A reschedule design for disrupted liner ships considering ports demand and CO₂ emissions: the case study of Islamic Republic of Iran Shipping Lines (IRISL)

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Abstract

This study presents a MILP model to retrieve or get close to the early schedule of disrupted container vessels. The model is applied on a real case study of Islamic Republic of Iran Shipping Lines (IRISL) considering container demands, and CO₂ emissions. Sensitivity analysis on fuel inventory level shows the inevitable influence of the fuel price and primary inventory level on cost of the ships. Variations in unviability duration of the ships suggest that substantial changes in the objective function are caused by omitting the port calls rather than speeding up the ships.

Keywords: Liner shipping, disruption management, rescheduling, transshipment, fuel costs, container demand.

1- Introduction

In countries abutting open seas, maritime transportation plays a conspicuous role in both national and domestic economy. It is noteworthy that maritime transportation share in Iran international trade is about 90 percent which can count to millions of US dollars (www.boursenews.ir/fa/news/146834). Moreover, the economy of landlocked countries is far dependent on neighbors with blue border, and this affiliation brings unmatched opportunity for Countries adjacent to the open seas in order to achieve considerable transit income. In 2013 transported freight was over 9.3 billion tons, in which 16 percent were containership cargos, 30 percent were petroleum products and 54 percent were bulk cargos, and according to the statistics the monetary value of worldwide cargo trade in 2020 would be more than its double in 2010, which indicates Maritime growth in years ahead (Liu et al., 2014).

Liner shipping transport cargoes (mostly containers) with high-capacity and ocean-going ships provide regular services on fixed routes and schedules (Plum et al., 2014). There are approximately 400 liner services in operation today and most of them provide weekly services for ports all around the world. The very basic characteristic of liner shipping is to provide contracted ports or consignees with regular services (Qi, 2015). In one year, a container ship might carry over 200,000 containers. Many container ships can transport up to 8,000 containers on a single voyage. Currently there are hundreds of container shipping lines around the world which differ from giant corporations (like Maersk and China Shipping Container Lines), to medium regional lines, and even much smaller companies.

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Although those companies provide different range of services, they all serve a common mission: transporting cargo containers around the world. With regard to the regular service provision for liner shipping, the significance of ship scheduling cannot be overlooked. Sometimes a slight deviation from predetermined schedule can impose extravagant fines on ship owners or shipping companies, especially when they are carrying highly time sensitive cargoes (Wang et al., 2015). Despite the importance of ship scheduling, a large number of ships cannot reach to their destination at the predetermined schedule every day. Disturbance factors which deflect a ship from its primary schedule are mostly categorized into two sets, port and sea contingencies (Pantuso et al., 2014). Port contingencies include situations like personnel’s strike, political decisions and unfavorable berthing or un-berthing conditions; however there are many factors at sea like bad weather conditions, ship malfunctioning and piracy attack which can affect the scheduling of a ship (Biewirth et al., 2014). In this case study, ship rescheduling or contriving some recovery actions in order to bounce back to the previous schedule or to minimize the ramifications of the deviations would be a critical decision in order to remain in the competitive market of container sea freight. Furthermore, as for fuel costs count for much of the expenses of ships, decision adaptation for speed optimization and bringing in the fuel inventory balance in decision making is vital, especially in determining recovery strategy in disruption situation (Christiansen et al., 2004).

According to the literature in the context of maritime transportation, issues typically include fleet deployment, fleet size and mix problems, port management, port allocation, network design, routing, scheduling and ship speed optimization problems which will be described in detail in section 2. Among conducted studies, a few of them devoted to ship rescheduling problems, which is one of featured outcome of uncertainties in the scheduling of a ship (Psaraftis et al., 2015). In case of disruptions, shipping companies usually apply some recovery actions in order to mitigate or eliminate the fluctuations of scheduling. These recovery actions mostly categorized as: accepting the consecutive delays, omitting the port calls, swapping the port calls and speeding up the ships (Gurning et al., 2009). The first two strategies not only result in higher expenses, but also lower the service level; though, the two latter mostly try to keep the service level and result in higher bunker prices. The main purpose of this paper is to facilitate decision making process in disruption occurrence while maintaining the weekly services with regard to the bunker costs as they increase by third function of speed of the ships, and conducting loading and unloading operation in two superficies: transshipment at some specific ports, and general loading/unloading operations considering ports demand and the capacity of the ships. There are too many disruptive factors in a path that schedulers are not aware till they occur (Gurning et al., 2011). On the other hand, disruptions not only impact on the time table of the ships, but also the scheduling of all the ships of a company through providing and maintaining weakly services. Hereupon, it is a momentous job to design a decision support system in case of disruptions for vessels rescheduling.

The primary contribution of this paper is to present a mathematical model in order to reschedule disrupted ships and the ones affected indirectly at any point of the routes and to take them to the predetermined schedule at most in fifteen days considering CO2 emissions and ports demands, so the total cost including lost sales, bunker price and operational costs could be minimized. In order to mitigate the influence of disruption on the schedules of the ships, recovery actions are considered; moreover, a real case study of Islamic Republic of Iran Liner Shipping is going to be considered to show the applicability of the proposed model. The reminder of this paper is organized as follows. Section 2 briefly describes literature review on the previous researches. Section 3 presents a novel mathematical model. In section 4 and 5 computational results for a real life case study of IRISL, and the conclusion and future work will be discussed respectively.

2- Literature Review
The very first study on maritime transportation in the field of operation research goes back to 1954, but it was Lawrence who pioneered in categorizing ships according to their operations in 1972, and according to his division there are three types of commercial shipping Liner, Tramp and Industrial shipping (Lawrence, 1972). In many studies it is mentioned that liner shipping is like bus lines, it follows a tight schedule and time table to reach which is predetermined from 3 to 6 month (Tsang, 2015). For tramp shipping it is mostly like taxi cab services in which they can carry optional cargos as well as the contracted ones. In industrial shipping, ship owners own the cargo as well, and their major
consignments are high volume fluid, petroleum products and dry bulk cargos (Ronen, 1993). With continues growth in international economic trades in recent decades and the irrevocable need for fast and safe delivery, transmission of high volume products and even time sensitive ones, cargo carrying capacity needed to be increased and container transportation broadened. This new means of transportation even pierced in maritime, and liner shipping companies began to invest huge amounts of money on container shipping (Wang et al., 2015). Although, container shipping companies were enterprise large sums of money, they did not trust academic methods to solve their problems; hence they lost money which could have been saved easily (Ronen, 1993). Recently, however, several leading liner shipping companies have sought to use Operation Research methods to make better decisions and eschew to lose their market share because of the increased market competition and high bunker prices in this industry. Compared to other modes of transportation, maritime has solicited less attention. Christiansen et al. (2004) presented a new classification based on planning horizon and divided studies to strategic, tactical and operational level. In strategic level, decisions are mostly concerned with long term problems such as fleet size & mix, alliance strategy, network design, ship design, locating and port design problems. Some of the planning horizons of the decision are even more than 30 years (Christiansen et al., 2004). In tactical level, studies are concerned with frequency determination, fleet deployment, optimization of ship sailing speed, schedule design, empty container repositioning, quay crane and port scheduling & ship routing problems. Last but not least, operational level of decision making problems are including cargo booking and routing, environmental routing, ship rescheduling and disruption management (Christiansen et al.,2006).There are no signs of studying the determination of container shipping frequency among early studies of 1980s and 1990s; while Bendall and Stent (2001) and Meng et al. (2011) presented models in order to find the optimal number of ships that a container shipping company should deploy according to their service frequency.

The sailing speed of ships has a significant impact on the total operating costs because the increase in just a couple of nautical knots could result in dramatic increase in bunker consumption. As it is mentioned in Meng et al. (2014), the speed of container ships not only determines the bunker consumption rate, but also the pollutant emissions. It should be kept in mind that container ships are among the top fuel-consuming, hence air-polluting categories of ships, and the main reason is their high speed service. Since speed optimization accounts for a large amount of money, and its environmental impact is getting more pronounced for company owners, studies in this field are abundant. Notteboom et al. (2009) investigated the sailing speed effect on schedule stability in liner services. Du et al. (2011) also considered vessel emission and fuel consumption in a berth allocation problem. Jansson et al. (1987) & Jepsen et al. (2011) contributed to the green liner shipping network design. Qi et al. (2012) also aimed to minimize fuel emissions in a liner vessel schedule optimization considering uncertain port times. Song et al. (2012) represented an activity based method to reduce liner ship emission carrying empty containers. Wang et al. (2012) regarded the problem of speed optimization in a liner ship network. The main target in appearance of liner shipping was the urgent need for fast, safe and on time services, from this perspective scheduling play the main role in service level and consequently in supply chain synchronization. Once the schedule is designed, the speed of the container ships is largely determined; therefore schedule design is usually interwoven with speed optimization (Meng et al., 2013). A few studies on ship scheduling problems considered transshipment at ports. Karlafitis et al. (2009) render mathematical models for direct cargo delivery without considering transshipment, as well as the Meng et al. (2011). Bausch et al. (1998), Appelgren et al. (1971) and Agarwal et al. (2008) have considered transshipment operation in their liner ship scheduling problem. Boffey et al. (1979), Brown et al. (1987) and Cho et al. (2001) have considered transshipment in their shipping operation for both crude oil ocean-going ships and the container carrying ones. In later studies Kjeldsen et al. (2012), Bruer et al. (2013), Wang et al. (2012b) bring transshipment considerations into their liner shipping services scheduling problems. Bruer et al. (2013) proposed a vessel schedule recovery problem to deal with a given disruption scenario, and to select a recovery action compromising among increased bunker consumption, the impact on cargos, and the customer service level. In this study, it is assumed that at the very start of each leg, the ship is aware of delay it is going to be faced and the author propose three recovery actions- increasing speed, omit a port call and swap a port call- to bounce back to the previous schedule. Although, Bruer took the very first step on disruption management in ship rescheduling; his study cannot answer the
situation in which the ship is facing disruption everywhere in each leg and considering the ports rotation, ship would not be allowed to swap the port calls. Most importantly, providing a regular service to ports which is the basic characteristic of liner shipping is waived and all ships must visit every port calls in a rotation. The presented model does not provide ship tracking and simultaneous rescheduling of all ships affected by disruption; moreover, unforeseen effects of increasing speed, like bunkering in un-pre-determined ports, fluctuations in bunker price at each port and ports demand are not considered as well.

It is hoped that the following research could answer the aforementioned gaps, therefore a MILP mathematical model presented and solved for IRISL real problem. The very initial assumption in this case study is that the ships in all loops should reschedule simultaneously according to the disruption and as the liner shipping company committed to provide weekly services, there are some recovery actions to be taken in order to bounce back to the primary schedule or to minimize tardiness; Moreover, it is spotted that there are two ports of transshipment which transfer cargoes between ships, both internal ships and external ones. Considered recovery actions are speeding up of the ships, so that they can reach their previous schedule; omit a port call in case of an intense disruption, and for small disruption acquiesce to the tardiness and arrive at port in allowed time window. Unlike the study conducted by Brouer et al. (2013), Disruption can happen everywhere in each leg so it would be unexpected to the ship, and planning horizon embarks on right after disruption happens. Furthermore, a time line is considered so the ships provide weekly services to the ports. All ships must not visit every port and different ships are assigned to different routes. Moreover; as the IRISL have been unsuccessful to obtain the weekly services to the contracted ports in case of a disruption, swapping between ships to visit a port is allowed merely to keep the service level.

3- Problem definition

Based on a thorough study on literature review, it has been noticed that one of the serious problems that container-cargo shipping companies dealing with daily is the operational scheduling of the ships. As the company committed to provide regular (most of the time weekly) services, failing in this mission would extremely influence on national economy, level of service and vitiates the credit of the company; hence the scheduling of the containerships is a hypersensitive decision making process. The rescheduling of the containerships during disruption occurrence and devising recovery action is even more intense regarding to the decisions comprising fuel purchasing, re-bunkering and maintaining the weekly services to satisfy ports demand. We noticed that the most possible recovery actions for containerships in routes are omitting a port call or speeding up the ships, so that the ship could service the most prioritized port in its rout (mostly transshipment ports) or reach the following port. For this purpose, we propose a novel MILP model to assist the planners of the shipping company in terms of disruption outbreak and in selecting the best afterward recovery action. In this problem, the ships can encounter disruptions in any coordinate in routes and rescheduling begins just after disruptions happen for one or more ships of the company. By estimating the duration of disruptive factor (as a parameter) rescheduling strive to bounce ships back to their previous schedule at most in two weeks while it preserves weekly services to the ports. The priority of ports differ from one another as they are assumed to have different operations, general loading/unloading operation or transshipment, and their operation periods differ as well. The principal target of the model is to delineate where skipping the port calls, speeding up the ships or the combination of both must be applied so the objective function could minimize the costs of the company and customers satisfaction hold at a maximum level considering ports demand and CO₂ emissions. On the other hand, subsidiary consequences must be taken into account, such as fuel inventory level which diminishes by utilizing speed accretion and different fuel consumption rate which considered in the objective function. In this problem ships could be tracked at any coordinate and according to the time window of each port they can either reach the contracted ports on time or late, in both case studies they can omit the port call or stay for operation in case of on time arrival. Ships can also increase their speed to make up the delay, so they could perform loading/unloading or transshipment operation at ports.

1-1- Mathematical Model
The problem is formulated as a mixed integer linear programming problem in this section. Let,

**Sets**
- $V$ Set of Speed phase
- $K$ Set of ports
- $R,S$ Indices for routes
- $i,j$ Indices for ships

**Parameters**
- $c_{ls_{r,k}}$ Lost sale cost
- $\alpha_{r,k}$ Port existence in a route
- $\mu_v$ Fuel consumption at speed phase $v$
- $\lambda_{i,r}$ Initial location of ship $i$ in route $r$
- $\theta_{i,r}$ Existence of ship $i$ in route $r$
- $\pi_{r,k}^{min}, \pi_{r,k}^{max}$ Port time window
- $\delta_i$ The unavailability duration of ship $i$
- $\tau_{r,k}$ Transshipment duration
- $Vol_i$ Containership capacity
- $dem_{k}^c$ The container demand of the port
- $\epsilon_{fuel}$ CO$_2$ output from fuel (kilogram per ton)
- $\kappa, \kappa'$ The time needed to load or unload set of containers
- $\eta_{r,s,k}$ Transshipment port
- $C_f$ Fuel cost
- $\rho_{i,r}$ number of rounds track
- $\chi_r$ Route length
- $\phi_{r,k}$ The location of Port $k$ in route $r$
- $\gamma_i$ Disrupted ships
- $\sigma_v$ Speed phase
- $\zeta_{r,k}$ General operation duration
- $Vol_k$ Port container depot capacity
- $IE_{k,c,t}$ External ships operation at port $k$
- $\epsilon_{CO_2}$ Environmentally price of CO$_2$ output (US $ per kilogram)

**Variables**
- $del_{i,k,t}$ Number of unloaded containers to port $k$ from ship $i$
- $pic_{i,k,t}$ Number of loaded containers from port $k$ by ship $i$
- $v_{i,v,t}$ The speed phase of ship $i$ in $v$ level of speed (minimum, economy or maximum) in day $t$
- $vel_{i,t}$ The speed of the ship $i$ in day $t$
- $Inv_{i,t}$ The Fuel inventory level of Ship $i$ in day $t$
- $at_{i,k}$ The Arrival time of Ship $i$ at port $k$
- $\gamma_{i,r,t}$ The location of the ship $t$ in route $r$ in day $t$
- $g_{i,r,s,k,t}$ \( \{1 \) if ship $i$ from route $r$ and ship $j$ from route $s$ arrive at port $k$ in day $t$ \( 0 \) otherwise
- $O_{i,r,j,s,k}$ \( \{1 \) if ship $i$ from route $r$ and ship $j$ from route $s$ arrive at port $k$ \( 0 \) otherwise
- $p_{i,r,j,s,k,t}$ \( \{1 \) if both ship $i$ and $j$ be present at port $k$ in day $t$ \( 0 \) otherwise
- $tttrs_{k}$ \( \{1 \) if transshipment in port $k$ could not be conducted due to the disruptions \( 0 \) otherwise
- $trs_{i,j,k,t}$ \( \{1 \) if transshipment conducted by ships $i$ and $j$ at port $k$ in day $t$ \( 0 \) otherwise

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and according to the speed of the ships their location can be determined. of all is the ship location at any point; consider that we are aware of the primal location of each ship. There are many considerations regarding this problem which can be assorted into six categories. Frist, the main goal of this model is to minimize the rescheduling costs of liner shipping companies. The relative variable costs regarding to the described rescheduling problem are lost sales and bunkering, hence the objective function could be formulized as below:

$$
\text{Min } z = \sum_r \sum_k c_l r_k \cdot q_{r,k} \cdot \alpha_{r,k} + \sum_r \sum_k c_l r_k \cdot t_{tr} s_k \cdot \sum_r \eta_{r,s,k} + \sum_r \sum_k c_l r_k \cdot x_{l,k} + \\
\sum_i \sum_t \sum_k c_f \cdot v^p_{i,,r,t} \cdot v_\mu + \sum_i \mu_{i,v} \cdot c_{\text{fuel}} \cdot \xi_{CO_2}$$

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$$\begin{align*}
y_{l,r,1} &= \lambda_{l,r} \quad \forall l, r \\
y_{l,r,t} &= y_{l,r,t-1} + \text{vel}_{l,t} - x_r \cdot \rho_{l} \\
y_{l,r,t} &\leq x_r 
\end{align*}$$
\[ y_{i,r,t} \leq M_2 \cdot \theta_{i,r} \quad \forall i, r, t \] (5-3)

\[ y_{i,r,t} - \phi_{r,k} = (yg_{i,r,k,t}^+ - yg_{i,r,k,t}^-) \cdot \theta_{i,r} \cdot \alpha_{r,k} \quad \forall i, r, k, t \] (6-3)

\[ (1 - d_{i,r,k,t}) \cdot \theta_{i,r} \cdot \alpha_{r,k} \leq (yg_{i,r,k,t}^+ + yg_{i,r,k,t}^-) \cdot \theta_{i,r} \cdot \alpha_{r,k} \quad \forall i, r, k, t \] (7-3)

\[ (yg_{i,r,k,t}^+ + yg_{i,r,k,t}^-) \cdot \theta_{i,r} \cdot \alpha_{r,k} \leq M_2 \cdot (1 - d_{i,r,k,t}) \cdot \theta_{i,r} \cdot \alpha_{r,k} \quad \forall i, r, k, t \] (8-3)

\[ yg_{i,r,k,t}^+ \leq M_2 \cdot \text{art}_{i,r,k,t} \cdot \theta_{i,r} \cdot \alpha_{r,k} \quad \forall i, r, k, t \] (9-3)

\[ yg_{i,r,k,t}^- \leq M_2 \cdot (1 - \text{art}_{i,r,k,t}) \cdot \theta_{i,r} \cdot \alpha_{r,k} \quad \forall i, r, k, t \] (10-3)

It should be noticed that ships can be located before, after or in a port, on the other hand it is assumed that their location is weighted by a specific port \((7-3)-(10-3)\), so if they are visiting a port more than once, total length of the route will be subtracted from the current location of the ship.

The second class of constraints is designated to the arrival of the ships at ports. For a ship In order to berth at a port, it must arrive at the predetermined time window.

\[ d_{i,r,k,t} \leq a_{i,r,k} \quad \forall i, r, k, t \] (11-3)

\[ a_{i,r,k} \leq \sum_t d_{i,r,k,t} \quad \forall i, r, k, t \] (12-3)

\[ d_{i,r,k,t} \leq \alpha_{r,k} \cdot \theta_{i,r} \quad \forall i, r, k, t \] (13-3)

\[ \sum_t m_{i,r,k,t} = a_{i,r,k} \quad \forall i, r, k \] (14-3)

\[ m_{i,r,k,t} \leq d_{i,r,k,t} \quad \forall i, r, k, t \] (15-3)

\[ \sum_t m_{i,r,k,t} \cdot t \leq d_{i,r,k,t} \cdot t + M_3 (1 - d_{i,r,k,t}) \quad \forall i, r, k, t \] (16-3)

\[ a_{r,k}^r \leq \sum_i a_{i,r,k} \quad \forall r, k \] (17-3)

\[ \sum_i a_{i,r,k} \leq a_{r,k}^r \quad \forall r, k \] (18-3)

\[ a_{i,r,k} - \pi_{r,k}^\text{min} \leq M_3 \cdot b_{i,k}^+ \quad \forall i, r, k \] (19-3)

\[ -M_3 (1 - b_{i,k}^+) \leq a_{i,r,k} - \pi_{r,k}^\text{min} \quad \forall i, r, k \] (20-3)
\(-at_{ik} + \pi^{max}_{r,k} \leq M_3 \cdot b_{ik}^- \quad \forall i, r, k \) (21-3)

\(-M_3 \cdot (1 - B_{ik}^+) \leq -at_{ik} + \pi^{max}_{r,k} \quad \forall i, r, k \) (22-3)

\(e_{ik} = b_{ik}^+ - b_{ik}^- \quad \forall i, k \) (23-3)

\(at_{ik} = \sum_r \sum_t m_{i,r,k,t} \cdot t \cdot \theta_{i,r} \cdot \alpha_{r,k} \quad \forall i, k \) (24-3)

It is assumed that each port have a specific time window for ships to arrive and violating this time window would result in missing ports, since they would no longer permit ship's berthing. On the other hand, arriving at a port would not necessarily mean that the ships could berth there, but they can omit or miss the port because of their sooner or later arrival time. For each day berthing at the port k the variable \(d_{i,r,k,t} \) would take 1.

The next class of constraints refers to the speed of the ship, as one of the goals of this paper is to find optimized speed, it is assumed that during disruption and the cessation of the ships at ports, speed variable cannot be valued, so it is assumed to be equal to zero. This is also noteworthy that in most of the time, the speed of the ships are categorized into three classes, minimum, economy and maximum speed; therefore, in this paper these three speed classes are considered so that the speed can take one value at the time. Regarding constraints can be formulized as below:

\[ \sum_{t \leq \tau_i} vel_{i,t} \leq M_2 \cdot (1 - \gamma_i) \quad \forall i \] (25-3)

\[ \sum_v v_{i,v,t} \cdot \sigma_v = vel_{i,t} \quad \forall i, t \] (26-3)

\[ \sum_v v_{i,v,t} \leq 1 \quad \forall i, t \] (27-3)

\[ \sum_v v_{i,v,t} \geq 1 - \sum_r \sum_k P_{i,r,k,t} \cdot \theta_{i,r} \quad \forall i, r, k, t \] (28-3)

The fourth class of constraints refers to the ships’ fuel and inventory level. As for fuel costs contributed for much of liner ships expenses, considering the fuel inventory level of the ships in order to determine optimized speed is necessary, since fuel consumption raise as nonlinear function of speed increment.

\[ Inv_{i,t} = Inv_{i,t-1} - \sum_v v_{i,v,t} \cdot \mu_v \quad \forall i, t > 1 \] (29-3)

\[ Inv_{i,t} \leq \beta_i \quad \forall i, t \] (30-3)
\( \text{Inv}_{l1} = \text{Inv}(0)_l \) \quad \forall i \quad (31-3)

According to the recovery strategies, ships can encounter four different situations while they arrive at a port. If they do not arrive in the predetermined time window, they must miss the port and pay the consecutive lost sale. If they arrive on time they can either decide to stay or omit the port call. Staying at the port will result in performing general loading/unloading or transshipment operation. The stand time of the ships at ports can vary with respect to their operation type. In this paper they are considered as given parameters.

\[
x_{i,k} \leq \sum_r a_{i,r,k} \cdot \theta_{i,r} \cdot \alpha_{r,k} \quad \forall i, k \quad (32-3)
\]

\[
\sum_t \text{OP}_{i,k,t} + x_{i,k} + \sum_j \sum_t \text{trs}_{i,j,k,t} \leq 1 \\
\forall i, k \quad (33-3)
\]

\[
\sum_t \text{OP}_{i,k,t} + x_{i,k} + \sum_j \sum_t \text{trs}_{i,j,k,t} \geq e_{i,k} \\
\forall i, k \quad (34-3)
\]

\[
\text{OP}_{i,k,t} \leq \sum_r m_{i,r,k,t} \cdot \theta_{i,r} \cdot \alpha_{r,k} \quad \forall i, k, t \quad (35-3)
\]

\[
1 - q_{r,k} \leq \sum_i \sum_t \text{OP}_{i,k,t} \cdot \theta_{i,r} \\
\forall r, k \quad (36-3)
\]

\[
\sum_t \text{OP}_{i,k,t} \leq e_{i,k} \\
\forall i, k \quad (37-3)
\]

\[
\sum_t d_{i,r,k,t} = \zeta_{r,k} \cdot \sum_t \text{OP}_{i,k,t} + \sum_j \sum_t \text{trs}_{i,j,k,t} \cdot \tau_{r,k} \\
\forall i, r, k \quad (38-3)
\]

The next class of constraints is devoted to the transshipment operation. There are some specific considerations regarding to its fulfillment. Involved ships must arrive at port on time and they must be allowed to stay at port till they finish transshipment. Otherwise they miss the port and pay the consecutive costs.

\[
x_{i,k} \geq \sum_r a_{i,r,k} \cdot \theta_{i,r} - e_{i,k} \quad \forall i, k \quad (39-3)
\]

\[
g_{i,r,s,k,t} \leq \theta_{i,r} \cdot \theta_{j,s} \cdot \eta_{r,s,k} \\
\forall i, r, s, j, k, t \quad (40-3)
\]

\[
z_{i,r,s,j,k,t} \leq d_{i,r,k,t} \cdot \theta_{i,r} \cdot \theta_{j,s} \cdot \eta_{r,s,k} \\
\forall i, r, j, s, k, t \quad (41-3)
\]

\[
z_{i,r,s,j,k,t} \leq d_{j,s,k,t} \cdot \theta_{j,s} \cdot \eta_{r,s,k} \\
\forall i, r, j, s, k, t \quad (42-3)
\]
\[ g_{i,r,j,s,k,t} \leq z_{i,r,s,j,k,t} \cdot \theta_{i,r,j,s,\eta_{r,s,k}} \quad \forall i, r, j, s, k, t \] (43-3)

\[ \sum_t g_{i,r,j,s,k,t} = o_{i,r,j,s,k} \quad \forall i, r, j, s, k \] (44-3)

\[ \sum_t g_{i,r,j,s,k,t} \cdot t \leq z_{i,r,j,s,k,t} \cdot t + (1 - z_{i,r,j,s,k,t}) \cdot M_3 \quad \forall i, r, j, s, k, t \] (45-3)

\[ z_{i,r,s,j,k,t} \leq o_{i,r,j,s,k} \cdot \theta_{i,r,j,s,\eta_{r,s,k}} \quad \forall i, r, j, s, k, t \] (46-3)

\[ \sum_t t r s_{i,r,j,s,k,t} \leq \sum_r \sum_s \sum_t g_{i,r,j,s,k,t} \cdot \theta_{i,r,j,s,\eta_{r,s,k}} \quad \forall i, r, j, s, k \] (47-3)

\[ 1 - tt s_k \leq \sum_i \sum_j \sum_t t r s_{i,j,k,t} \quad \forall k \] (48-3)

\[ p_{i,r,j,s,k,t} = d_{i,r,k} \cdot d_{j,s,k,t} \quad \forall i, r, j, s, k, t \] (49-3)

\[ \sum_t p_{i,r,j,s,k,t} \geq r_{r,k} - \left(1 - \sum_t t r s_{i,j,k,t}\right) \quad \forall i, r, j, s, k \] (50-3)

\[ y, vel, at, yg^+, yg^-, Inv \geq 0 \] (51-3)

\[ z_{i,r,s,j,k,t}, o_{i,r,j,s,k}, g_{i,r,j,s,k,t}, x_{l,k}, a_{i,r,k}, d_{l,r,k,t}, a_{r,k}, v_{l,v}, e_{l,k}, b_{l,k}^+, b_{l,k}^- \]

\[ art_{l,r,k,t}, q_{r,k}, t r s_{i,j,k,t}, OP_{l,k,t}, p_{i,r,j,s,k,t}, tt s_k \quad Binary \ variables \] (52-3)

In case of on time arrival and ship berthing at a port, they can load or unload containers according to their container capacity and preset demand. The inventory level of the containers in a ship can be determined according to the port operation and its previous inventory level, so the related constraints can be:

\[ IL_{i,c,t} = IL_{i,c,t-1} + \sum_k pic_{i,k,t}^c - \sum_k del_{i,k,t}^c \quad \forall i, c, t \] (53-3)

\[ \sum_t IL_{i,c,t} \cdot Vol_c \leq Vol_i \quad \forall i, c \] (54-3)

\[ del_{i,k,t}^c + pic_{i,k,t}^c \leq M \cdot \left(OP_{l,k,t} + \sum_l tr s_{i,j,k,t}\right) \quad \forall i, k, t, c \] (55-3)

In order to load or unload containers, the inventory level of the ports must be considered as well. The inventory level of the ports can change by the operations of other incoming ships which can be delineated as:

\[ ILP_{k,c,t} = ILP_{k,c,t-1} + \sum_t del_{i,k,t}^c - \sum_t pic_{i,k,t}^c \quad \forall k, c, t \] (56-3)

\[ \sum_t ILP_{k,c,t} \leq Vol_k \quad \forall c, k \] (57-3)
Finally, the time each ship spends at a port can vary according to its operation and the demand of the port, so:

\[ p_{r,k} = \kappa \delta_{i,k,t} + \kappa \Pi_{i,k,t} + \text{setup time} \]  
\[ p_{ss,r,k} = \kappa' \delta_{i,k,t} + \kappa' \Pi_{i,k,t} + \text{setup time} \]  

**Linearization:** The following transformations can linearize the nonlinear constraints (23-3), (49-3):

\[ e_{i,k} = b_{i,k}^+ \cdot b_{i,k}^- \]  
This can be in form of:

\[ e_{i,k} \leq b_{i,k}^+ \forall i,k \]  
\[ e_{i,k} \leq b_{i,k}^- \forall i,k \]  
\[ e_{i,k} \geq b_{i,k}^+ + b_{i,k}^- - 1 \] \[ \forall i,k \]  

And for \( p_{i,r,s,k,t} = d_{i,r,k,t} \cdot d_{j,s,k,t} \) the same rule can be confirmed:

\[ p_{i,r,s,k,t} \leq d_{i,r,k,t} \forall i,r,j,s,k,t \]  
\[ p_{i,r,s,k,t} \leq d_{j,s,k,t} \forall i,r,j,s,k,t \]  
\[ p_{i,r,s,k,t} \geq d_{i,r,k,t} + d_{j,s,k,t} - 1 \forall i,r,j,s,k,t \]  

**PROPOSITION:** The two proposed recovery actions can fully cover the disruption of the ships problem, and there would be no need to swap the ports.

**PROOF:** Consider two ports \( P_1 \) and \( P_2 \) with specified time windows \( (\alpha_1, \beta_1) \) and \( (\alpha_2, \beta_2) \) respectively. Assume that ships distance to \( P_1 \) is \( d_1 \) and distance between two ports is \( d_2 \). Logically if the ship is supposed to reach \( P_1 \) at the predetermined schedule, then \( \alpha_2 \) must be at least equal to \( \beta_1 + d_2/v_{\text{max}} \).

Objective function is mainly composed of two terms, one is lost sales and the other is bunker price in this proof we are going to compare these two terms in case of swapping the port call.

We know that \( \alpha_1 \leq \alpha_2 \) and \( \beta_1 \leq \beta_2 \) so:

\[ \alpha_2 \leq \frac{d_1 + d_2}{v} \leq \beta_2 \]  
And \( \alpha_1 \leq \frac{d_1 + 2d_2}{v} \leq \beta_1 \) so if the ship swap the ports \( \frac{d_1 + d_2}{v} \leq \frac{d_1 + 2d_2}{v} \)

On the other hand, \( \alpha_2 \leq \frac{d_1 + 2d_2}{v} \) and \( \frac{d_1 + d_2}{v} \leq \beta_1 \),

Also \( \frac{d_1 + 2d_2}{v} \leq \beta_1 \leq \beta_2 \) and \( \alpha_1 \leq \alpha_2 \leq \frac{d_1 + d_2}{v} \)
So swapping the port calls would never change the objective function as the ship arrives in predeterminated time window, but there is still the term for bunker price left to check:

We proved that arrival time to port $P_1$ is $\frac{d_1+2d_2}{v}$ and to port $P_2$ is $\frac{d_1+d_2}{v}$. As they arrive in their time windows by not swapping the port calls we can now switch the arrival times in their worst case study, which is:

\[
at_1 = \frac{d_1+d_2}{v} \quad \text{and} \quad at_2 = \frac{d_1+2d_2}{v}.
\]

In order to arrive at $P_1$ in $at_1$ the ship must pass $d_1$ at the speed of $\frac{d_1}{d_1+d_2} \cdot v$ and go through the $d_2$ at the same speed of $v$. Hence, the speed of the ship would be less and as a result fuel consumption will diminish as ships overlook the swapping strategy.

### 3- Numerical Results

In order to test the implementation and applicability of the developed model, a real case study of IRISL have been considered and its corresponding answers are discussed in the following.

#### 3-1- Case study

In this paper a real case study of Islamic Republic of Iran Shipping Lines (IRISL) is going to be considered. Islamic Republic of Iran possesses an exceptional location in the world as it neighbors all but 10 landlocked countries which can use to transmit their cargos through open seas by ships. IRISL established in 1967 and its cargo transition started in 1968 with four ocean-going ships and with total capacity of 40338 DWT (Dead Weight Ton). According to the sessions of interviews with IRISL experts in containerized operation departments, all of the interviewees express that the most serious problem IRISL schedule planners are dealing with is providing weakly services for Liner Ships. They all do believe that the major setback is unforeseen contingencies and regarding to the fuel price which count for a great quota of the costs of the ship, rescheduling of the disrupted ships is harder than ever. Hence, their avowal impels us to address this real case study problem. Currently this company contracted with sixteen ports in three closed loops, there are 8 ships sailing in HDM loop with 6500 container capacity, 4 in SCP loop with 3300 container capacity and 4 others in SPL&ISC loop with 2100 container capacity. Each of these routes includes ports depicted in Figure 1.

**Table 1.** Routes and Service calls

<table>
<thead>
<tr>
<th>Route</th>
<th>Port calls</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 HDM</td>
<td>Bandar Abbas(1)- Assaluyeh(2)- Singapore(5)- Qingdao(14)- Tianjin(15)- Dalian(16)- Lianyungang(13)- Shanghai(12)- Ningbo(11)- Jebel Ali(3)- Bandar Abbas(1)</td>
</tr>
<tr>
<td>2 SCP</td>
<td>Bandar Abbas(1)- Singapore(5)- Kaohsiung(8)- Xiamen(10)- chiwan(9)- Jebel Ali(3)- Bandar Abbas(1)</td>
</tr>
<tr>
<td>3 ISC &amp; SPL</td>
<td>Bandar Abbas(1)- Colombo(4)- Singapore(5)- Port Kelang(7)- Mundari(6)- Jebel Ali(3)- Bandar Abbas(1)</td>
</tr>
</tbody>
</table>
According to the case, ships must arrive at the predetermined time window to the ports, otherwise they have to pass the ports and pay the respective fines; furthermore in order to keep the service level standards, other ships must make up to their tardiness. However, in case of arriving at the specified time window, they can perform either general operation, transshipment or even omit the port call. Figure 2 represents a synopsis report on the operation of ships. Table 2 represents the container demand at each port for 20 TEU containers for two types of containers, laden and empty. As the IRISL provided us with the container demand of 9 ports the solution for this part of the model is represented for 2 routes and 9 ports. It is noteworthy that in all solutions it is assumed that two ships (ships number 2 and 9) are disrupted and the rescheduling is executed for all shipping lines of the company.

Table 2. Ports Demand (20TEU)

<table>
<thead>
<tr>
<th>Port</th>
<th>Laden</th>
<th>Empty</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1200</td>
<td>750</td>
</tr>
<tr>
<td>2</td>
<td>1950</td>
<td>780</td>
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<td>1600</td>
<td>860</td>
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<tr>
<td>4</td>
<td>-200</td>
<td>-1050</td>
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<tr>
<td>5</td>
<td>1300</td>
<td>-300</td>
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<tr>
<td>6</td>
<td>3000</td>
<td>1500</td>
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<tr>
<td>9</td>
<td>3500</td>
<td>-1000</td>
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</tbody>
</table>
As the Figure 2, ship 1, 2, 4, 6, 8, 9, 12, 14 and 16 could not arrive at the predetermined time window at each port in decision horizon which is a two week period sailing. However, other ships could fulfill their weekly services to all ports except Kaohsiung which was omitted.

On the other hand, in Table 3 the strategy of changing speed can be understood clearly, since disrupted ships or the indirectly impacted ones need to increase their speed in order to catch up to the contracted ports. Some days like day 9, 11 and 12 are the days that the ship 3 is either facing disruption or has stopped at the port so the speed is equal to zero while in other days the speed is categorized into three speed phase.
Table 3. Optimal solution for ships’ speed (knots per day)

<table>
<thead>
<tr>
<th>Day</th>
<th>1</th>
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<th>4</th>
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</tbody>
</table>

According to the fuel price and the cost of environmental hazards published by the United Nations reports in 24th week of 2014, the cost of the ships is delineated in Table 4. As the objective function represents, the ships costs is divided into three classes the bunker costs, CO2 emissions costs and the costs from skipping a port call, lost sale and the demurrage of the ships. Sustainable transportation systems consider environmental indicators like CO2 emissions and as it can be seen in Table 4 and Figure 3 its costs are subjected to substantial contribution of the cost of the ship, which again remind us of the dramatic impact of the bunker cost and its environmental consequences. The efficiency ratio of the ships is represented in table 4 as well.

Since ships 2 and 9 are disrupted their efficiency ratio is significantly reduced. Due to this situation, in order to maintain the service level, other ships from all routes are obliged to reschedule their voyage, therefore they indicate modest efficiency ratio.
<table>
<thead>
<tr>
<th>Route</th>
<th>Ship</th>
<th>Omitted Port Calls</th>
<th>Ship costs</th>
<th>Bunker costs</th>
<th>CO₂ Emissions Costs</th>
<th>Efficiency Ratio</th>
<th>Excluding port omission</th>
<th>Excluding ships speeding up</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>9.62E+15</td>
<td>2.16E+15</td>
<td>4.20E+15</td>
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<td>1.14E+14</td>
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<td>5.01E+13</td>
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<tr>
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<td>6.01E+12</td>
<td>0.477</td>
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</tr>
</tbody>
</table>
Since the speed of the ships follows three levels as slow, economy and high class, considering the primary location of the ships, the location of the ships at any day is demonstrated in Table 5. As the ships finish their round trip and get back to the first port which is Bandar Abbas in our problem, their location will be updated for their next round. See ship 7 in Table 5. Moreover, according to the speed of the ships, while a ship stops at ports in order of fulfilling loading/unloading or transshipment operation, its location would be similar to the port during operation days.

**Table 5. Location of the Ships (nm)**

<table>
<thead>
<tr>
<th>Ship</th>
<th>Primary Location</th>
<th>Voyage Day</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1000</td>
<td>1000 1300 1600 1900 2200 2500 2800</td>
</tr>
<tr>
<td>2</td>
<td>4200</td>
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</tr>
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<td>6</td>
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</tr>
<tr>
<td>7</td>
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</tr>
<tr>
<td>8</td>
<td>900</td>
<td>900 1000 1200 1400 1600 1800 2000</td>
</tr>
<tr>
<td>9</td>
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<td>2700 2800 2900 3100 3300 3400 3600</td>
</tr>
<tr>
<td>10</td>
<td>4100</td>
<td>4100 4400 4600 4900 5200 5400 5700</td>
</tr>
<tr>
<td>11</td>
<td>5600</td>
<td>5600 5700 5800 5900 6000 6200 6300</td>
</tr>
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<table>
<thead>
<tr>
<th>Ship</th>
<th>Primary Location</th>
<th>Voyage Day</th>
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<tbody>
<tr>
<td>1</td>
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<td>1000 1300 1600 1900 2200 2500 2800</td>
</tr>
<tr>
<td>2</td>
<td>4200</td>
<td>4200 4400 4600 4700 4800 5000 5200</td>
</tr>
<tr>
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<td>900 1000 1200 1400 1600 1800 2000</td>
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<td>2700</td>
<td>2700 2800 2900 3100 3300 3400 3600</td>
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<tr>
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<td>4100 4400 4600 4900 5200 5400 5700</td>
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<tr>
<td>11</td>
<td>5600</td>
<td>5600 5700 5800 5900 6000 6200 6300</td>
</tr>
</tbody>
</table>

Figure 3. Ships Costs Quota
Table 6 represents the results for loading and unloading operation of the ships at each port in each day according to the capacity and inventory level of the both ports and ships. As discussed before countering disruption by ship number 2 and 9 impel other ships to make up for their tardiness or omitting the ports. In this way the efficiency of ships capacity which is a major yardstick in evaluating the proficiency of the whole company can be compromised because they need to atone for the disrupted ships and try not to disregard any port call.

Table 6. Loading/Unloading Operation (20 TEU)

<table>
<thead>
<tr>
<th>Day</th>
<th>Ship</th>
<th>Port</th>
<th>Delivery Laden</th>
<th>Delivery Empty</th>
<th>Pick Up Laden</th>
<th>Pick Up Empty</th>
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<td>1300</td>
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</tr>
</tbody>
</table>

1-1- Sensitivity analysis
According to the problem and its consideration including fuel price, fuel inventory level and the two recovery strategies, sensitivity analysis can be discussed as followings.

a) Variations in primitive fuel inventory level
Increasing in the primary fuel inventory level would logically seem to diminish the objective function, although in this survey, the variation strictly depends on the amount of increase in fuel cost per ton and the increase in primary fuel inventory level. In the diagram below objective function versus increase in the primary fuel inventory level and increase in the fuel price per ton is delineated.

![Figure 4: Variation in inventory level & Bunker price](image)

As can be seen in Figure 4, just 0.25% increase in the primary fuel inventory level (dark gray curve) against any percent increase in fuel price would not help to decrease the ship’s cost, on the other hand by increasing the initial fuel level by 0.5% (light gray curve) ship’s cost could cease
raising or in better case studies diminishes versus increase in fuel price per ton in any scale, and this analysis indicates the impressive impact of fuel price on Iran’s Liner ships. However, it must be kept in mind that ships usually start their voyage with extra storage of fuel and increasing in initial fuel inventory level by even a 1% would not be possible in most case studies.

b) Variation in ships unavailability duration

According to the proposed model it is considered that the unavailability time of the disrupted ships could alter and the effect on the objective function is delineated in Figure 5. As it is shown below, if the disrupted ships’ unavailability time augmented from one day to two, there is a mutation in objective function and the main reason can be regarded to omitting one or more port call. By increasing their unavailability time through 6 days, the objective function variation is not that remarkable and the reason could be the ships striving to reach the port calls by increasing speed, while they are omitting the same number of port calls. This interpretation is extendable through the rest of the diagram.

![Figure 5. Ships unavailability Duration variation](image)

c) Dominance point of two strategies

One of the most conspicuous results in this problem is the dominance point of two strategies. In this problem fuel costs and lost sales constitutes the objective function. Finding dominance point means to find the balance point between these two terms. As fuel costs increase at the rate of $C_f' = C_f(2^q-1 - 1)$ the shipping company prefers to speed up the ships, but according to Figure 6 there is a point in fuel cost incretion that skipping a port could be more beneficial than increasing the speed of ships which could be really determinant in utilizing these strategies. On the other hand, according to the ships’ costs in 26th week of 2014, the importance of preserving a service at ports is distinctive in which ships are more obliged to save their weekly services to the ports rather than expunging the port calls and saving more fuel.
d) Average Ports demand versus disruption period

In this analysis the contrast between an increase in the average demand of ports and a reduction in the number of disruption days is delineated. The average demand will increase and the disruption duration will reduce like 12, 11, 10… 1.

\[ \text{Demand}' = \text{Demand}(1 + q \times 0.1\%) \]

The Sinusoidal behavior of the chart can contribute to an increase in lost sale because of an increase in average demand and the unavailability of the ships, while can cause a reduction in lost sale because of the reduction in unavailability time of the ships and more ports get visited.

2- Concluding marks

In this paper a novel MILP model have presented. The model addresses liner ships rescheduling problem in case of disruption outbreak, while it is necessary to maintain weekly services and speed designation. Two recovery strategies have considered so the disrupted ships could bounce back to their previous schedule or to minimize their succulent costs. The model is solved in less than 90 seconds using CPLEX solver. An analysis of a real case study problem of IRISL shows that the model can update ships schedule at most in every two weeks (planning horizon), and by omitting a port call or speed increment, disrupted ships can service their contracted ports considering ports demand and CO₂ emissions. It is also proved that these two recovery strategies can fully cover the aforementioned problem. However, study on disruption management on liner shipping seems to have marvelous extensions. Pricing approaches in fuel purchase and fuel sustenance at ports could be investigated.
Considering uncertainties in the duration of operations at ports, using game theory approach in selecting the best recovery strategy and scheduling prioritization is another outspread which worth to be considered.

References


