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A Stackelberg game model for optimizing price, service level and inventory decisions in Vendor Managed Inventory system

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Abstract

Vendor management inventory (VMI), as one of the inventory management methods, reduces the cost of inventory in the chain, quick response to customers, increased service level, customer satisfaction, and improve collaboration between the members of the supply chain. In this study, a bi-level supply chain model with one manufacturer and one retailer under a vendor management inventory policy is investigated. The manufacturer produces a single product by the production capacity constraint at wholesale price and delivers the product to the retailer who then sells the product in the dispread market at retail price. The demand rate for the product in the retailer market is assumed to be a decreasing function of the price and an increasing function of the service level. The manufacturer determines its wholesale price, product's replenishment cycle time, and backorder quantity to maximize its profit. Retailer determines the optimal retail price and service level to maximize its profit. This problem formulated as a Stackelberg game model with consideration manufacture as a leader and retailer as a follower. To search the Stackelberg game equilibrium, a solution algorithm has been proposed. A numerical study has been conducted to demonstrate how the algorithm works and to understand the influences of decision variables and parameters. Also, in order to validate the proposed model, sensitivity analysis has been performed on some parameters.

Keywords: Supply chain management, Vendor Managed Inventory (VMI), Stackelberg game, price, service level

1-Introduction

Vendor managed inventory is an inventory management strategy in which the retailer gives its sales information and inventory level to the supplier and the supplier determines the replenishment rate in each period based on the information received. In this way, the supplier can offer a good replenishment plan and the retailer can receive the order quantity on time (Kaipia et al. 2002, and Lee et al. 2000). The VMI system has been widely accepted by many industries for many years. The partnership between Walmart and P&G is a successful example of this approach (Ortmeyer and Buzzell, 1995).

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The supply chain in this paper includes a manufacturer and a retailer, and the manufacturer uses the VMI approach. The manufacturer produces and delivers a type of product to the retailer despite the limited production capacity.

The retailer buys the product from the manufacturer at a wholesale price and sells it at a retail price. In many studies on VMI, the demand rate is considered as a function of price product, (see Yu et al. (2009), Rasay et al. (2013), Yu et al. (2013), Zare Mehrjerdi et al. (2014), Niknamfar and Pasandideh (2014), Rasay et al. (2015), Taleizadeh et al. (2015), Hemmati et al. (2017), Haji et al. (2018), Bahrami and Batarfi et al. (2019) Pasandideh (2019)). But since considering demand as a function of selling price alone, it cannot adequately explain customer behavior, hence, in this study, in addition to product price, the service level of retailer is also considered. Therefore, the demand rate in retailers' market is assumed a decreasing and convex function of the retail price and increasing and convex function of the service level. In this supply chain, the retailer is able to determine the retail price and service level of the product to maximize its own profit and the manufacturer is able to determine its wholesale price, replenishment cycle, backorder quantity to maximize its own profit.

2-Problem description and formulation

In this section, a mixed-integer non-linear programming model is developed including the one manufactures and one retailer as a Stackelberg game with the manufacturer as the leaders and the retailer as the follower. The demand rate in retailers' market is assumed a decreasing (increasing) and convex function of the retail price (service level). Hence, the notation of the problem, the mathematical model and its description are explained.

The basic assumptions of the problem are provided as the following:

- The model is one-manufacture, one-retailer, and one-item
- The demand is certain.
- The backorder is considered.

Parameters

$D(p,s)$	Demand rate of retailer from manufacture, a decreasing and convex function of p and an increasing and concave function of service level.
a	Demand slope in the retailer demand function
k	A constant in the demand function of retailer which represents her market scale
Cap	Production capacity of the manufacture
Cm	Manufacturing cost of the product by manufacture
Hr	Holding cost of the product at retailer's warehouse
Hm	Holding cost of the product at the manufacturer's warehouse
Br	Backorder cost of the product of retailer
Sr	Fixed replenishment cost of the product for retailer
Sm	Production setup cost of the product for the manufacturer
φ	direct transportation cost for shipping one unit product from the manufacturer to retailer
ξ	inventory cost paid by retailer to the manufacturer for one unit product according to the VMI policy
η	Service cost factor for the retailer $\eta > 0$, $1/\eta$ reflects the service investment efficiency.

Decision variables for manufacture

C	replenishment cycle of the product
y	Fraction of backlogging time in a common replenishment cycle from manufacture i for retailer j
w_p	Wholesale price of product set by manufacture

Decision variables for retailer

p	Retailer' s price
s	The service level of the retailer

Stackelberg game model (SG model)

$$Max\ NP_m(y,C,w_p) = D(p,s)\left(Wp+\xi\right) - D(p,s)\left(Cm+\phi\right) - \frac{1}{C}\left[Sr+Sm\right] - \frac{C}{2}\left[D(p,s)\left(Hr(1-y)^2+Bry^2\right) + \frac{Hm\,D(p,s)^2}{Cap}\right] \quad (1)$$

s.t: $D(p,s) \le Cap$ (2)

$0 \le y \le 1$ (3)

$C \succ 0, w_p \ge 0$ (4)

$$Max\ NP_r(p,s) = (p - w_p - \xi)D(p,s) - \frac{\eta s^2}{2} \quad (5)$$

$(p - w_p - \xi) \succ 0$ and $p, s \ge 0$ (6) and (7)

Equations (1) and (5) are the objective functions of the manufacturer and its retailer, respectively. Constraint (2) is the production capacity constraint. Constraint (3) shows that retailer's backorder fraction is between 0 and 1. Constraint (6) indicates that the basic existing condition of the retailer. And finally, positive variables are mentioned in constraints (4) and (7).

3-Solution procedure

In order to calculate the equilibrium, we calculate the best reaction function of the retailer first and then analyze the manufacture's optimal decisions that consider the trailer's best reactions.

3-1-Optimal strategy of the retailer

If the retailer cannot meet $p \succ w_p + \xi$, it has to suffer from a loss (negative net profit) in the supply chain, and will ultimately leave the system. Our following discussion will ignore this case.

As shown in Model SG, the decision model of the retailer depends on w_p from the manufacture. To maximize $(p - w_p - \xi)D(p,s) - \frac{\eta s^2}{2} = (p - w_p - \xi)(K - ap + s) - \frac{\eta s^2}{2}$, which a concave function is of p and s the unique optimal retail price and service level, in reaction to the wholesale price w_p are given by:

Theorem 1: Assume $\eta \succ 1/2a$ the optimal value of retail price is equal to

$$p^*(w_p) = \frac{K\eta + (w_p+\xi)(a\eta - 1)}{2a\eta - 1} \quad (8)$$

And the service level is

$$s^*(w_p) = \frac{K - (w_p+\xi)a}{2a\eta - 1} \quad (9)$$

Proof of theorem 1: From (5), we know that the Hessian matrix over (p,s) is $\begin{pmatrix} -2a & 1 \\ 1 & -\eta \end{pmatrix}$, which is negatively definite following from $\eta \succ 1/2a$. Solving the first-order condition $\frac{\partial NP_r(p,s,w_p)}{\partial p} = 0$ for (p,s), we obtain the optimal reactions for the retailer in equations (8) and (9).

By inserting (8) and (9) into (5), we obtain the retailer's profit

$$NP_r^*(w_p) = \frac{\eta(K - a(w_p+\xi))^2}{2(2a\eta - 1)} \quad (10)$$

Thus, the corresponding optimal demand rate of retailer is given by

$$D^*(w_p) = D^*(p^*(w_p), s^*(w_p)) = \frac{a\eta(K - (w_p+\xi)a)}{2a\eta - 1} \quad (11)$$

The assumption of $\eta \succ \frac{1}{2a}$ means that service investment should not be too inexpensive, which is a common assumption in the economics literature [15, 16]. Here it assures that there exists a unique optimum. We assume that $\eta \succ \frac{1}{2a}$ is satisfied throughout this paper.

3-2-Optimal strategy of the manufacturer

The manufacturer's decisions include its replenishment cycle of the finished product C, wholesale price w_p and backlogging fraction y. Substituting equations (8), (9) and (11) into model SG, we obtain the decision model of the manufacturer as follows.

$$MaxNP_m(y,C,w_p) = D^*(w_p)(Wp+\xi) - D^*(w_p)(Cm+\phi) - \frac{1}{C}[Sr+Sm] - \frac{C}{2}\left[D^*(w_p)\left(Hr(1-y)^2+Bry^2\right)+\frac{Hm\,D^*(w_p)^2}{Cap}\right] \tag{12}$$

S.t: $D^*(w_p) \le Cap$ and $0 \le y \le 1$, and $C \succ 0, w_p \ge 0$ (13)

Because the second derivative of equation (12) with respect to y is

$$\frac{\partial^2 NP_m(y,C,w_p)}{\partial y^2} = -C.D^*(w_p)(Hr+Br) < 0 \tag{14}$$

Therefore, $NP_m(y,C,w_p)$ is a concave function of y for any other given C and w_p. Thus, from the $\frac{\partial NP_m(y,C,w_p)}{\partial y} = 0$, the optimal value of y can be obtained as

$$y^* = \frac{Hr}{Hr+Br} \tag{15}$$

By substituting equation (15) into equation (12) and rearranging the result, we obtain

$$NP_m(C,w_p) = D^*(w_p)(w_p+\xi) - D^*(w_p)(Cm+\phi) - \frac{1}{C}[Sr+Sm] - \frac{C}{2}\left[D^*(w_p)\left(\frac{HrBr}{(Hr+Br)}\right)+\frac{Hm\,D^*(w_p)^2}{Cap}\right] \tag{16}$$

The second derivative of equation (16) with respect to C is

$$\frac{\partial^2 NP_m(C,w_p)}{\partial C^2} = -\frac{2}{C^3}(Sr+Sm) < 0 \tag{17}$$

Therefore, $NP_m(C,w_p)$ is a concave function of C for any given w_p. Thus, from $\frac{\partial NP_m(C,w_p)}{\partial C} = 0$, the optimal value of C^* is

$$C^* = \sqrt{\frac{2(Sr+Sm)}{\frac{D^*(w_p)HrBr}{Hr+Br}+\frac{Hm\;D^*(w_p)^2}{Cap}}} \tag{18}$$

Substituting equation (18) into equation (16), the manufacturer's net profit becomes a function of variable w_p:

$$NP_m(w_p) = D^*(w_p)(w_p+\xi) - D^*(w_p)(Cm+\phi) - \sqrt{2(Sr+Sm)\left[\frac{D^*(w_p)HrBr}{Hr+Br}+\frac{Hm\,D^*(w_p)^2}{Cap}\right]} \tag{19}$$

S.t: $D^*(w_p) \le Cap$ and $w_p \ge 0$ (20) and (21)

According to equation (19), there is one continuous variable (w_p) in the model. Hence, we use the Kuhn-Tucker condition to calculate the optimal values of model. λ is considered as the Lagrange multiplier and the Lagrange function is defined as equation (22).

$$Max\ L_m(w_p,\lambda) = NP_m(w_p) + \lambda\left(Cap - D^*(w_p)\right) \tag{22}$$

Then the Kuhn-Tucker condition for the model is

$$\frac{\partial NP_m(w_p)}{\partial w_p} - \lambda \frac{\partial D^*(w_p)}{\partial w_p} = 0 \quad and \quad \lambda\left(Cap - D^*(w_p)\right) = 0 \tag{23}$$

For $\lambda = 0$, we have

$$\frac{\partial NP_m(w_p)}{\partial w_p} = 0 \tag{24}$$

And give the corresponding critical point w_p . For $\lambda > 0$, we obtain the corresponding critical point w_p and λ by calculating

$$\frac{\partial NP_m(w_p)}{\partial w_p} - \lambda \frac{\partial D^*(w_p)}{\partial w_p} = 0 \quad and \quad D^*(w_p) = Cap \tag{25}$$

By comparing the objective values corresponding to all points w_p calculated above, we can obtain the optimal w_p by selecting the one that has a bigger NP_m . From the above analysis, it can be concluded that the equilibrium of Stackelberg game is obtained from solutions that meet all the optimal conditions (equations (8), (9), (15), (18) and (23)).

3-3- Proposed solution algorithm

The solution algorithm with the following steps is suggested to find the equilibrium of Stackelberg game.

Step 1. Calculate the optimal w_p :

Step 1.1. Calculate the w_p with equation (24) ($\lambda = 0$) and with equation (25) ($\lambda > 0$), respectively with Wolfram Mathematica 12.1.

Step 1.2. Calculate and compare the manufacturer's net profit with equation (19) with the calculated w_p at $\lambda = 0$ and $\lambda > 0$.

Step 1.3. Set w_p that make equation (19) bigger as the optimal w_p . Set $w_p = w_p^*$ and the maximum net profit NP_m^* of the manufacture can be calculated with equation (19) and go to the next step calculation.

Step2. Calculate the optimal y^* and C^* with equations (15) and (18), respectively.

Step 3. Calculate the optimal values p^* and s^* by equations (8) and (9).

Step 4. Calculate the optimal NP_r^* by taking the optimal and w_p^* into equation (10) to get the maximum profit for retailer.

Following the above steps, we will get the optimal solution (equilibrium) p^* and s^* from step 3; y^* and C^* from step 2; w_p^* from step 1; the net profits of the retailer and manufacturer NP_r^* and NP_m^* from step 4 and step 1.3, respectively.

4-Numerical example

In this section, a numerical example is provided to evaluate the proposed model and its solution algorithm. Table 1, shows the values of the input parameters of the numerical example. Table 2 shows the optimal decisions for the manufacturer and retailer.

Table 1. Input parameters values

Parameters	a	k	Cap	Cm	Hr	Hm	Br	Sr	Sm	φ	ξ	η
Value	0.69	1020	150	20	7.7	4	716.6	238.3	200	10.8	18.7	100

Table 2. Optimal values of variable

Parameters	w_p^*	p^*	s^*	C^*	y^*	D^*	$NP_m^*(10^3)$	$NP_r^*(10^3)$
Base example	1027.92	1264.02	2.17	0.71	0.01	150	15.14	64.74

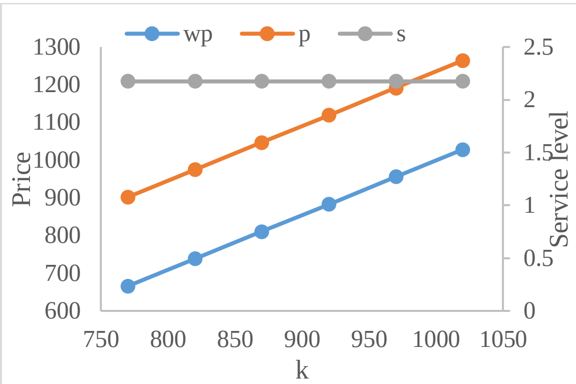
5-Sensitivity analysis of some selected parameters

Sensitivity analysis has been conducted with the model for parameters in two groups: manufacturer-related parameter (e.g. Cap) and retailer-related parameters (e.g. K, a, η, ξ, ϕ). Figure 1 (a-l), shows the impact of the parameters on the wholesale price, price and service level of retailer, manufacturer profit and retailer profit. Figure (1-a) shows the effect of the Cap parameter on supply chain performance. As expected, in Figures (1-a and 1-b), by increasing the manufacturer's production capacity, the wholesale price and retailer price decrease, but the retailer service level, manufacturer profit, and retailer profit increase. In general, we can say: $Cap \uparrow \Rightarrow Wp \downarrow \; p \downarrow s \uparrow NP_m \uparrow NP_r \uparrow$

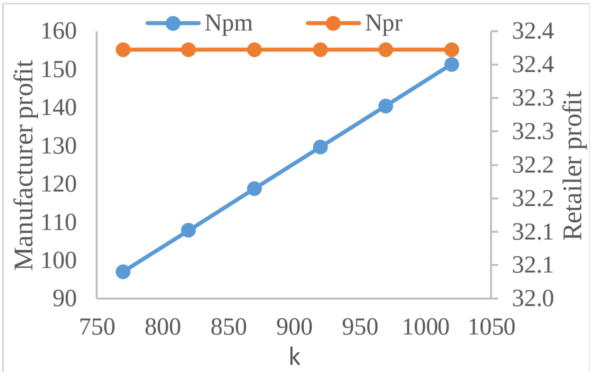
In summary, the impact of retailer-related parameters on the optimal values of variables can be expressed as follows: $K \uparrow \Rightarrow Wp \uparrow \; p \uparrow s \equiv Const \;\; NP_m \uparrow \; NP_r \equiv Const$

$a \uparrow \Rightarrow Wp \downarrow \; p \downarrow s \downarrow NP_m \downarrow NP_r \downarrow$ $\qquad \eta \uparrow \Rightarrow Wp \,, \; p \equiv Const \;\; s \downarrow NP_m \equiv Const \;\; NP_r \uparrow$

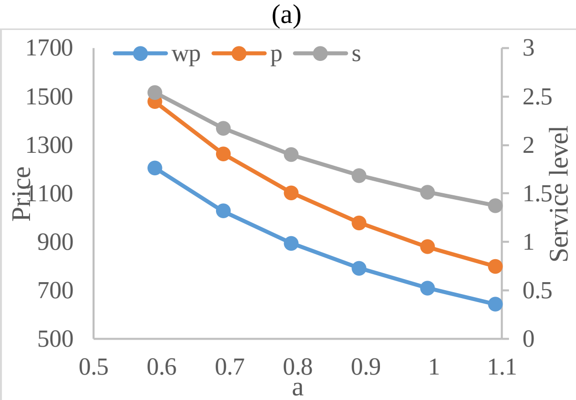
$\xi \uparrow \Rightarrow Wp \downarrow \; p \uparrow s \downarrow NP_m \uparrow NP_r \downarrow$ $\qquad \phi \uparrow \Rightarrow Wp \,, \; p \,, s, NP_r \equiv Const \; NP_m \equiv \downarrow$



(a)



(b)



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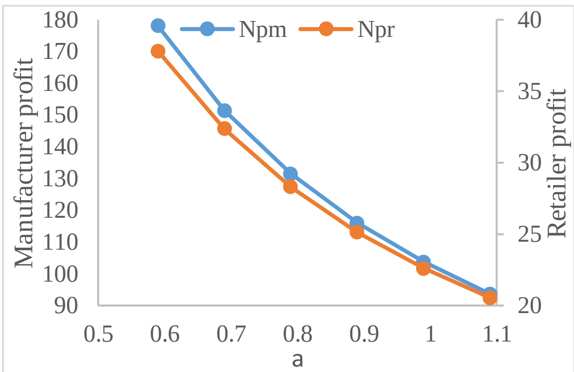

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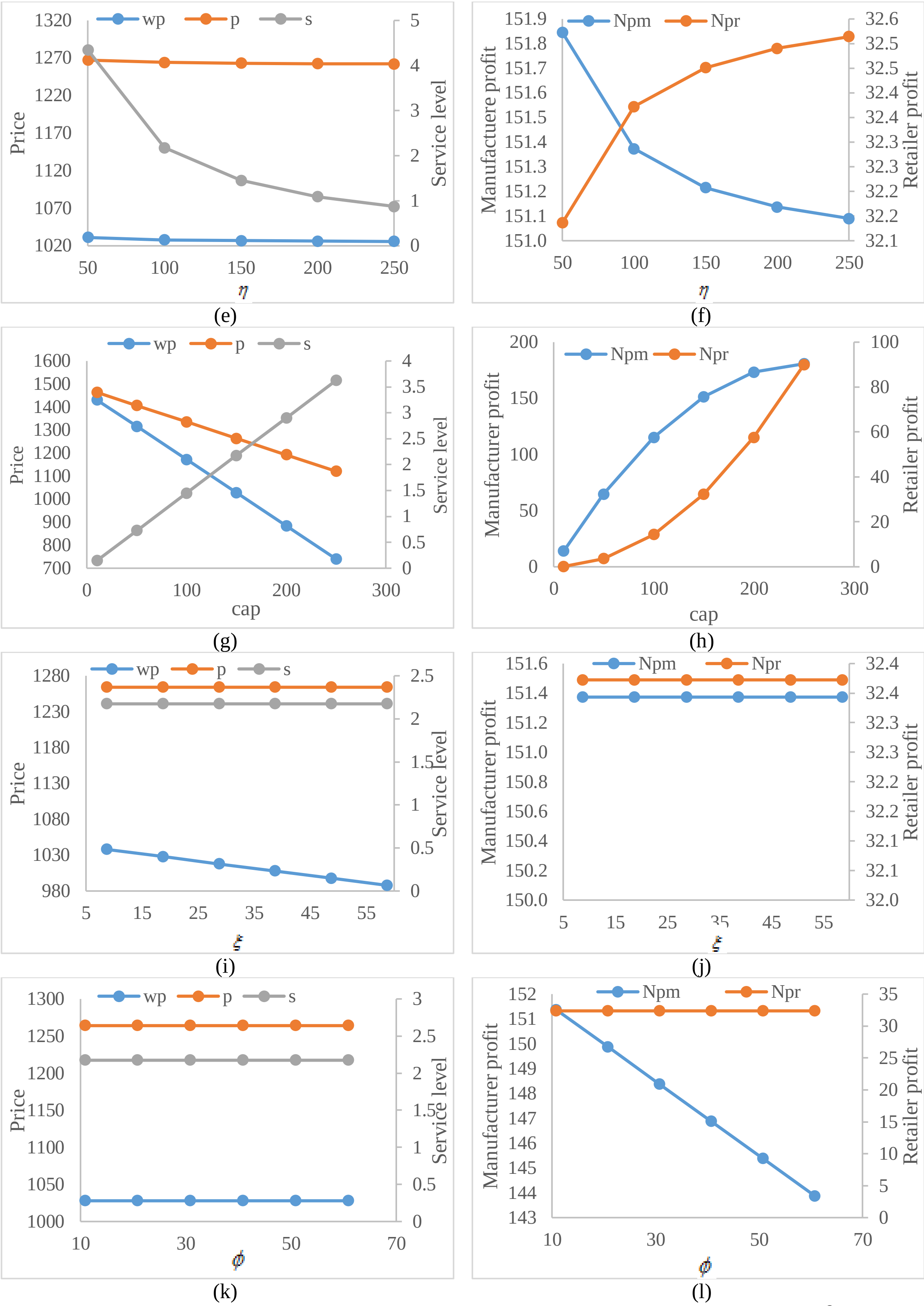

Fig 1. The impact of some selected parameter on optimal values of variables (Profits (10^3))

6-Conclusion

In this study, a bi-level supply chain model with one manufacturer and one retailer under a vendor management inventory strategy is presented. The manufacturer produces a single product by the production capacity constraint at wholesale price and delivers the product to the retailer who then sells the product in dispread market at retail price. The demand rate for the product in the retailer market, is assumed to be a decreasing function of the price and an increasing function of the service level. This problem formulated as a Stackelberg game model with consideration manufacture as a leader and retailer as a follower The proposed Stackelberg game model, is solved by a computational algorithm based on the theoretical analysis of the best response function. Finally, a numerical example is provided to evaluate the proposed model and its solution algorithm and examine impact of manufacturer-related parameter (production capacity) and the parameters related to the retailer.

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