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Heuristic approach for a pessimistic robust closed loop supply chain network considering commercial, end-of-use and end-of-life returns and quality constraint

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Abstract

Nowadays, due to the environmental issues, governmental regulations and economic benefits, focus on collecting and recovery of products has increased. Recovered products can be reused or sold in secondary markets. In this paper, we consider a given structure for a closed loop supply chain, including a manufacturer, distributer and retailer in the forward logistic; the original products are given to the primary market. In the reverse logistic of the given structure, the returned products are disassembled and some obtained parts are used in the manufacturer. We assume that the produced products from returned parts can be given to a secondary market. A minimum quality level is considered for the returned parts. A collection site, and a repair site is added to the initial structure and it is assumed that the disassembled parts to be categorized into end-of-use, end-of-life and disposals. Some products called commercial returns are not assembled and can be given to the secondary market after a simple repair. Furthermore, uncertainty on the demand and return rates are considered and the operational decision variables of the models which are mainly the flow values in the chain and opening some facilities are determined. Electronic devices such as mobile phones and printers are suitable examples for the studied supply chain. The robust counterpart of the model is developed and a solution approach based on the Lagrangian relaxation is developed for solving the problem. Two heuristics based on partial derivations are developed to solve the sub problems and results are analyzed.

Keywords: Closed loop supply chain, end-of-use, end-of-life, robust optimization, quality level, Lagrangian relaxation

1- Introduction

Nowadays, closed loop supply chain (CLSC) has attracted the attentions of many researchers. In the CLSC, both forward and reverse supply chains are considered jointly. In the forward supply chain, material and product flow is from suppliers to manufacturers, distributors, retailers and customers, while in many industries there is another flow in supply chains formed in the reverse side. In this flow, products are returned from customers to the higher echelons of the chain. In this paper, we study a CLSC structure, including a manufacturer, distributer, retailer and customers in the primary and secondary markets, a collection site, a repair site, a disassembly site, a parts warehouse, external suppliers, recycling sites and a disposal site. Returned products in the CLSC can be remanufactured and sold in the secondary market.

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The full descriptions of the supply chain structure and constraints of the model will be given in the nest sections. Fleischmann et al. (2001) presented a generic facility location model for recovery network configuration. They analyzed product return flows impact on logistic networks. Beamon and Fernandes (2004) proposed a multi-period integer programming model for CLSC design. Jayaraman (2006) presented an analytical approach for production planning and control of CLSC considering product recovery and reuse. Meade et al. (2007) mentioned two reasons for the increased interest and investment in the reverse supply chains: environmental factors and business factors. Environmental factors include environmental impacts of used products, environmental legislations and growing environmental consciousness of customers. Business factors include returned product's benefits and liberal return policies for customer satisfaction attraction. Chung et al. (2008) proposed a multi-echelon inventory system considering remanufacturing capability; then, they developed a CLSC inventory model to maximize profit of the retailer, the third party recycle dealer, the manufacturer and the supplier jointly.

Guang-zhi et al. (2009) formulated a mathematical model for a CLSC design problem supposing normal distribution for demand and return. Gong et al. (2009) studied CLSC problem considering demands as fuzzy variable and returns as stochastic variables; then, they presented a fuzzy chance constraint programming model. Salema et al. (2009) developed a strategic and tactical location-allocation model for CLSC. They also developed formulations in order to integrate strategic and tactical decisions together. They solved their problem using standard branch and bound technique.

Pishvaee et al. (2011) developed a deterministic mixed integer linear programming model for CLSC network design; then, they used robust optimization theory to consider the uncertainty of demand, returns, and shipping costs between facilities. Their results showed that the robust model dominates the deterministic one. Fazel Zarandi et al. (2011) studied CLSC distribution network design problem to emphasize the role of incorporating reverse parameters. Hassanzadeh Amin and Zhang (2012a) proposed a mathematical model for supplier selection and CLSC configuration with two phases. In the first phase, they proposed a framework for supplier selection and then used a fuzzy method to evaluate suppliers. In the second phase, they proposed a multi-objective mixed integer linear programming to maximize total profit, minimize defect rates and maximize the importance of external suppliers that earned in the first phase.

Mehrbod et al. (2012) presented a multi-objective mixed integer nonlinear programming model for closed loop logistics network. Model objectives are minimization of total cost, delivery time of new products and collection time of used products. They applied interactive fuzzy goal programming to solve the model. Wang and Hsu (2012) considered a closed loop supply chain with shortage and surplus arising from the uncertainty of demand, recovery and landfilling. Joochim (2012) developed a dynamic mathematical model to determine strategic decisions for the capacitated facility location problem in CLSC. The uncertainty is described by fuzzy sets. Hasanzadeh Amin and Zhang (2013) proposed a model for CLSC design to minimize total cost. They extended the model to consider environmental objective. They also developed their model by stochastic programming to examine effects of uncertainty on demand and return. Diabat et al. (2013) considered closed loop location inventory problem. In this problem, returns are remanufactured as spare parts. Also, demand and return are considered as uncertain parameters with normal distribution. They proposed a mixed integer nonlinear location-allocation model and an exact two-phase Lagrangian relaxation algorithm to solve it. Ramezani et al. (2013) considered a multi-echelon and multi-product closed loop logistic network under uncertainty and presented a robust design for that. Soleimani et al. (2013) developed a multi-echelon, multi-period and multi-product mathematical model for designing and planning a comprehensive closed loop supply chain network. Cardoso et al. (2013) developed a mixed integer linear programming model for closed loop supply chain design. They considered the uncertainty of demand using scenario tree approach. They showed that reverse logistics incorporation has economic benefits, although costly. Özkır and Başlıgil (2013) examined CLSC design problem. They supposed price and demands as uncertain parameters using fuzzy logic. They proposed a fuzzy multi-objective model to maximize the satisfaction level of trade, satisfaction degrees of customers and total CLSC profit.

Altmann and Bogaschewsky (2014) presented a multi-objective model based on robust optimization for CLSC design to minimize total costs and carbon dioxide equivalents. They considered demand and used product return ratio as uncertain parameters. Mirakhorli (2014) proposed an interactive fuzzy bi-objective model to design CLSC by minimizing total cost and total delivery time of the system. In this problem, demand and return are considered as uncertain parameters. Litvinchev et al. (2014) proposed two mathematical programming models to determine pricing strategy and optimal network design. The first model is proposed for multi-period case while the second one for stochastic demand. Govindan et al. (2015) presented a review of papers which published recently in reverse and CLSC. Jindal et al. (2015) proposed a fuzzy mixed integer linear programming model for CLSC design problem under uncertain environment. In this problem, demand, return, fraction of the recovered parts in different recovery processes and costs related to purchasing, transportation, inventory, processing and setup are considered as uncertain parameters and handled with fuzzy numbers. Garg et al. (2015) presented a bi-objective integer nonlinear programming model for CLSC network design to maximize total profit and minimize the transporting vehicles in forward direction.

Kaya and Urek (2016) presented a mixed integer nonlinear facility location-inventory-pricing model for a CLSC network design to decide on the optimal locations of the facilities, inventory amounts, prices for new products and incentive values for the collection of right amount of used products in order to maximize the total supply chain profit. They developed heuristics for the solution of this model. Ahmadzadeh and Vahdani (2017) considered a three-level location-inventory-pricing problem in a CLSC where demand across the customer zones was correlated, the inventory control at distribution centers followed a periodic review inventory policy, and shortage was allowed. They proposed a mathematical model for the mentioned network structure. Jangali et al. (2021) developed a CLSC network for engine oil in an uncertain environment. In this model, adverse environmental effects are also considered as well as the supply chain regular costs. The goal programming approach was deployed to solve the problem. The demand recyclable materials values were assumed to be of uncertainty, and a robust optimization approach was applied to tackle with the given uncertainty. Biçe and Batun (2021) studied the problem of CLSC network design with uncertainty in demand quantities, return rates, and quality of the returned items. They formulate the problem as a two-stage stochastic mixed-integer program which maximizes the total expected profit.

Regarding returns in CLSC which are collected in collection centers with different qualities, Guide and Van Wassenhove (2009) categorized returns, according to product life cycle as follows: a- Commercial returns, which are products that returned by customers during a certain period of time (for example, 30 days after purchasing); b- End-of-use returns, which are products that returned due to technological upgrade; c- End of life returns, which are products that returned due to obsolete technology. Hence, proper action with confronting each category is repairing, remanufacturing and recycling, respectively. Hassanzadeh Amin and Zhang (2012b) presented a mixed integer linear programming model to design CLSC network considering the product life cycle.

Due to inexact demand forecasting, demand variation around the time (Synder, 2006), and the ambiguity nature of quality and quantity of returns, demand and return are recognized as important source of uncertainty in CLSC. Hence, considering this issue CLSC network design is profitable. Researchers used different approaches to consider uncertainty including stochastic, scenario based, fuzzy based and robust optimization approaches. In this paper, we use robust optimization technique to describe the uncertainty of parameters. Also, considering the closed loop network structure, the manufacturer may use recycled, end-of-use and new parts to produce his products. This in turn affects quality of parts and correspondingly quality of products to be different. For this reason a quality index for parts is defined. This index obliges the manufacturer to reserve the mean quality of used parts higher than a certain limit. “The major novelties of this paper are as following:

- Closed loop network is designed with respect to the product life cycle by considering end-of-use and end-of-life modes
- Returns and demands are considered as uncertain parameters and robust optimization technique is used to tackle their uncertainty
- A quality index is considered to control used parts quality
- A heuristic solution approach based on the Lagrangian relaxation is proposed to solve the problem for large instances by converting it to two sub-problems.”

The rest of the paper is organized as follows: In section 2, notations are presented. In section 3, the problem is defined and the proposed deterministic model is described. Then the used robust optimization technique is described and the developed robust counterpart model is presented. In section 4, we propose a heuristic solution approach based on the Lagrangian relaxation for solving the large instances of problem. In section 5, sensitivity analysis is done with respect to different parameters, the robust model is validated and the heuristic solution approach is evaluated. Finally, we give concluding remarks and some possible future researches in Section 6.

2- Notation

In this section the indices, parameters and decision variables of the model are given.
The indices are as follows:
i: Index of parts, $i = 1, \dots., I$
j: Index of products, $j = 1, \dots., J$
k: Index of suppliers, $k = 1, \dots., K$
l: Index of recycling sites, $l = 1, \dots., L$

The parameters are as follows:
α_i: Quality index for end-of-use part i, $0 \leq \alpha_i \leq 1$
β_i: Quality index for recycled part i, $0 \leq \beta_i \leq 1$.
γ_i: Acceptable quality index for part i, $0 \leq \gamma_i \leq 1$.
M_{1i}: Percentage of end-of-use returns of part i after disassembly of returned products
M_{2i}: Percentage of end-of-life returns of part i after disassembly of returned products
N_j: Percentage of the demand of product j returned to the collection site
z_j: Percentage of commercial returns of product j
A: Maximum capacity of the manufacturer plant
t: Maximum number of recycling sites to be opened
SA_j: Unit selling price of product j at the primary market
SE_j: Unit selling price of product j at the secondary market
ha_j: Unit inventory holding cost of product j at the primary market
he_j: Unit inventory holding cost of product j at the secondary market
a_j: Resource usage to produce one unit of product j in the manufacturer plant
H_j: Unit inventory holding cost of product j at collection site
y_j: Unit direct manufacturing cost of product j
e_j: Resource usage to repair one unit of product j in the repair site
C_j: Maximum capacity of repair site for product j
DA_j: Demand of product j at the primary market
DE_j: Demand of product j at the secondary market
c_j: Unit collection cost of product j
d_j: Unit repair cost of product j
f_j: Set-up cost of disassembly site for product j
g_j: Set-up cost of repair site for product j
B_i: Maximum capacity of disassembly site assigned to part i
h_i: Unit disassembly cost of part i
m_i: Unit disposing cost of part i
r_i: Resource usage to disassemble one unit of part i at the disassembly site

n_{il}: Unit recycling cost of part i at recycling site l
o_{il}: Set-up cost of recycling site l for part$i$
s_{il}: Resource usage to recycle one unit of part i at recyclingsite l
O_{il}: Maximum capacity of recycling site$l$ to recycle part i
q_{ij}: Consumption rate of part i in one unit of product j
p_{ik}: The cost of purchasing part i from external supplier k
b_{ik}: Resource usage of supplier k to produce one unit of part i
T_k: Maximum capacity reserved from external supplier k
M: A very big positive number

Decision variables are as follows:
X_j: The amount of repaired commercial returns of product j
PA_j: The amount of product j produced for the primary market
PE_j: The amount of product j produced for the secondary market
Z_j: The amount of product j to be disassembled after collection
Y_j: The amount of product j imported at collection site
QA_{ik}: The amount of part i purchased from external supplier k for the products of primary market
QE_{ik}: The amount of part i purchased from external supplier k for the products of secondary market
E_i: The amount of part i obtained from disassembly site
F_{il}: The amount of end-of-life returns of part i to be recycled at recycling site l
G_i: The amount of part i to be disposed after disassembly of products
R_i: The amount of end-of-use returns of part i to be imported to the warehouse
U_{il}: Binary variable for installation of recycling site l for part i
V_j: Binary variable which is 1 if the disassembly site is set up for processing product j; otherwise, is 0
W_j: Binary variable which is 1 if the repair site is set up for processing product j; otherwise, is 0
To give better understanding, we have shown decision variables on figure 1.

3- Problem description and formulation

In this section, we will give the under study supply chain structure, problem description, and robust counterpart of the problem.

3-1- The supply chain structure

In figure 1, the CLSC network is given. It includes a manufacturer, distributer, retailer and finally customers in primary market. We add a secondary markets to this structure in which the products which are produced using returned parts, are sold. There is a collection site which is added to the initial structure. The collected products are divided into two groups. The first group are those products that are usually returned from retailer's shelves or returned by customers because of some minor quality problems. The products in group one which are called commercial products are usually repaired and given to the secondary market by the manufacturer. The second group of returned products are disassembled. Disassembled products are converted into their initial parts. The addressed parts are assumed to be end-of-use, end-of-life or disposal items. End-of-use parts in this research are those parts which can't be used in the original products but can be used in the products that are supposed to be distributed to the secondary market. End-of-life parts should be processed according to the legislative obligations; in this chain, we have assumed to be recycled. The remaining parts are those which can't be recycled and must be disposed because of the environmental effects or other managerial reasons. All the returned parts and purchased parts from the external suppliers which are used for the secondary market products are collected in the part warehouse. The manufacturer uses new parts purchased from the suppliers in order to produce products for the primary market. Furthermore, the secondary market customer demands can be satisfied by products which are produced using returned parts or purchased ones. The cell phone industry is a good example for this network.

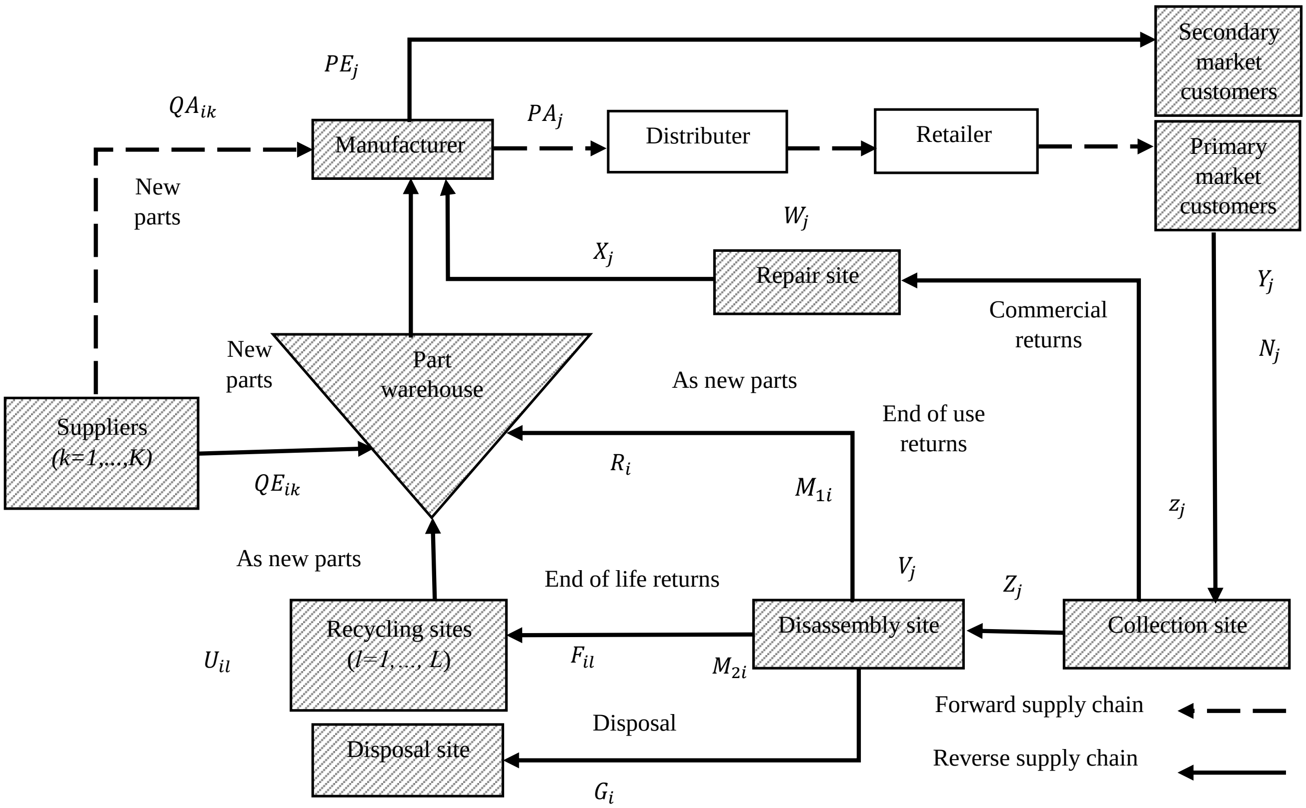


Fig 1. The closed loop supply chain network structure (highlighted area)

3-2- Deterministic model

The structure of the CLSC network was described in the previous section. Since quality of returned parts are of importance for the supply chain owner, we consider a minimum quality level for the returned parts. This affects the quality of products given to the secondary market. Therefore, we define a quality index for returned parts. This index makes the manufacturer to reserve the quality of products higher than an acceptable level. The deterministic model considering the quality constraint can be stated as:

$$MaxZ = \sum_{j=1}^{J}\left(SA_j . DA_j + SE_j . DE_j\right) \quad (1a)$$

$$-(\textstyle\sum_{i=1}^{I}\sum_{k=1}^{K} p_{ik}(QA_{ik} + QE_{ik}) \quad (1b)$$

$$+\sum_{i=1}^{I} h_i E_i \quad (1c)$$

$$+\sum_{l=1}^{L}\sum_{i=1}^{I} n_{il} F_{il} \quad (1d)$$

$$+\sum_{i=1}^{I} m_i G_i \tag{1e}$$

$$+\sum_{j=1}^{J} y_j (PA_j + PE_j) \tag{1f}$$

$$+\sum_{j=1}^{J} \left(c_j + H_j\right) Y_j \tag{1g}$$

$$+\sum_{j=1}^{J} d_j X_j \tag{1h}$$

$$+\sum_{l=1}^{L}\sum_{i=1}^{I} o_{il} U_{il} + \sum_{j=1}^{J} f_j V_j + \sum_{j=1}^{J} g_j W_j \tag{1i}$$

$$+\sum_{j=1}^{J} ha_j.\left(PA_j - DA_j\right) + \sum_{j=1}^{J} he_j.\left(PE_j + X_j - DE_j\right)\Bigg) \tag{1j}$$

$s.t$:

$$\beta_i \sum_{l=1}^{L} F_{il} + \sum_{k=1}^{K} QE_{ik} + \alpha_i R_i \geq \gamma_i \sum_{j=1}^{J} q_{ij} PE_j \quad \forall i \in \{1, \dots, I\} \tag{2}$$

$$\sum_{k=1}^{K} QA_{ik} = \sum_{j=1}^{J} q_{ij} PA_j \qquad \forall i \in \{1, \dots, I\} \tag{3}$$

$$\sum_{l=1}^{L} F_{il} + \sum_{k=1}^{K} QE_{ik} + R_i = \sum_{j=1}^{J} q_{ij} PE_j \quad \forall i \in \{1, \dots, I\} \tag{4}$$

$$R_i + \sum_{l=1}^{L} F_{il} + G_i = E_i \qquad \forall i \in \{1, \dots, I\} \tag{5}$$

$$E_i = \sum_{j=1}^{J} q_{ij} Z_j \qquad \forall i \in \{1, \dots, I\} \tag{6}$$

$$X_j + Z_j = Y_j \qquad \forall j \in \{1, \dots, J\} \tag{7}$$

$$\sum_{j=1}^{J} a_j (PA_j + PE_j) \leq A \tag{8}$$

$$\sum_{i=1}^{I} b_{ik} (QA_{ik} + QE_{ik}) \leq T_k \qquad \forall k \in \{1, \dots, K\} \tag{9}$$

$$r_i E_i \leq B_i \quad \forall i \in \{1, \dots, I\} \tag{10}$$

$$s_{il} F_{il} \leq O_{il} U_{il} \quad \forall i \in \{1, \dots, I\}, \forall l \in \{1, \dots, L\} \tag{11}$$

$$e_j X_j \leq C_j \quad \forall j \in \{1, \dots, J\} \tag{12}$$

$$PA_j \geq DA_j \quad \forall j \in \{1, \dots, J\} \tag{13}$$
$$PE_j + X_j \geq DE_j \quad \forall j \in \{1, \dots, J\} \tag{14}$$
$$X_j = z_j Y_j \quad \forall j \in \{1, \dots, J\} \tag{15}$$
$$Z_j = (1 - z_j) Y_j \quad \forall j \in \{1, \dots, J\} \tag{16}$$
$$R_i = M_{1i} E_i \quad \forall i \in \{1, \dots, I\} \tag{17}$$
$$\sum_{l=1}^{L} F_{il} = M_{2i} E_i \quad \forall i \in \{1, \dots, I\} \tag{18}$$
$$G_i = (1 - M_{1i} - M_{2i}) E_i \quad \forall i \in \{1, \dots, I\} \tag{19}$$
$$Y_j = N_j DA_j \quad \forall j \in \{1, \dots, J\} \tag{20}$$
$$\sum_{l=1}^{L} \sum_{i=1}^{I} U_{il} \leq t \tag{21}$$
$$Z_j \leq MV_j \quad \forall j \in \{1, \dots, J\} \tag{22}$$
$$X_j \leq MW_j \quad \forall j \in \{1, \dots, J\} \tag{23}$$
$$U_{il}, V_j, W_j \in \{0,1\} \quad \forall i \in \{1, \dots, I\}, \forall j \in \{1, \dots, J\}, \forall l \in \{1, \dots, L\} \tag{24}$$
$$PA_j, PE_j, Z_j, QA_{ik}, QE_{ik}, E_i, F_{il}, G_i, R_i, Y_j, X_j \geq 0 \quad \forall i \in \{1, \dots, I\}, \forall j \in \{1, \dots, J\}, \forall k \in \{1, \dots, K\}, \forall l \in \{1, \dots, L\} \tag{25}$$

The objective function Z represents the total profit. Term (1a) gives the revenue from selling products in the primary and secondary markets. Terms (1b)-(1e) represent the total cost of purchasing parts from suppliers, disassembly costs of all parts, the costs of recycling and disposal costs, respectively. Terms (1f)-(1h) give the manufacturing cost, operation and holding cost in collection site and the cost of repairing, respectively. Term (1i) gives the set up costs of recycling sites, disassembly and repair costs of products. Finally, Term (1j) represents the product holding costs for the primary and secondary markets.

Constraint (2) guarantees that the quality of used parts for the secondary market products to be equal to or higher than an acceptable quality level. Constraint (3) shows that the required parts for producing primary market products equals to the original purchased parts from suppliers. Constraint (4) ensures that the required parts for producing the secondary market products to be equal to the number of recycled parts, end of use parts and purchased parts from the suppliers. Constraint (5) shows the equality between disassembled parts and the summation of end of use, end of life and disposed parts. Constraint (6) gives the relationship between parts and products in the disassembly site. Constraint (7) makes sure that the collected products in collection site are dispatched to disassembly or repair sites. Constraints (8)-(12) are related to the maximum capacity of the manufacturer, suppliers, disassembly site, recycling sites and repair site, respectively. Constraint (13) represents that the demand of the primary market should be satisfied with manufactured products. Constraint (14) shows that the demand of the secondary market is satisfied with manufactured or repaired products. Constraint (15) gives the amount of commercial returns of each product sent to the repair sites. Constraint (16) gives the amount of returns sent to the disassembly site.

Constrains (17)-(19) give the amount of wastes, end of use and end of life returns for each part, respectively. Constraint (20) gives the amount of total returns for each product. Constraint (21) shows the restriction of the number of recycling sites. Constraints (22)-(23) are control constraints for opening disassembly and repair sites for different products. Constraints (24)-(25) give the status of the decision variables.

3-3- Robust optimization

Robust optimization approach was initially introduced by Soyster (1973). In the robust optimization, the decision maker is looking for a solution which remains optimal or near to optimal and also feasible for all or majority of uncertain values in a given interval. Alem and Morabito (2012) mentioned two following

reasons for using robust optimization:
1. The uncertainty described based on interval sets is interesting.
2. The model obtained from robust optimization is comfortable to solve.
On the other hand, in the robust optimization, having clear knowledge about the probability distribution of non-deterministic data is not required. Historical data and decision maker experiences could be used for determining the non-deterministic intervals. In this paper, we use robust optimization technique based on Ben-tal and Nemirovski (2009) in which, it is assumed that all non-deterministic parameters vary in a specified closed bounded box. The general form of this box is given as in equation (26) (Ben-tal et al., 2005 and 2009, Pishvaee et al., 2011):

$$U_{Box} = \{\xi \in R^n : |\xi_t - \overline{\xi_t}| \le \rho G_t, t = 1, \dots, n\}, \quad (26)$$

In which, $\overline{\xi_t}$ is the nominal value of ξ_t as tth parameter of vector ξ which is an n-dimension vector and the positive parameters G_t and ρ represent uncertainty scale and uncertainty level, respectively. A special case of interest is $G_t = \overline{\xi_t}$ where relative deviation from the nominal value is up to a coefficient of ρ multiplied by the nominal value. With respect to the nature of the given deterministic model in this paper, consider the basic model as in equation (27).

$$\begin{aligned}
&max\, sd - cx - fy - hd \\
&S.t: \\
&Ax \ge d \\
&Bx = rx \\
&Mx \ge 0 \\
&Nx = 0 \\
&Ox \le b \\
&Px \le Cy \\
&y \le e \\
&x \ge 0, y \in \{0,1\}
\end{aligned} \quad (27)$$

In the aforementioned model, *s, c, f, h* and *d* represent the sale prices, unit operational costs in different parts of the supply chain, opening cost of facilities, unit inventory holding costs and the demand. In the constraints *r* represents the return rate. It should be noted that *A, B, M, N, O, P, C* represent the coefficient matrices of the constraints and *e* and *b* are right hand side values in the given constraints.

We will assume that the demand and return rate will be uncertain parameters; because of this U^d_{BOX} and U^r_{BOX} are defined. According to the above descriptions, the robust counterpart of model (27) can be stated as in (28) - (37).

$$max\, z \quad (28)$$
$$S.t:$$
$$sd - cx - fy - hd \ge z \quad \forall d \in U^d_{BOX} \quad (29)$$
$$Ax \ge d \quad \forall d \in U^d_{BOX} \quad (30)$$
$$Bx = rx \quad \forall r \in U^r_{BOX} \quad (31)$$
$$Mx \ge 0 \quad (32)$$
$$Nx = 0 \quad (33)$$
$$Ox \le b \quad (34)$$
$$Px \le Cy \quad (35)$$
$$y \le e \quad (36)$$
$$x \ge 0, y \in \{0,1\} \quad (37)$$

In the given model, z represents the objective function which should be maximized. The robust counterpart of the basic model is written considering the uncertainty of demand and return rate. Ben-tal et al. (2005) indicate that in a closed bounded box, the robust counterpart model can be converted into a tractable equivalent model. To indicate the tractable form of the robust model, equations. (29)-(31) should be converted to their equivalent tractable ones. For constraint (29) we have:

$$sd - hd \geq z + cx + fy, \quad \forall d \in U_{Box}^{d} | U_{Box}^{d} = \left\{ d \in R^{n_d} : \left| d_j - \bar{d}_j \right| \leq \rho_d G_j^d, j = 1, \ldots, n_d \right\}. \tag{38}$$

In the given inequality, $\bar{d}_j$represents the nominal value of d_j; d_j represents the *j*th value of demand considering n_d as the number of demand values. ρ_d and G_j^d represent the uncertainty level and scale of *j*th demand. The left hand side of inequality (38) contains the vector of uncertain parameters. The tractable form of inequality (38) can be written as in (39):

$$\begin{aligned} &\sum_j (s\bar{d}_j - h\bar{d}_j + \eta_j) \geq Z + cx + fy, \\ &s\rho_d G_j^d \leq \eta_j, \quad \forall j = 1, \ldots, n_d, \\ &s\rho_d G_j^d \geq -\eta_j, \quad \forall j = 1, \ldots, n_d. \end{aligned} \tag{39}$$

In this paper, since we have considered the pessimistic robust approach, we will consider the least value for the demand in the objective function of the model as $\bar{d}_j - \rho_d G_j^d$.
As Pishvaee et al. (2011) for constraints (31), we have :

$$Bx = r_k x, \quad \forall k \in \{1, \ldots, n_r\}, \forall r \in U_{Box}^{r} | U_{Box}^{r} = \{ r \in R^{n_r} : |r_k - \overline{r_k}| \leq \rho_r G_k^r, k = 1, \ldots, n_r \}. \tag{40}$$

Using the extreme points of U_{Box}, equation (40) can be converted into inequalities (41)-(43):

$$Bx \geq \overline{r_k} x - \eta_k^r \quad \forall k \in \{1, \ldots, n_r\}, \tag{41}$$

$$Bx \leq \overline{r_k} x + \eta_k^r \quad \forall k \in \{1, \ldots, n_r\}, \tag{42}$$

$$|\rho_r G_k^r x| \geq \eta_k^r. \tag{43}$$

Where, $\eta_k^r \geq 0$ is an auxiliary variable. Inequalities (41)-(43) can be rewritten as (44) and (45).

$$Bx \geq \overline{r_k} x - |\rho_r G_k^r x| \quad \forall k \in \{1, \ldots, n_r\}, \tag{44}$$

$$Bx \leq \overline{r_k} x + |\rho_r G_k^r x| \quad \forall k \in \{1, \ldots, n_r\}. \tag{45}$$

Noting that $\rho_r G_k^r x \geq 0$, equations (44)-(45) are converted into (46)-(47).

$$Bx \geq \overline{r_k} x - \rho_r G_k^r x \quad \forall k \in \{1, \ldots, n_r\}, \tag{46}$$

$$Bx \leq \overline{r_k} x + \rho_r G_k^r x \quad \forall k \in \{1, \ldots, n_r\}. \tag{47}$$

Similarly, for Constraint (30) we have:

$$Ax \geq d \quad, \forall d \in U_{box}^{d} | U_{box}^{d} = \left\{ d \in R^{n_d} : \left| d_j - \bar{d}_j \right| \leq \rho_d G_j^d, j = 1, \ldots, n_d \right\}. \tag{48}$$

Thus, Constraint (48) can be rewritten as (49):

$$Ax \geq \bar{d}_j + \rho_d G_j^d, \quad \forall j \in \{1, \dots, n_d\}. \tag{49}$$

3-4- Robust counterpart of the problem

In this section, we consider the proposed CLSC structure with uncertain parameters of percentage of return, percentage of commercial returns, percentage of end-of-use return and percentage of end-of-life returns, and demand of the primary and secondary markets. These parameters are considered to vary in some intervals independently. The addressed uncertain parameters are given in Constraints (13)-(20) and the objective function; the addressed constrains are replaced with their robust counterparts. Thus, according to the above descriptions, the robust counterpart of the deterministic model is as follows defining some new parameters:

$\overline{DA}_j$: Nominal value of DA_j

ρ_{DA}: Uncertainty level of DA_j for all j

G_j^{DA}: Uncertainty scale of DA_j

$\overline{DE}_j$: Nominal value of DE_j

ρ_{DE}: Uncertainty level of DE_j for all j

G_j^{DE}: Uncertainty scale of DE_j

$\bar{z}_j$: Nominal value of z_j

ρ_z: Uncertainty level of z_j for all j

G_j^z: Uncertainty scale of z_j

$\bar{M}_{1i}$: Nominal value ofM_{1i}

ρ_{M_1}: Uncertainty level ofM_{1i} for all j

$G_i^{M_1}$: Uncertainty scale of M_{1i}

$\bar{M}_{2i}$: Nominal value ofM_{2i}

ρ_{M_2}: Uncertainty level ofM_{2i} for all j

$G_i^{M_2}$: Uncertainty scale of M_{2i}

$\bar{N}_j$: Nominal value of N_j

ρ_N: Uncertainty level of N_j for all j

G_j^N: Uncertainty scale of N_j

$$Max\ Z = \sum_{j=1}^{J}\left(SA_j.\overline{DA}_j - SA_j.\rho_{DA}G_j^{DA} + SE_j.\overline{DE}_j - SE_j.\rho_{DE}G_j^{DE}\right)$$
$$- \sum_{i=1}^{I}\sum_{k=1}^{K} p_{ik}(QA_{ik} + QE_{ik}) - \sum_{i=1}^{I} h_i E_i - \sum_{l=1}^{L}\sum_{i=1}^{I} n_{il}F_{il} - \sum_{i=1}^{I} m_i G_i$$
$$- \sum_{j=1}^{J} y_j(PA_j + PE_j) - \sum_{j=1}^{J}(c_j + H_j)Y_j - \sum_{j=1}^{J} d_j X_j - \sum_{l=1}^{L}\sum_{i=1}^{I} o_{il}U_{il} \quad (50)$$
$$- \sum_{j=1}^{J} f_j V_j - \sum_{j=1}^{J} g_j W_j - \sum_{j=1}^{J}\left(ha_j.\left(PA_j - \overline{DA}_j\right) + ha_j.\rho_{DA}G_j^{DA}\right)$$
$$- \sum_{j=1}^{J}\left(he_j.\left(PE_j + X_j - \overline{DE}_j\right) + he_j.\rho_{DE}G_j^{DE}\right)$$

$s.t$:

(2)-(12)

$$PA_j \geq \overline{DA}_j + \rho_{DA}G_j^{DA} \quad \forall j \in \{1, \dots, J\} \quad (51)$$
$$PE_j + X_j \geq \overline{DE}_j + \rho_{DE}G_j^{DE} \quad \forall j \in \{1, \dots, J\} \quad (52)$$
$$X_j \leq \left(\bar{z}_j + \rho_z G_j^z\right)Y_j \quad \forall j \in \{1, \dots, J\} \quad (53)$$
$$X_j \geq \left(\bar{z}_j - \rho_z G_j^z\right)Y_j \quad \forall j \in \{1, \dots, J\} \quad (54)$$
$$Z_j \geq \left[1 - \left(\bar{z}_j + \rho_z G_j^z\right)\right]Y_j \quad \forall j \in \{1, \dots, J\} \quad (55)$$
$$Z_j \leq \left[1 - \left(\bar{z}_j - \rho_z G_j^z\right)\right]Y_j \quad \forall j \in \{1, \dots, J\} \quad (56)$$
$$R_i \leq \left(\overline{M}_{1i} + \rho_{M_1}G_i^{M_1}\right)E_i \quad \forall i \in \{1, \dots, I\} \quad (57)$$
$$R_i \geq \left(\overline{M}_{1i} - \rho_{M_1}G_i^{M_1}\right)E_i \quad \forall i \in \{1, \dots, I\} \quad (58)$$
$$\sum_{l=1}^{L} F_{il} \leq \left(\overline{M}_{2i} + \rho_{M_2}G_i^{M_2}\right)E_i \quad \forall i \in \{1, \dots, I\} \quad (59)$$
$$\sum_{l=1}^{L} F_{il} \geq \left(\overline{M}_{2i} - \rho_{M_2}G_i^{M_2}\right)E_i \quad \forall i \in \{1, \dots, I\} \quad (60)$$
$$G_i \geq \left[1 - \left(\overline{M}_{1i} + \rho_{M_1}G_i^{M_1}\right) - \left(\overline{M}_{2i} + \rho_{M_2}G_i^{M_2}\right)\right]E_i \quad \forall i \in \{1, \dots, I\} \quad (61)$$
$$G_i \leq \left[1 - \left(\overline{M}_{1i} - \rho_{M_1}G_i^{M_1}\right) - \left(\overline{M}_{2i} - \rho_{M_2}G_i^{M_2}\right)\right]E_i \quad \forall i \in \{1, \dots, I\} \quad (62)$$
$$Y_j \leq \left(\overline{N}_j + \rho_N G_j^N\right)\left(\overline{DA}_j + \rho_{DA}G_j^{DA}\right) \quad \forall j \in \{1, \dots, J\} \quad (63)$$
$$Y_j \geq \left(\overline{N}_j - \rho_N G_j^N\right)\left(\overline{DA}_j - \rho_{DA}G_j^{DA}\right) \quad \forall j \in \{1, \dots, J\} \quad (64)$$

(21)-(25)

In this model, the objective function is as in (50). Constraints (51)-(64) are the robust counterpart of constraints (13)-(20), respectively.

4- Proposed heuristic solution

With regard to the NP-hard nature of the problem, the problem can't be solved for large instances by regular optimization packages; thus, we propose a solution approach based on the Lagrangian relaxation technique. Considering the given robust approach, we relax constraints (8) and (9) using coefficients of $\lambda_1, \lambda_{2k} \geq 0$; thus, the model can be stated as follows:

$$MaxZ' = Z - \lambda_1(\sum_{j=1}^{J} a_j(PA_j + PE_j) - A) - \sum_{k=1}^{K} \lambda_{2k}.\left(\sum_{i=1}^{I} b_{ik}(QA_{ik} + QE_{ik}) - T_k\right) \qquad (65)$$

$s.t$:

(2)-(12) except for (8) and (9)

(21)-(25)

(51)-(64)

Term (65) represents objective function of the new problem. With these relaxations, the problem is converted into two sub problems; Sub problem 1 is for the forward supply chain to satisfy the primary market demands. Sub problem 2 is for the reverse supply chain for collecting and recovery of returns to satisfy the secondary market demands.

4-1- Sub problem 1

This sub problem represents the forward supply chain to satisfy the primary market demands. It can be stated as follows:

$$\begin{aligned} Max\ z_1 = & \sum_{j=1}^{J}\left(SA_j.\overline{DA}_j - SA_j.\rho_{DA}G_j^{DA}\right) - \sum_{i=1}^{I}\sum_{k=1}^{K} p_{ik}.QA_{ik} - \sum_{j=1}^{J} y_j.PA_j \\ & - \sum_{j=1}^{J}\left(ha_j.\left(PA_j - \overline{DA}_j\right) + ha_j.\rho_{DA}G_j^{DA}\right) - \lambda_1(\sum_{j=1}^{J} a_j.PA_j - A) \\ & - \sum_{k=1}^{K} \lambda_{2k}.\left(\sum_{i=1}^{I} b_{ik}.QA_{ik} - T_k\right) \end{aligned} \qquad (66)$$

$s.t:$

(3) and (51)

$$PA_j, QA_{ik} \geq 0 \quad \forall i \in \{1, \dots, I\}, \forall j \in \{1, \dots, J\}, \forall k \in \{1, \dots, K\} \qquad (67)$$

Term (66) represents the objective function of sub problem 1 pertinent to the involved costs and incomes regarding the forward supply chain. Constraint (67) represents the non-negativity of decision variables. For solving this sub problem, we find optimum value of PA_j using first-order derivation with respect to PA_j as in (68):

$$\frac{dz_1}{dPA_j} = -y_j - ha_j - \lambda_1 a_j < 0 \qquad (68)$$

With regard to Term (68) and maximization type of the objective function, the optimum value of $PA_j$is the least possible value with respect to Constraint (51). So, PA_j^* can be given as in (69):

$$PA_j^* = \overline{DA}_j + \rho_{DA}G_j^{DA} \forall j \in \{1, \dots, J\} \qquad (69)$$

On the other hand considering Constraint (3), the amount of parts to be purchased is as in (70):

$$\sum_{k=1}^{K} QA_{ik}^{\ *} = \sum_{j=1}^{J} q_{ij} PA_j^* \quad \forall i \in \{1, \ldots, I\} \tag{70}$$

General scheme of the solution algorithm for sub problem 1 is illustrated as in figure 2.

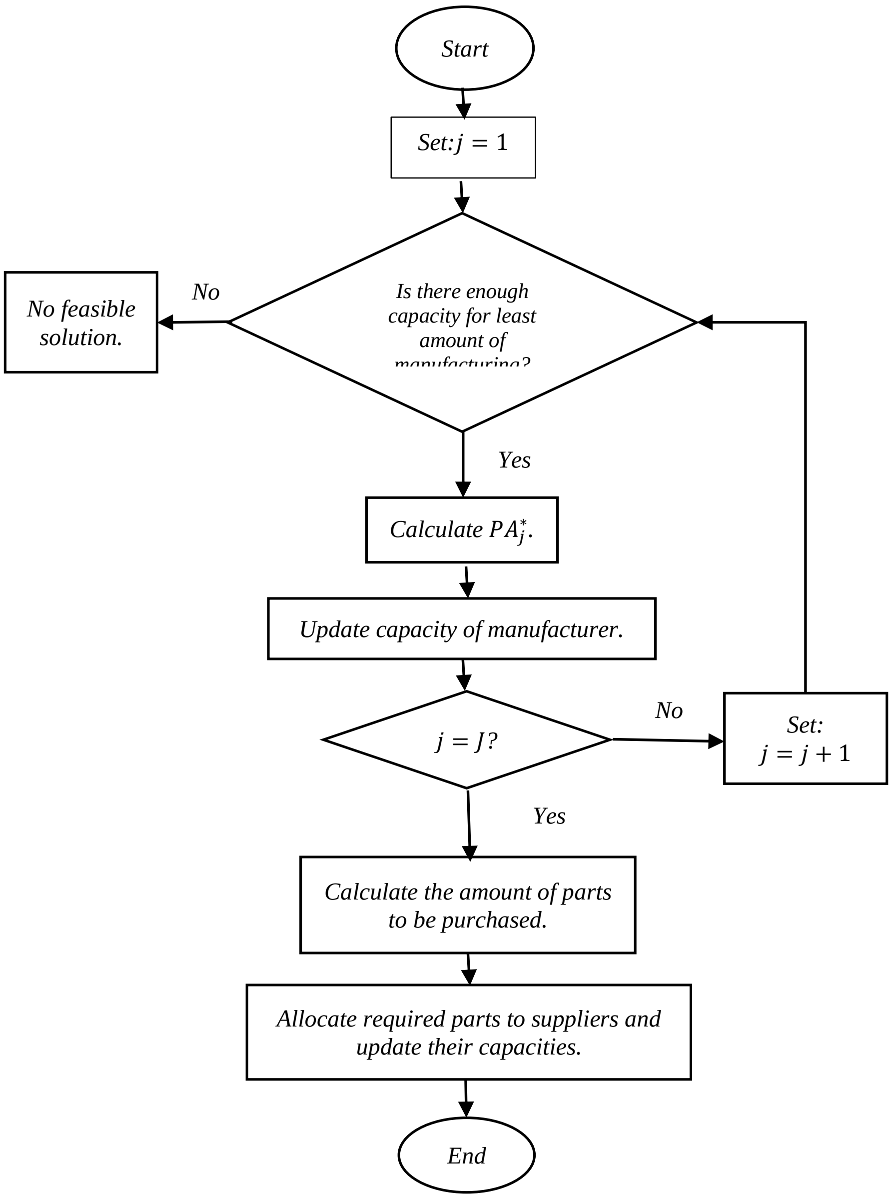


Fig. 2. General scheme of solution algorithm of sub problem 1

4-2- Sub problem 2

This sub problem represents the reverse supply chain for collecting and recovery of returns to satisfy the secondary market demands. It can be stated as follows:

$$Max\ Z_2 = \sum_{j=1}^{J}\left(SE_j.\overline{DE}_j - SE_j.\rho_{DE}G_j^{DE}\right) - \sum_{i=1}^{I}\sum_{k=1}^{K} p_{ik}.QE_{ik} - \sum_{i=1}^{I} h_i E_i - \sum_{l=1}^{L}\sum_{i=1}^{I} o_{il}U_{il} - \sum_{j=1}^{J} f_j V_j - \sum_{j=1}^{J} g_j W_j - \sum_{j=1}^{J}\left(he_j.\left(PE_j + X_j - \overline{DE}_j\right) + he_j.\rho_{DE}G_j^{DE}\right) - \lambda_1\left(\sum_{j=1}^{J} a_j.PE_j\right) - \sum_{k=1}^{K} \lambda_{2k}.\left(\sum_{i=1}^{I} b_{ik}.QE_{ik}\right) \quad (71)$$

$s.t$:

(2)

(4)-(12) except for (8) and (9)

(52)-(64)

(21)-(25)

Terms (71) represents the objective function of sub problem 2. For solving this sub problem, first we relax Constrains (2) and (4) using positive coefficients $\lambda_{3i}$and λ_{4i}. Then, we add Constraint (73) to reinforce the model. So, the new objective function is given as in (72):

$$Max\ Z_2 = \sum_{j=1}^{J}\left(SE_j.\overline{DE}_j - SE_j.\rho_{DE}G_j^{DE}\right) - \sum_{i=1}^{I}\sum_{k=1}^{K} p_{ik}.QE_{ik} - \sum_{i=1}^{I} h_i E_i) - \sum_{l=1}^{L}\sum_{i=1}^{I} n_{il}F_{il} - \sum_{i=1}^{I} m_i G_i - \sum_{j=1}^{J} y_j.PE_j - \sum_{j=1}^{J}\left(c_j + H_j\right)Y_j - \sum_{j=1}^{J} d_j X_j - \sum_{l=1}^{L}\sum_{i=1}^{I} o_{il}U_{il} - \sum_{j=1}^{J} f_j V_j - \sum_{j=1}^{J} g_j W_j - \sum_{j=1}^{J}\left(he_j.\left(PE_j + X_j - \overline{DE}_j\right) + he_j.\rho_{DE}G_j^{DE}\right) - \lambda_1\left(\sum_{j=1}^{J} a_j.PE_j\right) - \sum_{k=1}^{K} \lambda_{2k}.\left(\sum_{i=1}^{I} b_{ik}.QE_{ik}\right) - \sum_{i=1}^{I} \lambda_{3i}.\left(\sum_{l=1}^{L} F_{il} + \sum_{k=1}^{K} QE_{ik} + R_i - \sum_{j=1}^{J} q_{ij}PE_j\right) - \sum_{i=1}^{I} \lambda_{4i}.\left(\beta_i \sum_{l=1}^{L} F_{il} + \sum_{k=1}^{K} QE_{ik} + \alpha_i R_i - \gamma_i \sum_{j=1}^{J} q_{ij}PE_j\right) \quad (72)$$

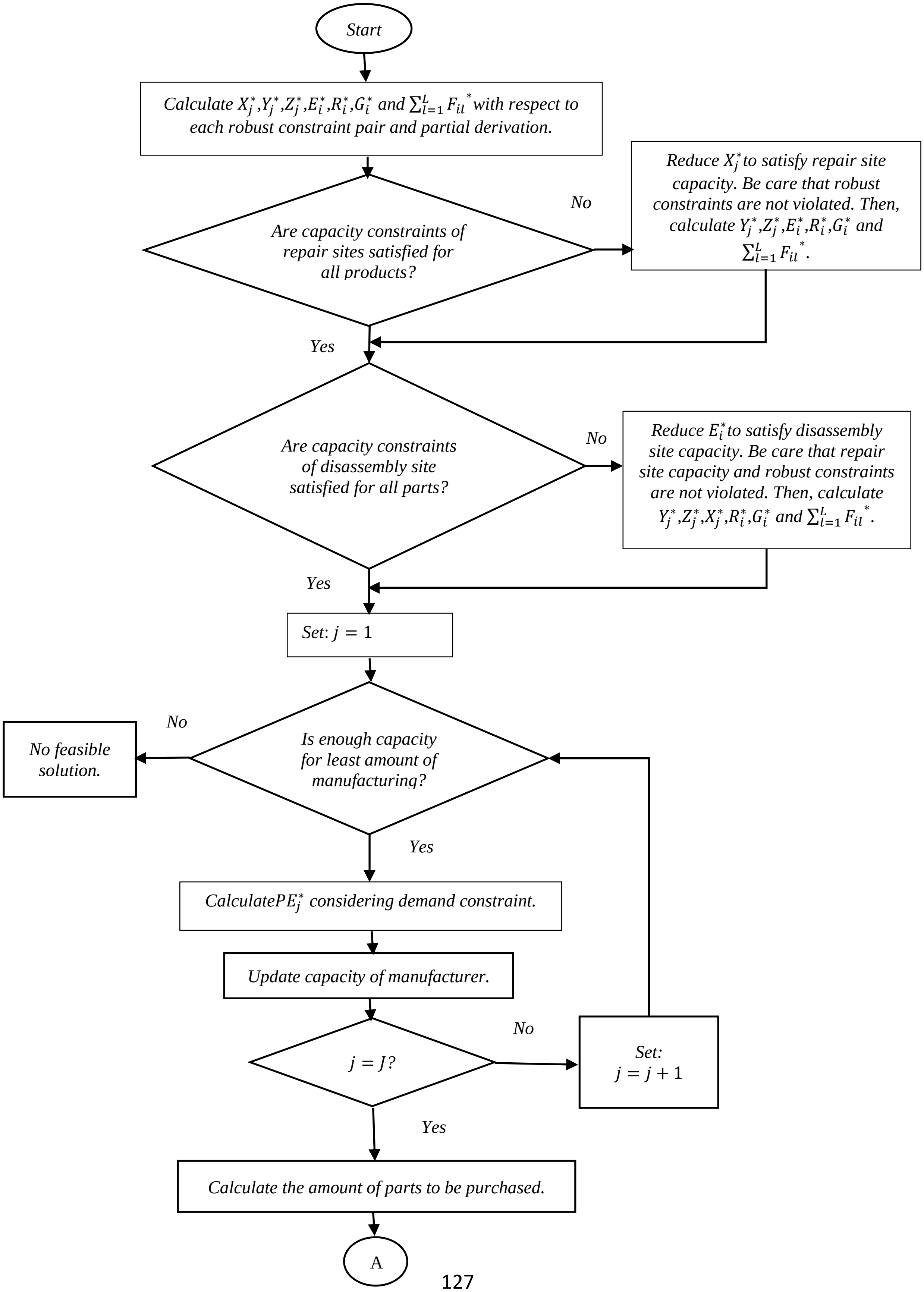
$S.t$:

(5)-(12) except for (8) and (9)
(52)-(64)
(21)-(25)

$$\sum_{l=1}^{L} F_{il} + \sum_{k=1}^{K} QE_{ik} + R_i \geq \sum_{j=1}^{J} q_{ij} PE_j \;\; \forall i \in \{1, \dots, I\} \tag{73}$$

It is clear that for satisfying the secondary market demands, repairing is more economic than manufacturing and disassembly. We propose an algorithm based on partial derivations for solving sub problem 2. This algorithm is illustrated in figure 3.

By implementing the algorithm of sub problem 1, as well as the decision variables PA_j and QA_{ik}, the reminder capacity of manufacturer and suppliers (A and T_k) are specified. By substituting this values in the algorithm of sub problem 2, we can obtain other decision variables includingPE_j,Y_j,X_j,Z_j,QE_{ik},E_i,F_{il},G_i,R_i,U_{il},$V_j$and W_j. We update the capacity of the manufacturer and suppliers in each stage of manufacturing which causes the solution to be feasible for the problem. Robust model solution algorithm is illustrated in figure 4.


Start
Calculate $X_j^*, Y_j^*, Z_j^*, E_i^*, R_i^*, G_i^*$ and $\sum_{l=1}^{L} F_{il}^*$ with respect to each robust constraint pair and partial derivation.
Are capacity constraints of repair sites satisfied for all products?
No
Reduce X_j^* to satisfy repair site capacity. Be care that robust constraints are not violated. Then, calculate $Y_j^*, Z_j^*, E_i^*, R_i^*, G_i^*$ and $\sum_{l=1}^{L} F_{il}^*$.
Yes
Are capacity constraints of disassembly site satisfied for all parts?
No
Reduce E_i^* to satisfy disassembly site capacity. Be care that repair site capacity and robust constraints are not violated. Then, calculate $Y_j^*, Z_j^*, X_j^*, R_i^*, G_i^*$ and $\sum_{l=1}^{L} F_{il}^*$.
Yes
Set: $j = 1$
Is enough capacity for least amount of manufacturing?
No
No feasible solution.
Yes
Calculate PE_j^* considering demand constraint.
Update capacity of manufacturer.
$j = J$?
No
Set: $j = j + 1$
Yes
Calculate the amount of parts to be purchased.
A

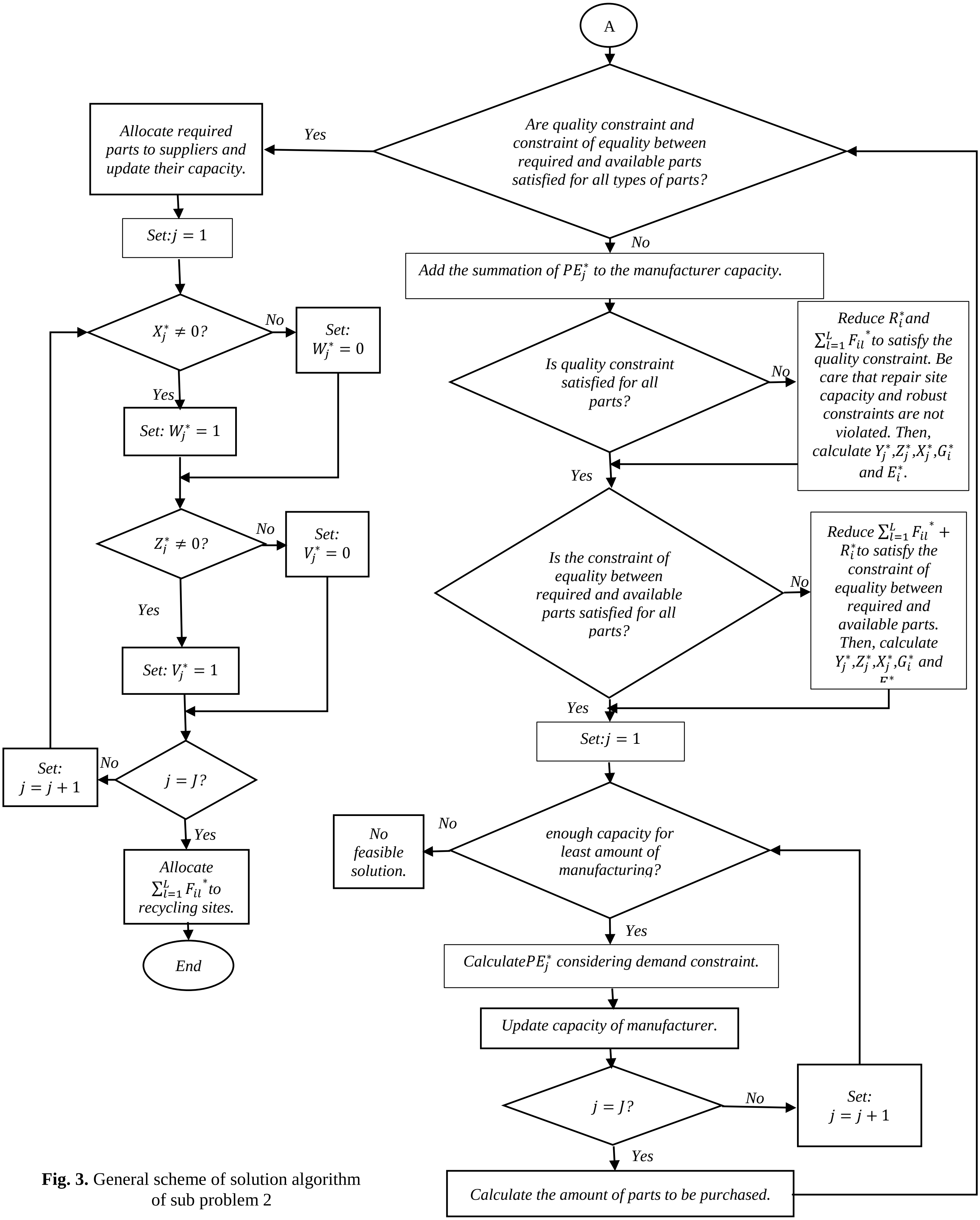


Fig. 3. General scheme of solution algorithm of sub problem 2

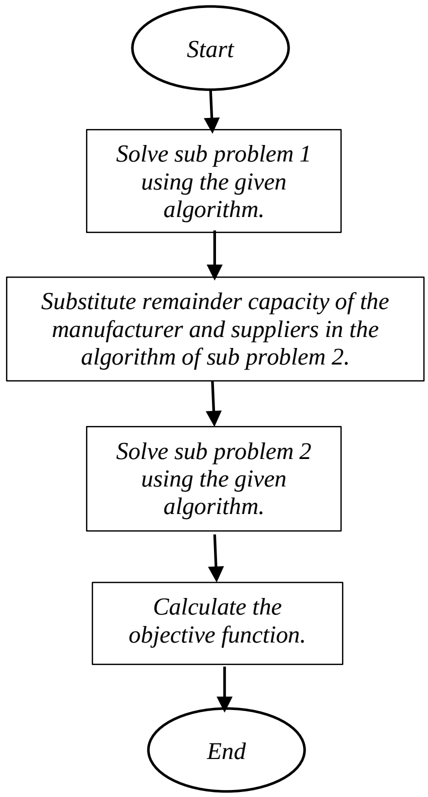


Fig. 4. General scheme of robust model solution algorithm

We set $\rho_{DA} = \rho_{DE} = \rho_N = \rho_z = \rho_{M_1} = \rho_{M_2} = 0$ to use the proposed algorithm for solving the deterministic model.

5- Computational experiments

In this section, we have designed some numerical examples in order to assess the performance of the given models. Ten different problems are designed. The first five are for small size instances and the second five are for large size instances. Details of the problems are illustrated by table 1.

Table 1. The characteristics of designed scenarios

Problem No.		1	2	3	4	5	6	7	8	9	10
Number of	Products	5	5	10	15	20	100	150	200	250	300
	Parts	5	7	7	10	15	100	150	200	250	300
	Suppliers	5	10	10	15	20	100	150	200	250	300
	Recycling sites	5	10	10	15	20	100	150	200	250	300

5-1- Sensitivity analysis

In this section, a sample sensitivity analysis of the robust model is performed considering Problem (1). Figure 5 shows the changes of the objective function with respect to the capacity of the disassembly site for part 1. It shows that the maximum objective function occurs in a certain capacity of the disassembly site (i.e. 3456) and after this value by increasing the capacity, the objective function remains stable (on the value of 377365).

Figure 6 indicates the effect of variations in the nominal value of total return's percentage. It is obvious that the maximum amount of objective function occurs in $\overline{N} \cong 0.8$. In other words, when the nominal value of total return's percentage is higher than 0.8, the amount of returns is higher than the secondary market demand; thus, in this condition the costs increase and the objective function decreases.

Similar effects are observed in figure 7 for the nominal value of commercial return's percentage, in figure 8 for the nominal value of end-of-use part's percentage and the nominal value of end-of-life part's percentage. In Fig. 8 we observe that the effect of variations in the nominal value of end-of-use part's percentage is higher than that of the end-of-life part's percentage, because the end- of-life parts for remanufacturing are more expensive than the end-of-use parts.

Now we analyze the effects of the quality parameters in the robust model. Figure 9 shows the effect of variations in parameters α_1 (i.e. quality index for end-of-use part 1), β_1 (i.e. quality index for end-of-life part 1) and γ_1 (i.e. acceptable quality index for part 1). By increasing α_1 and $\beta_1$to 0.53 and 0.58, the objective function increases; but after reaching this amount, the objective function becomes stable at the value of 359624; furthermore, by increasing γ_1 to 0.84, the objective function does not change, but after that the objective function decreases.

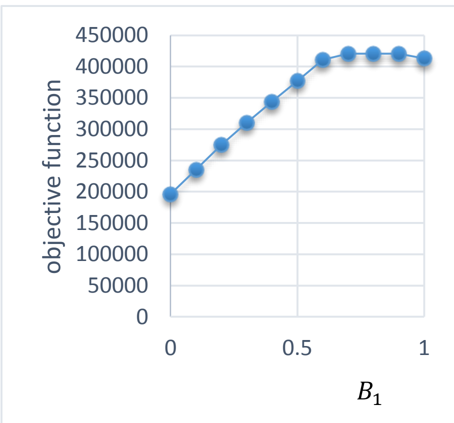


Fig 5. Sensitivity analysis of parameter B_1(capacity of disassembly site for part 1)

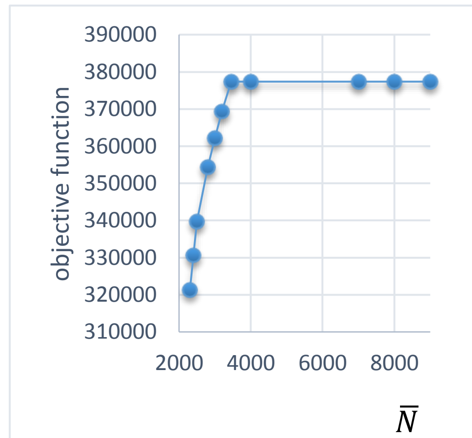


Fig 6. Sensitivity analysis of parameter $\bar{N}$ (nominal value of returns percentage)

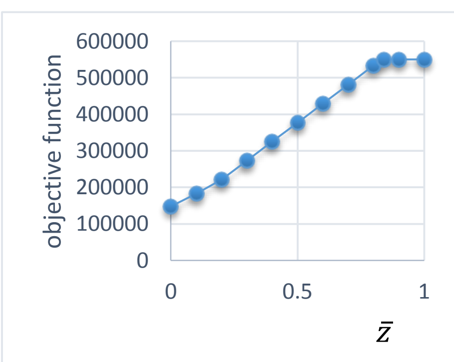


Fig 7. Sensitivity analysis of parameter $\bar{Z}$ (nominal value of commercial returns

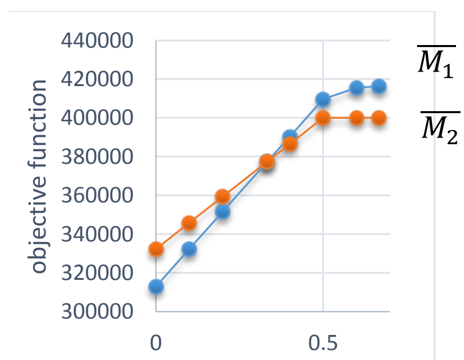


Fig 8. Sensitivity analysis of parameter$\overline{M_1}$and $\overline{M_2}$ (nominal value of EOU and EOL parts percentage)

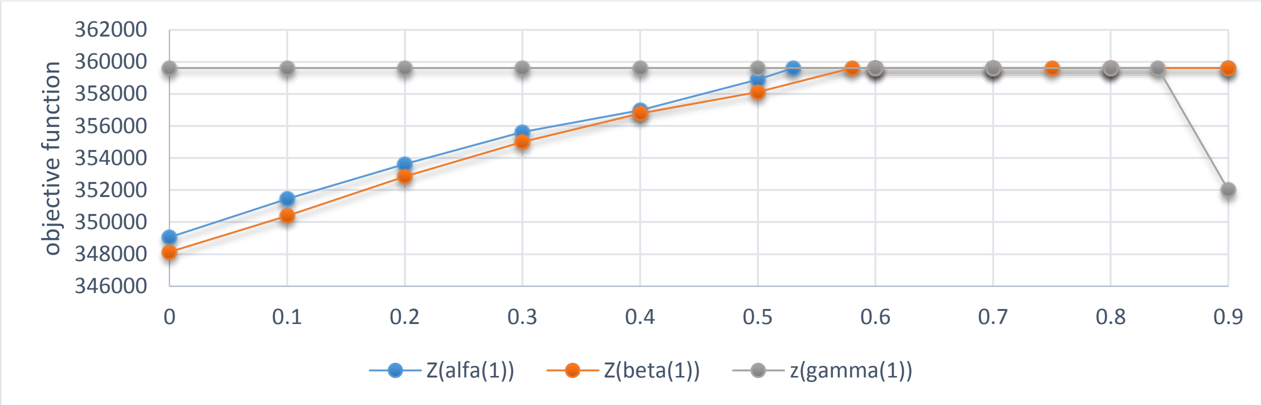


Fig 9. Sensitivity analysis of the quality parameters

5-2- Robust model validation

In this section, to assess the performance of the robust model, several numerical examples are implemented. To this aim, the deterministic and robust models are compared under nominal data and under random realization of uncertain parameters. Both the deterministic and robust models are solved by GAMS 24.1.3 optimization software. Table 2 shows the results using nominal data. It is clear from table 3 that the objective function of the robust model is higher than the deterministic one for all problems 1-5.

Table 2. Summary of results under uncertainty on returns and demands.

Problem No.	$\rho_{DA} = \rho_{DE}$	$\rho_N = \rho_z = \rho_{M_1} = \rho_{M_2}$	Objective function values under nominal data	
			deterministic model	robust model
1	0.05	0.2	277295	312788
	0.1	0.5	277295	372696
	0.15	0.8	277295	4408922
2	0.05	0.2	290013	328667
	0.1	0.5	290013	402696
	0.15	0.8	290013	492468
3	0.05	0.2	341999	455216
	0.1	0.5	341999	635141
	0.15	0.8	341999	833108
4	0.05	0.2	244461	458306
	0.1	0.5	244461	844936
	0.15	0.8	244461	1306395
5	0.05	0.2	3803017	4108232
	0.1	0.5	3803017	4841126
	0.15	0.8	3803017	5768435

To validate the robust model, first uncertain parameters including primary and secondary market demands, percentage of total returns, percentage of commercial returns, percentage of end-of-use return and percentage of end-of-life returns are randomly generated in the related sets. The related set for each uncertain parameter is a symmetrical interval with a neighborhood radius around the nominal value of the parameter. To evaluate the models, we assume that the uncertainty level of demand parameters are equal together. The uncertainty level of return parameters are supposed to be equal. After generating uncertain parameters, we consider a penalty cost for unsatisfied constraints. Every test includes 100 iterations. Random generations and calculations of the Mean and the standard deviation of the objective function for each one in the deterministic and uncertain cases are programmed by MATLAB 8.5.0.197613.

Table 3. Test results for the designed problems

Problem No.	$\rho_{DA} = \rho_{DE}$	$\rho_N = \rho_z = \rho_{M_1} = \rho_{M_2}$	Deterministic model			Robust model		
			mean	Standard deviation	CV	mean	Standard deviation	CV
1	0.05	0.2	272938	11521	0.042	306893	12299	0.040
	0.1	0.5	264181	24714	0.093	357782	26093	0.073
	0.15	0.8	266719	33197	0.124	433203	35426	0.082
2	0.05	0.2	284200	14220	0.050	320504	14892	0.046
	0.1	0.5	279426	28112	0.101	382606	29657	0.077
	0.15	0.8	268926	40576	0.151	473951	42500	0.090
3	0.05	0.2	329138	22533	0.068	437300	23780	0.054
	0.1	0.5	309981	45235	0.146	599152	47545	0.080
	0.15	0.8	294836	66599	0.226	798975	70092	0.088
4	0.05	0.2	353460	39840	0.113	409101	42392	0.104
	0.1	0.5	329666	72904	0.221	772699	75684	0.098
	0.15	0.8	318207	119368	0.375	1256255	122696	0.098
5	0.05	0.2	3788116	97056	0.026	4024596	99026	0.025
	0.1	0.5	3750788	167574	0.045	4703455	169479	0.036
	0.15	0.8	3700412	255445	0.069	5611124	260279	0.046

It is clear that with respect to the maximization type of the objective function, the desirable model has higher mean of the objective function value and lower standard deviation. Thus, to compare two models, we use the coefficient of variations criterion (CV). This criterion is a combination of centralization and disperse criteria (the standard deviation/the mean). Therefore, the desirable model has lower CV. Table 3 shows the results of experiments under random realizations for the uncertain parameters. Since for all problems and uncertainty levels, the robust model has higher mean and lower CV than the deterministic model, the robust model dominates the deterministic one.

5-3- Lagrangian heuristic solution approach validation

To assess the performance of the Lagrangian heuristic solution approach, by running the proposed heuristic utilizing MATLAB 8.5.0.197613 package, we compare the results obtained from GAMS optimization software and the proposed heuristic for all the ten problems.

Table 4. Comparison of results from GAMS optimization software and the proposed heuristic

Problem No.	Optimization software (GAMS)		Heuristic approach		Error percent
	Objective function	Time (second)	Objective function	Time (second)	
1	247968	0.016	247014	0.397	0.38%
2	251647	0.19	250821	0.704	0.33%
3	288446	0.09	286393	1.039	0.71%
4	108786	0.13	104255	2.567	4.16%
5	3307712	0.2	3257113	15.450	0.53%
6	1518414394	43.143	1514559806	21.445	0.25%
7	-	-	1982844389	175.195	-
8	-	-	2255913493	211.831	-
9	-	-	2181558271	208.494	-
10	-	-	1790208073	395.004	-

Results from the heuristic solution approach are close to the optimization software. On the other hand, running times of the heuristic solution approach are lower than the optimization software's. Moreover for problems 7-10, the optimization software can't solve the model; but the heuristic can give good solutions in acceptable times.

6- Conclusions and future researches

In this paper, we considered a CLSC network design with respect to product life cycle. First, we presented a deterministic model with profit maximization objective function. In this model different qualities for parts used by the manufacturer including end-of-use and end-of-life parts were considered. Electronic devices such as mobile phones and printers are suitable examples for the studied supply chain. In the addressed example, there may be commercial returns; e.g., a returned mobile phone due to a minor defect in the apparent or functioning. End-of-use returns, may be due to technological upgrades which frequently happen and end of life returns due to obsolete technology.

We utilized robust optimization approach in order to tackle uncertainties of demand and return parameters. We designed ten problems to evaluate the proposed model performance. In order to validate the proposed model, sensitivity analysis for various parameters was done for the problems. Results showed that the maximum profit of the manufacturer occurred in a certain capacity of disassembly site. This can help managers to decrease the costs of investment.

When uncertainty parameters presented only in the equality constraints, the objective function of the robust model was higher than of the deterministic model; on the other hand, by increasing the uncertainty level of the return parameters, the objective function increased; but, by increasing the uncertainty level of demand parameters, the objective function decreased. The robust model validation showed that this model dominates the deterministic one. With regard to the NP-hard nature of the problem, it could not be solved for large instances by optimization software; thus, we proposed a heuristic approach based on the Lagrangian relaxation. We relaxed two constraints in order to simplify the problem. With these relaxations, the problem was converted into two sub problems. Sub problem 1 represented the forward supply chain to satisfy the primary market demands. Sub problem 2 represented the reverse supply chain for collecting and recovery of returns to satisfy the secondary market demands. We also proposed two algorithms based on partial derivations to solve the sub problems.

From managerial point of view, the results of this research shows the benefits of establishing reverse logistic structure and its integration with the forward logistics. Making such a structure, shows the organizational commitment to environmental issues as well as cost saving in the regular operations of the supply chain.

Several future research ideas can be given: other approaches can be used for tackling the uncertainty of parameters like probability theory and fuzzy approaches. Proposed models in this paper were single period and can be extended to multi-period. In addition, meta-heuristic approaches can be used for solving the problem for large sizes; furthermore, considering purchasing price for return products and defining other objective functions like environmental factors can be other research directions.

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