

## An Analytical Approach for Single and Mixed-Model Assembly Line Rebalancing and Worker Assignment Problem

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### Abstract

In this paper, an analytical approach is used for assembly line rebalancing and worker assignment for single and mixed-model assembly lines, based on a heuristic-simulation algorithm. This approach helps managers to select a better marketing strategy while different combination of demands is suitable. Furthermore, it can be used as a guideline to know which worker assignment is better for each combination. We show the efficiency of the proposed approach for single and mixed-model assembly lines using different benchmarked standard test problems with different number of tasks, stations, skilled workers and demands. Comparisons show that the heuristic-simulation algorithm is faster than the GAMS software; and its results are rather optimum, or very close to the optimum values.

**Keywords:** Assembly line rebalancing problem, worker assignment, heuristic-simulation algorithm, task time dependent on the worker's skill.

### 1-Introduction

Nowadays, high variety of products and different customer demands has led many factories to produce or assemble several models of one basic product. Producing mixed-models can reduce inventories, eliminate transfer costs among models and meet ever-changing customer demands more efficiently (Hu et al. 2011). But, sometimes demand variations cause factories and managers to have several important problems. For example, they should know how they can assign the tasks and workers to the stations so the cycle time is reduced and the products are delivered to the customers sooner than their competitors; and the exact time of using a marketing strategy to encourage the customers to buy a special model of one product. The problems such as these cases motivated the academic researchers, engineers and managers to consider the assembly line balancing, rebalancing and worker assignment important subjects for their studies.

Assembly Line Balancing Problem (ALBP) means the structure of task assignments to the stations. This problem was first introduced in 1955. Because the importance of the subject, several extensions were considered to make better decisions in real-world situations. Several reviewing papers in this field can be studied, such as Boysen, Fliedner and Scholl(2007); Hu et al. (2011); Battaïa and Dolgui (2013) and Kumar and Mahto (2013). Most of researchers in this field assume that some tasks contain a fixed task time which is independent from the worker's skill.

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But, this assumption is not really applicable when there are human workers in the assembly lines, because a high skill worker usually executes a special task faster than a low skill worker. Therefore, several researchers investigated the impact of worker assignment in assembly line balancing problems. For example, Miralles et al. (2007) verified ALBP for disabled workers.

Moreover, Miralles et al. (2008) defined a mathematical model for worker assignment and ALBP to minimize the cycle time. Their model assigned the tasks to the workers, and the workers to the stations. They presented a station-oriented, depth-first branch and bound approach for solving very small cases while every worker could be assigned to only one workstation.

After one year, Costa and Miralles (2009) proposed a mixed integer linear model and a heuristic decomposition method to solve Job rotation in assembly lines that employ disabled workers. Chen et al. (2012) developed a grouping genetic algorithm for ALBP of sewing lines with different labor skill levels where the number of workstations was not fixed.

Oliveira et al. (2012) presented a case study involving a problem of mixed-model rebalancing of automotive assembly line. They proposed a procedure based on the branch and bound algorithm and compared the obtained results with a heuristic method. Their solely focus was on assembly line rebalancing problem and they did not pay attention to worker assignment.

Manavizadeh et al. (2013) applied a multi-objective simulated annealing algorithm for a mixed-model assembly U-line balancing problem considering human efficiency where permanent and temporary operators were available, and they could work in regular and overtime periods.

Vilà and Pereira (2013) solved worker assignment and ALBP by an exact enumeration algorithm. After one year, again Vilà and Pereira (2014) used a branch and bound remember algorithm with a cyclic best-first search strategy for their earlier problem.

Egilmez, Erenay and Süer (2014) addressed to a stochastic skill-based manpower allocation problem in a cellular manufacturing system for a specified risk level, where the task times and customer demand had a normal distribution. Recently, Moreira, Miralles and Costa (2015) presented mathematical models and heuristic methodologies for ALBP and worker assignment problem for disabled workers.

Since the simple assembly line balancing problem is NP-hard (Karp (1972)) and by increasing the size of problem, the necessary time to obtain the optimal results increase exponentially, using heuristic or metaheuristic algorithms for solving them are very common. Furthermore, if one considers ALBP and worker assignment, it will be more time consuming than solving only ALBP. Moreover, to the best knowledge of authors, there is no paper that investigates a heuristic-simulation approach for single or mixed-model assembly line balancing or rebalancing problems and worker assignments considering the workers' skills. Also there is no paper that presents an analytical approach for this problem. Therefore, this article presents an analytical approach based on a heuristic-simulation algorithm for single and mixed-model assembly line rebalancing and worker assignment problems with different combinations of demand.

The remaining sections of this paper are organized as follow. Section 2 presents the problem definition, including assumptions, mathematical model and using lower bound and rules in this paper. Section 3 is related to a simulation-heuristic algorithm. Section 4 investigates different test problems, analysis, and discussions. Finally, in the last section, the conclusions and future research's direction are presented.

## **2-Problem Definition**

In this paper, we have one or  $m$  types of one basic product that should be assigned on a one-sided straight assembly line. In this line, all tasks are done by human workers with different skill levels (low, medium, high) and all tasks can be done by each worker. Furthermore, it is possible to have more than one worker in each skill category. For example, we may have two high-skill workers with similar speeds for doing the tasks.

In rebalancing assembly line, we know the structure of the line such as the number of stations and number of available workers with the same skill levels, and we are going to change workers and task assignments to reduce the cycle time and deliver the products sooner. This problem is more highlighted for the conditions that there are variations in demand. Because of changing these values if we keep the earlier worker and task

assignment we may obtain another cycle time. Therefore, paying attention to these variations will be necessary to make better decisions for rebalancing the assembly line.

Following assumptions is considered in this paper:

1. Single or different models of one basic product with similar production characteristics are produced on a one-sided straight flexible assembly line.
2. Combined precedence diagram is known.
3. Each task is permitted to assign only once to only one station.
4. Task time is dependent on the worker who executes the task.
5. The portion demand value of each model in the production planning horizon is given.
6. There are several workers with different skill levels.
7. Number of stations (operators) is known.

## 2-1- Mathematical model

The mathematical model for minimizing the cycle time for rebalancing assembly line and worker assignment with the following notations is presented as below:

Indices

$i, h$	Task
$j, g$	Station
$m$	Product model
$k, q$	Worker's skill level

Sets

$K$	Set of skills (low, medium, high)
$I$	Set of tasks in the combined precedence diagram
$J$	Set of stations $J = \{1, 2, \dots, j, \dots, NS\}$
$W(i)$	Set of immediate predecessors of task $i$
$W_0$	Set of tasks which do not have any immediate predecessors

Parameters

$n$	Number of tasks
$M$	Number of models (for single-model: $M=1$ )
$NS$	Number of stations
$t_{imk}$	Task time $i$ for model $m$ when a worker with skill $k$ executes it
$O_k$	Number of available workers with skill $k$ ( $\sum_k O_k = NS$ )
$P_m$	Portion demand of product $m$ in production planning horizon

Variables

$y_{jk}$	Binary variable; 1: when a worker with skill $k$ is assigned to station $j$ ; 0: otherwise.
$x_{ijk}$	Binary variable; 1: when task $i$ is assigned to station $j$ where $i$ is equipped by a worker with skill $k$ ; 0: otherwise.
$C$	Cycle time

$$\text{Min } C \quad (1)$$

S.to:

$$\sum_{j=1}^{NS} \sum_{k=1}^K x_{ijk} = 1 \quad \forall i = 1, \dots, n \quad (2)$$

$$\sum_{g=1}^{NS} \sum_{q=1}^K g \cdot x_{hgq} \leq \sum_{j=1}^{NS} \sum_{k=1}^K j \cdot x_{ijk} \quad \forall i \in I - W_0, h \in W(i) \quad (3)$$

$$\sum_{j=1}^{NS} y_{jk} = O_k \quad \forall k = 1, \dots, K \quad (4)$$

$$\sum_{k=1}^K y_{jk} = 1 \quad \forall j = 1, \dots, NS \quad (5)$$

$$x_{ijk} \leq y_{jk} \quad \forall i = 1, \dots, n; j = 1, \dots, NS; k = 1, \dots, K \quad (6)$$

$$\sum_{i=1}^n t_{imk} \cdot P_m \cdot x_{ijk} \leq C \quad \forall m = 1, \dots, M; j = 1, \dots, NS; k = 1, \dots, K \quad (7)$$

$$x_{ijk} \in \{0, 1\} \quad \forall i = 1, \dots, n; j = 1, \dots, NS; k = 1, \dots, K \quad (8)$$

$$y_{jk} \in \{0, 1\} \quad \forall j = 1, \dots, NS; k = 1, \dots, K \quad (9)$$

Objective function (1) is related to cycle time minimization. Constraint (2) determines each task should be assigned to exactly one station. Constraint (3) represents the precedence relations between the tasks. Constraint (4) ensures the number of in use workers with a special skill should not be more than its maximum available number. Constraint (5) does not permit to have more than one worker in one station.

Constraint (6) shows that a task can be done in a station, only if this station has a worker. Constraint (7) guarantees that the load of stations should meet the cycle time. Finally, Constraint (8) and (9) express  $y_{jkr}$ ,  $x_{ijk}$ , are binary variables.

## 2-2-lower bounds on the cycle time and earliest and latest stations for a task

We use the following lower bound which is the maximum value of two modified lower bounds on cycle time:

$$LC = \max \left\{ \frac{\sum_{i=1}^n \sum_{m=1}^M t_{im1} \cdot P_m}{NS}, \max_{i,m} \{ t_{im1} \cdot P_m \} \right\} \quad (10)$$

Where  $t_{im1}$  is task time  $i$  for model  $m$  when a high-skill worker executes it.

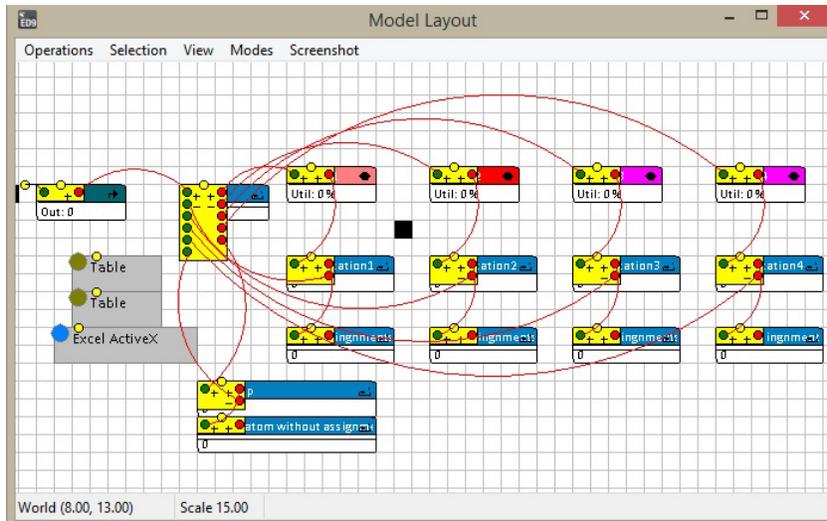
For determining the earliest and the latest stations that a task can be assigned to, we use the rules of Scholl and Becker (2006). Like the LC we use the task times of a high-skill worker.

## 3- Simulation-Heuristic Algorithm

This algorithm has two phases: in the first phase two processes, including determining 1-the initial trial cycle time (which is obtained by the presented lower bound), 2-the earliest and the latest stations that tasks can be assigned to (Scholl and Becker (2006)) are necessary. The first phase leads to the cut of unnecessary trial cycle time and it helps to have a faster algorithm. In the second phase, as well as the results of phase 1, a heuristic-simulation algorithm is used for task assignment and a complete enumeration method for worker assignment to get a good cycle time.

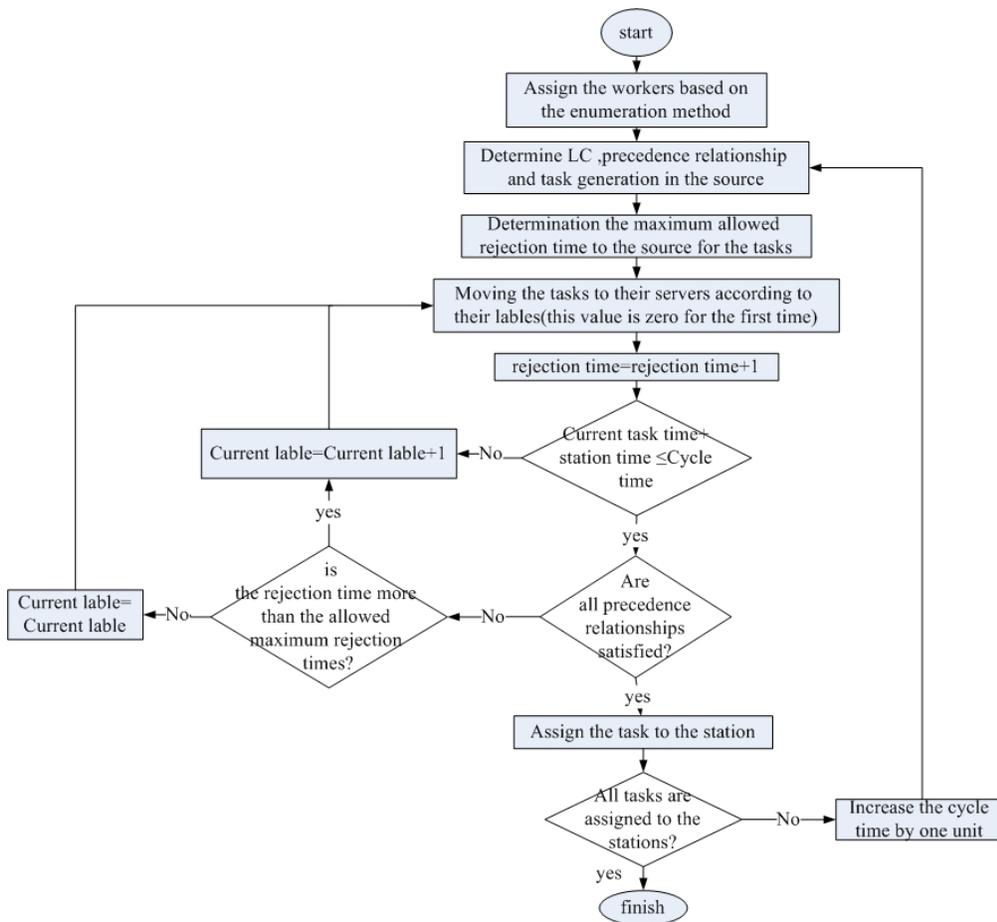
Complete enumeration method for worker assignment means that if there are three stations with different levels; one high-skill, one medium-skill and one low-skill workers then we should investigate  $3! = 6$  scenarios. Similarly, if there are four stations, two high-skill, one medium-skill and one low-skill workers, we should verify  $(4!/2!) = 12$  scenarios for worker assignment. Then we can select the best cycle time among these scenarios.

Figure 1 show a layout of a problem with four stations, which is modeled in Enterprise Dynamics9 (ED9) software. In this model, each task has several properties such as its task time according to the operators, the name of predecessors' tasks and the earliest and the latest stations. The entrance rule for each task is generated randomly. After entering task  $i$  to its earliest station, if the remaining time at this station permits doing this task, and all predecessors of this task in this station had been done in this station or previous stations, we assign this task to it. But if the remaining time in this station does not permit to assign this task to this station, and if all predecessors of this task had been done before, we will assign this task to the next station. If we want to assign this task to a station that all predecessors of this task had not been assigned before, this task rejects to the source again. Thus, we assign a task to a station if and only if all predecessors of this task had been executed before and this station has enough time to do it. In this heuristic-simulation algorithm, each task can be rejected to the source  $n$  times. If after  $n$  times of rejection, a task cannot be assigned to a station, we will stop the algorithm and restart it again with *trial cycle time* = *trial cycle time* + 1. This process will be repeated until there is no task for assignment, and all combinations of worker assignment are verified.



**Figure 1.** The layout of a problem with four stations in Enterprise Dynamics 9 software

The flow chart of this approach is presented in Figure 2.



**Figure 2.** The flow chart of the proposed approach

#### 4- Numerical Experiments and Discussion

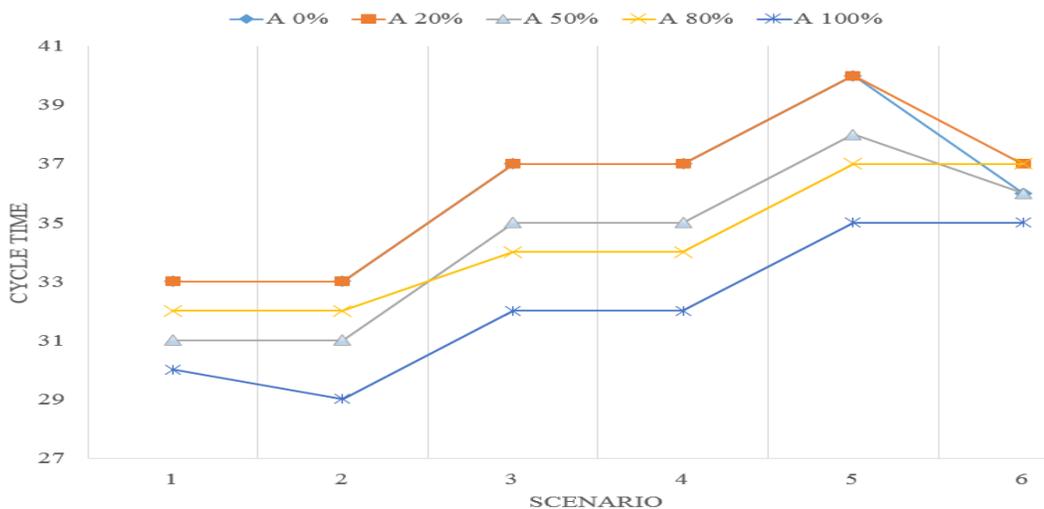
For numerical experiments, we use the combined precedence diagram and the task times of the first and the second models of different standard data sets from eight tasks to 148 tasks presented in [www.assembly-line-balancing.de](http://www.assembly-line-balancing.de), and then we benchmark them for assembly line rebalancing and worker assignment problem. For the problems with eight tasks to 83 tasks, we assume that we have three stations. For other problems, we use four stations for task and worker assignments. For the problems with three stations, we have one high-skill, one medium-skill and one low-skill workers. These values for problems with four stations are two high-skill, one medium-skill and one low-skill workers. Furthermore, we investigate five combinations of demands from 0% to 100% for each product and each problem.

Table 1 shows different scenarios of worker assignment for problems with three stations.

**Table 1.** Different scenarios for problems with 8 tasks to 83 tasks

	Worker of station 1	Worker of station 2	Worker of station 3
Scenario 1	high	medium	low
Scenario 2	high	low	medium
Scenario 3	medium	high	low
Scenario 4	medium	low	high
Scenario 5	low	high	medium
Scenario 6	low	medium	high

Figure 3 shows that the worst cycle times are usually obtained for the situations that we produce only product B or A 20% and B 80%. It means that, we can deliver our products sooner in other combinations than this combination. It leads to a better decision for marketing strategy. For example, if we encourage the customers to buy product more than 20 % A, as well as selling our products and obtaining profits, we can have a higher-speed balanced assembly line. Furthermore, this figure shows the best worker assignment is for conditions that we have a high-skill worker at the first station, and the worst worker assignment is assigning a low-skill worker to the first station. Additionally, it shows if we assign a medium-skill worker to the first station, the obtained cycle time is not hesitated to have a high or a low-skill worker in the second or the third station.



**Figure 3.** The obtained cycle time by the simulation-heuristic algorithm for the problem with 8 tasks (P8)

Figure4 demonstrates that the best cycle time is obtained by producing product A, and the worst cycle times are for the situations that we produce A 20% and B 80%. For other combinations, the cycle times are between these two values. Also, it shows that if we have to produce A 20% and B 80%, we should not assign a low-skill worker in Station 1. Moreover, it shows that if we want to produce only product A the best worker assignment is based on Scenario 1 but for other combinations, scenario 5 is the best one.

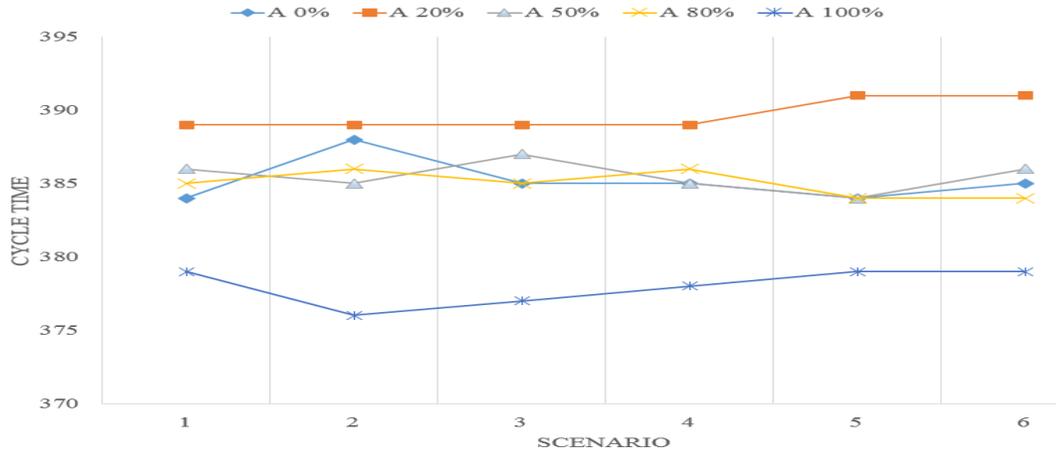


Figure 4. The obtained cycle time by the simulation-heuristic algorithm for the problem with 28 tasks (P28)

Figure 5 shows the best cycle time for all situations is for the condition that we only produce product A, and use Scenario 4 for worker assignment. Also, the variations in this figure show if we want to produce A 100% or A 80% and B 20% the role of worker assignment is important.

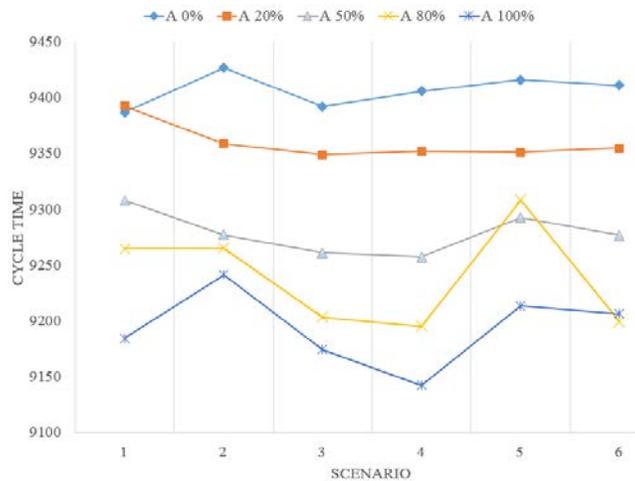
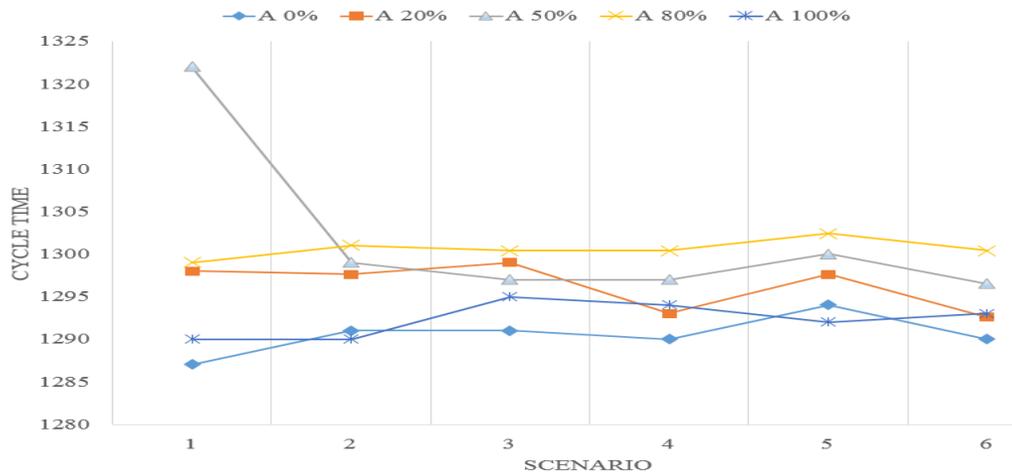


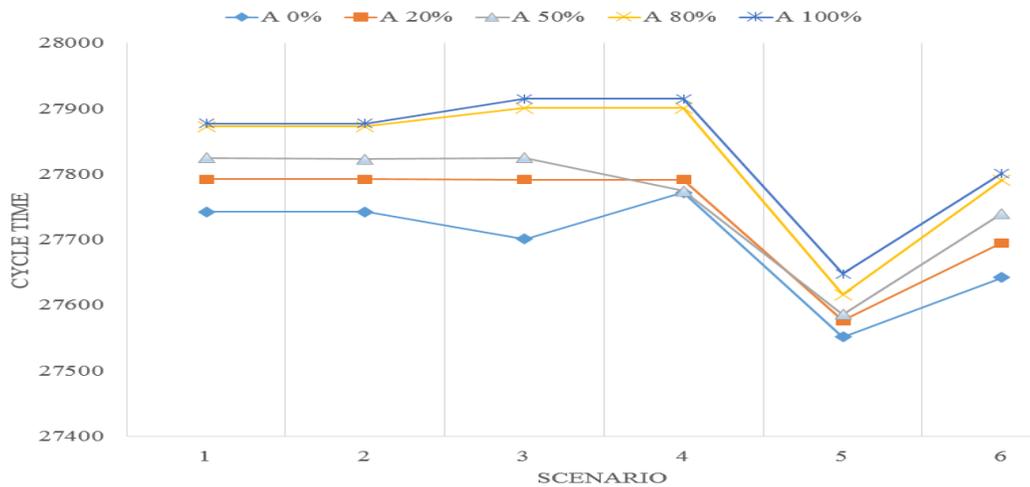
Figure 5. The obtained cycle time by the simulation-heuristic algorithm for the problem with 50 tasks(P50)

Figure 6 demonstrates if we want to produce A 50% and B 50% we should not select Scenario 1 for worker assignment, as this selection leads to obtain a bad cycle time.



**Figure 6.** The obtained cycle time by the simulation-heuristic algorithm for the problem with 70 tasks (P70)

Figure 7 shows the best scenario for all combinations is Scenario 5. Furthermore, it shows the minimum and the maximum cycle times are for conditions that we only assemble product B and A, respectively.



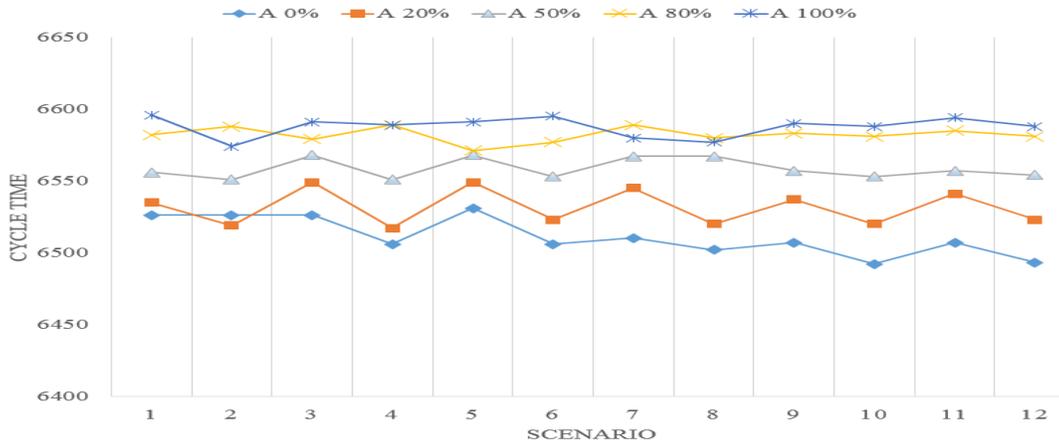
**Figure 7.** The obtained cycle time by the simulation-heuristic algorithm for the problem with 83 tasks P83

As well as the above examples that have three stations, we test our method for two larger problems with 100 and 148 tasks. In these problems, we have four stations, two high-skill workers, one medium-skill worker and one low-skill worker. Therefore, it causes to have 12 scenarios for worker assignments that they are presented in Table 2.

**Table 2.**The scenarios for problem with 100 tasks and 148 tasks

	Worker of Station 1	Worker of Station 2	Worker of Station 3	Worker of Station 4
Scenario 1	high	high	medium	low
Scenario 2	high	high	low	medium
Scenario 3	high	medium	low	high
Scenario 4	high	low	medium	high
Scenario 5	high	medium	high	low
Scenario 6	high	low	high	medium
Scenario 7	medium	low	high	high
Scenario 8	low	medium	high	high
Scenario 9	medium	high	low	high
Scenario 10	low	high	medium	high
Scenario 11	medium	high	high	low
Scenario 12	low	high	high	medium

Figure 8 shows that in some cases, the cycle time is hesitated to the position of workers who work next to each other. Additionally, it shows that by producing only product B, we can have the minimum cycle time.



**Figure 8.** The obtained cycle time by the simulation-heuristic algorithm for P100

Figure 9 demonstrates the obtained cycle times for different worker assignment scenarios. It shows that the minimum cycle time is for conditions that we only produce product B, and the second minimum cycle time will be obtained if we only produce product A. Other combinations of A and B have higher cycle times. The worst cycle time is for the situation that we produce A 80% and B 20%. Also, this figure shows that the best worker for the first station is a high-skill worker. Thus, if we assign a medium or a low-skill worker to the first station, we will get higher cycle time.

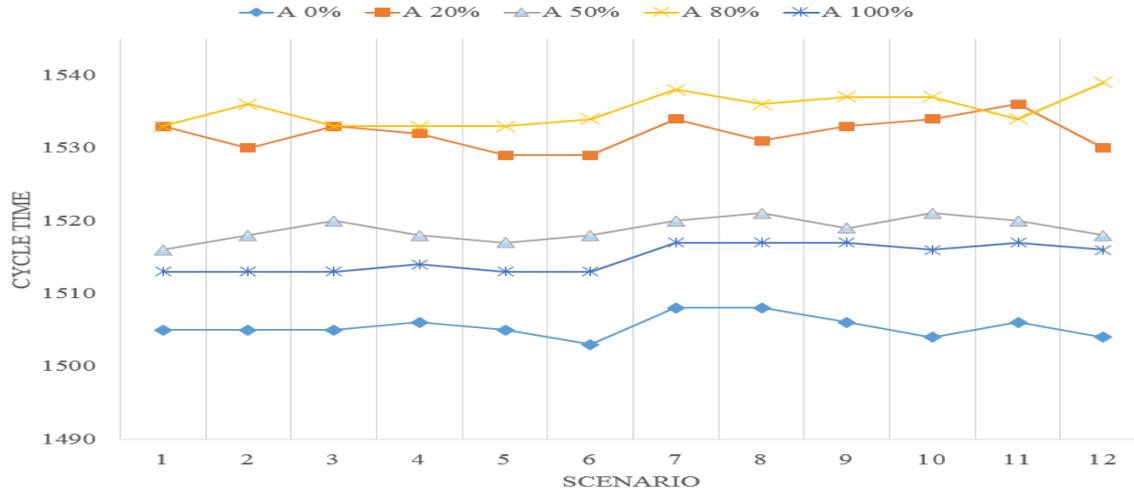


Figure 9. The obtained cycle time by the simulation-heuristic algorithm for P148

As well as the above verification, we use an index that is presented as follows:

$$Index\ 1 = \frac{\sum_{j=1}^{NS} (C - LS_j)}{NS \times C} \quad (11)$$

Where  $LS_j$  is the load station of station  $j$ . This index shows a ratio for the total idle time per total available time. Clearly, a lower value of this index usually shows high efficiency and low variation of load stations. We use this index for all problems and combinations of demand. These results are shown in Figure 10. It shows that for all problems except the problem with eight tasks (P8) the value of index 1 is lower than 0.01 that it can present the efficiency of the proposed approach. It is expected that the structure of P8 does not permit to reduce this value.

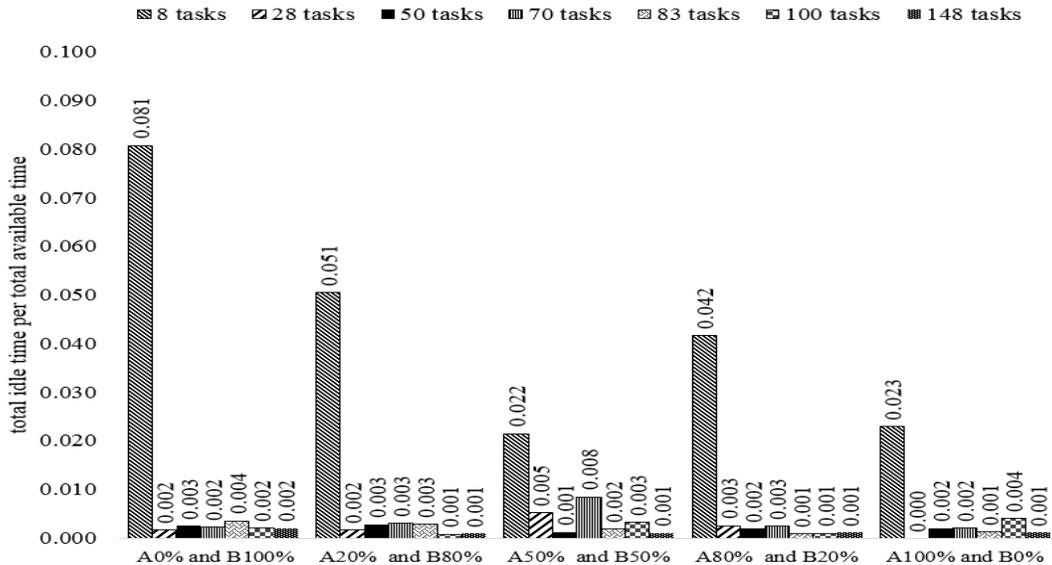


Figure 10. Total idle time per total available time

For validation of the proposed approach, we compare the best obtained cycle time with the GAMS software results in Table 3.

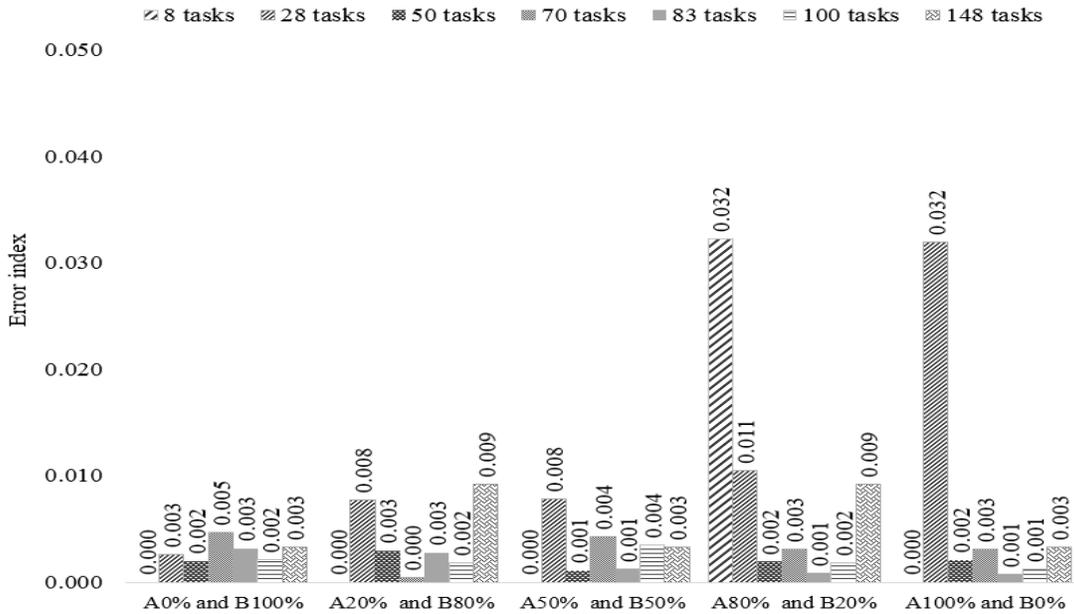
**Table 3.** A comparison between the obtained results of the proposed approach and optimum solution

Combination of demands		8 tasks		28 tasks		50 tasks		70 tasks		83 tasks		100 tasks		148 tasks	
A	B	C*	C'	C*	C'	C*	C'	C*	C'	C*	C'	C*	C'	C*	C'
0%	100%	33	33	383	384	9368	9387	1281	1287	27464	27551	6478	6492	1498	1503
20%	80%	33	33	386	389	9321	9349	1292	1292.6	27499	27576	6505	6517	1515	1529
50%	50%	31	31	381	384	9247	9257	1291	1296.5	27551	27586	6528	6551	1511	1516
80%	20%	31	32	380	384	9177	9195	1295	1299	27592	27615.8	6559	6571	1519	1533
100%	0%	29	29	375	387	9123	9142	1286	1290	27627	27648	6566	6574	1508	1513

C\*: optimal solution, C': the obtained cycle time by the proposed approach

For further analysis, we use the following index that we call it error index. The results of using this index are presented in Figure 10.

$$Error\ index = \frac{(c^l - c^*)}{c^*} \quad (12)$$



**Figure 11.** Error index for the problems

This figure shows that the error index is between 0 and 0.032. Furthermore, in most of the cases this value is lower than 0.009. If we accept the maximum value (0.032) as a risk value, it presents the high efficiency of the proposed approach. Furthermore, if we compare Figure 10 and Figure 11, we find that the optimum results for the problem with 8 tasks is obtained by the proposed approach and reducing the cycle time is impossible.

## 5-Conclusion

In this paper, an analytical approach based on a simulation-heuristic algorithm for single and mixed-model assembly line rebalancing and worker assignment problems is presented. The problem is modeled in ED9 software for task assignment, and a complete enumeration method is used for worker assignment. Since, it is possible to have demand variation in the mixed-model assembly lines; five combinations of demands are verified. Furthermore, two indices for verification of our approach are used. The results show that a low variation from the optimum cycle time and small idle times in the system are occurred. Additionally, by our approach one can understand which task and worker assignment are necessary for each condition, and can find when should encourage the customers to buy a special product to have a balanced line and selling products.

Therefore, the proposed approach can be used as an effective method for assembly line rebalancing, worker assignment, and decision-making on the marketing strategy when having demand variations.

For future studies, researchers can present better lower bounds for this problem or use another strategy for increasing the trial cycle times.

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