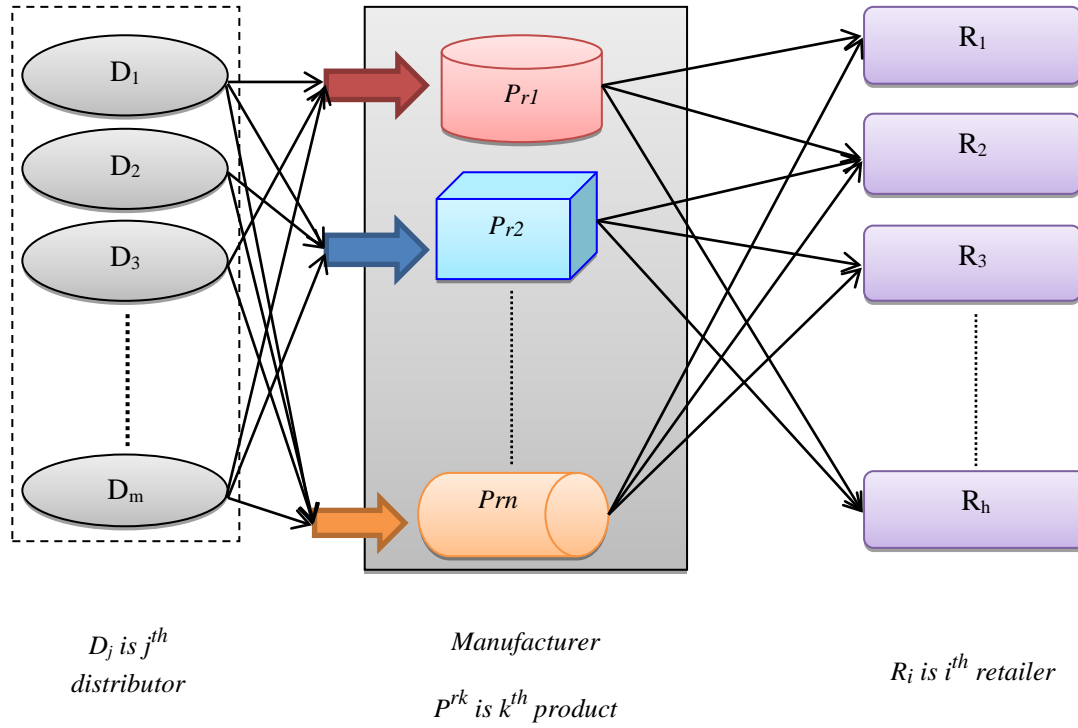


Figure1. Configuration of supply chain

In this chain, the distributors sell raw materials to the pharmaceutical factory and pharmaceutical factory transfers them to the multi-finished products such as Cefexime, Cephalexine Sodium, Amoxiclov 156, Amoxiclov 228, Amoxiclov 312, Amoxiclov 457, Co-Amoxiclov, etc., and then sends them to the drug distributors companies and they satisfy their customers' demand. We assume that each distributor supplies one kind of raw material and pharmaceutical factory needs to the combination of different kinds of raw material in a defined percentage for each product. According to customers' demand, the distributors companies' demand are different and the ordering lot size of raw material and the selling price of pharmaceutical factory and the selling price of drug distributors companies are decision variables.

The aim of this paper is to determine the optimal values of order quantity of distributors to their supplier, the selling price of pharmaceutical factory and the selling price of drug distributors companies in a three echelons supply chain such that the total profit of supply chain network is maximized.

In order to model the problem following notations are used. The number of distributors is assumed to be m and its index is $j = 1, 2, \dots, m$, the number of products is n and its index is $k = 1, 2, \dots, n$, and there are h retailers and its index is $h = 1, 2, \dots, k$.

- I_j^d The inventory level of j^{th} distributor,
- Q_j The demand of j^{th} raw material from the manufacture to the j^{th} distributors,
- h_j^d The ordering lot size of j^{th} distributor of raw material to his supplier,
- A_j^d The holding cost of j^{th} distributor of raw material,
- HC_j^d The ordering cost of j^{th} distributor of raw material,
- OC_j^d The total holding cost of j^{th} distributor of raw material,
- C_j^d The total ordering cost of j^{th} distributor of raw material,
- w_j^d The purchasing price of j^{th} distributor of raw material,
- T_j^d The selling price of j^{th} distributor of raw material,
- θ_{kj} The cycle time of j^{th} distributor of raw material,
- I_k^m The percentage of j^{th} raw material for producing k^{th} product,

	The inventory level of manufacturer for k^{th} finished product,
D_k^m	The demand of h^{th} retailer for k^{th} product, $D_{ik}^m = a - bw_k^m$
P_k	The production rate of k^{th} product,
h_k^m	The holding cost of manufacturer for k^{th} finished product,
A_k^m	The ordering cost of manufacturer for k^{th} finished product,
HC_k^m	The total holding cost of manufacturer for k^{th} finished product,
OC_k^m	The total ordering cost of manufacturer for k^{th} finished product,
$C(P_k)$	The production cost of manufacturer for k^{th} finished product,
L_k	The fixed cost of production for k^{th} finished product,
ϕ_k	The variable cost of production for k^{th} finished product,
w_k^m	The selling price of manufacturer for k^{th} finished product,
T_{P_k}	The cycle time of production for k^{th} finished product,
T_k^m	The cycle time of manufacturer,
μ_{ik}	The percentage of demand of k^{th} product for satisfying the demand of i^{th} retailer,
D_{ik}^r	The end user's demand of k^{th} product to which the i^{th} retailer is faced, $D_{ik}^r = a - bw_{ik}^r$
h_{ik}^r	The holding cost of i^{th} retailer for k^{th} product,
A_{ik}^r	The ordering cost of i^{th} retailer for k^{th} product,
HC_{ik}^r	The total holding cost of i^{th} retailer for k^{th} product,
OC_{ik}^r	The total ordering cost of i^{th} retailer for k^{th} product,
w_{ik}^r	The selling price of i^{th} retailer for k^{th} product,
T_{ik}	The required time of i^{th} retailer for collecting k^{th} product,
T_{ik}^r	The cycle time of i^{th} retailer for selling k^{th} product,
TP_j^d	The total profit of j^{th} distributor,
TP^m	The total profit of manufacturer,
TP_i^r	The total profit of i^{th} retailer,

The proposed model is developed under following assumptions:

1. Demands are constant and retailers and customers' demands are price-sensitive.
2. This model is extended to multi finished products and multi raw materials.
3. Holding costs, ordering costs and selling prices of raw material and products for members of each echelon are different.
4. Shortage is not permitted.
5. Each distributor supplies one kind of raw material for the single manufacturer meaning the number of supplier is equal to the kinds of raw materials.
6. For producing each product, manufacturer uses a certain percentage of different kinds of raw materials.
7. Production rate of manufacturer is bigger than the retailers' demand.
8. Replenishment rates of retailers are bigger than the market demand.
9. Lead time is negligible.
10. Demand of each retailer is different due to essence of various demands of customers.
11. All the parameters of the model are constant.

3- Mathematical Model

Here, we formulate a multi-product production and inventory model of three echelons supply chain where the members of chain are multiple distributors, one manufacturer and multiple retailers.

3-1- Distributors model

Since we assume that the number of distributors is equal to the kind of raw materials, so j^{th} distributor delivers j^{th} raw material at a rate of D_j^d to the manufacturer. The differential equation of the inventory level of j^{th} distributor over the time is:

$$\frac{dI_j^d(t)}{dt} = -D_j^d, \quad 0 \leq t \leq T_j^d, \quad j = 1, 2, \dots, m \quad (1)$$

According to Figure (2), for the j^{th} distributor $I_j^d(0) = Q_j$ and $I_j^d(T_j^d) = 0$. Then, the inventory level of j^{th} distributor at time t during $[0, T_j^d]$ is:

$$I_j^d(t) = Q_j - D_j^d t, \quad 0 \leq t \leq T_j^d \quad (2)$$

Clearly, we have:

$$I_j^d(T_j^d) = 0 \Rightarrow T_j^d = \frac{Q_j}{D_j^d} \quad (3)$$

The total profit of j^{th} distributor is equal to subtraction of total holding cost and ordering cost of raw material from its revenue such that its total holding cost is:

$$HC_j^d = \frac{1}{T_j^d} \left[h_j^d \int_0^{T_j^d} (Q_j - D_j^d t) dt \right] = h_j^d Q_j - h_j^d \left(\frac{D_j^d T_j^d}{2} \right) = \frac{h_j^d Q_j}{2} \quad (4)$$

And the ordering cost is:

$$OC_j = \frac{A_j^d}{T_j^d} = \frac{D_j^d A_j^d}{Q_j} \quad (5)$$

Hence, the total profit of j^{th} distributor is:

$$TP_j^d(Q_j) = (w_j^d - C_j^d) D_j^d - \left(\frac{h_j^d Q_j}{2} + \frac{D_j^d A_j^d}{Q_j} \right) \quad (6)$$

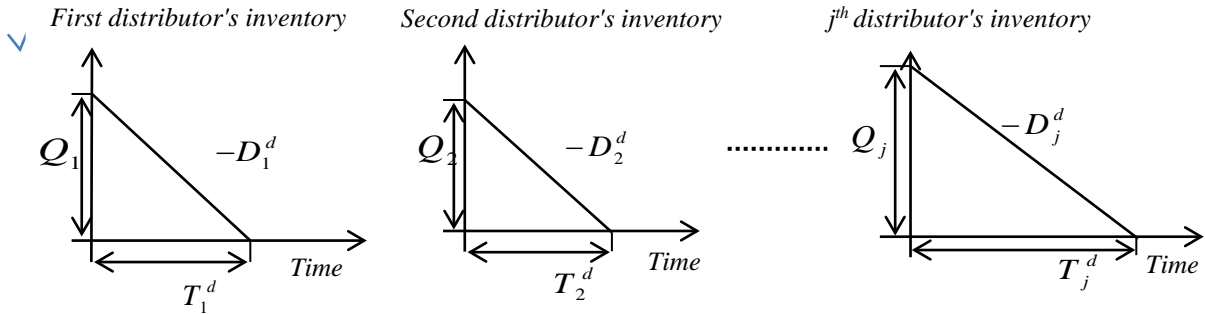


Fig2. Distributors' inventory profile

3-2- Manufacturer Model

The manufacturer orders different kinds of raw material to distributors, and in order to produce several products, combines certain percentage of each material together, where production rate of k^{th} product is as the following:

$$P_k = \sum_{j=1}^m \theta_{kj} D_j^d, \quad 0 \leq \theta_{kj} \leq 1, \quad \sum_{k=1}^n \theta_{kj} = 1 \quad (7)$$

And the production uptime for k^{th} product is:

$$T_{P_k} = \frac{\sum_{j=1}^m \theta_{kj} D_j^d T_j^d}{P_k} = \frac{\sum_{j=1}^m \theta_{kj} Q_j}{P_k} \quad (8)$$

According to Figure (3), during the cycle time of production (T_{P_k}), the inventory of k^{th} product of manufacturer is increasing at the rate of $P_k - D_k^m$ and he completely consumes his inventory during $T_k^m - T_{P_k}$. The differential equation of this level is:

$$\frac{dI_k^m}{dt} = P_k - D_k^m, 0 \leq t \leq T_{P_k}, k = 1, 2, \dots, n \quad (9)$$

At the start of each cycle the manufacturer's inventory is zero, so $I_k^m(0) = 0$.

$$\frac{dI_k^m}{dt} = -D_k^m, T_{P_k} \leq t \leq T_k^m, k = 1, 2, \dots, n \quad (10)$$

Also we have $I_k^m(T_{P_k}) = (P_k - D_k^m)T_{P_k}$ and $I_k^m(T_k^m) = 0$. Then, the manufacturer's inventory is:

$$I_k^m(t) = (P_k - D_k^m)t, 0 \leq t \leq T_{P_k} \quad (11)$$

Manufacturer's inventory

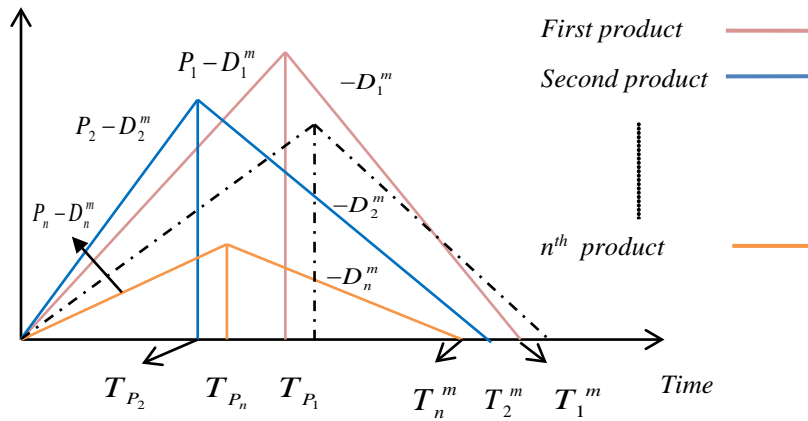


Figure3. Manufacturer's inventory profile

And

$$I_k^m(t) = D_k^m(T_k^m - t), T_{P_k} \leq t \leq T_k^m \quad (12)$$

According to Equations (10) and (11), the manufacturer's cycle time is:

$$T_k^m = \frac{P_k T_{P_k}}{D_k^m} = \frac{\sum_{j=1}^m \theta_{kj} Q_j}{D_k^m} \quad (13)$$

The total holding cost of manufacturer for k^{th} product is:

$$\begin{aligned} HC_k^m &= \frac{1}{T_k^m} \left[h_k^m \left[\int_0^{T_{P_k}} (P_k - D_k^m) t dt + \int_{T_{P_k}}^{T_k^m} D_k^m (T_k^m - t) dt \right] \right] = \frac{1}{T_k^m} \left[h_k^m \left[\frac{1}{2} P_k T_{P_k}^2 + \frac{1}{2} D_k^m T_k^m{}^2 - D_k^m T_k^m T_{P_k} \right] \right] \\ &= h_k^m \left[\frac{1}{2 T_k^m} P_k T_{P_k}^2 + \frac{1}{2} D_k^m T_k^m - D_k^m T_{P_k} \right] = h_k^m \frac{\sum_{j=1}^m \theta_{kj} Q_j}{2} \left[1 - \frac{D_k^m}{P_k} \right] \end{aligned} \quad (14)$$

And his total ordering cost is:

$$OC_k^m = \frac{A_k^m}{T_k^m} = \frac{D_k^m A_k^m}{\sum_{j=1}^m \theta_{kj} Q_j} \quad (15)$$

The revenue of the manufacturer is equal to $\sum_{k=1}^n (w_k^m - C(P_k) - \sum_{j=1}^m w_j^d) D_k^m$ in which

$C(P_k) = \frac{L_k}{P_k} + \varphi_k P_k$ is the production cost of k^{th} product. Finally, the total profit of manufacturer is:

$$\begin{aligned} TP^m(w_k^m) &= \sum_{k=1}^n (w_k^m - C(P_k) - \sum_{j=1}^m w_j^d) D_k^m - (HC_k^m + OC_k^m) \\ &= \sum_{k=1}^n \left(w_k^m - C(P_k) - \sum_{j=1}^m w_j^d \right) (a - bw_k^m) - \left(h_k^m \frac{\sum_{j=1}^m \theta_{kj} Q_j}{2} \left[1 - \frac{(a - bw_k^m)}{P_k} \right] + \frac{(a - bw_k^m) A_k^m}{\sum_{j=1}^m \theta_{kj} Q_j} \right) \end{aligned} \quad (16)$$

3-3- Retailers' Model

The retailers put orders for different kinds of products to the manufacturer, and manufacturer sends proportion of k^{th} product to i^{th} retailer in his cycle (T_k^m). According to Figure (4), i^{th} retailer's inventory starts increasing at a rate of $\mu_{ik} D_k^m - D_{ik}^r$ ($0 \leq \mu_{ik} \leq 1$ and $\sum_{i=1}^h \mu_{ik} = 1$) during the collecting time of the k^{th} product ($[0, T_{ik}]$) and then decreases to satisfy the market demand.

The differential equations of this level are:

$$\frac{dI_{ik}^r}{dt} = \mu_{ik} D_k^m - D_{ik}^r, \quad 0 \leq t \leq T_{ik}, i = 1, 2, \dots, h \quad (17)$$

$$\frac{dI_{ik}^r}{dt} = -D_{ik}^r, \quad T_{ik} \leq t \leq T_{ik}^r, i = 1, 2, \dots, h \quad (18)$$

According to Figure (4), $I_{ik}^r(0) = 0$, $I_{ik}^r(T_{ik}) = (\mu_{ik} D_k^m - D_{ik}^r) T_{ik}$ and $I_{ik}^r(T_{ik}^r) = 0$. So, the inventory level of i^{th} retailer is:

$$I_{ik}^r(t) = (\mu_{ik} D_k^m - D_{ik}^r) t, \quad 0 \leq t \leq T_{ik} \quad (19)$$

$$I_{ik}^r(t) = D_{ik}^r (T_{ik}^r - t), \quad T_{ik} \leq t \leq T_{ik}^r \quad (20)$$

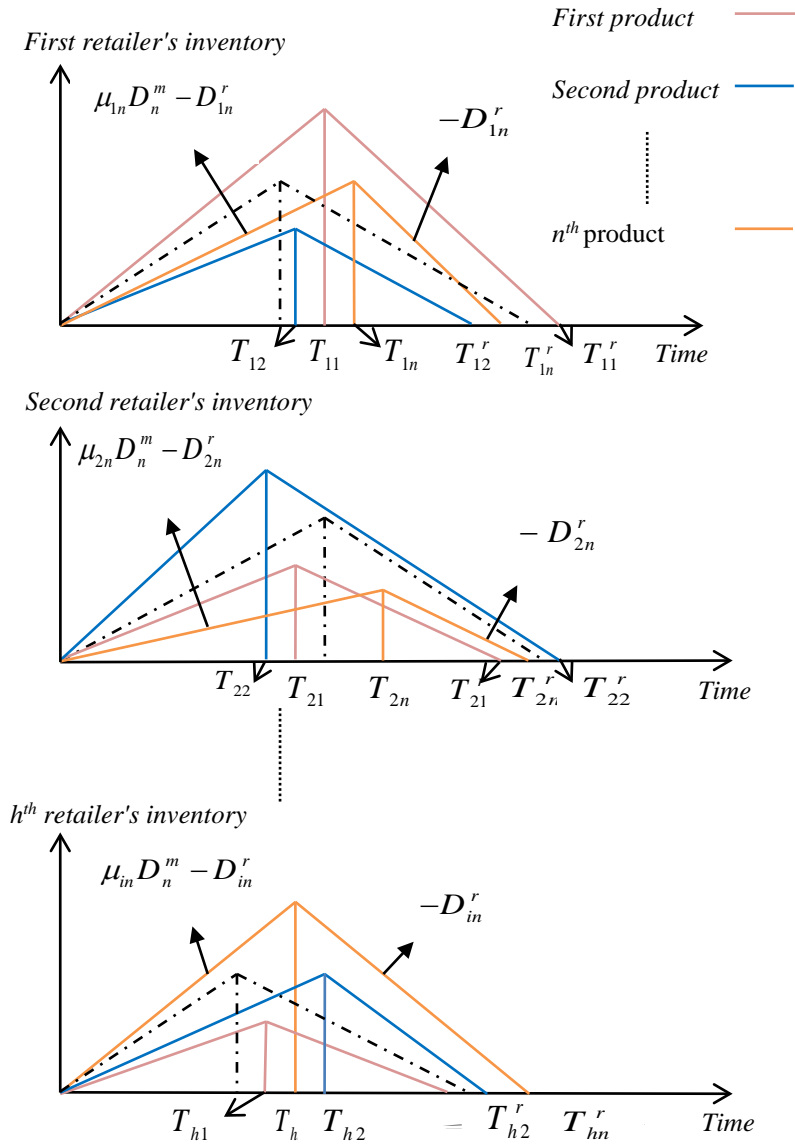


Figure4. Retailers' inventory diagram

Then, the period length of i^{th} retailer is:

$$T_{ik}^r = \frac{\mu_{ik} D_k^m T_{ik}}{D_{ik}^r} = \frac{\mu_{ik} P_k T_{P_k}}{D_{ik}^r} = \frac{\mu_{ik} \sum_{j=1}^m \theta_{kj} Q_j}{D_{ik}^r} \quad (21)$$

Total holding cost of i^{th} retailer for k^{th} product is shown by:

$$HC_{ik}^r = \frac{1}{T_{ik}^r} \left[h_{ik}^r \left[\int_0^{T_{ik}^r} (\mu_{ik} D_k^m - D_{ik}^r) t dt + \int_{T_{ik}^r}^{T_{ik}^r} D_{ik}^r (T_{ik}^r - t) dt \right] \right] = \frac{h_{ik}^r \mu_{ik}}{2} \left[1 - \frac{D_{ik}^r}{\mu_{ik} D_k^m} \right] \sum_{j=1}^m \theta_{kj} Q_j \quad (22)$$

And the total ordering cost is:

$$OC_{ik}^r = \frac{A_{ik}^r}{T_{ik}^r} = \frac{D_{ik}^r A_{ik}^r}{\mu_{ik} \sum_{j=1}^m \theta_{kj} Q_j} \quad (23)$$

Finally, the total profit of i^{th} retailer is:

$$\begin{aligned} TP_i^r(w_{ik}^r) &= \sum_{k=1}^n \left[(w_{ik}^r - w_k^m) D_{ik}^r - HC_{ik}^r - OC_{ik}^r \right] \\ &= \sum_{k=1}^n \left((w_{ik}^r - w_k^m)(a - bw_{ik}^r) - \frac{h_{ik}^r \mu_{ik} \sum_{j=1}^m \theta_{kj} Q_j}{2} \left[1 - \frac{(a - bw_{ik}^r)}{\mu_{ik}(a - bw_k^m)} \right] - \frac{(a - bw_{ik}^r) A_{ik}^r}{\mu_{ik} \sum_{j=1}^m \theta_{kj} Q_j} \right) \end{aligned} \quad (24)$$

4- Solution method

As mentioned, when the chains partners' have different market power, clearly, powerful partners prefer to perform as the dominant member in supply chain and optimize their decisions based on the other dominated partners. In other words, they work individually instead of joining to it. So, in this study, to model the relation between powerful and powerless partners, we consider a decentralized supply chain in which the partners with different market power optimize own profit under the Stackelberg game approach where the retailers are leaders and the distributors and the manufacturer are followers.

In this chain, at the first stage, the manufacturer is leader and the distributors are followers in which the distributors firstly determine their decision variables. Afterwards, the manufacturer makes decision about his decision variable. In the second stage, the retailers are leaders and the distributors and the manufacturer are followers. In this stage, also the retailers optimize their decision variables after characterizing the optimal values of the decision variables by the followers. The ordering lot size of j^{th} raw material (Q_j) is a decision variable of the total profit function of j^{th} distributor (TP_j^d). Moreover the selling prices of the manufacturer (w_k^m) is the decision variable of the total profit function of manufacturer and the selling prices of the i^{th} retailer (w_{ik}^r) for the k^{th} product are decision variables of the i^{th} retailers' total profit function (TP_i^r). In order to optimize the total profit of each member of supply chain, we should prove that their total profit functions are concave.

Theorem1. The j^{th} distributor's total profit function ($TP_j^d(Q_j)$) is concave.

Proof. Concavity of the j^{th} distributor can be proved by taking the second order derivative of $TP_j^d(Q_j)$ (Equation (6)) respect to Q_j which is strictly negative.

$$\frac{\partial^2 TP_j^d}{\partial^2 Q_j} = -\frac{2D_j^d A_j^d}{Q_j^3} < 0 \quad (25)$$

The root of the first order derivative of j^{th} distributors' objective function respect to Q_j is the optimal value of Q_j and we have:

$$\frac{\partial TP_j^d}{\partial Q_j} = -\frac{h_j^d}{2} + \frac{D_j^d A_j^d}{Q_j^2} = 0 \rightarrow Q_j^* = \sqrt{\frac{2D_j^d A_j^d}{h_j^d}} \quad (26)$$

Theorem2. The manufacturer's total profit function ($TP^m(w_k^m, Q_j^*)$) is concave.

Proof. Similarly, by taking second order derivative of $TP^m(w_k^m, Q_j^*)$ (Equation (16)) respect to w_k^m , the concavity of the manufacturer's total profit function will be proved because it is strictly negative.

$$\frac{\partial^2 TP^m}{\partial^2 w_k^m} = -2b < 0 \quad (27)$$

The root of the first order derivative of total profit function, $a - 2bw_k^m + bC(P_k)$

$$-\frac{bh_k^m \sum_{j=1}^m \theta_{kj} Q_j^*}{2P_k} + \frac{bA_k^m}{\sum_{j=1}^m \theta_{kj} Q_j^*} = 0, \text{ respect to } w_k^m \text{ is its optimal value and after substituting } Q_j^* \text{ with Equation}$$

(26), the optimal price will be:

$$w_k^{m*} = \frac{a}{2b} + \frac{C(P_k)}{2} + \sum_{j=1}^m \frac{w_j^d}{2} - \frac{h_k^m}{4P_k} \sum_{j=1}^m \theta_{kj} \sqrt{\frac{2D_j^d A_j^d}{h_j^d}} + \frac{A_k^m}{2 \sum_{j=1}^m \theta_{kj} \sqrt{\frac{2D_j^d A_j^d}{h_j^d}}} \quad (28)$$

Theorem3. The i^{th} retailer's total profit function, $TP_i^r(w_{ik}^r, w_k^{m*}, Q_j^*)$, is concave too.

Proof. Similar to the previous cases, concavity of the total profit function of the i^{th} retailer will be proved too, because the second order derivative of $TP_i^r(w_{ik}^r, w_k^{m*}, Q_j^*)$ (Equation (24)) respect to w_{ik}^r is strictly negative as shown in Equation (29).

$$\frac{\partial^2 TP_i^r}{\partial^2 w_{ik}^r} = -2b < 0 \quad (29)$$

The optimal selling price of i^{th} retailer (w_{ik}^r) will be obtained by solving the first order derivative of the total profit function of i^{th} retailer respect to w_{ik}^r ,

$$a - 2bw_{ik}^r + bw_k^m - \frac{bh_{ik}^r}{2(a - bw_k^m)} \sum_{j=1}^m \theta_{kj} Q_j^* + \frac{bA_{ik}^r}{\mu_{ik} \sum_{j=1}^m \theta_{kj} Q_j^*} = 0, \text{ which is:}$$

$$w_{ik}^{r*} = \frac{a}{2b} + \frac{w_k^m}{2} - \frac{h_{ik}^r}{4(a - bw_k^m)} \sum_{j=1}^m \theta_{kj} \sqrt{\frac{2D_j^d A_j^d}{h_j^d}} + \frac{A_{ik}^r}{2\mu_{ik} \sum_{j=1}^m \theta_{kj} \sqrt{\frac{2D_j^d A_j^d}{h_j^d}}} \quad (30)$$

In order to solve the model, the following solution algorithm is developed.

Solution algorithm

Step1. Input the desired price sensitive demand function of the manufacturer for raw material j^{th} .

Step2. Determine Q_j^* using Equation (26).

Step3. Determine Q_k^* using Equation (28).

Step4. Input the desire price sensitive demand function of h^{th} retailer for k^{th} product.

Step5. Determine w_{ik}^{r*} using Equation (30).

5- Numerical Example and Sensitivity Analysis

5-1- Numerical Example

In this section, a numerical example is presented for a chain with five distributors, five raw materials, one manufacturer, four products and three retailers in a proposed three-echelon supply chain. Consider $a=10000$, $b=45$. The other data of the distributors, the manufacturer and the retailers are shown in Tables (1), (2) and (3), respectively. The results of the leaders and the followers are presented in Tables (4) and (5).

Table1. Data of the distributors

Distributors j	Raw Material j	C_j^d	h_j^d	A_j^d	w_j^d	D_j^d
1	1	8	0.50	2	18	550
2	2	7	0.30	3	16	680
3	3	11	0.60	5	22	450
4	4	9	0.55	4	20	500
5	5	8	0.40	1.5	17	600

Table2. Data of the manufacturer

Product k	θ_{k1}	θ_{k2}	θ_{k3}	θ_{k4}	θ_{k5}	L_k	φ_k	h_k^m	A_k^m
1	0.3	0.6	0	0.3	0	4000	0.01	0.9	30
2	0	0.2	0.8	0	0.2	4800	0.02	1	35
3	0.3	0.2	0	0.5	0.4	4500	0.01	0.8	33
4	0.4	0	0.2	0.2	0.4	4200	0.015	0.7	37

Table3. Data of the retailers

Retailer i	μ_{i1}	μ_{i2}	μ_{i3}	μ_{i4}	h_{i1}^r	h_{i2}^r	h_{i3}^r	h_{i4}^r	A_{i1}^r	A_{i2}^r	A_{i3}^r	A_{i4}^r
1	0.4	0.25	0.3	0.35	1	1	1.1	0.9	40	47	43	42
2	0.3	0.3	0.4	0.35	1.05	0.98	1.08	0.92	43	46	45	48
3	0.3	0.45	0.3	0.3	1.1	0.95	1.12	0.9	45	45	46	40

Table4. Results of the followers

Q_1^*	Q_2^*	Q_3^*	Q_4^*	Q_5^*
66.33	116.61	86.60	85.28	67.08

Table5. Results of the leaders

Product k	w_k^{m*}	w_{1k}^{r*}	w_{2k}^{r*}	w_{3k}^{r*}
1	164.08	193.57	193.76	193.79
2	167.78	195.88	195.71	195.46
3	164.52	193.99	193.86	194.04
4	165.90	194.73	194.83	194.81

5-2- Sensitivity Analysis

In this section, the effects of followers' parameters changes' versus the leaders' decision variables, is studied. So a sensitivity analysis is carried out by increasing or decreasing parameters, at a time, by %25, %50 and %75 changes. We study the effects of distributors' holding cost, ordering cost and purchasing cost changes on the manufacturer and the retailers' selling price.

According to Table (6), the manufacturer's selling price is increased by increasing the holding cost and decreasing ordering cost of followers, but manufacturer's selling price is not sensitive respect to the changes

of followers' purchasing cost. Diagram of manufacturer's selling price changes versus the changes of distributors' holding cost and ordering cost are shown in Figures (5) and (6), respectively.

Table 6. Effects of the changes of followers' costs on the selling price of manufacturer

<i>Parameters</i>	<i>% Changes</i>					
h_j^d	-75	-50	-25	+25	+50	+75
w_1^{m*}	-0.061	-0.032	-0.014	+0.011	+0.021	+0.030
w_2^{m*}	-0.074	-0.039	-0.017	+0.014	+0.026	+0.038
w_3^{m*}	-0.061	-0.033	-0.014	+0.012	+0.023	+0.033
w_4^{m*}	-0.077	-0.043	-0.019	+0.016	+0.031	+0.044
A_j^d	-75	-50	-25	+25	+50	+75
w_1^{m*}	+0.090	+0.039	+0.015	-0.010	-0.019	-0.026
w_2^{m*}	+0.111	+0.048	+0.018	-0.013	-0.023	-0.032
w_3^{m*}	+0.097	+0.041	+0.016	-0.011	-0.020	-0.027
w_4^{m*}	+0.134	+0.056	+0.021	-0.015	-0.026	-0.035
C_j^d	-75	-50	-25	+25	+50	+75
w_1^{m*}	0	0	0	0	0	0
w_2^{m*}	0	0	0	0	0	0
w_3^{m*}	0	0	0	0	0	0
w_4^{m*}	0	0	0	0	0	0

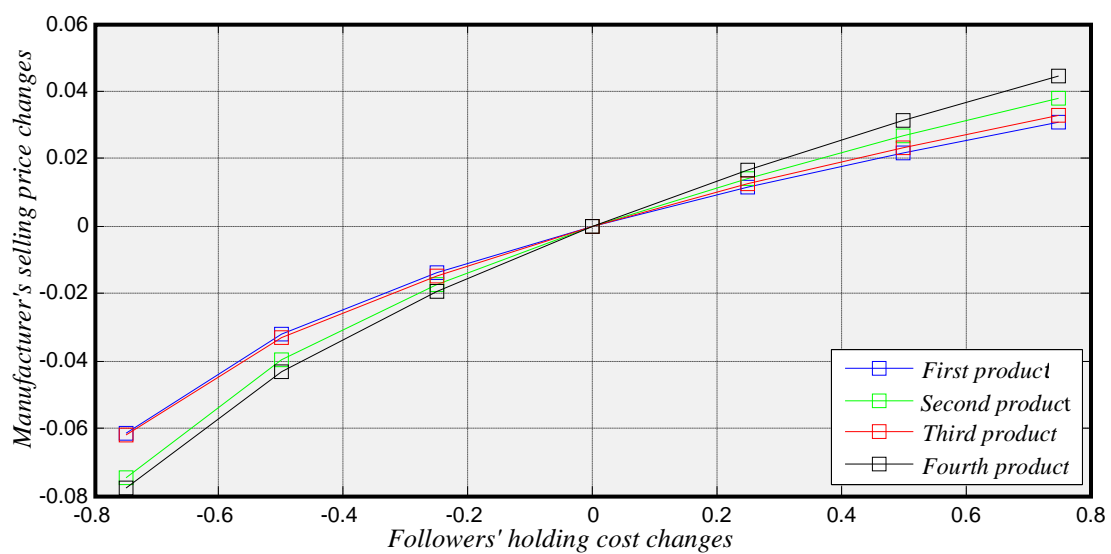


Figure5. The changes of manufacturer's selling price versus the changes of followers' holding cost

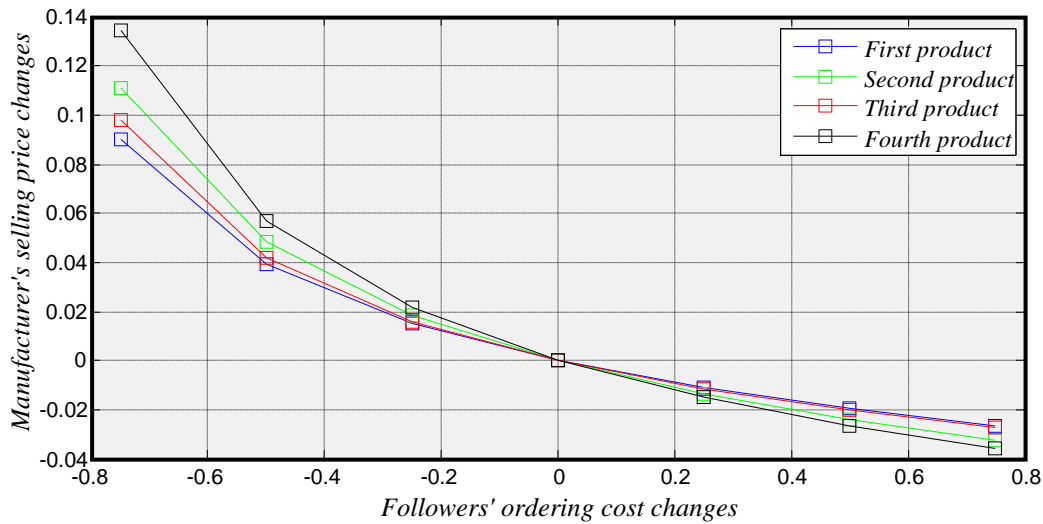


Figure6. The changes of manufacturer's selling price versus the changes of followers' ordering cost

Evidently, increasing the holding cost of raw materials influences on the manufacturer's order quantity and the manufacturer compels to order less of each type of raw materials in order to avoid additional costs. In turn, his selling price increases. In contrast, increasing ordering cost leads to increase the order quantity of the manufacturer. This change will decrease the number of replenishment times and consequently, his selling price of finished products decreases. Although price of each product and its scale of changes depend on their operational costs which are different for each item.

Note that, as it is shown in the above Figures, the first and the fourth product have the most changes of all, in term of selling price, while the second and the third items' selling price changes are moderately, depended on the holding and the ordering cost of the different kinds of raw materials combined to produce each product.

In the other hand, the retailers' price changes' versus the operational costs of the followers is examined in Tables (7), (8), and (9). Based on Table (7), the first retailer' selling price is increased by increasing the holding cost and decreasing the ordering cost of followers, but their price is not sensitive respect to the purchasing cost of leaders, too. Diagram of the changes of the first retailer's selling price versus the changes of the holding cost and ordering cost of followers are presented in Figures (7) and (8), respectively.

In this stage, as the first stage, changes on the ordering and holding cost of the distributors' raw materials as the followers affect on the order quantity and the number of replenishment times of retailers as the leaders, respectively.

In other word, the first retailer decreases his order quantity of finished products by increasing the raw material holding costs because it leads to increase the selling prices of finished products in previous stage. Moreover, he obligates to increase his order quantity in order to decrease replenishment times when the raw materials ordering costs increase.

According to Figures (7) and (8), the first retailer's selling prices for the first and the second item have the most changes while the selling prices of the third and the fourth product change mildly.

Table7. Effects of followers' costs on the selling price of the first retailer

<i>Parameters</i>	<i>% Changes</i>					
h_j^d	-75	-50	-25	+25	+50	+75
w_{11}^{r*}	-0.143	-0.081	-0.036	+0.031	+0.060	+0.086
w_{12}^{r*}	-0.263	-0.151	-0.068	+0.060	+0.114	+0.163
w_{13}^{r*}	-0.196	-0.112	-0.051	+0.044	+0.084	+0.121
w_{14}^{r*}	-0.212	-0.122	-0.055	+0.048	+0.092	+0.133
A_j^d	-75	-50	-25	+25	+50	+75
w_{11}^{r*}	+0.264	+0.111	+0.041	-0.028	-0.050	-0.067
w_{12}^{r*}	+0.503	+0.209	+0.078	-0.054	-0.094	-0.126
w_{13}^{r*}	+0.372	+0.155	+0.058	-0.040	-0.070	-0.093
w_{14}^{r*}	+0.410	+0.170	+0.064	-0.044	-0.076	-0.102
C_j^d	-75	-50	-25	+25	+50	+75
w_{11}^{r*}	0	0	0	0	0	0
w_{12}^{r*}	0	0	0	0	0	0
w_{13}^{r*}	0	0	0	0	0	0
w_{14}^{r*}	0	0	0	0	0	0

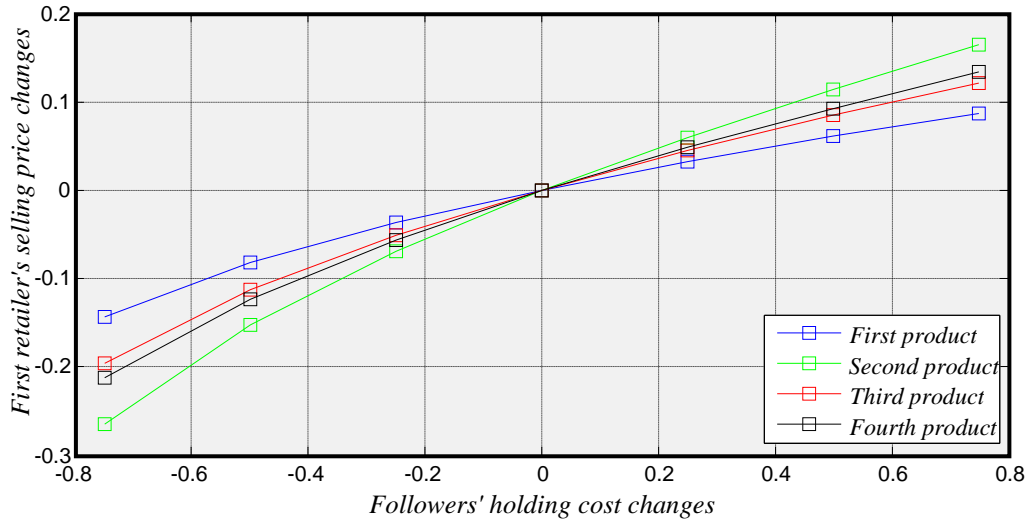


Figure7. The changes of the first retailer's selling price versus the changes of the followers' holding cost

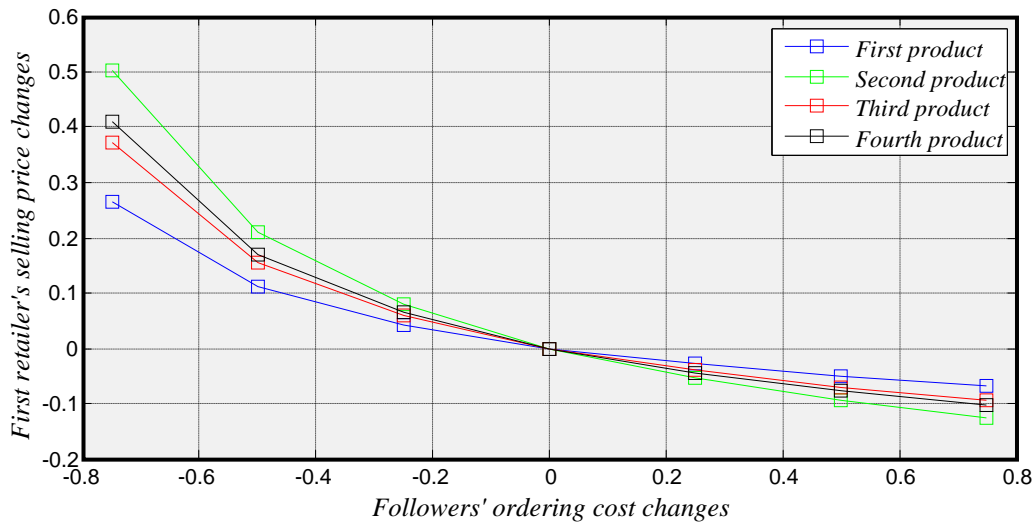


Figure8. The changes of the first retailer's selling price versus the changes of the followers' ordering cost

For the second retailer, changes of selling price on the holding, ordering and purchasing cost changes are demonstrated in Table (8) which it seems to be a rational behavior and follow the logic presented for the first retailer. In addition, Figures (9) and (10) indicate these changes.

Table8. Effects of followers' cost on the selling price of the second retailer

Parameters	% Changes					
h_j^d	-75	-50	-25	+25	+50	+75
w_{21}^{r*}	-0.192	-0.110	-0.049	+0.043	+0.082	+0.118
w_{22}^{r*}	-0.222	-0.127	-0.057	+0.050	+0.095	+0.136
w_{23}^{r*}	-0.161	-0.092	-0.041	+0.036	+0.068	+0.098
w_{24}^{r*}	-0.237	-0.137	-0.062	+0.054	+0.104	+0.149
A_j^d	-75	-50	-25	+25	+50	+75
w_{21}^{r*}	+0.361	+0.151	+0.056	-0.039	-0.068	-0.091
w_{22}^{r*}	+0.419	+0.175	+0.065	-0.045	-0.079	-0.105
w_{23}^{r*}	+0.301	+0.126	+0.047	-0.032	-0.057	-0.076
w_{24}^{r*}	+0.460	+0.191	+0.071	-0.049	-0.085	-0.114
C_j^d	-75	-50	-25	+25	+50	+75
w_{21}^{r*}	0	0	0	0	0	0
w_{22}^{r*}	0	0	0	0	0	0
w_{23}^{r*}	0	0	0	0	0	0
w_{24}^{r*}	0	0	0	0	0	0

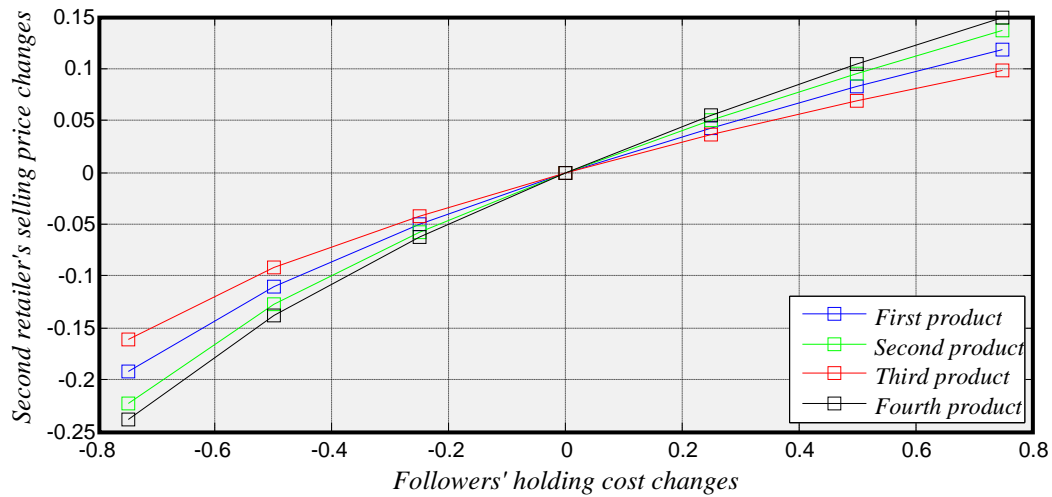


Figure9. The changes of the second retailer's selling price versus the changes of the followers' holding cost

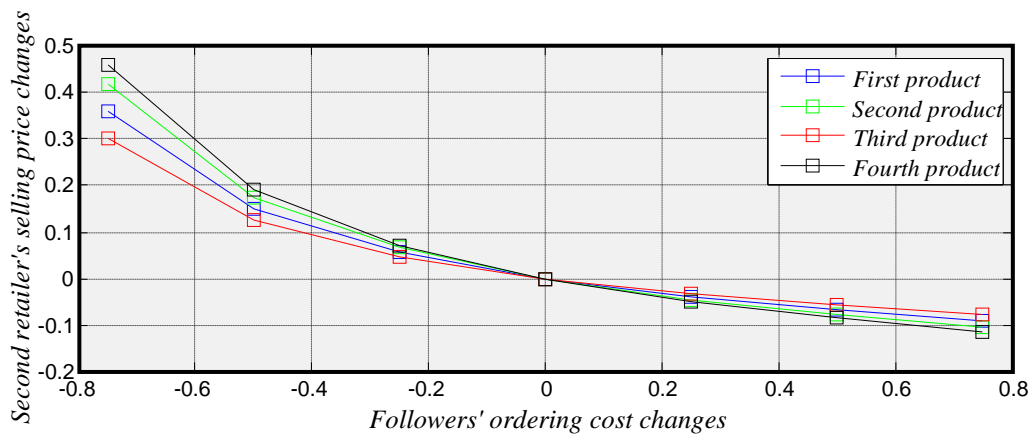


Figure10. The changes of the second retailer's selling price versus the changes of the followers' ordering cost

Also, the selling price changes of the second retailer for the third and the fourth product are more remarkably than the first and the second one, as indicated in Figures (9) and (10).

Furthermore, for the third retailer, as the other ones, the selling price changes versus the distributors' costs are surveyed presented in Table (9) and also the changes diagrams are drawn in Figures (11) and (12).

Note that based on Figures (11) and (12), the selling prices changes of the second and the fourth product at the third retailer are more noticeable.

Consequently, since dominant members as the leader of supply chain decide based on the best response of the dominated members (followers), so, the optimal decisions of leaders (the manufacturer and the retailers) are sensitive to the changes of followers' operational costs so that the least changes in the dominated members costs influence on the leaders' decisions.

Table9.Effects of followers' cost on the selling price of the third retailer

<i>Parameters</i>	<i>% Changes</i>					
h_j^d	-75	-50	-25	+25	+50	+75
w_{31}^{r*}	-0.199	-0.114	-0.051	+0.045	+0.085	+0.122
w_{32}^{r*}	-0.158	-0.089	-0.040	+0.035	+0.066	+0.095
w_{33}^{r*}	-0.207	-0.119	-0.054	+0.047	+0.089	+0.128
w_{34}^{r*}	-0.232	-0.134	-0.061	+0.053	+0.101	+0.145
A_j^d	-75	-50	-25	+25	+50	+75
w_{31}^{r*}	+0.376	+0.157	+0.059	-0.040	-0.071	-0.095
w_{32}^{r*}	+0.291	+0.122	+0.046	-0.031	-0.055	-0.074
w_{33}^{r*}	+0.395	+0.164	+0.061	-0.042	-0.074	-0.099
w_{34}^{r*}	+0.449	+0.186	+0.070	-0.048	-0.083	-0.111
C_j^d	-75	-50	-25	+25	+50	+75
w_{31}^{r*}	0	0	0	0	0	0
w_{32}^{r*}	0	0	0	0	0	0
w_{33}^{r*}	0	0	0	0	0	0
w_{34}^{r*}	0	0	0	0	0	0

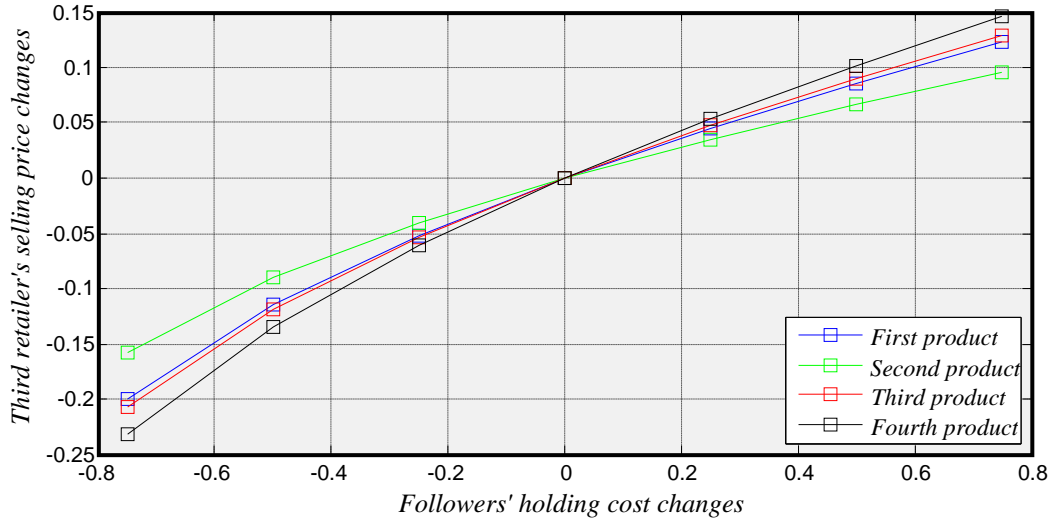


Figure11. The changes of the third retailer's selling price versus the changes of the followers' holding cost



Figure12. The changes of the third retailer's selling price versus the changes of the followers' ordering cost

6- Conclusion

This paper studied a production-inventory model for multiple products in a three-echelon supply chain of pharmacological industry under both pricing and ordering decision policies, considered by none of the previous researches in supply chain area, including non-competing multiple distributors, one manufacturer and non-competing multi-retailer. Each distributor delivers one kind of raw material to the manufacturer and it combines certain percentage of different kinds of raw material to produce each product. Finished products are sent to the retailers and the retailers satisfy end users' demand. The model is developed for pharmaceutical raw material and products under a real case in pharmacological industry. In addition, it is assumed that demand is price-sensitive and shortage is not allowed. The closed form solutions of decision variables are presented so that the concavity of the total profit functions is satisfied. The order quantity of the distributors and the selling prices of the pharmaceutical manufacturer and the retailers are the decision variables of the proposed model.

We employ a Stackelberg game theoretic-approach among members of the chain in which the distributors and manufacturer, considered as the followers, identify the optimal values of their decision variables and the retailers, considered as the leaders, determine their selling prices. Eventually, a real case is studied to illuminate the applicability of the proposed model and then some sensitivity analyses are performed to survey the effect of the changes of followers' operational costs on the optimal decisions of leaders.

We conclude that, in the first echelon of the supply chain, decreasing ordering cost of followers (distributors) and increasing the holding cost lead to increase the manufacturer's selling price while the changes of followers' purchasing cost is not influence on the manufacturer's selling price. Furthermore, in the second one, the retailers' selling price is increased by increasing the holding cost and decreasing the ordering cost of followers, but their price is not sensitive respect to the purchasing cost of leaders. Also, it is found that the selling prices of retailers for each product depend on their operational costs and they are changeable. The presented model has some limitations. For example, it was assumed that lead time is negligible and shortage is not allowed. It is evident that by considering lead time and each kind of shortage, the practical applicability of the model would be increased. If market demand is deemed in stochastic setting, the model will become more realistic. In addition, here, we assume that the production rate of the manufacturer is infinite while it can be considered as a fixed parameters or a decision variable (finite variable) and develop the model under this constraint. We leave these extensions for future researches.

Acknowledgments

The authors thank the three anonymous reviewers for their helpful suggestions which have strongly enhanced this paper. The authors would like to thank the financial support of the University of Tehran for this research under Grant Number 30015-1-02.

References

- Abad, P.L., (2003). Optimal pricing Lee, H., Rosenblatt, M.J., (1986). A generalized quantity discount pricing model to increase and lot-sizing under conditions of perishability, finite production and partial backordering and lost sale. *European Journal of Operational Research*, 144 (3), 677–685.
- Arkan, A., Hejazi, S.R., (2012). Coordinating orders in a two echelon supply chain with controllable lead time and ordering cost using the credit period. *Computers & Industrial Engineering*, 62, 56–69.
- Ben-Daya, M., Darwish, M., Ertogral, K., (2008). The joint economic lot sizing problem: Review and extensions. *European Journal of Operational Research*, 185, 726–742.
- Boyaci, T., Gallego, G., (2002). Coordinating pricing and inventory replenishment policies for one wholesaler and one or more geographically dispersed retailers. *International Journal of Production Economics*, 77, 95–111.
- Cachon, G., Netessine, S., (2004). Game theory in supply chain analysis. In: *Simchi-Levi, E.b.D., Wu, S.D., Shen, M. (Eds.), Supply Chain Analysis in the e-Business Era*. Kluwer Academic, Norwell, MA (Chapter 2).
- Cai, G., Zhang, Z.G., Zhang, M., (2009). Game theoretical perspectives on dual-channel supply chain competition with price discounts and pricing schemes. *International Journal of Production Economics*, 117, 80–96.
- Cai, G., Chiang, W.C., Chen, X., (2011). Game theoretic pricing and ordering decisions with partial lost sales in two-stage supply chains. *International Journal of Production Economics*, 130, 175–185.
- Cardenas-Barron, L.E., Sana, S.S., (2014). A production-inventory model for a two-echelon supply chain when demand is dependent on sales teams' initiatives. *International Journal of Production Economics*, doi: 10.1016/j.ijpe.2014.03.007 (In Press).
- Chen, X., Li, L., Zhou, M., (2012). Manufacturer's pricing strategy for supply chain with warranty period-dependent demand. *Omega*, 40, 807–816.
- Dai, Y., Chao, X., Fang, S.C., Nuttle, H.L.W., (2005). Pricing in revenue management for multiple firms competing for customers. *International Journal of Production Economics*, 98, 1–16.
- Ekici, A., Altan, B., and Özener, Ö. Ö. (2015). Pricing decisions in a strategic single retailer/dual suppliers setting under order size constraints. *International Journal of Production Research*, doi:10.1080/00207543.2015.1054451 (In Press).
- Giri, B.C., Sharma, S., (2014). Manufacturer's pricing strategy in a two-level supply chain with competing retailers and advertising cost dependent demand. *Economic Modelling*, 38, 102–111.
- Guan, R., Zhao, X., (2011). Pricing and inventory management in a system with multiple competing retailers under (r, Q) policies. *Computers & Operations Research*, 38, 1294–1304.
- Hill, R.M., (1999). The optimal production and shipment policy for the-single vendor single-buyer integrated production-inventory problem. *International Journal of Production Research*, 37, 2463–2475.
- Hua, G., Wang, S., Cheng, T.C.E., (2012). Optimal order lot sizing and pricing with free shipping. *European Journal of Operational Research*, 218, 435–441.
- Huang, Y., Huang, G.Q., Newman, S.T., (2011). Coordinating pricing and inventory decisions in a multi-level supply chain: A game-theoretic approach. *Transportation Research Part E*, 47, 115–129.

- Jorgensen, S., Kort, P.M., (2002). Optimal pricing and inventory policies: Centralized and decentralized decision making. *European Journal of Operational Research*, 138, 578–600.
- Khouja, M., (2003). Optimizing inventory decisions in a multi-stage multi-customer supply chain. *Transportation Research part E*, 39, 193–208.
- Kovacs, A., Egri, P., Kis, T., Vancza, J., (2013). Inventory control in supply chains: Alternative approaches to a two-stage lot-sizing problem. *International Journal of Production Economics*, 143(2): 385-394.
- Lee, H. L., & Rosenblatt, M. J. (1986). A generalized quantity discount pricing model to increase supplier's profits. *Management science*, 32(9), 1177-1185.
- Mutlu, F., Cetinkaya, S., (2013). Pricing decisions in a carrier–retailer channel under price-sensitive demand and contract-carriage with common-carriage option. *Transportation Research Part E*, 51, 28–40.
- Nagarajan, M., Sobic, G., (2008). Game-theoretic analysis of cooperation among supply chain agents: Review and extensions. *European Journal of Operational Research*, 187, 719–745.
- Pal, B., Sana, S.S., Chaudhuri, K., (2012). A three layer multi-item production–inventory model for multiple suppliers and retailers. *Economic Modelling*, 29, 2704–2710.
- Qi, Y., Ni, W., Shi, K., (2015). Game theoretic analysis of one manufacturer two retailer supply chain with customer market search. *International Journal of Production Economics*, doi:10.1016/j.ijpe.2015.02.005 (In Press).
- Rosenthal, E.C., (2008). A game-theoretic approach to transfer pricing in a vertically integrated supply chain. *International Journal of Production Economics*, 115, 542–552.
- Ru, J., Wang, Y., (2010). Consignment contracting: Who should control inventory in the supply chain. *European Journal of Operational Research*, 201, 760–769.
- SeyedEsfahani, M.M., Biazaran, M., Gharakhani, M., (2011). A game theoretic approach to coordinate pricing and vertical co-op advertising in manufacturer–retailer supply chains. *European Journal of Operational Research*, 211, 263–273.
- Soon, W., (2011). A review of multi-product pricing models. *Applied Mathematics and Computation*, 217, 8149–8165.
- Szmerekovsky, J.G., Zhang, J., (2009). Pricing and two-tier advertising with one manufacturer and one retailer. *European Journal of Operational Research*, 192, 904–917.
- Taleizadeh, A.A., Noori-daryan, M., (2015a). Pricing, manufacturing and inventory policies for raw material in a three-level supply chain. *International Journal of System Science*, 47(4), 919-931.
- Taleizadeh, A.A, Noori-daryan, M., Cárdenas-Barrón, L.E., (2015a). Joint optimization of price, replenishment frequency, replenishment cycle and production rate in vendor managed inventory system with deteriorating items. *International Journal of Production Economics*. 159, 285–295.
- Taleizadeh, A.A, Noori-daryan, M., Tavakkoli-Moghaddam, R., (2015b). Pricing and ordering decisions in a supply chain with imperfect quality items and inspection under buyback of defective items. *International journal of Production Research*. 53(15), 4553-4582.

- Xiao, T., Jin, J., Chen, G., Shi, J., Xie, M., (2010). Ordering, wholesale pricing and lead-time decisions in a three-stage supply chain under demand uncertainty. *Computers & Industrial Engineering*, 59, 840–852.
- Yu, Y., Liang, L., Huang, G.Q., (2006). Leader–follower game in vendor-managed inventory system with limited production capacity considering wholesale and retail prices. *International Journal of Logistics*:
Yu, Y., Huang, G.Q., (2010). Nash game model for optimizing market strategies, configuration of platform products in a Vendor Managed Inventory (VMI) supply chain for a product family. *European Journal of Operational Research*, 206, 361–373.
- Zhao, X., Atkins, D., (2009). Transshipment between competing retailers. *IIE Transactions*, 41(8), 665-676.
- Zhao, J., Wang, L., (2015). Pricing and retail service decisions in fuzzy uncertainty environments. *Applied Mathematics and Computation*, 250, 580–592.
- Whitin, T.T., (1955). Inventory control and price theory. *Management Science*, 2, 61–68.