

Taguchi Design optimization using multivariate process capability index

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Abstract

The Taguchi method is a useful technique to improve the performance of products or processes at a lower cost and in lesser time. This procedure can be categorized as the static and dynamic quality characteristics. The optimization of multiple responses has received increasing attention over the last few years in many manufacturing organizations.

Several approaches dealing with multiple static quality characteristic problems have been reported in the literature. However, little attention has been made on optimizing the multiple dynamic quality characteristics.

In this paper, we investigate multivariate signal response systems (Dynamic Taguchi) and propose a method based on multivariate process capability. Simulated data shows that the proposed method can increase robustness of dynamic Taguchi method. Furthermore, the proposed method is capable to find the optimal value of controllable factors in a continuous space.

Keywords: Signal response system, Robust parameter design (RPD), Dynamic Taguchi design, Process capability index.

1- Introduction

1-1 Robust parameter design

The robust design has been successfully applied to a variety of industry problems for upgrading product quality since Taguchi (1987) first introduced this method. The objective of robust design is to reduce response variation in products or processes by selecting the settings of control factors, it provides the best performance and the least sensitivity to noise factors which is done by using interaction between control and noise factors.

To apply the robust design, Taguchi employs an orthogonal array (OA) to arrange the experiments and uses the signal-to-noise ratio (SNR) to measure the performance of each experimental run.

A two-step optimization procedure is then used to determine the optimal factor combination to simultaneously reduce the response variation and bring the mean close to the target value.

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Taguchi divided the RPD methodology into two categories: static and dynamic characteristics. Static systems are defined as those desired output of the system that has a fixed target value and one attempts to obtain the value of a quality characteristic of interest as close as possible to a single specified target value. Whereas dynamic systems are those that target value depends on the input signal set and there is a relationship between response (output) and signal factor (input). This signal-response relationship is of primary importance to the performance of the system. Figure 1(a-b) denotes dynamic and static systems.

Miller and Wu (2002) criticized the terminology of Taguchi and labeled the static system as *simple-response system* and the dynamic system as *signal-response system*. They said that static and dynamic systems are applied for those systems which concerned with time whereas the simple response system and signal-response systems can concern with other input variables. They divided signal-response system into two categories: multiple target systems and measurement systems. Wu and Hamada (2001) introduced the third category as the control systems.

Because the signal-response concept plays an important role in product/process development, robust parameter design of signal-response systems (also called dynamic parameter design in Taguchi's terminology) is an effective and powerful tool for quality improvement.

Signal-response systems ideally suppose that there is a linear relationship between the response and the signal factor. Moreover, dispersion and sensitivity are two important aspects of the signal-response system which are considered with an index such as SNR (Wu and Hamada, 2001). According to the linear assumption of relationship function, the simple linear regression model can be written as follows:

$$y = \alpha + \beta M + \varepsilon, \quad (1)$$

where, M is the signal factor with predefined p levels with the value of m_i in i th level, y_i is the response value in i th level of signal factor, $E(\varepsilon) = 0$ and $Var(\varepsilon) = \sigma^2$.

It is always desirable for any signal-response system to have a small dispersion, i.e., a small σ^2 value. A large sensitivity (i.e., a large β value) is also desirable. A performance measure, which converts σ^2 and β in to a single measure is

$$\omega = \ln\left(\frac{\beta^2}{\sigma^2}\right) \quad (2)$$

Taguchi proposed the dynamic SNR formula for a nominal the best response as follows:

$$SNR = 10 \log\left(\frac{\beta^2}{\sigma^2}\right) \quad (3)$$

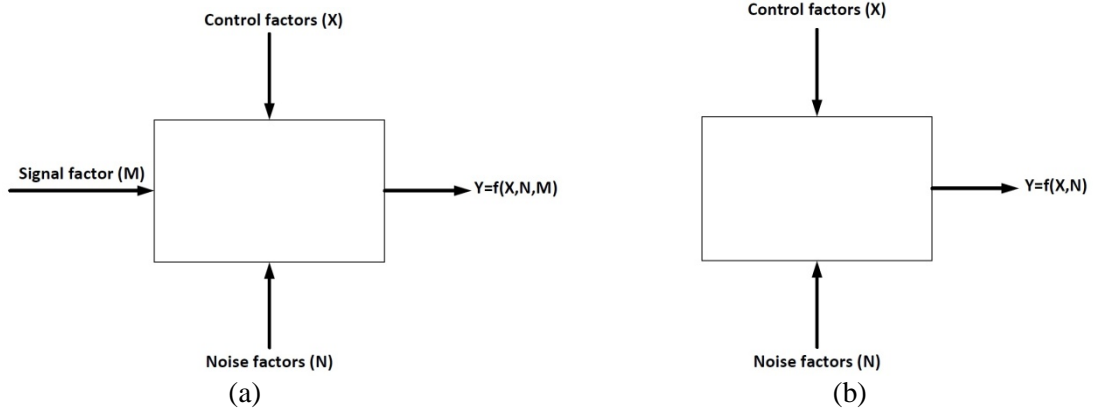


Figure 1. (a) Dynamic system (b) Static system

The dispersion and sensitivity are measured by σ^2 and β , respectively. β and σ are evaluated for different combinations of control factors. It is desired that the SNR to be maximized.

In contrast to Taguchi's dynamic SNR and two-step optimization procedure, Miller and Wu (2002) developed performance measure modeling (PMM) and response function modeling (RFM) to optimize signal-response systems. PMM requires a two stage modeling. The first stage includes estimation of performance measure (PM) and second stage consists the modeling of PM as a function of control factors using estimation of PM.

RFM uses the experimental data to model the signal-response relationship as a function of the control and noise factors. The specified performance measure is then evaluated with respect to the fitted regression models. This method is an extension of the response modeling approach first recommended by Welch et al. (1990) and Shoemaker et al. (1991) for simple response applications. Wasserman (1996) presented a case study of the parameter design with dynamic characteristics by using multiple regression models. He demonstrated Taguchi's SNR with a linear statistical model. Khattree (1996) provides a method for to estimate robust parameter design in the situation in which all of the noise variables cannot be studied simultaneously. He used the response surface approach.

Lunani, Nair and Wasserman (1997) noted that using SNR as a quality performance measure might produce inaccuracies due to a mistake to evaluate dispersion effect. They developed two graphical methods for identifying appropriate measures of dispersion, thereby avoiding interactions between the dispersion and sensitivity effects for a dynamic problem. Miller (2002) compared three methods of analyzing signal-response applications, including Miller and Wu's (2002) approach, Taguchi method and a graphical approach proposed by Lunani, Nair and Wasserman (1997). He introduces a new graphical technique, the joint effects plot and demonstrated usefulness of the proposed method.

Lesperance and Park (2003) proposed a joint generalized linear model to evaluate the robust design of dynamic characteristics, which is based on standard regression modeling techniques. Roshan and Wu (2002) described the application of SNR in the analysis of the multiple target systems (i.e., first category of signal-response systems). Again Roshan and Wu (2002) introduced a theoretical formulation for multiple target systems and developed a practical approach for optimization which overcomes some limitations.

Wu and Yeh (2005) presented an approach to optimize multiple dynamic problems based on quality loss function. The objective is to minimize the total average quality loss for the multiple dynamic quality characteristic's experiments. Zhiyu, Zhen and Xiangfen (2006), proposed a new desirability function method for multiple robust parameter design. Their proposed method can yield better results than traditional desirability function approach. Gupta, Kulahci, Montgomery and Borrer (2009), proposed a split-plot approach to the signal-response system characterized by two variance components. They demonstrate that explicit modeling of variance components using generalized linear mixed model (GLMM) leads to more precise point estimates of important model coefficients with shorter confidence intervals within-profile variance and between-profile variance.

Dasgupta, Miller and Wu (2010), presented a robust design of measurement systems (i.e., second category of signal-response systems). They developed an integrated approach for estimation and reduction of measurement variation through a single parameter design experiment. They considered a linear signal-response relationship. Arvidsson and Gremyr (2008) prepared a literature review of conflicts and agreements on the principles of robust design. Through this review four central principles of robust design are identified: awareness of variation, insensitivity to noise factors, application of various methods, and application in all stages of a design process.

Boylan and Cho (2013), proposed a multidisciplinary RPD methodology that provides an enhanced approach for modeling multiple, mixed type quality characteristics; uses the skew normal distribution to allow for a fuller and more accurate representation of asymmetric system properties.

Boylan, Goethals and Rao Cho (2013), proposed a trade-off analysis between the cost of replication and the desired precision of generated solutions. They considered several techniques in the early stages of experimental design, using Monte Carlo simulation as a tool, for revealing potential options to the decision maker.

Yadav, Bhamare and Rathore (2010), explore the possibilities of combining both approaches into a single model and proposes a hybrid quality loss function-based multi-objective optimization model.

Truong, Shin and Jeong (2011), focus on solving the robust design (RD) problem that occurs in pharmaceutical studies in which output responses are measured over time

In general, several researches have addressed multiple static quality characteristics problems (i.e. Dasgupta, Miller and Wu (2010), Khuri and Conlon (1981), Lin, Lin and Ko (2002), Su and Tong (1997), Lu and Antony (2002), Wu (2002), Vining (1997), Tong, Su and Wang (1997,1999).

Several publications have studied the robust design problem considering the dynamic systems (i. e. Wasserman (1996), Lunani, Nair and Wasserman (1997), Miller and Wu (1996), Su and Hsieh (1998), Tsui (1999), McCaskey (1997)).

However, few studies have been concerned with optimizing the parameter design for multiple dynamic quality characteristics. Chang (2008), proposed a procedure based on desirability function to optimize multiple dynamic quality characteristics. He used the Simulated Annealing (SA) to find the best factor setting. Desirability function cannot consider variation of the observations and only focuses on the distance between responses and their targets. This fact leads to effect of noise factor and the fluctuation of the design parameters are ignored in this approach (Zhiyu, Zhen and Xiangfen (2006)). Hence, it cannot be a precise index to evaluate the results completely.

Chang (2008), used SNR of responses instead of responses values to solve the mentioned problem, in addition he used geometric mean to construct a single index for evaluating responses. However, there are some problems with his proposed approach as follows.

- (i) Using geometric mean leads to face strong effect on the single aggregated index when there is a less SNR value, while other responses may have a high desirability values. Hence, analyzer ignores setting of control factors and process is rejected basically.
- (ii) The effect of correlation between responses is not considered, because each response is analyzed separately. Hence, Chang's (2008) procedure has an equal result for two processes which have similar mean and variance but different correlation. Note that, correlation between responses has a clear effect on the responses.

Similar to Chang (2008), the method proposed by Tong and Su (1997) considers TOPSIS method and focuses on SNR between responses and does not account for correlation between responses.

Tong, Wang and Hounq (2001) solved the mentioned problem with desirability function and proposed the dual response surface method based on the desirability function. They first provide a regression model of responses and then calculate desirability of mean and variance. Finally, a response surface model is generated to consider the problem in two aspects of mean and variance. Their proposed approach is more reliable because of considering the responses variances. However, it does not consider the correlation between responses.

Jalili, Bashiri and Amiri (2011), applied Data Envelopment Analysis (DEA) to optimize multivariate dynamic Taguchi design. The drastic difference between the method based on the DEA and proposed method of this paper is the correlation issue. The DEA cannot consider correlation of responses, but the multivariate process

capability index can account for the correlation between response variables. Moreover, the DEA is not a parametric method, hence normality assumption of the response variables is not required in this method while the multivariate process capability index used in this paper needs the normality assumption. In spite of the differences between the DEA and multivariate process capability index, both methods are appropriate for non linear relationship between response variable and controllable factors.

Amiri, Bashiri, Mogouie and Doroudyan (2012), proposed a non-normal response optimization method based on process capability index. Their method transforms the known non normal distribution to multivariate normal distribution and then uses process capability index to optimize a multi-response problem. The difference between that method and the proposed method in this paper is the ability of the proposed method to consider unilateral quality characteristics.

He, Zhu and Park (2012), proposed a robust desirability function approach to simultaneously optimize multiple responses. Their method takes account for all values in the confidence interval rather than a single predicted value for each response and then defines the robustness measure for the traditional desirability function using the worst case strategy. They used geometric mean to aggregate calculated desirability functions. This technique has some drawbacks as mentioned before.

Awad and Kovach (2011) utilized process capability index and proposed an optimization method for multi responses problems. Their multivariate process capability index, MC_{pm} , index is applicable if there is a target value for each response

Pal and Gauri (2010), proposed a new method that integrates multiple regression technique and Taguchi's signal-to-noise (SN) ratio concept. Two sets of experimental data were analyzed using this method. The proposed method considers total SN ratio as well as closeness of individual responses to their corresponding target values.

Hence, an efficient index which investigates a signal-response system should have two conditions. First, it should consider both location and dispersion of responses. Secondly, the proposed index should not have any relaxations about the relationship between responses and signal factor such as linearity. One of the important issues in optimization is about stage. Optimization can be applied in the both design and operation stage (for example of applying in operation stage see Sekhar and Tan (2009)). So, optimization methods which used process mean can be a suitable technique for optimization in operation stage.

1-2- Process Capability Index

Process Capability Indices (CPI) such as C_p , C_{pk} and C_{pm} are used in statistical process control to evaluate the capability of the processes in satisfying the customer's needs. Capability index C_p is defined as the ratio of tolerance range to the spread of the process as follows:

$$C_p = \frac{USL - LSL}{6\sigma}, \quad (4)$$

where USL and LSL are upper and lower specification limits. Also σ is standard deviation of quality characteristic. Sometimes, more than one quality characteristic describes the quality of a process. For analyzing the capability of these processes with correlated quality characteristics, multivariate CPIs are introduced by some researchers.

In this paper to evaluate responses of a multivariate signal-response system, we use MPC_{NCV} which have been proposed by Jalili, Bashiri and Amiri (2011). The proposed index is expressed as follows:

$$MPC_{NCV} = \left[\frac{\lambda \times PR + CV + \beta}{PR + NCV}, \frac{1}{D} \right], \quad (5)$$

where PR is process region, CV is conformance volume between the process region and modified tolerance region and NCV is non-conformance volume between the process region and modified tolerance region. λ is the sensitivity process capability parameter and in a bivariate response with bilateral specification limits is

proposed to be 0.27 (Jalili et al., 2011). Also β is added to the first component of the MPC_{NCV} to increase the sensitivity of the index to the processes which are completely within modified tolerance region. According to the simulations, it is better to apply $\beta = 0.1$. However, if volume of tolerance region is less than 1 it is advised to apply $\beta = 0.05 \times \text{Volume of Tolerance Region}$, because $\beta = 0.1$ leads to overestimations of the index. Also distance between process and target is computed as $D = \left(1 + (\mu - T)' \Sigma^{-1} (\mu - T)\right)^{\frac{1}{2}}$ (Jalili et al., 2011).

Advantage of the MPC_{NCV} is applicable to both unilateral and bilateral situations. Hence, when there is a unilateral response, MPC_{NCV} can consider the problem effectively. One of the other methods which is used to consider a unilateral response is desirability function. But as mentioned before desirability function cannot consider variance of responses and only take in to account the distance between each responses with corresponding target.

Another advantage of using MPC_{NCV} in process capability is the capability of considering correlation between responses and also its first component could consider distance between process mean and target and variation of observations simultaneously respect to traditional approaches (Jalili et al., 2011).

Hence, using process capability to evaluate the signal-response system can be useful to consider location (mean) and dispersion (variance) of observation as well as correlation of responses.

The rest of the paper is organized as follows: The proposed method is discussed in Section 2. In Section 3, a simulated dataset is applied and the performance of the proposed index is evaluated. Our concluding remarks and some future researches are given in the final section.

2- Proposed method

As mentioned before, to evaluate performance of the signal-response system, first a measurement index should be proposed. For example, Taguchi (1987), proposed SNR. This index considers dispersion and sensitivity of responses. Figure 2 depicts the processes of the Taguchi procedure. In this method as the first phase, variability of response decreases, and then in the second phase, the slope is brought to the target.

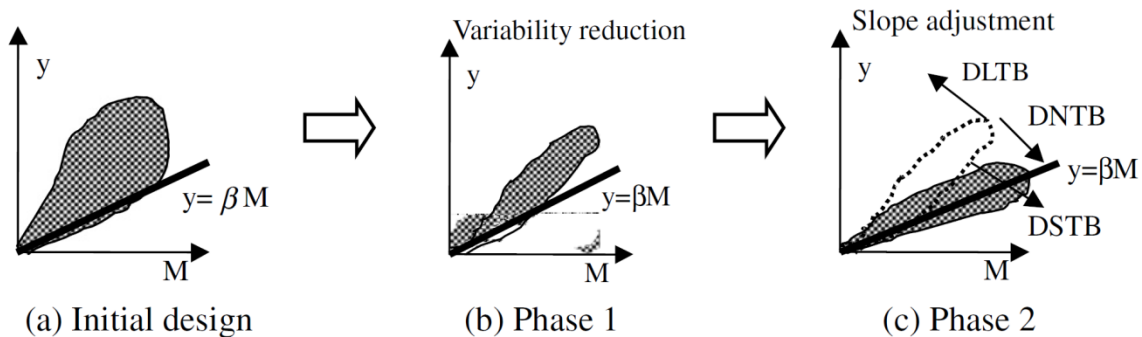


Figure 2. Taguchi procedure for dynamic problem (Extracted from Chang, 2008)

The target may be dynamic nominal the best (DNTB), dynamic larger the better (DLTB) or dynamic smaller the better (DSTB). In this paper, we propose a method based on process capability to evaluate performance of control factors combination. In a signal-response system, we assume there are n correlated responses in each level of signal factor. Furthermore, there are p levels for signal factor. So $n \times p$ response values should be considered.

The advantage of using process capability (PC) index in the signal-response system is that the n correlated responses in each level of signal factor can be converted to one response (process capability) which considers dispersion and sensitivity of responses without losing any properties of observations such as correlation. So

there are $1 \times p$ independent process capability indices and the optimization procedure can be done by using new independent responses.

Suppose that there are two responses for a signal-response system. It is obviously that in each level of signal factor, responses should have less variability (dispersion) and distances from the targets (sensitivity). As illustrated in Figure 3, PC index try to find a parameter combination which leads to decreasing in variability and distance from target, simultaneously. In Figure 3, observed data produce a region called the process region. The upper and lower bound of responses produce a region called the tolerance region. Observations which have less variability and less distance from the target have a larger PC value. Subsequently, with modeling PCs according to the control factors, the control factors setting, which has the maximum sensitivity and minimum dispersion are obtained.

Another advantage of using PC is that it considers the effects of noise factors on responses by considering the variation of responses. Hence, proposed combination of control factors in each level of signal factor is robust to the noise factor. In this paper, just the first component of MPC_{NCV} is considered because it includes both dispersion and sensitivity aspects.

The proposed index has the following three steps: In step 1 MPC_{NCV} of responses in each level of the signal factor is calculated. Then, in step 2, a general model for MPC_{NCV} according to the control factors is computed. Note that the numbers of MPC_{NCV} models are equal to the levels of signal factor. If the levels of signal factor increase, the generated models increases and then, there are more proposed setting for considered system. Consequently more improvement for responses is available. In step 3 by an optimization method, the best levels of the control factors are obtained.

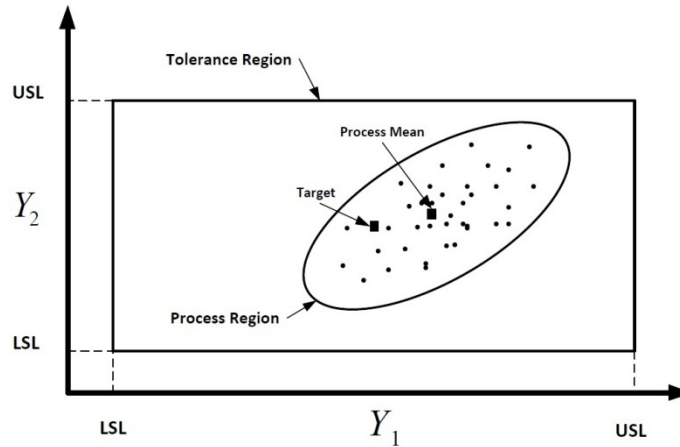


Figure 3. Presentation of tolerance and process regions

3- Numerical Example

In this section a simulated example is applied to show the performance of the proposed method. First a data set is simulated. In the simulation process, a predefined relationships between responses and control factors are used. Then, in the step 1, the process capability (MPC_{NCV}) index for each treatment and in every level of signal factors is calculated and in the step2, a regression equation between MPC_{NCV} and control factors is constructed for each level of signal factors. Finally, by optimizing regression equation, best level of control factors are estimated. Also the improvement of optimization will be compared by the best treatment of design matrix.

We investigate the proposed index by simulated dataset. Suppose that there are two responses Y_1 and Y_2 , five controllable factors x_1, x_2, x_3, x_4, x_5 , one noise factor N and a signal factor M . According to the Equation (2), relationship function between Y_1 and signal factor M is expressed as:

$$Y_1 = \alpha_1 + \beta_1 M + \varepsilon$$

In numerical example, it is assumed that α_1 and β_1 are as follows respectively:

$$\begin{aligned} \alpha_1 = & 2A + 3B + C - D + 0.3E + \\ & 0.2AB + 0.5AC - 0.7AD + \\ & 0.3BD - 0.2CD + 0.1DE + \\ & 0.4AN + 0.3BN - 0.5DN + 0.6EN \end{aligned}$$

$$\begin{aligned} \beta_1 = & A + 2B - C - 0.1D + 0.2E + \\ & 0.4AC + 0.3AD + 0.1BC + 0.5BE - \\ & 0.2CD - 0.7CE + 0.2DE + 0.1AN + \\ & 0.5BN - 0.5CN + 0.2EN \end{aligned}$$

Furthermore, relationship between Y_2 and signal factor M is expressed as:

$$Y_2 = \alpha_2 + \beta_2 M + \varepsilon$$

Also it is assumed that α_2 and β_2 is:

$$\begin{aligned} \alpha_2 = & 3A + 2B + C + D + 0.2E + \\ & 0.7AB + 0.2AC + 0.3AD + 0.1AE - \\ & 0.4BC + 0.1CD - 0.3CE + 0.1DE + \\ & 0.2AN - 0.2BN + 0.3DN + 0.7EN \end{aligned}$$

$$\begin{aligned} \beta_2 = & -A + B + 2C - D + 0.7E - \\ & 0.4AB + 0.2AC + 0.8AD + 0.1BC + \\ & 0.3BD + 0.2CD + 0.7CE + 0.4DE - \\ & 0.2AN + 0.3BN + 0.2CN + 0.1EN \end{aligned}$$

ε is a random variable with bivariate normal distribution and $E(\varepsilon) = (0, 0)$ and variance-covariance matrix

$$\Sigma = \begin{pmatrix} 1 & 0.1 \\ 0.1 & 1 \end{pmatrix}. \text{ Signal factor } M \text{ has three levels of } 0.1, 0.2 \text{ and } 0.3.$$

Figure 4 depicts the assumed relationship between Y_1 , Y_2 and M and target lines for them with a random setting for control factors. Target values for each level of the signal factor for Y_1 are 0.1, 0.2 and 0.1 respectively. And these values for Y_2 are -0.2, -0.1, 0.1.

To consider an example, a combined array with resolution VI is used by a 2_{VI}^{6-1} design (see Table 1). 10 replicates for Y_1 and Y_2 in each level of signal factor are simulated.

Table 1. Combined array 2_{VI}^{6-1} for the numerical example

No.	A	B	C	D	E	N
1	-1	-1	-1	-1	-1	-1
2	1	-1	-1	-1	-1	1
3	-1	1	-1	-1	-1	1
4	1	1	-1	-1	-1	-1
5	-1	-1	1	-1	-1	1
6	1	-1	1	-1	-1	-1
7	-1	1	1	-1	-1	-1
8	1	1	1	-1	-1	1
9	-1	-1	-1	1	-1	1
10	1	-1	-1	1	-1	-1
11	-1	1	-1	1	-1	-1
12	1	1	-1	1	-1	1
13	-1	-1	1	1	-1	-1
14	1	-1	1	1	-1	1
15	-1	1	1	1	-1	1
16	1	1	1	1	-1	-1
17	-1	-1	-1	-1	1	1
18	1	-1	-1	-1	1	-1
19	-1	1	-1	-1	1	-1
20	1	1	-1	-1	1	1
21	-1	-1	1	-1	1	-1
22	1	-1	1	-1	1	1
23	-1	1	1	-1	1	1
24	1	1	1	-1	1	-1
25	-1	-1	-1	1	1	-1
26	1	-1	-1	1	1	1
27	-1	1	-1	1	1	1
28	1	1	-1	1	1	-1
29	-1	-1	1	1	1	1
30	1	-1	1	1	1	-1
31	-1	1	1	1	1	-1
32	1	1	1	1	1	1

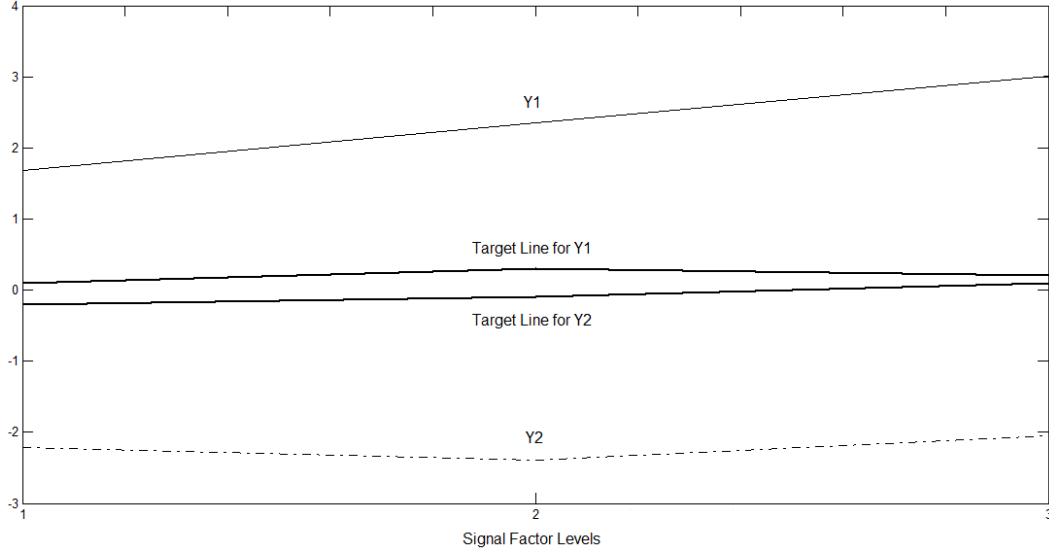


Figure 4. Relationship between Y_1 , Y_2 and M with a random setting for control factors with simulated data

Steps of the proposed procedure are as follows:

Step 1. Calculate the MPC_{NCV} index for each level of signal factor M in each treatment.

The mean and variance-covariance matrix of Y_1 and Y_2 are computed and the value of first component of MPC_{NCV} for each level of signal factor M is reported in table 2. Note that the values of MPC_{NCV} should be maximized. Hence, the factor setting which leads to maximum value of MPC_{NCV} in each level of signal factor is desired. For example treatment 31 which contains larger value of MPC_{NCV} in each level of signal factor can be a preferable factor setting.

Step 2. Construct the regression function between controllable factors and the MPC_{NCV} .

In this step according to the values of MPC_{NCV} , a general linear model for each level of signal factor is generated. The constructed models of MPC_{NCV} for the numerical example are as follows:

$$PC_1 = -0.09A + 0.09B - 0.25AB - 0.07AD - 0.07AE + 0.09BD + 0.46 \quad \text{where } M = 0.1$$

$$PC_2 = -0.09A + 0.09B - 0.24AB - 0.1AC - 0.06AD + 0.1BD - 0.08BE + 0.44 \quad \text{where } M = 0.2$$

$$PC_3 = -0.08A + 0.08B - 0.27AB + 0.09BD - 0.07CE + 0.48 \quad \text{where } M = 0.3$$

Where PC_i is the process capability regression function in the i th level of signal factor.

Step 3. Optimize the problem in each level of signal factor.

In this step, each of the former regression models is optimized as an objective function. The process capability model in other levels of signal factor is considered as constraints and should be capable. Hence, their values should be at least 1. For example, the optimization model for PC_1 is expressed as:

$$\text{Max} : -0.09A + 0.09B - 0.07AD - 0.07AE + 0.09BD + 0.46$$

$$\text{S.T} : -0.09A + 0.09B - 0.24AB - 0.1AC$$

$$-0.06AD + 0.1BD - 0.08BE + 0.44 > 1$$

$$-0.08A + 0.08B - 0.27AB + 0.09BD - 0.07CE + 0.48 > 1$$

$$-1 \leq A \leq 1$$

$$-1 \leq B \leq 1$$

$$-1 \leq C \leq 1$$

$$-1 \leq D \leq 1$$

$$-1 \leq E \leq 1$$

Upper and lower bounds for control factors are 1 and -1, respectively.

Table 2. The experimental results and calculated MPC_{NCV} index for the numerical example

No.	M1=0.1			M2=0.2			M3=0.3			M1=0.1	M2=0.2	M3=0.3
	Mean (Y1)	Mean (Y2)	Var.-Cov. Matrix	Mean (Y1)	Mean (Y2)	Var.-Cov. Matrix	Mean (Y1)	Mean (Y2)	Var.-Cov. Matrix	MPC(NCV)	MPC(NCV)	MPC(NCV)
1	2.25	-1.83	1.12 0.60 0.60 0.96	2.87	-2.04	1.51 0.46 0.46 1.21	2.61	-1.53	1.15 0.27 0.27 0.63	0.14	0.14	0.14
2	2.18	0.70	0.45 -0.12 -0.12 1.23	1.47	0.97	0.87 0.07 0.07 1.22	1.12	1.27	0.44 -0.17 -0.17 0.87	0.42	0.45	0.35
3	-1.49	-2.89	0.92 0.35 0.35 1.38	-1.38	-3.01	0.50 0.08 0.08 0.70	-0.81	-3.40	1.56 0.05 0.05 1.13	0.34	0.35	0.45
4	-3.53	-3.63	1.02 0.48 0.48 1.26	-4.18	-2.40	0.63 0.01 0.01 0.83	-5.05	-1.79	0.70 -0.01 -0.01 0.92	0.41	0.29	0.25
5	0.91	-1.68	1.08 -0.22 -0.22 2.58	0.62	-2.23	1.23 0.14 0.14 0.46	0.09	-3.16	0.37 0.05 0.05 0.97	0.16	0.18	0.17
6	11.32	5.56	1.30 -0.09 -0.09 0.94	10.77	5.57	1.15 -0.44 -0.44 0.41	10.78	5.83	1.69 0.04 0.04 0.46	0.98	0.14	1.19
7	-5.94	-7.26	0.63 0.11 0.11 1.86	-6.09	-7.55	0.86 0.27 0.27 1.15	-6.15	-7.18	0.66 0.01 0.01 0.59	0.90	0.92	1.28
8	-6.04	-1.54	1.33 0.53 0.53 1.34	-5.56	-0.97	0.96 0.07 0.07 0.69	-6.28	-2.54	0.86 0.05 0.05 0.34	0.14	0.14	0.14
9	-5.75	-3.40	0.74 0.46 0.46 1.14	-5.78	-3.87	0.37 -0.14 -0.14 0.91	-6.20	-4.64	1.69 0.21 0.21 0.74	0.14	0.14	0.14
10	1.59	-2.85	0.69 -0.23 -0.23 0.75	1.04	-2.24	0.44 0.03 0.03 1.97	1.47	-1.52	0.58 0.36 0.36 0.62	0.31	0.36	0.23
11	5.54	2.3	0.94 0.14 0.14 1.44	6.15	0.99	1.10 -0.23 -0.23 0.65	6.12	1.50	0.76 0.12 0.12 1.22	0.79	0.92	0.70
12	-7.70	-2.58	0.45 -0.20 -0.20 0.92	-7.85	-3.71	1.02 0.22 0.22 0.53	-8.61	-3.26	0.75 -0.20 -0.20 0.33	0.51	0.50	0.56
13	4.69	7.70	0.63 0.38 0.38 1.76	5.25	6.76	0.84 -0.33 -0.33 0.77	5.70	6.64	0.59 0.08 0.08 0.99	0.14	0.14	0.14
14	-1.82	5.15	0.77 -0.15 -0.15 1.01	-1.75	5.08	0.75 0.55 0.55 1.42	-2.86	5.58	0.69 0.49 0.49 1.78	0.34	0.33	0.23
15	1.96	-0.55	1.49 1.29 1.29 1.88	1.93	-0.14	0.63 -0.26 -0.26 0.78	1.69	-0.39	1.33 0.05 0.05 0.11	0.89	1.26	0.97
16	4.37	7.95	1.10 0.27 0.27 0.63	5.04	7.93	1.65 -0.10 -0.10 0.41	5.24	7.91	1.48 -0.41 -0.41 1.33	0.14	0.14	0.14
17	7.08	10.63	1.63 0.21 0.21 0.37	8.36	10.69	0.25 -0.04 -0.04 0.72	8.72	11.23	2.08 -0.37 -0.37 0.92	0.24	0.26	0.27
18	-6.16	-5.26	1.57 -0.01 -0.01 1.18	-6.58	-5.44	0.98 0.03 0.03 0.99	-6.33	-4.92	1.00 0.30 0.30 1.10	0.90	1.27	0.91
19	-1.63	-5.00	0.82 0.35 0.35 0.80	-0.98	-4.95	0.97 0.23 0.23 1.18	-1.55	-5.14	1.31 0.15 0.15 0.35	1.05	0.67	0.93
20	4.30	-1.65	1.34 -0.15 -0.15 0.65	3.88	-0.74	1.43 0.62 0.62 1.61	3.80	0.07	1.03 0.45 0.45 1.50	0.14	0.14	0.14
21	-3.59	2.75	0.42 0.12 0.12 1.83	-3.29	2.23	1.67 -0.05 -0.05 1.14	-3.23	1.85	0.92 0.60 0.60 0.75	0.47	0.50	0.42
22	1.69	-0.37	0.79 0.12 0.12 0.53	2.00	0.39	1.75 0.11 0.11 0.95	2.38	0.13	0.43 -0.18 -0.18 2.16	0.40	0.54	0.57
23	-4.33	3.28	0.52 0.00 0.00 0.49	-4.84	2.4	1.21 0.63 0.63 1.05	-5.90	2.93	0.77 -0.03 -0.03 1.22	0.54	0.56	0.63
24	9.63	5.10	0.99 0.19 0.19 1.21	10.74	5.16	0.84 0.26 0.26 0.51	11.05	4.37	0.65 0.23 0.23 0.91	0.14	0.14	0.14
25	4.78	2.25	1.56 -0.08 -0.08 0.66	4.32	2.20	0.32 -0.04 -0.04 0.42	3.70	2.13	1.59 1.02 1.02 1.59	0.27	0.14	0.20
26	-6.64	-0.33	0.33 0.01 0.01 0.83	-7.77	-0.39	1.32 -0.28 -0.28 1.13	-8.56	0.00	0.65 -0.09 -0.09 0.43	0.52	0.59	0.85
27	-0.78	-5.48	1.12 -0.23 -0.23 0.82	-0.06	-5.39	1.93 0.09 0.09 1.13	0.58	-5.03	1.04 -0.02 -0.02 1.73	1.27	0.97	1.08
28	-10.03	-6.39	0.63 0.24 0.24 1.05	-10.53	-7.75	0.69 0.12 0.12 0.43	-10.77	-7.70	1.90 0.05 0.05 2.08	0.15	0.14	0.14
29	-2.06	-1.63	0.77 0.50 0.50 2.00	-1.89	-1.58	0.79 -0.09 -0.09 0.33	-2.28	-1.30	0.59 0.41 0.41 1.54	0.15	0.17	0.14
30	1.72	4.05	1.22 0.56 0.56 0.60	2.29	3.68	0.81 0.53 0.53 1.98	2.77	3.42	0.83 0.95 0.95 3.14	0.40	0.39	0.35
31	-3.76	-4.96	0.50 -0.16 -0.16 0.85	-3.87	-5.01	0.15 0.18 0.18 0.64	-4.83	-5.00	2.30 0.48 0.48 1.14	1.27	1.21	1.27
32	9.19	3.22	1.22 -0.03 -0.03 0.95	9.28	3.44	0.87 0.46 0.46 1.71	10.44	3.31	1.00 -0.04 -0.04 1.16	0.14	0.14	0.14

By optimizing the above model, the best levels of the control factors are calculated and named as model 1 optimal setting.

$$A = -1, B = 1, C = 1, D = 1, E = 0$$

$$PC_1 = 1.05, PC_2 = 1.12, PC_3 = 1$$

Similar to model of PC_1 , the optimizing models of PC_2 and PC_3 are conducted and the best levels of optimizing model PC_2 and PC_3 is obtained as follows. We call them as model 2 and 3 optimal settings respectively:

$$A = -1, B = 1, C = 1, D = 1, E = -0.714$$

$$PC_1 = 1, PC_2 = 1.18, PC_3 = 1.05$$

$$A = -1, B = 1, C = -0.1, D = 1, E = 0.125$$

$$PC_1 = 1.06, PC_2 = 1, PC_3 = 1$$

According to the proposed levels for control factors, Y_1 and Y_2 have been simulated 1000 times for three models. Note that the noise factor changes randomly between its upper and lower values. In addition, according to the results obtained by initial simulations, we select treatments 7, 15, 18, 19, 27 and 31 from experimental design which have the best results of process capability in simulated data. Then, the mean and variance of responses in optimal settings of model 1, 2 and 3 are compared with mean and variance of responses in selected treatments respect to the targets to show the better performance of the proposed method.

As Table 3 shows, the variance of responses for the selected treatments is a large value and the distance of means from targets are not acceptable. Hence, these treatments do not produce an efficient response with minimum variation and distance from the target.

The optimal results of the models 1, 2 and 3 have been used to calculate the response mean and variance in a continuous space. The results have been shown in Table 4. Mean and variance of Y_1 and Y_2 indicate that setting of control factors obtained from the proposed method has less variance value than the selected treatments and the method is capable to bring the mean of observations to the targets.

By comparing the results of Tables 3 and 4, we can conclude that distance between responses' means and targets and variation of responses decrease considerably based on the proposed method. To demonstrate this fact, Table 5, 6 and 7 give a comparison between the selected treatments (7, 15, 18, 19, 27, and 31) and models 1, 2 and 3 optimal setting which reported before.

In these tables the percent of improvement for responses in each model are compared with the best selected treatments. The results of proposed index in Equation (6) show efficiency of the proposed approach in dynamic multi-response robust optimization.

$$IR_{k,bt}^{i,m} = \frac{\left| Y_{bt}^{i,m} - T^{i,m} \right| - \left| Y^{*i,m} - T^{i,m} \right|}{\left| Y_{bt}^{i,m} - T^{i,m} \right|} \quad (6)$$

where $IR_{k,bt}^{i,m}$ is the improvement rate for the i th response in m th level of signal factor between k th model and b th treatment, $Y_{bt}^{i,m}$ is the best observed i th mean (variance) response value in m th level of signal factor for b th treatment, $Y^{*i,m}$ is the optimal mean (variance) value of i th response in m th signal level obtained by the proposed models and the $T^{i,m}$ is the mean (variance) target of i th response of m th signal level. Note that the target value of variance for responses in the entire signal factor levels is set to zero level.

Table 3. Mean and variance of selected treatments between 32 existing ones according to the MPC_{NCV} index value for simulated data

	Signal factor levels	Responses	Mean	Variance		Signal factor levels	Responses	Mean	Variance
Treatment 7	M1	Y_1	1.096	0.105	Treatment 19	M1	Y_1	2.438	0.203
		Y_2	3.142	0.259			Y_2	1.956	0.097
	M2	Y_1	1.208	0.103		M2	Y_1	3.069	0.218
		Y_2	2.717	0.223			Y_2	1.818	0.106
	M3	Y_1	1.122	0.109		M3	Y_1	3.486	0.241
		Y_2	2.328	0.222			Y_2	1.780	0.098
Treatment 15	M1	Y_1	1.343	0.214	Treatment 27	M1	Y_1	1.528	0.107
		Y_2	1.159	0.146			Y_2	-0.091	0.129
	M2	Y_1	1.629	0.236		M2	Y_1	2.164	0.089
		Y_2	1.078	0.147			Y_2	0.049	0.138
	M3	Y_1	1.671	0.244		M3	Y_1	2.615	0.103
		Y_2	1.078	0.126			Y_2	0.229	0.147
Treatment 18	M1	Y_1	1.206	0.230	Treatment 31	M1	Y_1	1.542	0.096
		Y_2	1.044	0.152			Y_2	0.683	0.137
	M2	Y_1	1.266	0.247		M2	Y_1	1.577	0.101
		Y_2	1.551	0.133			Y_2	1.411	0.152
	M3	Y_1	1.154	0.230		M3	Y_1	1.431	0.099
		Y_2	2.195	0.124			Y_2	2.218	0.159

Table 4. Results from models

	Levels of signal factor	Responses	Mean	Variance
Model 1	M1	Y_1	0.190	0.110
		Y_2	-0.448	0.096
	M2	Y_1	0.240	0.098
		Y_2	0.068	0.092
	M3	Y_1	0.002	0.121
		Y_2	0.652	0.100
Model 2	M1	Y_1	-0.793	0.188
		Y_2	-1.317	0.091
	M2	Y_1	-0.818	0.187
		Y_2	-0.864	0.123
	M3	Y_1	-1.054	0.173
		Y_2	-0.416	0.123
Model 3	M1	Y_1	0.311	0.129
		Y_2	-1.109	0.100
	M2	Y_1	0.544	0.116
		Y_2	-0.803	0.089
	M3	Y_1	0.637	0.146
		Y_2	-0.519	0.108

Consider Model 1. For example, according to the Table 4, mean of Y_1 in the first level of signal factor is 0.190. This value for treatment 7 is 1.096 (see Table 3). So improvement rate for Y_1 of model 1 respect to treatment 7 in first level of signal factor calculated as follows:

$$IR_{1, tr=7}^{1,1} = \frac{|1.096 - 0.1| - |0.190 - 0.1|}{|1.096 - 0.1|} = 0.91$$

Note that in some levels of signal factor for responses, the distance between mean and targets is increased. For example, mean of Y_2 in the first level of the signal factor in model 1 is -0.448 . This value for treatment 27 is -0.091 . Hence, its improvement rate is -1.28 . Tables 5, 6 and 7 show that in most of cases, the obtained improvement rates are positive and large.

Table 5. Comparison between mean and variance of the best selected treatments respect to the model 1 according to the proposed *IR* index

	Mean					
	M1		M2		M3	
	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2
Treatment 7	0.91	0.93	0.96	0.94	0.90	0.75
Treatment 15	0.93	0.82	0.97	0.86	0.94	0.44
Treatment 18	0.92	0.80	0.96	0.90	0.91	0.74
Treatment 19	0.96	0.88	0.99	0.91	0.97	0.67
Treatment 27	0.94	-1.28	0.98	-0.13	0.96	-3.28
Treatment 31	0.94	0.72	0.97	0.89	0.93	0.74

	Variance					
	M1		M2		M3	
	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2
Treatment 7	-0.05	0.63	0.05	0.59	-0.10	0.55
Treatment 15	0.49	0.34	0.59	0.37	0.51	0.20
Treatment 18	0.52	0.37	0.60	0.31	0.48	0.20
Treatment 19	0.46	0.01	0.55	0.13	0.50	-0.02
Treatment 27	-0.03	0.26	-0.10	0.33	-0.17	0.32
Treatment 31	-0.14	0.30	0.03	0.39	-0.21	0.37

Table 6. Comparison between mean and variance of the best selected treatments respect to the model 2 according to the proposed *IR* index

	Mean					
	M1		M2		M3	
	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2
Treatment 7	0.10	0.67	-0.01	0.73	-0.13	0.77
Treatment 15	0.28	0.18	0.29	0.35	0.27	0.47
Treatment 18	0.19	0.10	0.04	0.54	-0.10	0.75
Treatment 19	0.62	0.48	0.65	0.60	0.66	0.69
Treatment 27	0.37	-9.24	0.48	-4.13	0.54	-3.01
Treatment 31	0.38	-0.27	0.26	0.49	0.13	0.76

	Variance					
	M1		M2		M3	
	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2
Treatment 7	-0.79	0.65	-0.81	0.45	-0.58	0.45
Treatment 15	0.12	0.38	0.21	0.16	0.29	0.02
Treatment 18	0.18	0.40	0.24	0.08	0.25	0.01
Treatment 19	0.07	0.06	0.14	-0.16	0.28	-0.25
Treatment 27	-0.76	0.29	-1.10	0.11	-0.67	0.17
Treatment 31	-0.96	0.34	-0.86	0.19	-0.74	0.23

Table 7. Comparison between mean and variance of the best selected treatments respect to the model 3 according to the proposed *IR* index

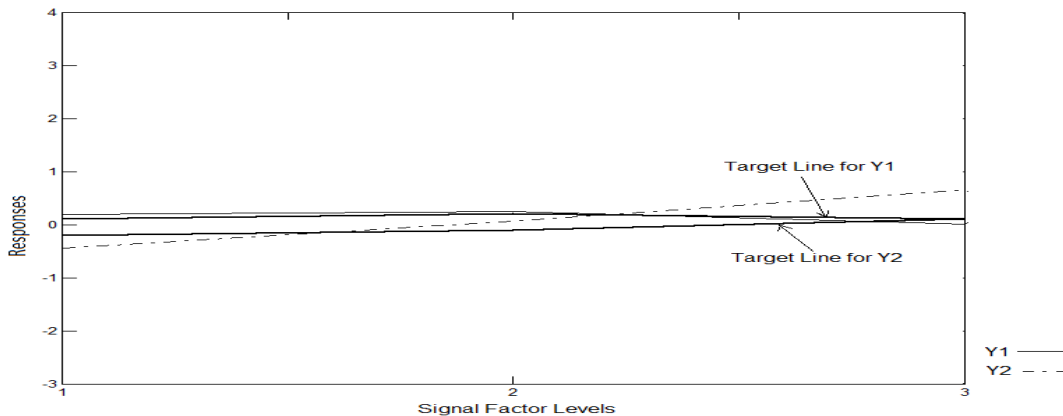
	Mean					
	M1		M2		M3	
	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2
Treatment 7	0.79	0.73	0.66	0.75	0.47	0.72
Treatment 15	0.83	0.33	0.76	0.40	0.66	0.37
Treatment 18	0.81	0.27	0.68	0.57	0.49	0.70
Treatment 19	0.91	0.58	0.88	0.63	0.84	0.63
Treatment 27	0.85	-7.33	0.82	-3.72	0.79	-3.81
Treatment 31	0.85	-0.03	0.75	0.53	0.60	0.71

	Variance					
	M1		M2		M3	
	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2
Treatment 7	-0.23	0.61	-0.12	0.60	-0.33	0.51
Treatment 15	0.40	0.31	0.51	0.39	0.40	0.14
Treatment 18	0.44	0.34	0.53	0.33	0.37	0.13
Treatment 19	0.37	-0.03	0.47	0.16	0.39	-0.10
Treatment 27	-0.21	0.22	-0.30	0.36	-0.41	0.27
Treatment 31	-0.35	0.27	-0.15	0.41	-0.47	0.32

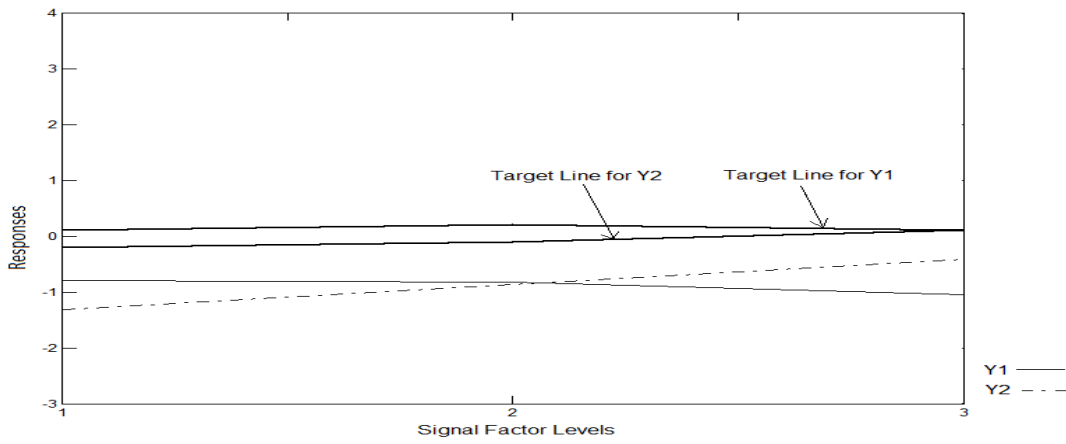
Table 8 includes responses' mean and optimal setting of control factors in three models. Figure 5 depicts the mean of Y_1 and Y_2 according to the optimal setting. As Figure 5 shows, profiles of Y_1 and Y_2 have less distance from target in randomly levels of the noise factor for three models. So, optimum setting which are obtained from three models could decrease distance of mean from the target in all of the signal factor levels.

Table 8. Optimum setting obtained from three models

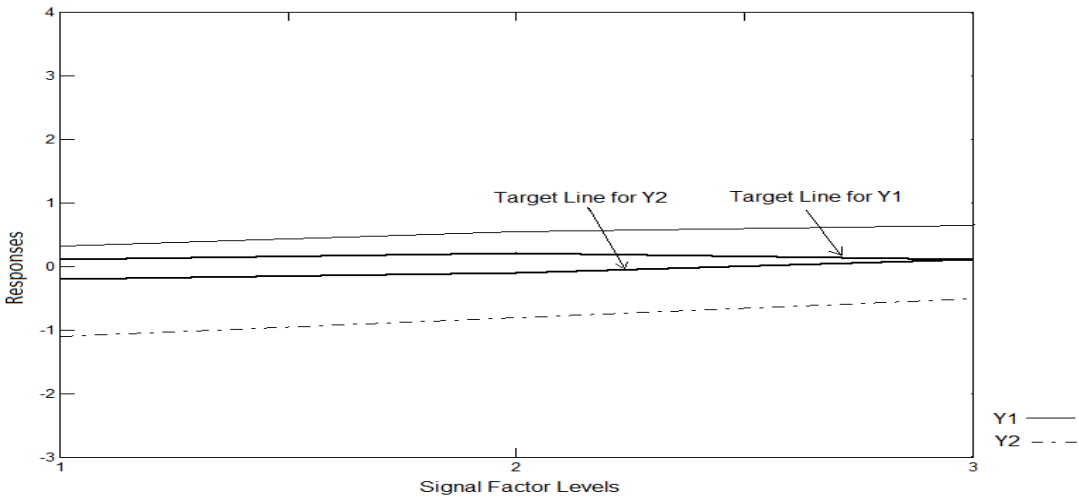
	Levels of Control factors					Mean Y_1			Mean Y_2		
	A	B	C	D	E	M1=0.1	M2=0.2	M3=0.3	M1=0.1	M2=0.2	M3=0.3
Model 1	-1	1	1	1	0	0.190	0.240	0.002	-0.448	0.068	0.652
Model 2	-1	1	1	1	-0.714	-0.793	-0.818	-1.054	-1.317	-0.864	-0.416
Model 3	-1	1	-0.1	1	0.125	0.311	0.544	0.637	-1.109	-0.803	-0.519



(a)



(b)



(c)

Figure 5. Profile of Y_1 and Y_2 according to the proposed setting (a) Model 1 (b) Model 2 (c) Model 3

Figure 6 presents a comparison between process mean, targets and variations of model 1 as the best model and treatment 27 as the best initial treatment. In the mentioned figure, Y_{ij} is mean for i th response in j th level of signal factor. Also V_{ij} is variance for i th response in j th signal factor level.

Results show the efficiency of the proposed method to obtain the setting of control factors. However, distance between process mean and its target and variance for Y_2 are increased a little, but variation and distance between process mean and the target for Y_1 are decreased considerably.

4- Conclusions

In this paper, a new method based on the process capability index is proposed to optimize multivariate signal-response systems. By using multivariate process capability index, location and dispersion of responses can be considered with a single measurement (PC) in each level of the signal factor. Finally, PC is modeled as a general linear model in each level of signal factor according to the control factors. Optimum levels of control factors are obtained by solving optimization models of control factors in each level of signal factor. The

simulated results show the efficiency of the proposed approach considering the improvement ratio index. Determination of Pareto optimal solutions for all levels of signal factor using multi objective optimization method can be studied as a related future research.

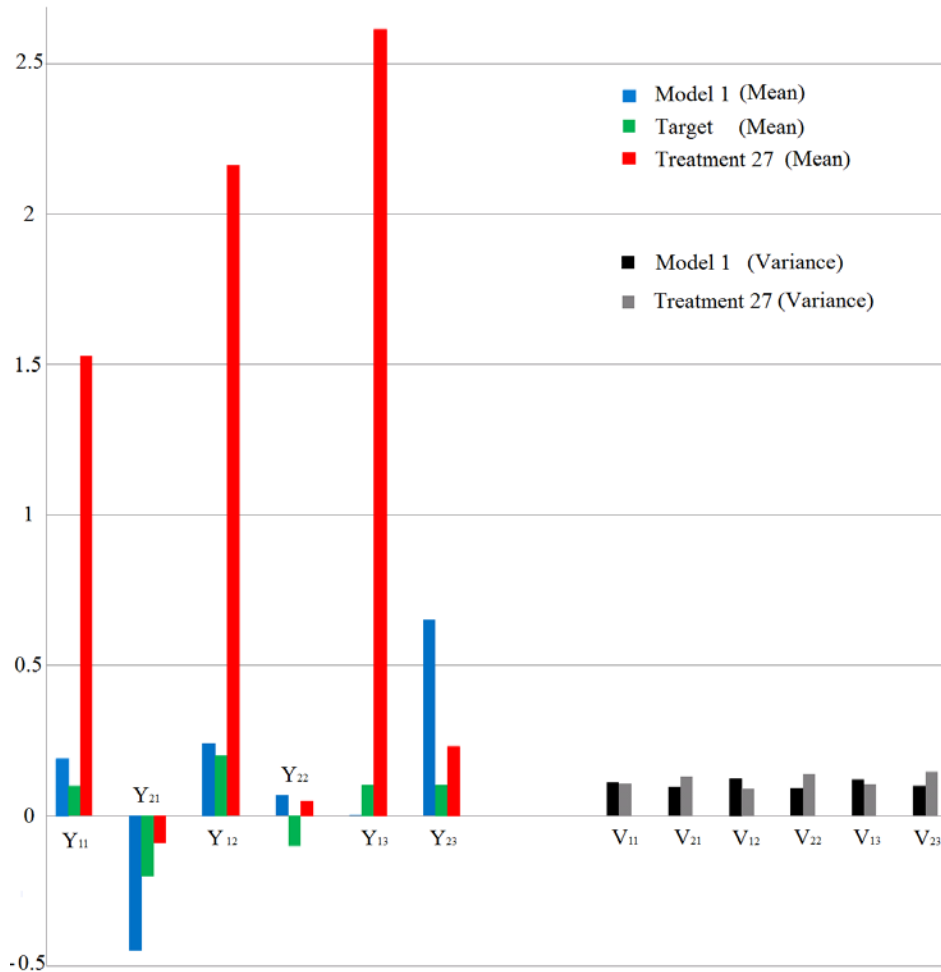


Figure 6. Comparison between process mean and targets of Y_1 and Y_2 as well as variance of them for model 1 and treatment 27

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