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Three new heuristic algorithms for the fleet size and mix green vehicle routing problem

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Abstract

In recent years, great efforts have been made to reduce greenhouse gas emissions by vehicles. Petroleum products produce green house gas emissions; therefore reducing the use of these products can make a major contribution to reducing pollution. The Fleet Size and Mix Vehicle Routing Problem is one of the most widely used routing branches.In this problem, there are vehicles with different capacities and there is the possibility of choosingvehicles of different types. In this paper, Fleet Size and Mix Vehicle Routing Problem is death(?)consideringthe reduction of fuel consumption. Since this problem is NP-hard, three novel heuristic methods entitled GROS-I, GROS-II, GGTare presented for the problem. In order to evaluate the proposed heuristics a number of small, medium and large problems are solved. The results show that proposed algorithms have good performances.

Keywords: Mix vehicle routing, Reduction of fuel consumption, Heuristics, Green vehicle routing problem

1- Introduction

Vehicle Routing Problem (VRP) is a familiar concept in the field of operations research, andgreat efforts and developments have been made in the last three decades in this area. This problem was first formulated and solved by Dantzig and Ramser based on mathematical techniques (Dantzig and Ramser, 1959). A classic vehicle routing problem can be defined as a complete graph; $G=(V,A)$, $V=\{0,1,2,...,n\}$, as a set of nodes, and A as a collection of arcs between two nodes in which nodes are the customers and edges show the path between the clients. In this case, node 0 represents the depot. All vehicles must begin their service from the depot and after surveying the determined path return there again. The main objective inclassical VRP is to minimize travel costs. In the Fleet Size and Mix Vehicle Routing Problem, there are several types of vehicles and there is the possibility of choosing the vehicle

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type. Transportation is one of the main sources of greenhouse gases emission such as carbon dioxide (CO_2) and its value is directly dependent on the amount of fuel consumed by the vehicle (Kirby et al.). Transportation sector as responsible for transporting millions of tons of cargo and many passengers is an irreplaceable foundation for economic and industrial development.

Given the importance of the issue, three heuristic algorithmsare presented to solve the Green Fleet Size and Mix Vehicle Routing Problem and their performances are examined.

The structure of the paper is as follows: after the introduction in the second sectiona literature review is given. In the third part, mathematical model of Fleet Size and Mix Vehicle Routing Problem is presented due to reducing fuel consumption. In the fourth part,solution approaches are discussed. The fifth section,deals with the results and in the final section, conclusions are presented.

2- Literature Review

Shao et al have developed fuel consumption formulation. They offered the rate of fuel consumption for a vehicle routing problem with limited capacity. In their study, the load of the vehicle as well as the distance traveled was presented as factors to determine the cost of fuel.It is assumed that the rate of fuel consumption is a function of vehicle load and it changes linearly (Shao and Huang, 2014).

Besides the distance traveled and the load, Kucukoglu added the vehicle speed to fuel consumption model,and developed it for vehicle routing problem dependent on time and used simulated annealing algorithm to solve the model (Kucukoglu et al. 2013). Palmer integrated vehicle routing and carbon dioxide emissions and calculatedthe amount of carbon dioxide produced, time and distance traveled. He studied the effect of speed in reducing carbon dioxide in various traffic conditions and time limitation. The results show that five per cent reduction in carbon dioxide production is accessible (Palmer, 2007). Fagerholt et al offered a model for reducing fuel consumption and greenhouse gas emissions by optimizing the speed. They assumed transport routes and time windows as fixed and optimized speed in every section of the course considering the savings in fuel consumption (Fagerholt et al. 2010). According to Campbell's research vehicle type has a significant impact on fuel consumption; if for products distribution, large vehicles are replaced with a larger number of small vehicles fuel consumption and thus CO_2 emissions will significantly increase.On the other hand,the type of vehicle also affects factors such as engine wear, engine speed, enginedisplacement, aerodynamics friction, total vehicle weight and cargo transporting ability and so on and as a result fuel consumption.

Kara et al. modeled minimizing the energy of vehicle routing problem like CVRP through a new objective function, which contains the product of the multiplication of total load (including empty load and vehicle) and arc length.He determined the relationship between minimizing the energy consumed and the factors related to the vehicle variable. According to the authors, this model minimizes the total energy requirements and consequently fuel consumption; but the details of the formulation of fuel consumption were not provided (Kara et al.2007).

Yong Peng modeled vehicle routing problem with regard to fuel consumption and associated fuel consumption only to the cargo of the vehicle. In their target function, they considered both goals of minimizing the distance traveled and fuel of the vehicle. To have lower fuel consumption, he proposed that the vehicles, at first serve the customers with higher demandand then the ones with lower demand (Yong Peng, 2009).

Ubeda et al. examined the VRP with carrying cargo on the return to minimize greenhouse gas (Ubeda et al. 2011). Faulin et al studied CVRP issue with regard to environmental issues, regardless of traditional costs. They concentrated on environmental costs stemming from noise and traffic congestion (Faulin et al.2011). Figliozzi focused on the analysis of CO_2 emissions for different levels of density and absolute demands of customer (Figliozzi,2011). Omidvar and Tavakkoli Moghadam introduced a model of vehicle routing for vehicles with alternative fuel AFV (hybrid, electric and fuel cell vehicles, etc.) in order to minimize CO_2 emissions and fuel consumption (Omidvar et al. 2012).

Saberi and Robass examined minimizing the emission of pollutants in the concept of TDVRP (Saberi, 2012). Erdogan and Miller-Hooks formulated GreenVRP as a mixed integer linear program and developed two construction heuristics, the Modified Clarke and Wright Savings heuristic and the Density Based Clustering Algorithm, and a customized improvement technique (Erdoğan and Miller-Hooks, 2012). Kopfer studied reducing emission of pollutants in VRP with regard to mix fleet(Kopfer and Kopfer,2013). Kwonalso proposed the Mix Vehicle Routing Problem with the aim of minimizing CO2 emissions (Kwon et al.,2013). Kocet al. modeled Mix Vehicle Routing Problem with regard to reducing fuel consumption and for solving the model proposed a search-based algorithm model (Koç et al.,2014). Lin et al presented a review of green routing problem in which past and future trends are discussed(Lin et al., 2014).

3- Defining the problem model

There are many models for the calculation of greenhouse gas emissions and fuel consumption, whichare different in modeling procedure, structure and data requirements. One of these models is microscopic model that estimates greenhouse gas emissions and fuel consumption moment by moment. These models are known as force-based models.

One of the most widely used microscopic models is comprehensive modal emission model (CMEM) which can be used to calculate greenhouse gas emissions and fuel consumption. This model is discussed in this paper according to research by Kocet al. In this model, speed and vehicle load and road inclination are examined (Koç et al.). In accordance with CMEM, the fuel consumption rate of vehicle type h is calculated from thebelow relationship (1).

$$FR^{h} = \xi\left(k^{h} N^{h} V^{h} + P^{h}/\eta\right)/\kappa \qquad (1)$$

Where, ξ mass rates of fuel to the air. k^{h} is vehicle type h engine friction. N^{h} is engine speed and V^{h} is engine displacement vehicle type h. η and κ are constant values and the diesel engines efficiencyparameter and thermal value of diesel fuel, respectively. P^{h} is momentary engine power output vehicle type h (in kW) and is calculated in equation (2).

$$P^{h} = P^{h}_{tract}/\eta^{h}_{tf} + P_{acc} \qquad (2)$$

Where, η^{h}_{tf} is the vehicle drive train efficiency. P_{acc} is the power needed for vehicle accessories such as air conditioning and etc., this parameter is assumed zero. P_{tract} is the required tensile force in vehicle wheels (in kW) and is calculated in equation (3).

$$P^{h}_{tract} = (M^{h}a + M^{h}g\sin\theta + 0.5C^{h}_{d}\rho A^{h}v^{2} + M^{h}gC_{r}\cos\theta)v/1000 \qquad (3)$$

Where, M^{h} is the weight of the vehicle type h (including the weight of the empty vehicle and load) in terms of kg. M^{h} is divided into two parts, w and f that are the weight of the empty vehicle and the vehicle's load weight are the weight of the empty vehicle and the vehicle's load weight. a is acceleration of the vehicle (m/ s2) . v ,θ and g are the vehicle speed (m/ s) , road slope and the gravitational constant. C^{h}_{d} and C_{r} are coefficient of aerodynamic drag and coefficient of rolling resistance, respectively. ρ and A^{h} are air density (kg/ m^3) and frontal surface area of the vehicle type h (m^2) . For arc (i, j) with d length, v is speed of vehicle that crosses this arc. If all the variables in the equation (1), except for the speed during arc are assumed constant, the fuel consumption (in liters) in the arc is calculated using equations (4) and (5).

$$F^{h} = k^{h}N^{h}V^{h}\lambda d/v + P^{h}\lambda\gamma^{h}d/v \qquad (4)$$

Where, in equation (4) λ and γ^{h} are calculated using (5) and (6) equations;

$$\lambda = \xi/\kappa\psi \qquad (5)$$

$$\gamma^{h} = 1/1000n^{h}_{tf}\eta \qquad (6)$$

Where, ψ is fuel conversion factor of g/s to lit/s.α and $\beta$are coefficients calculated using equations (7) and (8).

$$\alpha = a + g sin\theta + gC_r cos\theta \qquad (7)$$

$$\beta^{h} = 0.5C^{h}_{d}\rho A^{h} \qquad (8)$$

Index of (i, j) arc placed on speed, distance, vehicle load and □□of the same arc. Equation (4) can be written as equation (9) (Koç et al.).

$$F^{h} = \lambda(k^{h}N^{h}V^{h}d/v + M^{h}\gamma^{h}\alpha d + \beta^{h}\gamma^{h}v^{2}d) \qquad (9)$$

Vehicle common parameters and vehicle specific parameters are shown in Table1 & Table2.

Table 1.vehicle common parameters (Koç et al.)

Notation	Description	Typical values
ξ	fuel-to-air mass ration	1
g	Gravitational constant(m/s^2)	9.81
ρ	Air density(kg/m^3)	1.2041
C_r	Coefficient of rolling resistance	0.01
η	Efficiency parameter for diesel engines	0.45
f_c	Fuel cost per liter	1
κ	Heating value of a typical diesel fuel	44
ψ	Conversion factor (g/s to L/s)	737
a	acceleration	0

Table 2. Vehicle specific parameters

Notation	Description	Light duty (h=1)	Medium duty (h=2)	Heavy duty (h=3)
w^h	Curb weight (kg)	4672	6328	13154
Q^h	Capacity vehicle	2600	5000	17000
f^h	Fixed cost vehicle	41.68	59.9	93.92
k^h	engine friction factor (kJ/rev/L)	0.25	0.2	0.15
N^h	engine speed (rev/s)	39	33	30.2
V^h	engine displacement	2.77	5	6.66
C_d^h	coefficient of aerodynamic drag	0.6	0.6	0.7
A^h	frontal surface area (m^2)	9	9	9.8
n_{tf}^h	vehicle drive train efficiency	0.4	0.45	0.5

Model parameters are listed in Table 3. The notes written in front of some of the parameters are parameter index (indices) in the model.

Table 3. Model parameters

Notation	Description
N(i, j, p)	Set of customers and depot(nodes)
N′(i, j)	Set of customers
H (h)	Set of vehicles type
cap_i	i-th Customer capacity
Q_h	Vehicle capicity
d_{ij}	Distancebetween the customer i and j
f_h	Vehicle fixed coste
q_i	i-th customer demand
f_d	Driver Wage
v_h	Optimal speed for the vehicle of type h
M	Big number

3-1- The proposed model

The variables x_{ij}^{h} and f_{ij}^{h} are used in this modeling where i, j are customers indices and h is an index for the type of vehicle. x_{ij}^{h} of binary variable is equal to 1 if the vehicle of type h travels route i-j, and is equal to zero otherwise, and
variable f_{ij}^{h} shows the flow of vehicle type h in the route i to j. In this section, the proposed model and its description are presented. The proposed mathematical model for the Fleet Size and Mix Green Vehicle Routing Problem is as follows.

$$
\begin{aligned}
\min Z = & \sum_{i\in N}\sum_{j\in N}\sum_{h\in H}\lambda f_c k^h N^h V^h d_{ij} x_{ij}^{h} / v^h \\
& + \sum_{i\in N}\sum_{j\in N}\sum_{h\in H}\lambda f_c \gamma^h \alpha d_{ij} \left(w_h x_{ij}^{h} + f_{ij}^{h}\right) \\
& + \sum_{i\in N}\sum_{j\in N}\sum_{h\in H}\lambda f_c \beta^h \gamma^h d_{ij} \left(v^h\right)^2 x_{ij}^{h} \qquad (10) \\
& + \sum_{h\in H}\sum_{j\in N'} f^h x_{0j}^{h} \\
& + \sum_{i\in N}\sum_{j\in N}\sum_{h\in H} f_d d_{ij} x_{ij}^{h} / v^h
\end{aligned}
$$

Subject to:

$$\sum_{h\in H}\sum_{j\in N,j\neq i} x_{ij}^{h} = 1 \qquad \forall\, i \in N' \qquad (11)$$

$$\sum_{i\in N,i\neq p} x_{ip}^{h} = \sum_{j\in N,j\neq p} x_{pj}^{h} \qquad \forall\, p \in N'; h \in H \qquad (12)$$

$$\sum_{h\in H}\sum_{j\in N} f_{ji}^{h} - \sum_{h\in H}\sum_{j\in N} f_{ij}^{h} = q_i \qquad \forall\, i \in N' \qquad (13)$$

$$q_i * x_{ij}^{h} \leq f_{ij}^{h} \qquad \forall\, i,j \in N'; h \in H \qquad (14)$$

$$f_{ij}^{h} \leq \left(Q_h - q_i\right) * x_{ij}^{h} \qquad \forall\, i,j \in N'; h \in H \qquad (15)$$

$$x_{ij}^{h} \in \{0,1\}, f_{ij}^{h} \geq 0 \qquad \forall\, i,j \in N'; h \in H \qquad (16)$$

In the proposed model, target function is composed of the five parts.Parts one to threeare related to the cost of fuel consumption and emissions of pollutants, part 4 considers fixed vehicle costs and part 5 calculates driver wage. Constraint (11) ensures that each customer vertex has exactly one successor: a customer or depot vertex, restriction (12) guarantees by which the number of arrivals at a vertex must equal the number of departures for all.Constraint (13) determines the differences between the input side and the output side to each customer that are the goods delivered to the customer(?). Constraints (14) and (15) ensure that the input flow to a customer must be greater than customer demand and less than the capacity of theallocated vehicle. Constraint (16) defines the range of variables.

4-The proposed heuristic algorithms

In the section proposed heuristic algorithms are described.

4-1- Saving algorithm

As mentioned in the literature review, in 1964, Clark and Wright proposed an algorithm for solving homogeneous vehicle routing problems. This algorithm is based on the concept of saving (Clarke and Wright, 1964). The algorithm first calculates the savings from connecting two customers, then the two customers are allocated to a route by further savings, in algorithm process all non –allocated customers are placed only at start end(?) and cost function is calculated, the customer with the lowest cost is put in right place, this is repeated until the capacity of the vehicle allows and then customers not allocated will be allocated by other vehicles. The algorithm was then extended for fleet size and mix problems. Difference between fleet size and mix problem solving and homogeneous problem solving is the determining of saving. In every iteration of saving algorithm for fleet size and mix problem, saving is re-calculated (Golden et al.). Golden developed savings criteria of the algorithm in 1984 and four algorithms with different saving criteria were proposed. One of these algorithms is realistic opportunity algorithms that is developed in this paper for Green problem (Golden et al.,1984)

4-1-1-GROS-I algorithm

In this section the steps of the proposed GROS-I algorithm are explained. In order to correct ROS algorithm in the problem of green vehicle routing problem, the saving is corrected; the proposed saving is presented as relation (17).

$$s_{ij} = f(i) + f(j) - f(i,j) + \delta(w).F^{'}(P(z_i + z_j) - z_i - z_j)$$
$$w = P(z_i + z_j) - P(\max\{z_i, z_j\}) \tag{17}$$
$$\delta(w) = \begin{cases} 1 & \text{if} \quad w > 0 \\ 0 & \text{if} \quad w = 0 \end{cases}$$

In relation (17):

- $f(i)$ includes the cost of fuel, driver wage and fixed costs of the vehicle, and it is a tour where node i is the first or the last node of that tour. (Target function)
- $f(j)$ includes the cost of fuel, driver wage and fixed costs of the vehicle, and it is a tour where node j is the first or the last node of that tour.
- $f(i,j)$ includes the cost of fuel, driver wage and fixed costs of the vehicle, and it is a tour obtained from the integration of two tours by connecting to two nodes i and j.
- $P(z_i)$ is the capacity of the smallest vehicle that can serve the demand of a tourwhere i is its first or last node.
- $F'(z_i)$ represents the fixed cost of the largest vehicle which has the capacity less or equal to z.

- $\delta(w)$ gets one when the integration of twotours leads to use a vehicle with more capacity.

The steps of realistic saving of opportunityalgorithm are given below.
Step 1: A vehicle is allocated for each of the nodes (customers).
Step 2: Steps 3 to 6 are repeated.
Step 3: Saving obtained from connecting each pair of nodes that have the two following conditions simultaneously;calculate according to the relation (17).
1. Two nodes should be the starting and finishing points in the tours
2. Total demand of the two tours that these two nodes are its starting and finishing points must not exceed the capacity of the largest vehicle available.
Step 4: If there are nopositive savings among nodes, go to step 7.
Step 5: Two nodes that have the highest amount of savings should be selected
Step 6: Two tours where the two selected nodes are the starting and finishing points of them should be integrated.
Step 7: The end and displaying the answer.

4-1-2- GROS-II algorithm

Studying saving algorithms shows that thesealgorithms are greedy; meaning that in each step always the best pairregarding the savings is chosen. Therefore, in many cases suitable solutions are not investigated.Thus, in this paper, the selection procedure to cover the proposed disadvantage is that at every step nodes are not selected based on the maximum savings. For this purpose, tournament and roulette wheel selections are used in the selection of pair of the nodes. In this case, initially a random number between “a” and ” b” (“a” and “b” are the parameters of the algorithm and are the lowest and highest limit of the number of nodes that have the highest savings, respectively.) is produced.The pairs with the highest selective savings (t) are stored in another list. Now from this new list, considering the saving and using roulette wheel an edge is chosen to integrate. This method was implemented on ROS algorithm and is given as GROS-II in the calculations. In addition to the idea expressed in GROS-II algorithm, local 2-opt search is also used.
The proposed algorithm steps’ are as follows:
Step 1: A vehicle is allocated for each of the nodes (customers).

Step 2: Steps 3 to 11 should be repeated until creating the tours related to all customers.
Step 3: Saving results from connecting all pairs of nodesthathave the following two conditions should be calculated from Relation (17) and stored in List 1.
1. Two nodes should be the starting or ending point of the tours
2. Total demand for the two tours that these two nodes are the starting or ending point of them must not exceed the capacity of the largest vehicle available.
Step 4: If there were no positive savings among the nodes, go to step 12.
Step 5: List 1 must be ordered in a descending order
Step 6: From the intervals $\left[a,b\right]$,a(t) number must be selected randomly (in this paper a = 2 and b = 6);
Step 7: From the beginning of the List 1, t pairs of nodes must be selected and stored in List 2.
Step 8: In List 2, according to the amount of savings, roulette wheel must be used and a pair of nodes randomly chosen.
Step 9: Two tours where the pairs of nodes selected from the previous step are the starting and ending parts of them, must be integrated.
Step 10: Modified tours must be optimized using local search 2-opt.
Step 11: The end and displaying the answer.
It should be noted that the proposed algorithm is repeated several times (in this paper, 10 times repetition has been considered) and the results are reported.

4-2- Giant tour algorithm

Giant tour algorithms are examples of "route first-cluster heuristics. GT algorithm is a two-way algorithm. In the first stage, a tour that will visit all customers is produced, traveling salesman problem is solved for generating this tour; and in the second stage the giant tour is divided into sub-tours of which the start and end points are originated. In the first stage, solving the TSP guarantees that adjacent customers will be reasonably close as far as routing cost is concerned.Customerdemands and the fixed vehicle costs in the second stage will be considered. In this section, an algorithm based on the giant tour isproposed for green problem.

4-2-1- Green Giant Tour Algorithm (GGT)

Suppose TSP output is the sequence of customers as 0-L (1) -L (2) - ... -L (n) -0. COST (k, m) expresses the cost of placing customers L (k) to L (m-1) in a sub-tour that is defined as equation (18):

$$\begin{aligned} &COST\,(k,m) = c'_{0,s(k)} + \sum_{r=k}^{m-2} c'_{s(r),s(r+1)} + c'_{s(m-1),0} + F\left(\sum_{r=k}^{m-1} d_{s(r)}\right) \\ &c'_{ij} = \lambda f_c k^h N^h V^h c_{ij} / v^h + \lambda f_c \gamma^h \alpha c_{ij} \left(w_h + f_{ij}^h\right) + \lambda f_c \beta^h \gamma^h c_{ij} \left(v^h\right)^2 + f_d c_{ij} / v^h \end{aligned} \tag{18}$$

In equation (18) d_i is demand of node i, $F(\sum_{r=k}^{m-1} d_{L(r)})$ is the cost of the smallest vehicle that can carry demand $\sum_{r=k}^{m-1} d_{L(r)}$. $c_{0,L(k)}$ is the cost of travelbetween depotandnode k. $c_{L(i),L(j)}$ is the cost of travel between nodes i and j. Ifthedemand of a sub-tour is greater than thecapacityof the largest vehicle, then the sub-tour is infeasible. For allpossible states, equation (18) is calculated. L (1) is the firstcustomerin thesequenceobtainedfrom TSP. For better expression algorithm of figure(1) is presented.

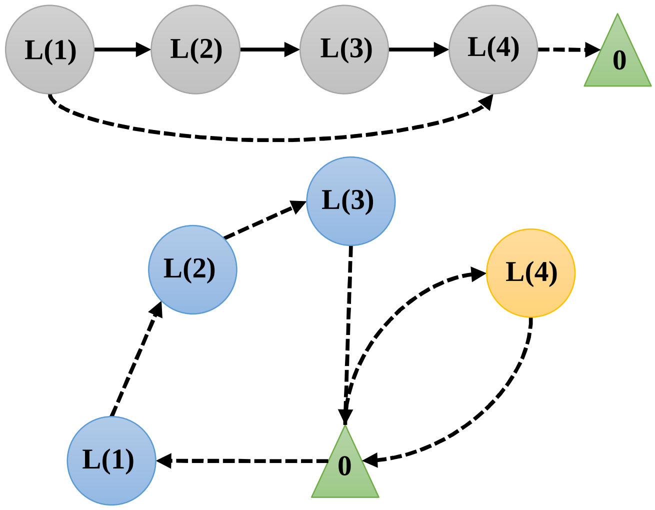


Fig 1. An example of Giant Tour algorithm

In Figure (1) the cost of the selected path (selected with dotted line) is as follows.

$$COST(1,4)+COST(4,0)=c'_{0,L(1)}+\sum_{r=1}^{2}c'_{L(r),L(r+1)}+c'_{L(3),0}+F(\sum_{r=1}^{3}d_{L(r)}) \\ +c'_{0,L(4)}+c'_{L(4),0}+F(d_{L(4)}) \qquad (19)$$

5- Computational results

In this section, modified heuristic algorithms are studied and their performance is evaluated. Fifteenmix vehicle routing problem with modified green realistic opportunity saving algorithm (GROS-II), green realistic opportunity saving algorithm (GROS-I), Green Giant Touring algorithm (GGT) are solved for the model of this paper. This section proposed heuristic algorithm are addressed in small, medium and large scales. To generate these problems, Augerat problem instances was used. For the production of small and medium-scale problems, for the intendednumber of nodes, from the beginning of nodes n = 16 has been removed.In large problem instances, from problem 1 to 4 for the number of nodes intended n=32 from the beginning of the nodes of the problem Augerat has been removed. For other large problem instances, from the beginning of the nodes of the problem n=80 from Augerat problems, the nodes are selected. The algorithm was implemented using MATLAB 2012 Software on a PC CPU Core i5 and 4GB RAM computer.In the output display of the program, the route and the type of vehicle and the objective function value are determined. Parameters related to fuel consumption reduction are given in Tables 1 and 2. Since the parameters related to fuel consumption reduction are real, in these problems, 3 types of small, medium and heavy vehicles are assumed whose parameters are considered according toTable (2). The speeds of vehicles are eql to their optimal speed according to Table (4).

Table 4. Optimal speed of different types of vehicles (m/s)**(**Koç et al., 2014)

Type of vehicle	Optimal speed
Light duty	13.83
medium duty	15.27
Heavy duty	18.5

The results of calculations are given in Tables 5 and 6. Table 5 is related to small and medium-sized problems in comparison with the exact solution, and Table 6 is related to large sized problems.In these tables, the percentage of error is calculated from Formula 20. It is notable that in the first column of the tables (the problem specifications), the first number is the problem number, and the second number indicates the number of customers. For example, the meaning of 1.3 is the first problem with three customers.

$$\frac{objective - BS}{BS} * 100 \qquad (20)$$

In relation (20), objective is the solution, and BS is the solution found by the algorithms studied.

5-1- The results in small and mediumsized problems

In this section, the results of proposed algorithms are compared with exact methods. Six problem are produced according to the descriptions given. The results of the small-scale problems are given in Table 5. The exact solution is given by using GAMZ software (Salver: CPLEX).

Table 5. Results ofheuristic algorithms in small and mediumsize problems

Problems	**Exact solution**		**GROS-I algorithm**			**GGT algorithm**			**GROS-II algorithm**		
	Objective function	time(s)	Objective function	Solving time(s)	Error percent	Objective function	time(s)	Error percent	Objective function	time(s)	Error percent
1/3	375.7	0.01	381.4	0.008	1.5	375.7	0.001	0	384.4	0.1	2.3
2/4	529.4	2.1	532.6	0.009	0.6	529.4	0.001	0	529.4	0.2	0
3/5	541.9	4.3	549.4	0.014	1.4	541.9	0.001	0	545.2	0.2	0.6
4/7	717.7	79.6	740.4	0.02	3.2	742.9	0.001	3.5	717.7	0.3	0
5/8	868.1	1411.2	921.2	0.02	6.1	913.4	0.001	5.2	874.8	0.3	0.7
6/10	1047.2	3456	1086.2	0.03	3.7	1094.6	0.002	4.5	1059.3	0.4	1.1
Average	**680**	**825.54**	**701.87**	**0.02**	**2.75**	**699.65**	**0.00**	**2.20**	**685.13**	**0.3**	**0.78**

According to the obtained result in small scale,all three algorithms have low error percentage. In general, the algorithm (GROS-II) has better mean of error and target function compared to GROS-I and GT algorithms.Although GROS-II algorithm, has increased solution time, it brings about obtaining better target function. The mean of error of the proposed algorithm GROS-II in small-scale problems is0.78%, which is low compared to the other two algorithms that have error of 2.75% and 2.2%, respectively.

5-2- The results in large scale

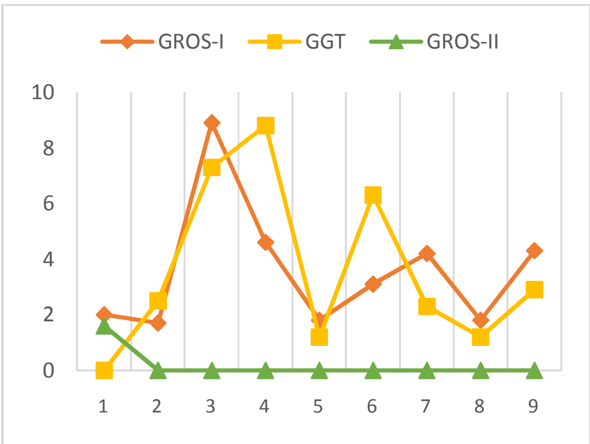
In order to evaluate the performance of large-scale heuristic algorithms, 9 problems were produced according to the process described and the results were evaluated. The results are given in Table (6).

Table 6. Results ofheuristic algorithms at large size problems

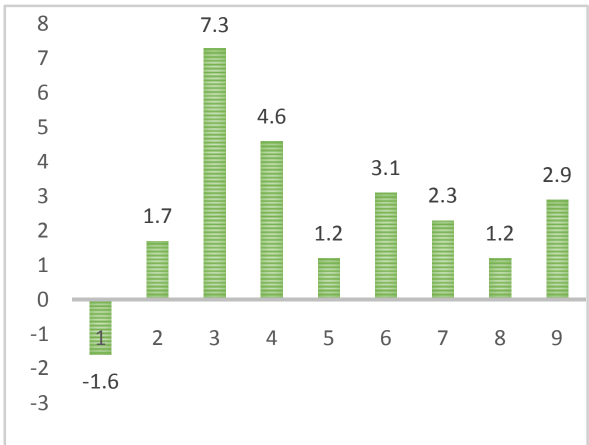
Problem	GROS-I algorithm			GGT algorithm			GROS-II algorithm		
	Objective function	Solving time(s)	Error percent	Objective function	time(s)	Error percent	Objective function	time(s)	Error percent
7/15	1399.4	0.04	2	1371.9	0.002	0	1394.2	0.5	1.6
8/20	1672.3	0.06	1.7	1686.7	0.002	2.5	1644.3	0.8	0
9/25	2212.1	0.09	8.9	2180.9	0.01	7.3	2031.2	1.3	0
10/30	2354.4	0.14	4.6	2447.2	0.01	8.8	2248.9	2.8	0
11/40	2824.3	0.54	1.8	2808.6	0.03	1.2	2773.5	6.1	0
12/50	3472.9	0.98	3.1	3578.4	0.05	6.3	3365.7	12.1	0
13/60	4356.2	1.8	4.2	4276.6	0.07	2.3	4177.3	22.3	0
14/70	4515.9	3.1	1.8	4488.1	0.09	1.2	4435.2	37.4	0
15/80	4917.2	4.9	4.3	4852.8	0.16	2.9	4714.1	57.1	0
Average	**308.5**	**1.3**	**3.6**	**3076.8**	**0.0**	**3.6**	**2976.0**	**16.1**	**0.2**

As shown in Table 6, GROS-II algorithm has 1.6% error only in one case,and in the rest of the sample problems, it could act better than the other two algorithms.

Two other algorithms have the mean error of 3.6% while algorithm GROS-II has a mean error of 0.2% error, and with respect to the results, it can be said that GROS-II algorithm has a better performance compared to the other two algorithms. Figure 2 shows the performance of the proposed algorithms in terms of error rate.



B:The error rate of the proposed algorithms



A: The rate of solutions improved by algorithm GROS-II

Fig 2. The performance of the proposed algorithms in terms of error percentage

As Figure (2) shows, in 8 out of 9 problems GROS-II algorithm has been able to improve the solutions of two other algorithms. The average improvement of solutions by this algorithm is 2.5%. Figure 3 shows the algorithm error in different problems.

6- Conclusion

Increasing consumption of oil products has created significant problem such as air pollution. Since the transportation sector isconsidered as one of the largest consumers of fuel and producing greenhouse gas emissions, studies focusing on the reduction of fuel consumption and greenhouse gas emissions can have positive effects on the environment. In this research three heuristic algorithmswere developed for Mix Green Vehicle Routing Problem. Small-scale computational results show that all three algorithms havelow percentage of error and GROS-II algorithms has bettermean of error andtarget function compared to GGT and GROS-I algorithms. The mean of the errors ofGROS-I, GGT and proposed GROS-II for large size problemsare 3.6%, 3.6% and 0.2% respectively. According to the obtained results, the proposed algorithmsare efficient**.**

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