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A novel approach for solving the fully fuzzy bi-level linear programming problems

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Abstract

Bi-level linear programming (BLP) is a problem with two decision makers and two levels: the Leader in the upper and the Follower in the lower levels. Decision on one level affects the other one. In this respect, finding an optimal solution for BLP problems with inexact parameters and variables (proposed in many real-world applications) is non-convex and very hard to solve regarding its structure. In the present study, Multi-Objective Linear Programming (MOLP) is applied to offer a new approach is proposed to find an optimal fuzzy solution for the BLP problems, in which all parameters and variables have fuzzy nature. The main contribution of this research can be described as follows. First based on lexicographic ordering and using triangular fuzzy numbers, the given fully fuzzy BLP problem is converted into its equivalent multi-objective BLP problem. Then, the lexicographic method is used to solve the obtained model in the previous step. Subsequently, the optimal solution of the multi-objective BLP problem is obtained. However the answer to the main question is given in Theorem 1 if the optimal solution of the multi-objective BLP problem can be considered an optimal solution of the fully fuzzy BLP problem. Finally, to demonstrate the applicability of the proposed approach, it is run to solve some examples, and its results are compared with one of the existing methods.

Keywords: Solving approach, fully fuzzy bi-level linear programming, multi-objective linear programming, Lexicographic method

1-Introduction

Bi-level Linear Programming (BLP) problem is a non-convex nested optimization problem with two decision makers called Leader and Follower. Each decision maker attempts to optimize its own objective function in the way that the leader first makes a selection and then the follower chooses a strategy based on the leader's selection. In BLP problems all of the coefficients and variables are assumed crisp values. However, in real world problems, as in logistic planning or human resource planning, it is difficult to determine an exact value for the coefficients due to the uncertain and ambiguous data. Due to the inherent fuzziness of such problems, fuzzy set theory can manage this uncertainty such that all coefficients are assumed to be fuzzy numbers in both objective functions and constraints (Jafarzadeh-Ghoushchi, 2018),(Jafarzadeh Ghoushchi et al., 2019), (Fardi et al., 2019) and (Akbari and Osgooei, 2020). Accordingly, the fuzzy BLP problem is formulated based on the BLP problem in uncertain environment (Ren, 2015) and (Ren, 2016).

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The fuzzy BLP problem has been widely applied to various fields such as supply chain management (Chalmardi and Camacho-Vallejo, 2019), electricity market and transportation (Einaddin and Yazdankhah, 2020), (Ayas et al., 2018), (Zhang et al., 2019) and (Jalil et al., 2019).

Recently, the applicability of BLP has received much consideration such that several attempts have been made to solve BLP problems considering uncertainty. Sakawa et al. (2000) the first who proposed the fuzzy BLP problem and considered a fuzzy programing method to solve this problem. Later, an extended Kuhn-Tucker approach based on the new definition of the optimal solution was developed by Zhang and Lu (2005) to solve this problem. Subsequently, Zhang and Lu (2005) presented a fuzzy bi-level decision making model for a general logistics planning problem. Also, they developed a fuzzy number based on the kth-best approach to finding an optimal solution for the proposed model. In another study, a membership function approach was presented by Ruziyeva (2013) to solve the fuzzy optimization problem. It is remarkable that in all of the mentioned problems, the decision variables are non-fuzzy. In recent years, researchers have considered the Fully Fuzzy BLP (FFBLP) issues in which all of the coefficients and variables are fuzzy numbers.

In 2015, Ren (2015) proposed a novel method for solving FFBLP problem by applying interval programming method. Also, one year later Ren (2016) solved FFBLP through Deviation degree measure and a ranking function method. Safaei and Saraj (2014) solved the FFBLP problem by decomposing the problem into three crisp Linear Programming (LP) problem with bounded variables constraints, then they solved three LP problem separately by using its optimal solutions. Hamidi and Mishmast (2013) studied FFBLP problem and proposed a new method to solve a fuzzy BLP using interval BLP. Tayebnasab et al., (2020) proposed a new approach to solve FFBLP based on crisp bi-level programming in the way that FFBLP was transformed to a deterministic bi-level programming problem using the ranking function method and the solution of the deterministic problem was achieved using the kth-best method. Recently, Khalifa (2019) investigated a new approach in finding an optimal solution of FFBLP problems in the base of ranking function. In 2020, Gurmu and Fikadu (2020), discussed a procedure for solving bi-level linear programming problem through linear fuzzy mathematical programming approach. Also, Chen et al., (2021) studied combined application and solution of bifuzzy variables and bi-level programming with the aim of solving the problem with bifuzzy information. Alessa (2021) solved bi-level linear fractional programming by proposing an interactive approach. Elsisy et al., (2021) proposed a novel algorithm for generating Pareto frontier of bi-level multi-objective rough nonlinear programming problem.

In most real-world applications, especially in the decision-making process or human resource planning, a decision is needed to make under uncertain information. Therefore, inaccurate optimization methods are used for giving environments under uncertainty .The study's principle objective and contribution are using fuzzy theory as a powerful tool for controlling this ambiguity or unreliability. To reach this objective, a novel approach is presented to solve the FFBLP problem based on the method of Ezzati et al. (2015) in solving Fully Fuzzy Linear Programming (FFLP) problem. This approach is implemented by converting the FFBLP problem to a multi-objective BLP problem and solving it by lexicographic method. In this method, the optimal solution of the multi-objective BLP problem is obtained. However, one may ask if this solution can be considered optimal for the fully fuzzy BLP problem. Therefore, finding an appropriate answer to this question is another objective of this paper that is proposed in theorem 1.

By finding a solution for the FFBLP problems, many kinds of applicable problems in transport network designs, principle-agent problems, price control, and electricity networks are solved.

It is remarkable that in converting fuzzy objective functions to crisp functions at each level, losing some data is unavoidable at the base of the ranking function; however, as one of the advantages of this approach, this issue does not happen in our proposed method.

The remainder of the present study is organized as follows: In section 2, some necessary and useful results of fuzzy set theory are reviewed. In section 3, the standard form of FFBLP and the Multi-Objective Linear Programming (MOLP) problem are described. This section proposes a new method to find an optimal fuzzy solution of the FFLBP by converting it to three BLP problems and solving them separately. In section 4, two numerical examples are presented to confirm the applicability of the proposed approach, and the results are compared with those of Safaei and Saraj's (2014) model. Finally, the conclusions of this research are presented in section 5.

2-Preliminaries

Nowadays, human decision-making is more dependent on information than ever before. However, since most information is not conclusive, man has to make rational decisions based on this uncertainty. Classical and traditional methods of modeling, reasoning, inference, and computation have a dual Yes/No value. However, it is very difficult to draw clear boundaries between phenomena in the real world, and in many cases, it is even impossible to make explicit judgments. Hence, such an issue poses a challenge for the decision-maker to design an intelligent system that makes decisions like humans. This view is the basis of fuzzy logic, which has been proposed by Bellman and Zadeh (1970) and Zadeh (1965).

Although less than four decades have passed since the advent of fuzzy theory, it has attracted the attention of scientists, experts, and managers in most academics, commercials, industrials, and even political centers. The fuzzy theory has numerous applications in various fields such as artificial intelligence, expert systems, information systems, computer sciences, electrical and electronic engineering, control engineering, planning, decision theory, logic, scientific management, operations research, and robotic and medical economics. These applications can be found in abundance in Tanaka (1973). This section introduces some necessary and notations of fuzzy set theory used in the rest of the study.

Definition 1. The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in *X*. This function can be generalized to a function μ_A such that the value assigned to the element of the universal set X falls within a specified range i.e. $\mu_A : X \to [0,1]$. The assigned value indicates the membership grade of the element in the set *A* (Dubois, 1980). The function μ_A is called the membership function and the set $A = \{(x, \mu_A(x); x \in X\}$ defined by $\mu_A(x)$ for each $x \in X$ is called a fuzzy set.

Definition 2. A fuzzy number $A = (a,b,c)$ is said to be a Triangular Fuzzy Number (TFN) if it's membership function is given based on equation (1).

$$\mu_A(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \le x \le b, \\ \dfrac{c-x}{c-b}, & b \le x \le c, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

The set of TFNs is denoted by *TF(.)* (Dubois, 1980).

Definition 3. A TFN $A = (a,b,c)$ is said to be a non-negative TFN if and only if $a \geq 0$. The set of all non-negative TFNs is denoted by $TF(\text{n})^+$ (Dubois, 1980)..

Definition 4. Two TFNs $A = (a,b,c)$ and $B = (e,f,g)$ are said to be equal if and only if $a = e, b = f, c = g$ (Dubois, 1980).

Definition 5. Let $A = (a,b,c)$ and $B = (e,f,g)$ be two TFNs, the algebraic operations between any two TFNs $\tilde{A}$ and $\tilde{B}$ can be defined as equations (2) to (7) (Allahviranloo et al., 2007).

$$\tilde{A} \oplus \tilde{B} = (a+e, b+f, c+g) \tag{2}$$
$$-\tilde{A} = -(a,b,c) = (-c,-b,-a) \tag{3}$$
$$\tilde{A} \ominus \tilde{B} = (a-g, b-f, c-g) \tag{4}$$
$$k(a,b,c) = (ka,kb,kc) \quad for \;\; k \geq 0 \tag{5}$$
$$k(a,b,c) = (kc,kb,ka) \quad for \;\; k < 0 \tag{6}$$

$$A \otimes B = \begin{cases} (ae, bf, cg), & a \geq 0, \\ (ag, bf, cg), & a < 0, c \geq 0, \\ (ag, bf, ce), & c < 0. \end{cases} \quad (7)$$

In equation (7), $A = (a,b,c)$ be an arbitrary TFN and $B = (e,f,g)$ be a non-negative TFN.

Definition 6. Let $A = (a,b,c)$ and $B = (e,f,g)$ be two TFNs. Then $\tilde{A}$ is relatively less than $\tilde{B}$, which is denoted by $A < B$ if:

i. $b < f$ *or*
ii. $b = f$ *and (c-a)>(g-e) or*
iii. $b = f$, $(c-a) = (g-e)$ *and* $(a+c) < (e+g)$

3-Fully fuzzy bi-level linear programming problem

One of the interesting concept in fuzzy optimization problems is to deal with Fuzzy Linear Programming (FLP) problems. The first FLP formula was proposed by Zimmerman (1978). Various methods have been proposed to solve the FLP problems (Tanaka, 1973), (Zimmermann, 1978), (Campos and Verdegay, 1989), (Buckley and Feuring, 2000), (Maleki et al., 2000), (Maleki, 2003|), (Nehi et al., 2004), (Allahviranloo et al., 2008), (Ebrahimnejad and Nasseri, 2009), (Ebrahimnejad et al., 2010), (Nasseri, 2008) and (Kumar et al., 2011). In the past few decades, the bi-level programming problems have been proposed and investigated from theoretical and computational points of view. In primary models of bi-level programming, all parameters were crisp and precise, however in real world applications this was not convincing since a decision on the basis of uncertain information was needed. Therefore, the fuzzy BLP problems in which the coefficients are fuzzy numbers and FFBLP models where both coefficients and variables are fuzzy numbers were introduced. Model (8) indicates the FFBLP problem of maximization-type objective functions at each level.

To solve the following problem, first by replacing $x_1 = ((x_1)^l, (x_1)^c, (x_1)^u)$ and $x_2 = ((x_2)^l, (x_2)^c, (x_2)^u)$ and considering the definitions of non-negative TFNs and the arithmetic operation between them the model (8) can be written as model (10). In the next step based on the method of Ezzati et al. (2015) model (10) can be transformed to the multi objective BLP problem with three crisp bi-level objective function as (11). Subsequently, by the help of lexicographic method, an optimal solution is can be obtained to multi-objective BLP problem. Finally in theorem 1 it is proved that this optimal solution can be considered as an optimal solution of model (8).

$$\begin{aligned} &\max_{x_1} z_1 = C_{11}^T \otimes x_1 \oplus C_{12}^T \otimes x_2 \\ &\max_{x_2} z_2 = C_{21}^T \otimes x_1 \oplus C_{22}^T \otimes x_2 \\ &s.t. \\ &\quad A \otimes x_1 \oplus B \otimes x_2 \ \{\leq = \geq\} t \\ &\quad x_1, x_2 \geq 0 \end{aligned} \quad (8)$$

Where

$A = (a_{ij})_{m \times n_1}$, $B = (b_{ij})_{m \times n_2}$, $t = (t_i)_{m \times 1}$.

$C_{11} = \left(C_{11_j}\right)_{1\times n_1}$, $C_{12} = \left(C_{12_j}\right)_{1\times n_2}$, $C_{21} = \left(C_{21_j}\right)_{1\times n_1}$, $C_{22} = \left(C_{22_j}\right)_{1\times n_2}$, $x_1 = (x_{1j})_{n_1\times 1}$, $x_2 = (x_{2j})_{n_2\times 1}$,
$n_1 + n_2 = n$, a_{ij} ,b_{ij} ,$t_i \in TF(\mathrm{n})$, $x_{1j}, x_{2j} \in TF(\mathrm{n})^+$ for $1 \le i \le m$, $1 \le j \le n$.

Remark 1. $x_1^* = ((x_1^*)^l, (x_1^*)^c, (x_1^*)^u)$ and $x_2^* = ((x_2^*)^l, (x_2^*)^c, (x_2^*)^u)$ are said to be exact optimal solutions of Model (8) if they satisfy in the following statements:
$x_1^* = (x_{1j}^*)_{n_1\times 1}, x_2^* = (x_{2j}^*)_{n_2\times 1}$ where x_{1j}^* , $x_{2j}^* \in TF(\mathrm{n})^+$, $j = 1,2,...,n$ $A \otimes x_1^* \oplus B \otimes x_2^* \{\le = \ge\} t$,
For each $x_1 = ((x_1)^l, (x_1)^c, (x_1)^u)$, $x_2 = ((x_2)^l, (x_2)^c, (x_2)^u) \in$
$S = \{x_1, x_2;\ \ A \otimes x_1 \oplus B \otimes x_2 \{\le = \ge\} t, x_1 = (x_{1j})_{n_1\times 1}, x_2 = (x_{2j})_{n_2\times 1}$
$where\ x_{1j}, x_{2j} \in TF(\mathrm{n})^+\}$
We have that

$$C_{11}^T \otimes x_1 \oplus C_{12}^T \otimes x_2 \le C_{11}^T \otimes x_1^* \oplus C_{12}^T \otimes x_2^*$$
$$C_{21}^T \otimes x_1 \oplus C_{22}^T \otimes x_2 \le C_{21}^T \otimes x_1^* \oplus C_{22}^T \otimes x_2^*,$$

(In case of minimization problem $\ge$ will be replaced).
In the following, the steps of new algorithm proposed to find an exact optimal fuzzy solution of the FFBLP problem are presented:
Step 1: Regarding to Definitions 3 and 5, model (8) can be written as model (9).

$$\begin{aligned}
&\max_{x_1}\left[(C_{11}^T x_1)^l, (C_{11}^T x_1)^c, (C_{11}^T x_1)^u\right] \oplus \left[(C_{12}^T x_2)^l, (C_{12}^T x_2)^c, (C_{12}^T x_2)^u\right]\\
&\max_{x_2}\left[(C_{21}^T x_1)^l, (C_{21}^T x_1)^c, (C_{21}^T x_1)^u\right] \oplus \left[(C_{22}^T x_2)^l, (C_{22}^T x_2)^c, (C_{22}^T x_2)^u\right]\\
&s.t.\\
&\left[(Ax_1)^l, (Ax_1)^c, (Ax_1)^u\right] \oplus \left[(Bx_2)^l, (Bx_2)^c, (Bx_2)^u\right] \le = \ge\\
&\left[(t)^l, (t)^c, (t)^u\right]
\end{aligned} \tag{9}$$

Where $C_{ij}^T x_j = ((C_{ij}{}^T x_j)^l, (C_{ij}{}^T x_j)^c, (C_{ij}{}^T x_j)^u)$ $Ax_i = ((Ax_i)^l, (Ax_i)^c, (Ax_i)^u)$,
$t = (t^l, t^c, t^u)$ $x_i = ((x_i)^l, (x_i)^c, (x_i)^u)$, $(x_i)^l \ge 0,\ i, j = 1,2$.
Equivalently, with respect to definition 4, model (9) can be written as follows (see model (10)):

$$
\begin{array}{l}
\max\limits_{x_1} \left[(C_{11}^T x_1)^l, (C_{11}^T x_1)^c, (C_{11}^T x_1)^u \right] \oplus \left[(C_{12}^T x_2)^l, (C_{12}^T x_2)^c, (C_{12}^T x_2)^u \right] \\
\max\limits_{x_2} \left[(C_{21}^T x_1)^l, (C_{21}^T x_1)^c, (C_{21}^T x_1)^u \right] \oplus \left[(C_{22}^T x_2)^l, (C_{22}^T x_2)^c, (C_{22}^T x_2)^u \right] \\
s.t. \\
\quad (Ax_1)^l + (Bx_2)^l \leq = \geq (t)^l \\
\quad (Ax_1)^c + (Bx_2)^c \leq = \geq (t)^c \\
\quad (Ax_1)^u + (Bx_2)^u \leq = \geq (t)^u \\
\quad (x_1)^l \geq 0, \quad (x_1)^c - (x_1)^l \geq 0, \quad (x_1)^u - (x_1)^c \geq 0 \\
\quad (x_2)^l \geq 0, \quad (x_2)^c - (x_2)^l \geq 0, \quad (x_2)^u - (x_2)^c \geq 0
\end{array}
\tag{10}
$$

Step 2: Based on the inspiration of the method of Ezzati et al. (2015) by Definitions 5 and 6, model (10) can be transformed to the multi-objective BLP problem with three crisp bi-level objective functions (see model (11)).

$$
\begin{array}{l}
\max\limits_{(x_1)^c} \left[(C_{11}^T x_1)^c + (C_{12}^T x_1)^c \right] \\
\max\limits_{(x_2)^c} \left[(C_{21}^T x_1)^c + (C_{22}^T x_1)^c \right] \\
\min\limits_{(x_1)^l,(x_1)^u} \left[(C_{11}^T x_1)^u + (C_{12}^T x_2)^u \right] - \left[(C_{11}^T x_1)^l + (C_{12}^T x_2)^l \right] \\
\min\limits_{(x_2)^l,(x_2)^u} \left[(C_{21}^T x_1)^u + (C_{22}^T x_2)^u \right] - \left[(C_{21}^T x_1)^l + (C_{22}^T x_2)^l \right] \\
\max\limits_{(x_1)^l,(x_1)^u} \left[(C_{11}^T x_1)^l + (C_{12}^T x_2)^l \right] + \left[(C_{11}^T x_1)^u + (C_{12}^T x_2)^u \right] \\
\max\limits_{(x_2)^l,(x_2)^u} \left[(C_{21}^T x_1)^l + (C_{22}^T x_2)^l \right] + \left[(C_{21}^T x_1)^u + (C_{22}^T x_2)^u \right] \\
s.t. \\
\quad (Ax_1)^l + (Bx_2)^l \leq = \geq (t)^l \\
\quad (Ax_1)^c + (Bx_2)^c \leq = \geq (t)^c \\
\quad (Ax_1)^u + (Bx_2)^u \leq = \geq (t)^u \\
\quad (x_1)^l \geq 0, \quad (x_1)^c - (x_1)^l \geq 0, \quad (x_1)^u - (x_1)^c \geq 0 \\
\quad (x_2)^l \geq 0, \quad (x_2)^c - (x_2)^l \geq 0, \quad (x_2)^u - (x_2)^c \geq 0
\end{array}
\tag{11}
$$

Step 3: According to the preference of bi-level objective functions, the lexicographic method is used as the following:

$$
\begin{aligned}
&\max_{(x_1)^c}\left[(C_{11}^T x_1)^c + (C_{12}^T x_2)^c\right]\\
&\max_{(x_2)^c}\left[(C_{21}^T x_1)^c + (C_{22}^T x_2)^c\right]\\
&s.t.\\
&\quad (Ax_1)^l + (Bx_2)^l \le = \ge (t)^l\\
&\quad (Ax_1)^c + (Bx_2)^c \le = \ge (t)^c\\
&\quad (Ax_1)^u + (Bx_2)^u \le = \ge (t)^u\\
&\quad (x_1)^l \ge 0,\quad (x_1)^c - (x_1)^l \ge 0,\quad (x_1)^u - (x_1)^c \ge 0\\
&\quad (x_2)^l \ge 0,\quad (x_2)^c - (x_2)^l \ge 0,\quad (x_2)^u - (x_2)^c \ge 0
\end{aligned}
\tag{12}
$$

If model (12) has unique optimal solutions, as $x_1^* = ((x_1^*)^l, (x_1^*)^c, (x_1^*)^u)$ and $x_2^* = ((x_2^*)^l, (x_2^*)^c, (x_2^*)^u)$, then it is an optimal solution of model (9) and the algorithm stops. Otherwise the algorithm goes to step 4.

Step 4: Over the optimal solutions that are obtained in step 3, model (13) is achieved.

$$
\begin{aligned}
&\min_{(x_1)^l,(x_1)^u}\left[(C_{11}^T x_1)^u + (C_{12}^T x_2)^u\right] - \left[(C_{11}^T x_1)^l + (C_{12}^T x_2)^l\right]\\
&\textit{where } x_2 \textit{ solve}\\
&\min_{(x_2)^l,(x_2)^u}\left[(C_{21}^T x_1)^u + (C_{22}^T x_2)^u\right] - \left[(C_{21}^T x_1)^l + (C_{22}^T x_2)^l\right]\\
&s.t.\\
&\quad (C_{11}^T x_1)^c + (C_{21}^T x_1)^c = m_1^*\\
&\quad (C_{21}^T x_1)^c + (C_{22}^T x_2)^c = m_2^*\\
&\quad (Ax_1)^l + (Bx_2)^l \le = \ge (t)^l\\
&\quad (Ax_1)^c + (Bx_2)^c \le = \ge (t)^c\\
&\quad (Ax_1)^u + (Bx_2)^u \le = \ge (t)^u\\
&\quad (x_1)^l \ge 0,\quad (x_1)^c - (x_1)^l \ge 0,\quad (x_1)^u - (x_1)^c \ge 0\\
&\quad (x_2)^l \ge 0,\quad (x_2)^c - (x_2)^l \ge 0,\quad (x_2)^u - (x_2)^c \ge 0
\end{aligned}
\tag{13}
$$

Where m_1^*, m_2^* are the optimal values of model (12). If model (13) has a unique optimal solution, as $x_1^* = ((x_1^*)^l, (x_1^*)^c, (x_1^*)^u)$ and $x_2^* = ((x_2^*)^l, (x_2^*)^c, (x_2^*)^u)$, then it is an optimal solution of model (9) and the algorithm stops. Otherwise it should go to step 5.

Step 5: Over the optimal solution that has been achieved in step 4, the model (14) is obtained as follows:

$$
\begin{aligned}
&\max_{(x_1)^l,(x_1)^u} \left[(C_{11}^T x_1)^l + (C_{12}^T x_2)^l\right] + \left[(C_{11}^T x_1)^u + (C_{12}^T x_2)^u\right] \\
&\max_{(x_2)^l,(x_2)^u} \left[(C_{21}^T x_1)^l + (C_{22}^T x_2)^l\right] + \left[(C_{21}^T x_1)^u + (C_{22}^T x_2)^u\right] \\
&s.t. \\
&\quad (C_{11}^T x_1)^c + (C_{21}^T x_2)^c = m_1^* \\
&\quad (C_{21}^T x_1)^c + (C_{22}^T x_2)^c = m_2^* \\
&\quad \left[(C_{11}^T x_1)^u + (C_{12}^T x_2)^u\right] - \left[(C_{11}^T x_1)^l + (C_{12}^T x_2)^l\right] = n_1^* \\
&\quad \left[(C_{21}^T x_1)^u + (C_{22}^T x_2)^u\right] - \left[(C_{21}^T x_1)^l + (C_{22}^T x_2)^l\right] = n_2^* \\
&\quad (Ax_1)^l + (Bx_2)^l \le = \ge (t)^l \\
&\quad (Ax_1)^c + (Bx_2)^c \le = \ge (t)^c \\
&\quad (Ax_1)^u + (Bx_2)^u \le = \ge (t)^u \\
&\quad (x_1)^l \ge 0, \quad (x_1)^c - (x_1)^l \ge 0, \quad (x_1)^u - (x_1)^c \ge 0 \\
&\quad (x_2)^l \ge 0, \quad (x_2)^c - (x_2)^l \ge 0, \quad (x_2)^u - (x_2)^c \ge 0
\end{aligned}
\tag{14}
$$

Where n_1^*, n_2^* are the optimal values of model (13). Now, solving the model (14), the optimal solution of model (9) is obtained as: $x_1^* = ((x_1^*)^l, (x_1^*)^c, (x_1^*)^u)$ and $x_2^* = ((x_2^*)^l, (x_2^*)^c, (x_2^*)^u)$. The following theorem shows that the achieved lexicographic optimal fuzzy solution of model (11) can be considered as an exact solution of model (9).

Theorem 1. If $x_1^* = ((x_1^*)^l, (x_1^*)^c, (x_1^*)^u)$ and $x_2^* = ((x_2^*)^l, (x_2^*)^c, (x_2^*)^u)$ be optimal solutions of models (12) to (14) and therefore a lexicographic optimal solution of model (11), then they are also considered as an exact optimal solution of model (9).

Proof. By contradiction, let $x_1^* = ((x_1^*)^l, (x_1^*)^c, (x_1^*)^u)$ and $x_2^* = ((x_2^*)^l, (x_2^*)^c, (x_2^*)^u)$ be optimal solutions of models (12) to (14), but they are not exact optimal solution of model (9). In this case, there exists feasible solutions of model (9), as $x_1^{\mathrm{n}} = ((x_1^{\mathrm{n}})^l, (x_1^{\mathrm{n}})^c, (x_1^{\mathrm{n}})^u) \ne x_1^*$ and $x_2^{\mathrm{n}} = ((x_2^{\mathrm{n}})^l, (x_2^{\mathrm{n}})^c, (x_2^{\mathrm{n}})^u) \ne x_2^*$ such that
$((C_{11}^T x_1^*)^l + (C_{12}^T x_2^*)^l, (C_{11}^T x_1^*)^c + (C_{12}^T x_2^*)^c, (C_{11}^T x_1^*)^u + (C_{12}^T x_2^*)^u)\,\mathrm{n}$
$((C_{11}^T x_1^{\mathrm{n}})^l + (C_{12}^T x_2^{\mathrm{n}})^l, (C_{11}^T x_1^{\mathrm{n}})^c + (C_{12}^T x_2^{\mathrm{n}})^c, (C_{11}^T x_1^{\mathrm{n}})^u + (C_{12}^T x_2^{\mathrm{n}})^u)$
and $((C_{21}^T x_1^*)^l + (C_{22}^T x_2^*)^l, (C_{21}^T x_1^*)^c + (C_{22}^T x_2^*)^c, (C_{21}^T x_1^*)^u + (C_{22}^T x_2^*)^u)\,\mathrm{n}$
$((C_{21}^T x_1^{\mathrm{n}})^l + (C_{22}^T x_2^{\mathrm{n}})^l, (C_{21}^T x_1^{\mathrm{n}})^c + (C_{22}^T x_2^{\mathrm{n}})^c, (C_{21}^T x_1^{\mathrm{n}})^u + (C_{22}^T x_2^{\mathrm{n}})^u)$.
So, regarding to Definition 6, three conditions are obtained as follows:

i. Let $(C_{11}^T x_1^*)^c + (C_{12}^T x_2^*)^c < (C_{11}^T x_1^{\mathrm{n}})^c + (C_{12}^T x_2^{\mathrm{n}})^c$
and $(C_{21}^T x_1^*)^c + (C_{22}^T x_2^*)^c < (C_{21}^T x_1^{\mathrm{n}})^c + (C_{22}^T x_2^{\mathrm{n}})^c$.

With respect to assumptions, it is concluded that:

$$(Ax_1^{\cap})^l \oplus (Bx_2^{\cap})^l \leq = \geq t^l$$
$$(Ax_1^{\cap})^c \oplus (Bx_2^{\cap})^c \leq = \geq t^c$$
$$(Ax_1^{\cap})^u \oplus (Bx_2^{\cap})^u \leq = \geq t^u$$
$$(x_1^{\cap})^l \geq 0,\ (x_1^{\cap})^c - (x_1^{\cap})^l \geq 0,\ (x_1^{\cap})^u - (x_1^{\cap})^c \geq 0$$
$$(x_2^{\cap})^l \geq 0,\ (x_2^{\cap})^c - (x_2^{\cap})^l \geq 0,\ (x_2^{\cap})^u - (x_2^{\cap})^c \geq 0.$$

Therefore, $((x_1^{\cap})^l, (x_1^{\cap})^c, (x_1^{\cap})^u)$, $((x_2^{\cap})^l, (x_2^{\cap})^c, (x_2^{\cap})^u)$ is a feasible solution of model (12) in which the objective value in $((x_1^{\cap})^l, (x_1^{\cap})^c, (x_1^{\cap})^u)$, $((x_2^{\cap})^l, (x_2^{\cap})^c, (x_2^{\cap})^u)$ is greater than the objective value in $((x_1^*)^l, (x_1^*)^c, (x_1^*)^u)$, $((x_2^*)^l, (x_2^*)^c, (x_2^*)^u)$. But this is a contradiction.

ii. Without losing the generality of the proof.

Let $(C_{11}^T x_1^*)^c + (C_{12}^T x_2^*)^c = (C_{11}^T x_1^{\cap})^c + (C_{12}^T x_2^{\cap})^c$, $(C_{21}^T x_1^*)^c + (C_{22}^T x_2^*)^c = (C_{21}^T x_1^{\cap})^c + (C_{22}^T x_2^{\cap})^c$ and

$$((C_{11}^T x_1^{\cap})^u + (C_{12}^T x_2^{\cap})^u) - ((C_{11}^T x_1^{\cap})^l + (C_{12}^T x_2^{\cap})^l) <$$
$$((C_{11}^T x_1^*)^u + (C_{12}^T x_2^*)^u) - ((C_{11}^T x_1^*)^l + (C_{12}^T x_2^*)^l)$$

and

$$((C_{21}^T x_1^{\cap})^u + (C_{22}^T x_2^{\cap})^u) - ((C_{21}^T x_1^{\cap})^l + (C_{22}^T x_2^{\cap})^l) <$$
$$((C_{21}^T x_1^*)^u + (C_{22}^T x_2^*)^u) - ((C_{21}^T x_1^*)^l + (C_{22}^T x_2^*)^l) .$$

Hence, $((x_1^{\cap})^l, (x_1^{\cap})^c, (x_1^{\cap})^u)$, $((x_2^{\cap})^l, (x_2^{\cap})^c, (x_2^{\cap})^u)$ is feasible solution of model (13) in which the objective value in $((x_1^{\cap})^l, (x_1^{\cap})^c, (x_1^{\cap})^u)$, $((x_2^{\cap})^l, (x_2^{\cap})^c, (x_2^{\cap})^u)$ is less than the objective value in $((x_1^*)^l, (x_1^*)^c, (x_1^*)^u)$, $((x_2^*)^l, (x_2^*)^c, (x_2^*)^u)$, which is a contradiction.

iii. Without losing the generality of the proof.

Let $(C_{11}^T x_1^*)^c + (C_{12}^T x_2^*)^c = (C_{11}^T x_1^{\cap})^c + (C_{12}^T x_2^{\cap})^c$

$$(C_{21}^T x_1^*)^c + (C_{22}^T x_2^*)^c = (C_{21}^T x_1^{\cap})^c + (C_{22}^T x_2^{\cap})^c,$$
$$((C_{11}^T x_1^{\cap})^u + (C_{12}^T x_2^{\cap})^u) - ((C_{11}^T x_1^{\cap})^l + (C_{12}^T x_2^{\cap})^l) =$$
$$((C_{11}^T x_1^*)^u + (C_{12}^T x_2^*)^u) - ((C_{11}^T x_1^*)^l + (C_{12}^T x_2^*)^l)$$
$$((C_{21}^T x_1^{\cap})^u + (C_{22}^T x_2^{\cap})^u) - ((C_{21}^T x_1^{\cap})^l + (C_{22}^T x_2^{\cap})^l) =$$
$$((C_{21}^T x_1^*)^u + (C_{22}^T x_2^*)^u) - ((C_{21}^T x_1^*)^l + (C_{22}^T x_2^*)^l)$$

and

$$((C_{11}^T x_1^*)^l + (C_{12}^T x_2^*)^l) + ((C_{11}^T x_1^*)^u + (C_{12}^T x_2^*)^u) <$$
$$((C_{11}^T x_1^{\cap})^l + (C_{12}^T x_2^{\cap})^l) + ((C_{11}^T x_1^{\cap})^u + (C_{12}^T x_2^{\cap})^u)$$
$$((C_{21}^T x_1^*)^l + (C_{22}^T x_2^*)^l) + ((C_{21}^T x_1^*)^u + (C_{22}^T x_2^*)^u) <$$
$$((C_{21}^T x_1^{\cap})^l + (C_{22}^T x_2^{\cap})^l) + ((C_{21}^T x_1^{\cap})^u + (C_{22}^T x_2^{\cap})^u) .$$

Therefore, $((x_1^{\cap})^l, (x_1^{\cap})^c, (x_1^{\cap})^u)$, $((x_2^{\cap})^l, (x_2^{\cap})^c, (x_2^{\cap})^u)$ is a feasible solution of model (14) in which the objective value in $((x_1^{\cap})^l, (x_1^{\cap})^c, (x_1^{\cap})^u)$, $((x_2^{\cap})^l, (x_2^{\cap})^c, (x_2^{\cap})^u)$ is greater than the objective value in

$((x_2^*)^l, (x_2^*)^c, (x_2^*)^u)$, $((x_2^*)^l, (x_2^*)^c, (x_2^*)^u)$, but this is also a contradiction. So, $x_1^* = ((x_1^*)^l, (x_1^*)^c, (x_1^*)^u)$, $x_2^* = ((x_2^*)^l, (x_2^*)^c, (x_2^*)^u)$ is exact solution of model (9).

4-Numerical example

In this section, two numerical examples are solved using the proposed approach of this study with the aim of demonstrating its ability in comparison with similar method.

Example 1. In this example based on the study of Safaei and Saraj (2014), it is considered the following FFBLP problem with non-negative variables (see model (15)).

$$
\begin{aligned}
&\max_{x_1} z_1 = (3,5,7) \otimes x_1 \oplus (2,4,8) \otimes x_2 \\
&\max_{x_2} z_2 = (3,5,10) \otimes x_2 \oplus (1,7,8) \otimes x_2 \\
&s.t. \\
&\quad (4,5,9) \otimes x_1 \oplus (2,7,8) \otimes x_2 \le (4,10,20) \\
&\quad (0,3,7) \otimes x_1 \oplus (1,2,10) \otimes x_2 \le (2,5,18) \\
&\quad x_1, x_2 \in TF(\text{n})^+
\end{aligned}
\tag{15}
$$

In above model, consider $x_1 = ((x_1)^l, (x_1)^c, (x_1)^u) = (x_1, y_1, t_1)$ and $x_2 = ((x_2)^l, (x_2)^c, (x_2)^u) = (x_2, y_2, t_2)$. Based on step 1 of the proposed approach, the FFBLP problem may be written as model (16).

$$
\begin{aligned}
&\max_{\tilde{x}_1} z_1 = (3x_1 + 2x_2, 5y_1 + 4y_2, 7t_1 + 8t_2) \\
&\max_{\tilde{x}_2} z_2 = (3x_1 + x_2, 5y_1 + 7y_2, 10t_1 + 8t_2) \\
&s.t. \\
&\quad (4x_1 + 2x_2, 5y_1 + 7y_2, 9t_1 + 8t_2) \le (4,10,20) \\
&\quad (x_2, 3y_1 + 2y_2, 7t_1 + 10t_2) < (2,5,18) \\
&\quad x_1 \ge 0,\ y_1 - x_1 \ge 0,\ t_1 - y_1 \ge 0 \\
&\quad x_2 \ge 0,\ y_2 - x_2 \ge 0,\ t_2 - y_2 \ge 0
\end{aligned}
\tag{16}
$$

So, inspiring the method of Ezzati et al. (2015), with respect to step 2, model (16) is converted to the multi-objective BLP problem (see model (17)).

$$
\begin{aligned}
&\max_{y_1}(z_1)^c = 5y_1 + 4y_2 \\
&\max_{y_2}(z_2)^c = 5y_1 + 7y_2 \\
&\min_{x_1,t_1}(z_1)^u - (z_1)^l = 7t_1 + 8t_2 - 3x_1 - 2x_2 \\
&\min_{x_2,t_2}(z_2)^u - (z_2)^l = 10t_1 + 8t_2 - 3x_1 - x_2 \\
&\max_{x_2,t_2}(z_1)^u + (z_1)^l = 7t_1 + 8t_2 + 3x_1 + 2x_2 \\
&\max_{x_2,t_2}(z_2)^u - (z_2)^l = 10t_1 + 8t_2 + 3x_1 + x_2 \\
&s.t. \\
&\quad 4x_1 + 2x_2 \le 4 \\
&\quad x_2 \le 2 \\
&\quad 5y_1 + 7y_2 \le 10 \\
&\quad 3y_1 + 2y_2 \le 5 \\
&\quad 9y_1 + 8t_2 \le 20 \\
&\quad 7y_1 + 10t_2 \le 18 \\
&\quad x_1 \ge 0 \\
&\quad x_2 \ge 0 \\
&\quad y_1 - x_1 \ge 0 \\
&\quad y_2 - x_2 \ge 0 \\
&\quad t_1 - y_1 \ge 0 \\
&\quad t_2 - y_2 \ge 0
\end{aligned}
\tag{17}
$$

Using steps 3 to 5, the optimal solutions of model (17) (and hence model (16)) is achieved by simplex method and using MATLAB software as follows:

$$(x_1^*)_{proposed\ method} = (x_1^*, y_1^*, t_1^*) = (0.772727, 1.363636, 1.647058),$$

$$(x_2^*)_{proposed\ method} = (x_2^*, y_2^*, t_2^*) = (0.454545, 0.454545, 0.647058).$$

By putting these optimal values in objective functions it is obtained that:

$$(z_1^*)_{proposed\ method} = (3.227271, 8.636360, 16.705870) \quad (z_2^*)_{proposed\ method} = (2.772726, 10.00000, 21.647044).$$

Now, the results on the basis of the model of Safaei and Saraj (2014) is achieved as follows:

$$(x_1^*)_{Safaei\ and\ Saraj} = (0.772727, 1.363636, 1.647058) \quad (x_2^*)_{Safaei\ and\ Saraj} = (0.454545, 0.454545, 0.647058)$$

$$(z_1)_{Safaei\ and\ Saraj} = (3.227271, 8.636360, 16.705870) \quad (z_2)_{Safaei\ and\ Saraj} = (2.772726, 10.000000, 21.647044)$$

Comparing the results of proposed method with the method of Safaei and Saraj (2014), the same results are obtained (table 1)

$(z_1)_{proposed\ method} = (z_1)_{Safaei\ and\ Saraj}$ $(z_2)_{proposed\ method} = (z_2)_{Safaei\ and\ Saraj}$).

Table 1. Summary of example 1 results

Solutions	Safaei and Seraj	Proposed method
Fuzzy optimal solution	$x_1^* = (0.772727, 1.363636, 1.647058)$	$x_1^* = (0.772727, 1.363636, 1.647058)$
	$x_2^* = (0.454545, 0.454545, 0.647058)$	$x_2^* = (0.454545, 0.454545, 0.647058)$
Fuzzy objective solution	$z_1 = (3.227271, 8.636360, 16.705870)$	$z_1 = (3.227271, 8.636360, 16.705870)$
	$z_2 = (2.772726, 10.000000, 21.647044)$	$z_2 = (2.772726, 10.000000, 21.647044)$

Example 2. In this example based on the study of Tayebnasab et al. (2020), it is considered the following FFBLP problem with non-negative variables (see model (18)).

$$
\begin{aligned}
&\max_{x_1} z_1 = (1,6,9) \otimes x_1 \oplus (2,3,8) \otimes x_2 \\
&\max_{x_2} z_2 = (1,6,3) \otimes x_2 \oplus (3,4,5) \otimes x_2 \\
&s.t. \\
&\quad (2,3,4) \otimes x_1 \oplus (1,2,3) \otimes x_2 = (6,16,30) \\
&\quad (-1,1,2) \otimes x_1 \oplus (1,3,4) \otimes x_2 = (1,17,30) \\
&\quad x_1, x_2 \geq 0
\end{aligned}
\tag{18}
$$

So, with respect to step 2, the above model is converted to the MOLP problem (see model (19)).

$$
\begin{aligned}
&\max_{y_1}(z_1)^c = 6y_1 + 3y_2 \\
&\max_{y_2}(z_2)^c = 6y_1 + 4y_2 \\
&\min_{x_1,t_1}(z_1)^u - (z_1)^l = 9t_1 + 8t_2 - x_1 - 2x_2 \\
&\min_{x_2,t_2}(z_2)^u - (z_2)^l = 3t_1 + 5t_2 - x_1 - 3x_2 \\
&\max_{x_1,t_1}(z_1)^u + (z_1)^l = 9t_1 + 8t_2 + x_1 + 2x_2 \\
&\max_{x_2,t_2}(z_2)^u - (z_2)^l = 3t_1 + 5t_2 + x_1 + 3x_2 \\
&s.t. \\
&\quad 2x_1 + x_2 = 6 \\
&\quad -x_1 + x_2 = 1 \\
&\quad 3y_1 + 2y_2 = 16 \\
&\quad y_1 + 3y_2 = 17 \\
&\quad 4t_1 + 3t_2 = 30 \\
&\quad 2t_1 + 4t_2 = 30 \\
&\quad x_1 \geq 0 \\
&\quad x_2 \geq 0 \\
&\quad y_1 - x_1 \geq 0 \\
&\quad x_1 \geq 0 \\
&\quad x_2 \geq 0 \\
&\quad y_1 - x_1 \geq 0 \\
&\quad y_2 - x_2 \geq 0 \\
&\quad t_1 - y_1 \geq 0 \\
&\quad t_2 - y_2 \geq 0
\end{aligned}
\tag{19}
$$

Using steps 3 to 5, the optimal solution of model (19) (and hence model (18)) is obtained as follows:

$(x_1^*)_{proposed\ method} = (x_1^*, y_1^*, t_1^*) = (1.67, 2, 3)$, $(x_2^*)_{proposed\ method} = (x_1^*, y_1^*, t_1^*) = (2.67, 5, 6)$.

By putting these optimal values in objective function it is obtained that:

$(Z_1^*)_{proposed\ method} = (7, 27, 75)$, $(Z_2^*)_{proposed\ method} = (9.67, 32, 39)$

Now, by the method of Safaei and Saraj (2014) the optimal value of model (18) is achieved which shows that the results are the same in both methods (table 2).

$(x_1^*)_{Safaei\ and\ Saraj} = (1.67, 2, 3)$, $(z_1)_{Safaei\ and\ Saraj} = (7, 27, 75)$

$(x_2^*)_{Safaei\ and\ Saraj} = (2.67, 5, 6)$, $(z_2)_{Safaei\ and\ Saraj} = (9.67, 32, 39)$

Table 2. Summary of example 2 results

Solutions	Safaei and Seraj	Proposed method
Fuzzy optimal solution	$x_1^* = (1.67, 2, 3)$	$x_1^* = (1.67, 2, 3)$
	$x_2^* = (2.67, 5, 6)$	$x_2^* = (2.67, 5, 6)$
Fuzzy objective solution	$z_1 = (7, 27, 75)$	$z_1 = (7, 27, 75)$
	$z_2 = (9.67, 32, 39)$	$z_2 = (9.67, 32, 39)$

5-Conclusions

According to introducing bi-level programming in the past few decades, the relevant problems have been investigated in terms of theoretical and computational points. One of the main shortcomings of its primary models is considering all parameters or some of them as crisp and precise numbers, while in real-world applications, this is not acceptable because of uncertain information. Therefore, it is required to use FFBLP models where both coefficients and variables are fuzzy numbers. However, regarding the difficulty involved in solving the FFBLP models, a heuristic approach is needed to deal with this issue: which can be addressed as a limitation. This study proposes a novel approach to solve these models by converting the FFBLP problem to multi-objective BLP problem in the base of TFNs definition. Next, using a lexicographic method and theorem 1, an optimal solution is obtained to FFBLP. The capability of the proposed approach was shown by solving two numerical examples and comparing their results with those of Safaei and Saraj (2014).

As some suggestions for future studies:

- The proposed problem can be considered in bi-polar, m-polar and spherical fuzzy environments to apply different uncertainties.
- The proposed model can be applied on solving some real world problems.
- The proposed method may be extended to handle fully fuzzy multi-level programming problems.
- The proposed model can also be applied to non-linear case.

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