New scheduling approach for freight and passenger inter-city trains considering blocking using queuing theory

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Abstract

We aim to plan better scheduling for movement of shuttle trains on the single-line route of inter-city railway network to decrease the delays due to the trains blocking in successive crowded stations. A MINLP model is examined to increase capacity efficiency of the stations using the queuing theory considering blocking. We propose an optimal schedule for moving trains which minimizes the blocking probability to raise the profit gained on a two-way railway. The results show that we have achieved the best scheduling with lowest delays considering the constraints of the track number inside stations. Queuing models are applicable because the trains’ departure scheduling can be evaluated with the aid of the performance criteria obtained by the queuing model. To validate the model, an optimal schedule is proposed for a real case-study in Iran. Finally, the benefits of the model and sensitivity analysis are conducted using GAMS v24.9.1, with BARON solver.

Keywords: inter-city network of railway, trains departure scheduling, railway stations capacity, queuing theory approach, blocking, train prioritization

1-Introduction

The main goal of an inter-city railway transport system is to transfer the cargo from one point to another. This cargo can be freight or human. The cargo transport operation consists of different stages during its life cycle including taking orders, lines planning, formation, train configuration and movement, program monitoring, and trains movement scheduling, loading and unloading and etc (Watanabe and Koseki, 2015).

In this article, the capacity of stations means: the number of passenger and freight trains in a certain period of time. Improving the capacity of a station means, for example, being able to serve more trains over a period of time. By increasing the capacity, more cargo can be carried and also a large number of passengers can be moved in a certain interval. To do that, we need better coordination between different parts of the railways (ie proper planning and scheduling). For example, planning for the proper distribution of intercity trains on the rail network is one of the tasks that will increase the capacity of stations on the rail network. So the costs related to trains travel in the railway network (including operational costs and time) will be minimized. The "station capacity" means the number of shuttle trains which receive a service in a certain time and leave the station.

Generally, the railway planning problem is divided into two main and important sub-problems including planning the stations capacity and trains departure scheduling, due to the complexity of the railway transport system.
Previous studies show that, according to the complexity and span of the lines planning and trains scheduling, different approaches have been utilized to face them. In addition, researchers have tried to investigate these issues using different ways which can be categorized into four different groups based on the works done so far. 1) Modeling based methods 2) methods based on an innovative solution of mathematical models 3) simulation methods 4) search-based methods.

Main obstacles and constraints that we face in the above-mentioned methods are as follows:

- Assuming some main parameters such as trains departure scheduling and the number of tracks to be constant or deterministic instead of uncertain,
- Solving a one-dimensional model due to the complexity of the problem such as neglecting the railways to be multi-track,
- Neglecting some vital parameters in the problem such as train type variety, train prioritization, and train speed.
- The optimization accuracy in simulation methods is low especially in large scales.
- Not solving the real scale problem and solution approaches become time consuming when the number of probabilistic variables in the model is great.
- Real costs for the railway network are not considered such as stopping and waiting times in the block and ideal times for stations. (Abuobidalla et al., 2015; Zhao et al., 2017). Waiting time occurs when a blocking state occurs for trains within the line between two stations. In other words, the amount of time that a train on the line between two stations waits until one of the track of the next station is emptied of the train, so that it can enter that station. These delays at all stations are transmitted in a chain mode, which leads to a large difference between the actual schedule and the ideal or planned scheduled schedule, and the rail network is forced to stop. Eventually, the costs of blocking delays and delays will lead to a reduction in rail profits and revenues.

The question now is: what is the suitable mathematical approach for solving the railway stations capacity-planning mode? Furthermore, is it possible to develop and improve the railway station capacity planning models using the queuing theory approach? (Motono et al., 2016).

To respond to the above questions, we should consider that the freight trains departure schedule is dependent on the passenger trains due to the priority of the passengers’ dissatisfaction. On the other hand, the delay may occur for freight trains departure schedule and this can lead to new costs. Therefore, the tradeoff between these costs is necessary (Chou et al., 2017; Abuobidalla et al., 2019; Sameni et al., 2016; Azadi Moghaddam Arani et al., 2019).

To overcome the above drawbacks, we have used the queuing theory approach because of the following three main reasons:

A) There are several factors causing uncertainty in a railway transport system and queuing theory approach is able to handle most of these uncertainties. These factors are the time of waiting of trains which are ready to receive a service at the station, delays in the movement of trains because of lines junction (the time percentage of lines occupation), the time of unwanted delays caused by the failure, blocking and unavailability of a free line, accident occurrence, train maneuver, electrical signs inspection, and locomotive failure and its consequences.

B) High-quality service to customers can be provided by minimizing the time of services to a customer at inter-city railway network stations and also the time of the entering and leaving the sequence of passenger and freight trains to/from stations. Such objectives can be gained by queuing theory.

C) According to the fact that the "queue capacity of each station" (including the total number of tracks in one station and the railway line between two stations) in the inter-city railway network is finite, a customer (train) enters the line between two stations (block) after receiving service from a station. If all the lines in the next station are occupied, the train must wait in Block because the possibility of space and service facilities does not exist (about the blocking phenomenon in queuing networks, Perros, 1994). In this case, blocking in the system occurs when all lines of a station are in service, while the train of the previous station is ready to enter the block. So we have to wait for the first train in the busy station to finish receiving services so that it can enter the second station. Finally, these delays also lead to increased train stops and increased delays and waiting times for trains within the line between two stations, as well as increased fuel consumption. The probability of blocking is a
side effect for the system wastes the resources and it causes dissatisfaction for customers. Such a drawback can be measured and optimize by only queuing theory approach.

In this research, for the first time, we use suitable planning and modeling the trains departure scheduling by invoking the queuing theory approach and considering blocking to reduce the costs of transportation in the inter-city railway network between successive stations and increase the profits gained from the railway transport in a one-day period.

2-Literature review

The application of decision-making models has a wide range in the railway network management. There exist several problems in the railway networks which need analysis and determination of a suitable (optimum) strategy. Assad (1980) gathered all the works done up to 1980 and divided them into 6 general groups based on their role. The most important groups are planning the trains departure schedule, rails capacity, and preparing the timetables to solve them. This problem can be divided into four different and important groups based on various decision-making parameters in the railway.

- Modeling based methods

In methods based on the modeling, Meirich and Nieben (2016) proposed a method to maximize the number of passing passenger trains in the railway network (Subway in European countries). The analytic and practical algorithms of queuing theory were used to analyze the railway tracks capacity where some sections of the railway tracks are mechanically adjusted using queuing models. Finally, they recognized the railway bottleneck (junctions with heavy rail traffic) with the aid of the proposed model and comparing it with the existing infrastructures (capacity) and they could eliminate them using the optimal rerouting. Xu and Liu (2014) designed a railway network model using the queuing approach to calculate the subway stations capacity. The proposed queuing model was built by M/G/C/C state dependent and discrete time Markov chain. Based on the definition and queuing theory, an optimized model was developed for subway stations capacity where the objective function of the model was the optimization of subway stations capacity with a satisfactory rate of remaining passengers. The model was at last integrated using the iterative generalized expansion method and the nodes with the greatest impacts were identified and introduced as the bottleneck station by analyzing the sensitivity.

Andersson et al. (2015), Herrigle et al. (2018), Peng et al. (2013), and Yang et al. (2018) could optimize the train timetable by using the mixed-integer linear programming approach and generally reduce the train journey delays. Just the critical points of the double-track railway transportation were examined. Yang et al. (2016) and Chen et al. (2013) presented a new method for improving the scheduling of high-speed trains in a single-line and reducing train stop planning at stations. They were able to create a new timetable for the departure of trains between stations by improving the scheduling of trains and minimizing the total delay at origin station and dwelling time at intermediate stations with using a complex integer linear programming model.

- Methods based on innovative solution of mathematical models

In methods based on the innovative solution of mathematical models, most researchers use innovative methods or the combination of innovative and classic methods of mathematical programming to solve these problems due to the high complexity and span of trains movement scheduling mathematical models. Qi et al. (2016) improved the railway stations capacity (which are mostly determined according to the multi-track configuration) especially in a crowded corridor with heterogeneous trains and reduced the total amount of trains travel time. In order to increase the stations capacity and the number of admissible cars at critical stations, considering the budget constraints, models with different constraints were formulated. One model was a single-state mixed-integer linear programming and the other one was a double-state model of large-scale and the purpose of the first one was to design loop lines at candidate stations and the aim of the second model was scheduling of trains by assigning a track to each train at each station. The commercial program, GAMS, with CPLEX solution algorithm and the innovative method of local search were integrated with CPLEX method was used to obtain near-optimal solutions. Finally, two examples were provided to show the performance and effectiveness of the proposed method where the first one was a case
study on the rail corridor in the inter-city railway network and the second one was a case study on a rail corridor used for Wuhan-Guangzhou high-speed trains. The main drawback of the proposed model was that it cannot be solved for large-scale cases and besides, the affecting parameter of the speed was assumed to be constant and can be implemented for two types of trains. Ye and Liu (2016) proposed an innovative method and a model to control and optimize the planning and train scheduling and reduce the fuel consumption in which important parameters such as train type diversity and speed, rail type and the number of tracks, train operational limit, and trains sequences are applied.

Xu et al. (2018), Corman al. (2017) and Hassanayebi et al. (2017) proposed some models in order to promote rail transport infrastructure, reducing costs and delays in the rail transport network. Xu et al. (2016) studied the disruption and causes of delays caused by passenger traffic in China in high-speed rail transport. They showed that the rate of arrival of trains to stations is subject to exponential distribution. They analyzed the effect of latency and disruption the stations by their proposed model. Şahin (2017) used a new method to reduce stochastic delays when trains move from a station and arrive in the next station. By adding the suggested time to the train timetable (as buffer times), which was obtained using the Markov chain model approach, he could reduce and improve the total train travel time.

- **Simulation methods**

Methods based on simulation models are the most common methods for solving complex problems due to their simplicity and great power in analyzing problems (Fan et al., 2018).

Most previous studies are performed based upon simulation methods because of the complexities in the problems associated with trains’ departure schedule. The advantage of these methods can be summarized in yielding reasonable responses in a relatively short time and they can be used for both preparing periodic timetables and for modifying and changing the trains timetable. Kabasakal et al. (2015), Scarduca (1997) and Jovanovic and Harker (1991) used simulation to solve the train movement scheduling problem.

Quaglietta (2014) could significantly improve the railway capacity and reduce the energy consumption by using optimization based upon block and signaling simulation in order to maximize the system productivity and shorten the block length. Xiaoming Xu et al. (2015) increased the capacity of railway utilization and reduced the delays caused by trains dwelling at stations. The rerouting for trains movements with the approach of raising the timetable efficiency is performed for the double-track line of the railway network. They could finally improve and develop trains’ departure schedule. They designed an optimized policy change for the first time by analyzing the delays due to the selection of different paths. Then, they examined three cases of integration in train routing in which the timetable is based on a discrete model (in the case of diversity in the departure for different trains). For efficiency evaluation in the railway industry, Wiegmans et al. (2018) introduced a methodology for analyzing and comparing the different aspects of efficiency of roads and railways in various geographical contexts.

Xu et al. (2019) used a simulation-based optimization approach to unscheduled trains with preplanned departure times instead of trains with a pre-scheduled timetable on a single-track railway line. They were able to minimize trains delay by simulating the trains running on a single-track railway line and mixed-integer programming models and Relaxation induced neighborhood search.

- **Search-based methods**

Search-based methods are able to discover the acceptable solution space completely considering the intended parameters and find relatively good solutions to the problem. New methods that are recently introduced in this field consider a portion of the solution space and try to obtain good results in the shortest time possible by using the available tools.

Yaghini et al. (2015) used a fuzzy railroad blocking model (by shortest path method) and proposed a method based on the developed genetic algorithm in order to diminish the railway costs and delays. They tested several simulation problems to evaluate and validate the solution method. The results show that the algorithm has an acceptable accuracy and computing speed in solving the problems in the railway-blocking plan. The fuzzy method is presented as the preferred method in this model to solve the problem and then the fuzzy model is transformed into a crisp railway model. Several different obstacles that exist in planning and simulating the railway blocking, including large numbers
of variables, lead to modeling constraints; and solving them would be devastatingly time-consuming using a software.

Cacchiani et al. (2016) studied the timetable optimization problem of trains running in the railway network. Their purpose was to determine the best schedule for a specific train set in a certain time horizon considering several railway operational limits. Finally, a highly congested railway network framework was proposed that could vanish the conflicts in timetable which included planning difference and optimal state. They used an integer linear program formulation to solve this problem. Moreover, an iterative heuristic algorithm was proposed which led to the presentation of a proper solution method in the real world for 1500 trains by calculating the shortest time possible for trains movement. Off-planning time delays caused in the model or in other words, the presence of the uncertainty causes the proposed model to be unsolvable.

Liu and Dessouky (2018) examined and improved the issue of scheduling integrated passenger transport trains in complex railway networks. In the proposed model, the objective was to improve the scheduling of passenger trains based on the time of freight train with considering the uncertainty in freight train departure times. On a large scale, they were able to improve the timetable of passenger trains using the BFS search and a branch and bound framework with a heuristic algorithm method.

Also, Sama and DAriano (2017), with the help of the heuristic algorithm and advanced Tabu search, were able to reduce trains delay at stations and suggest a better scheduling plan for trains.

According to the above literature survey, some important points are not considered yet and we aim to fulfill this gap in this research for example, most simulations contain queues as part of the model. Queuing theory refers to the mathematical models used to simulate these queues (Ferro et al, 2017). Calling populations are often assumed to be ‘infinite’ if the real population is large. This simplifies the model. For infinite populations the arrival rate is not affected by the number of people that left the population and entered the queue, on the other hand, with the help of queuing theory models, accurate and optimal solutions can be achieved, but it’s not in the simulation. Solving the problem of blocking the stations and accurate calculation of the system's idle time is not possible with the help of simulation in the long run. The most important points are considering blocking event in the railway transportation and minimizing its probability. Also, capacity utilization in the stations and prioritization of trains are the other important advantages of applying queuing theory for the railway transportation system in this paper considering uncertain conditions.

3-Model development

In this paper, we aim to achieve optimal and feasible solutions by providing the proposed model for scheduling the utilization of rail stations capacity and using the queuing theory approach (Qi et al., 2013).

Analyzing the queuing series systems in the whole railway transportation network is extremely complex when there are one or more crowded stations having limited capacity and experience blocking. Consequently, considering the vast span of the railway transportation network, and for implementing and analyzing the network queuing series model, we have chosen a single-track two-way main path. In addition, the most crowded station and its previous station are considered and we have formulated the proposed model based on those successive stations. Figure 1 shows the schematic of the model:

**Fig. 1.** Schematic of the proposed model.
3-1-Model assumptions
- The time distribution function for the train movement is examined in the steady state.
- Trains are grouped as passenger trains and freight trains and the number of them is assumed to be limited in the model.
- Train prioritization is considered in the model. Besides, trains cannot pass each other between two stations due to the single-track block assumption and overtaking is only allowed inside stations according to the train prioritization.
- In Iran's intercity rail network, slope information and arches and bridges are used to determine the speed of freight and passenger trains. First, due to safety factors, the speed of passenger and freight trains cannot exceed the set maximum. Second, the speed of the trains should not exceed the calculated value for the maximum passing speed, which is calculated according to the radius of the arc. Third, because passenger train traffic has a higher priority than freight trains, so the average speed of passenger trains is higher than the average speed of freight trains. So the speed of passenger and freight trains are assumed to be different.
- The track between two stations is single and two-way.
- Train stops for cases such as boarding, loading, and refueling are predetermined and fixed in the train’s departure schedule.
- Since the distance between two stations in the modeled case study is not long, we apply the following assumption. The railway line between two stations is called block and we aim to minimize the probability of blocking in order to increase utilization of the railway line. Only one train is allowed to be in a block at a time for safety reasons. Based on the mentioned point, queue capacity of each station (the total capacity of the queue is waiting for the trains to serve inside the station) is as follows:

The total capacity of the queue is waiting for the trains to serve inside the station = the number of tracks in a station for providing services to trains + 1 (block)

- In the proposed railway network queuing series model, the train entrance to stations is assumed to be the Poisson distribution with ( \( \bar{\lambda} \) ) parameter considering the case of blocking and limited queue capacity.

Note: It should be noted that according to the queuing theory theorems, once the entrance pattern to a station is Poisson and service time is exponential distribution, the departure pattern would be according to Poisson too. However, the departure rate for a station will be determined according to the capacity of the server in the station.
- In addition, because trains speeds are different when entering the station, average entrance rate variable (\( \bar{\lambda} \)) is used and its value is obtained by the proposed model as a decision variable. The number of tracks is limited at each station and the rate of providing services is different at each station and is considered to be a constant parameter in the model.

The calculation of evaluation criteria for the queuing system of the whole chosen movement line in the inter-city railway network with the assumption of stations being independent is as follows:

\[
\lambda_{total} = \sum_{k} \bar{\lambda}_k \\
\mu_{total} = \sum_{k} \mu_k \\
\pi_{1,2,...,k} = \prod_{k=1}^{K} \pi_k \\
L = L_1 + L_2 + ... + L_K \\
\bar{\lambda}_k \leq \mu_k \quad \text{System stability condition}
\]
3-2-Notations
In order to model the proposed problem, the following indexes, parameters, and decision variables are introduced.

<table>
<thead>
<tr>
<th>Notations</th>
<th>Detailed definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i$</td>
<td>Set of OD trains(^2) which includes passenger and freight trains $i = 1,2$</td>
</tr>
<tr>
<td></td>
<td>If $i = 1$, The train is a passenger type and If $i = 2$, The train is a freight type</td>
</tr>
<tr>
<td>$j$</td>
<td>Set of DO trains(^3) which includes passenger and freight trains $j = 1,2$</td>
</tr>
<tr>
<td></td>
<td>If $j = 1$, The train is a passenger type and If $j = 2$, The train is a freight type</td>
</tr>
<tr>
<td>$k$</td>
<td>Set of inter-city railway network stations</td>
</tr>
<tr>
<td>$n_{ik}$</td>
<td>Total number of $i^{th}$ OD trains ready for service at station $k$ which includes passenger and freight trains</td>
</tr>
<tr>
<td>$m_{jk}$</td>
<td>Total number of $j^{th}$ DO trains ready for service at station $k$ which includes passenger and freight trains</td>
</tr>
<tr>
<td>$V_{ik}$</td>
<td>The average arrival speed of $i^{th}$ OD train to station $k$ (Km/h) which includes passenger and freight trains</td>
</tr>
<tr>
<td>$V'_{jk}$</td>
<td>The average arrival speed of $j^{th}$ DO train to station $k$ (Km/h) which includes passenger and freight trains</td>
</tr>
<tr>
<td>$S_k$</td>
<td>Number of tracks at station $k$</td>
</tr>
<tr>
<td>$A_i$</td>
<td>Rate (coefficient) of $i^{th}$ OD train prioritization which includes passenger and freight trains</td>
</tr>
<tr>
<td>$A'_j$</td>
<td>Rate (coefficient) of $j^{th}$ DO train prioritization which includes passenger and freight trains</td>
</tr>
<tr>
<td>$C_i$</td>
<td>Cost of the delayed entrance of $i^{th}$ OD trains which includes passenger and freight trains</td>
</tr>
<tr>
<td>$C'_j$</td>
<td>Cost of the delayed entrance of $j^{th}$ DO trains which includes passenger and freight trains</td>
</tr>
<tr>
<td>$H_{ci}$</td>
<td>The total cost of $i^{th}$ OD train travel from the moment of departure until arriving at the next station which includes passenger and freight trains (per each hour)</td>
</tr>
<tr>
<td>$H_{cj}$</td>
<td>The total cost of $j^{th}$ DO train travel from the moment of departure until arriving at the next station which includes passenger and freight trains (per each hour)</td>
</tr>
<tr>
<td>$H_p$</td>
<td>The time interval between the entrance of two trains running in opposite directions at the block (per each hour)</td>
</tr>
<tr>
<td>$H_{sk}$</td>
<td>The time interval between the departure of two successive trains running in the same directions from station $k$ (per each hour)</td>
</tr>
<tr>
<td>$H_{sx}$</td>
<td>The time interval between the arrival of two successive trains running in the same directions at $k^{th}$ station (per each hour)</td>
</tr>
<tr>
<td>$STOP_k$</td>
<td>Maximum allowed stopping time for trains at station $k$ (per each hour)</td>
</tr>
<tr>
<td>$t_{sk}$</td>
<td>Time of planned stop for trains at station $k$ (per each hour)</td>
</tr>
<tr>
<td>$\mu_k$</td>
<td>The rate of service providing to the train at station $k$ (the average number of trains receiving a service from a service provider per each hour)</td>
</tr>
<tr>
<td>$t_{block}$</td>
<td>The average time between the train departure from one station and its arrival to the next one (per each hour)</td>
</tr>
<tr>
<td>$\text{Incom eod}$</td>
<td>The average income from the OD train travel between two successive stations for $\lambda = 3$ (Toman)</td>
</tr>
<tr>
<td>$\text{Incomo do}$</td>
<td>The average income from the DO train travel between two successive stations for $\lambda = 3$ (Toman)</td>
</tr>
</tbody>
</table>

\(^2\) OD train: movement of the trains that depart from origin station to destination station

\(^3\) DO train: movement of the trains that depart from destination station to origin station
### Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>The rate of train entrance to the station (the average number of trains entering to station $k$ per each hour)</td>
</tr>
<tr>
<td>$ttod$</td>
<td>Total time of an OD train travel from one station to the next station</td>
</tr>
<tr>
<td>$ttdo$</td>
<td>Total time of a DO train travel from one station to the next station</td>
</tr>
<tr>
<td>$todd_k$</td>
<td>Time of an OD train departure from station $k$</td>
</tr>
<tr>
<td>$toda_k$</td>
<td>Time of an OD train arrival at station $k$</td>
</tr>
<tr>
<td>$tdod_k$</td>
<td>Time of a DO train departure from station $k$</td>
</tr>
<tr>
<td>$tdoa_k$</td>
<td>Time of a DO train arrival at station $k$</td>
</tr>
<tr>
<td>$X$</td>
<td>Binary variable- if an OD train passes the block sooner than a DO train, its value is one, otherwise a DO train passes the block sooner than an OD train it is zero</td>
</tr>
<tr>
<td>$Y$</td>
<td>Binary variable- if an OD train leaves the station $k$ and enters the next station sooner than the other OD train its value is one, otherwise a DO train leaves the station $k$ and enters the next station sooner than the other DO train its value is zero</td>
</tr>
<tr>
<td>$\pi(\alpha, \beta)$</td>
<td>Shows the probabilistic situation of the system in which $\alpha$ is the number of trains at the first station and $\beta$ is the number of trains at the second station are receiving services</td>
</tr>
<tr>
<td>$L$</td>
<td>The average number of trains existing in the system in the long run</td>
</tr>
<tr>
<td>$L_q$</td>
<td>The average number of trains existing in the queue in the long run</td>
</tr>
</tbody>
</table>

### 3-3-Mathematical model statement

We formulate the problem for small scale assuming that the number of tracks inside each station is two, in order to improve the train movement schedule at crowded stations. Moreover, we will formulate the model using plotting the state transition diagram and then we indicate all queuing equilibrium equations. Finally, they are applied in the optimization model. Figure 2 shows the model schematic of small scale:

![Model Schematic](image)

**Fig. 2.** The schematic of the case study in small scale $S_k = 2$ and $k = 2$

This model, which is adapted from the continuous-time Markov chain model, assumes that the train entrance at station $k$ from out of the system occurs based on the Poisson process with the rate of $\lambda$. The times of providing services at both lines inside the station $k$ are dependent, and they are according to the exponential random variable with $\mu_k$ parameter. We now mathematically present the proposed model considering the assumptions:

- **Objective function**
The objective functions maximize the net profit gained from OD and DO trains traveling from one station to the next one. The proposed model is mixed-integer nonlinear programming. The costs of the railway transportation are determined according to the train movement scheduling from one station to the next. In this way, the first term in the objective function denotes the total income gained from OD and DOES trains. The other terms in the objective function are different types of costs as follows. The first and second cost terms are the total costs of OD and DO trains entering and leaving the stations.
which includes passenger and freight trains (per one day), respectively. Finally, the third and fourth cost terms demonstrate the total costs of OD and DO trains travel from one station to the next (per one day), respectively.

\[
\text{Max}_{\Lambda\text{ndo}, t\text{tod}, t\text{tdo}} Z(\Lambda\text{ndo, t\text{tod}, t\text{tdo}}) = (24 \cdot (\text{Incomeodo} + \text{Incomeodo}) \cdot \Lambda) - \\
(24 \cdot (\sum_{i=1}^{2} \sum_{k=1}^{2} A_{i} C_{j} n_{ik} V_{ik} \Lambda_{j} + \sum_{j=1}^{2} \sum_{k=1}^{2} A_{j} C_{j} m_{jk} V_{jk} \Lambda_{j} + (\sum_{j=1}^{2} \sum_{k=1}^{2} n_{ik} H_{i} t\text{tdo} + \sum_{j=1}^{2} \sum_{k=1}^{2} m_{jk} H_{j} t\text{tdo}))
\]

- **Constraints**

Constraints of the research model are divided into two groups. The first one is writing the set of constraints associated with queuing equilibrium equations with the aid of the state transition diagram plotting (about the blocking phenomenon in queuing networks, Perros, 1994). The second group constraints are scheduling and transportation limitations. Figure 3 shows the state transition diagram of the proposed model with the assumed dimensions.

![State Transition Diagram](image)

**Fig. 3.** The state transition diagram associated with two stations having two tracks

The nodes of the state transition diagram (figure 3) represent the probable situation of the system, in other words, all possible states express the number of trains that are serving at the two stations, for example, a train in the first station and two trains at the second station is being served. Arrows inserted into a node represent the previous situation of the system associated with that node, and the arcs extracted from a node represent the next possible situation of the system associated with that node.

To write the equilibrium equation of the probability of nodes, the following conditions should be met:
- The sum of probability states associated with arcs inserted into a node = the sum of possible probabilistic positions associated with arcs that are out of that node.
- The sum of system probable states is equal to one.
- The value of each of the system probable states is a positive.

Considering the fact that the number of tracks at each station is assumed two, the blocking in the system occurs when all lines at the second station are busy by providing services, while the train at the first station is ready for the departure and enters the block. Therefore, it should wait until the first train at the second station finishes receiving services so it can enter the second station. As shown in figure(3), two nodes \(\pi(1b2)\) and \(\pi(2b2)\) indicate the probable state of the system in a blocking mode. This problem indicates that the service to the trains on the two platform of the second station is
not finished yet, while the service of at least one of the trains at the first station is over, but due to the completion of the platform at the second station, a train leaves the first station and it stops in the block between the two stations to end serving a train at the second station, and that train would be dispatched from the second station. For instance, $\pi(1b2)$ situation shows the probability of a situation that a train leaves the first station and it stops in the block while the second station is completed busy by two trains. Therefore, to solve and improve the proposed model variables and to reduce the blocking state in the second station, we investigate the probable state of the system of the two node $\pi(1b2)$ and $\pi(2b2)$.

Equations related to the system probable situation equations ($\pi(\alpha, \beta)$) are as follow. It is noted that equations (2) - (12) are equilibrium equation associated with one node in figure 3.

\[
\begin{align*}
\pi_{00} + \pi_{01} + \pi_{02} + \pi_{10} + \pi_{11} + \pi_{12} + \pi_{20} + \pi_{21} + \pi_{22} + \pi_{1b2} + \pi_{2b2} &= 1 \\
(\lambda + \lambda \cdot )\pi_{00} &= \mu_1 \cdot \pi_{01} \\
(\lambda + \mu_2 + \lambda) \pi_{01} &= \mu_1 \cdot \pi_{10} + \mu_2 \cdot \pi_{02} + \lambda \cdot \pi_{00} \\
(\lambda + \mu_2) \pi_{02} &= \mu_1 \cdot \pi_{11} + \mu_2 \cdot \pi_{1b2} + \lambda \cdot \pi_{01} \\
(\lambda + \mu_2) \cdot \pi_{1b2} &= \mu_1 \cdot \pi_{12} \\
(\lambda + \lambda + \mu_4) \cdot \pi_{10} &= \lambda \cdot \pi_{00} + \mu_2 \cdot \pi_{11} \\
(\lambda + \lambda + \lambda + \mu_4) \cdot \pi_{11} &= \mu_1 \cdot \pi_{20} + \lambda \cdot \pi_{01} + \mu_2 \cdot \pi_{12} + \lambda \cdot \pi_{10} \\
(\lambda + \mu_1 + \mu_2) \cdot \pi_{12} &= \mu_1 \cdot \pi_{21} + \lambda \cdot \pi_{02} + \mu_2 \cdot \pi_{2b2} + \lambda \cdot \pi_{11} \\
\mu_2 \cdot \pi_{2b2} &= \mu_1 \cdot \pi_{22} + \lambda \cdot \pi_{1b2} \\
(\lambda + \mu_1 + \lambda) \cdot \pi_{20} &= \lambda \cdot \pi_{10} + \mu_2 \cdot \pi_{21} \\
(\mu_1 + \mu_2 + \lambda) \cdot \pi_{21} &= \mu_2 \cdot \pi_{22} + \lambda \cdot \pi_{11} + \lambda \cdot \pi_{20} \\
(\mu_1 + \mu_2) \cdot \pi_{22} &= \lambda \cdot \pi_{12} + \lambda \cdot \pi_{21} \\
L &= (0) \cdot \pi_{00} + (1) \cdot (\pi_{10} + \pi_{01}) + (2) \cdot (\pi_{11} + \pi_{20} + \pi_{02}) + (3) \cdot (\pi_{21} + \pi_{12} + \pi_{1b2}) + (4) \cdot (\pi_{22} + \pi_{2b2}) \\
Lq &= (0) \cdot \pi_{00} + \pi_{10} + \pi_{01} + \pi_{11} + \pi_{20} + \pi_{02} + \pi_{21} + \pi_{12} + \pi_{22} + (1) \cdot \pi_{1b2} + (2) \cdot \pi_{2b2} \\
0.33 \leq \lambda \leq \mu_4 \\
0 \leq \pi \leq 1 \\
L, Lq \geq 0 \\
\end{align*}
\]

In the above constraints, constraint (13) computes total average number of trains in the rail-way system. Constraint (14) shows the total average number of trains in the queue of the system. Constraint (15) indicates the allowed interval for each variable according to the queuing theory and assumptions.

The second one is constraints associated with trains’ departure scheduling which are defined as follow:

Constraints (16) and (17) show the time interval constraint of the crossing of two trains running in opposite directions for entering the block. These two constraints guarantee that the crossing of OD and DO trains running in the opposite directions or them passing from each other can only happen at stations due to the line being single-track:

\[
toda_2 \leq tdoa_2 + M \cdot (1 - X) - Hp \\
toda_2 \leq tdoa_1 + M \cdot X - Hp \\
\]

Constraints (18) and (19) indicate the time interval constraint for departing two successive OD and DO trains running in the same directions from one station to the next. These two constraints guarantee that the departure of the successive OD and DO trains from one station to the next will occur at certain time intervals for strict safety reasons:

\[
toda_2 \leq tdoa_1 + M \cdot (1 - Y) - Hs_1 \\
toda_1 \leq tdoa_2 + M \cdot Y - Hs_2 \\
\]
Constraints (20), (21), (22) and (23) illustrate the constraint of maximum and minimum time of allowed stops (planned) of OD and DO trains at the first and second stations. These four constraints guarantee that the time taken by trains’ stops and receiving a service at the first and second stations will not exceed a certain amount:

\[ st_1 \leq todl_1 - tdoa_1 \leq STOP_1 \]  

\[ st_2 \leq todl_2 - tdoa_2 \leq STOP_2 \]  

\[ st_1 \leq tdo_1 - tdoa_1 \leq STOP_1 \]  

\[ st_2 \leq tdo_2 - tdoa_2 \leq STOP_2 \]  

Constraints (24) and (25) explain the constraint of obtaining the time taken by the OD train for reaching the second station from the moment of its entrance at the first station

\[ todl_1 = tdoa_1 + (1/ \mu_1) \]  

\[ tdoa_2 = todl_1 + tblock \]  

Constraints (26) and (27) explain the constraint of obtaining the time taken by the DO train for reaching the first station from the moment of its entrance at the second station:

\[ tdo_1 = tdoa_1 + (1/ \mu_1) \]  

\[ tdoa_2 = tdo_1 + tblock \]  

Constraint (28) show the constraint of obtaining the total time travel from the first station to the second station for OD trains,

\[ ttod = tdoa_2 + (1/ \mu_2) \]  

Constraint (29) demonstrates the constraint of obtaining the total time travel from the second station to the first station for DO trains

\[ ttdo = tdoa_1 + (1/ \mu_1) \]  

\[ ttod, ttdo, todl_k, tdoa_k, tdo_1, tdo_2, tdoa_k \geq 0, X, Y \in \{0,1\} \]  

Constraint (30) show the constraint of total average income obtained from each OD and DO trains travel from one station to the next (for \( \lambda = 3 \)):

\[ Incomeod + Incomedo = cte \]  

4-Numerical calculations and model validation

According to the fact that the performance of the proposed method for solving the model depends on the problem dimensions, we intend to conduct the numerical calculations in this section to validate the model and identify its interests. For this, we first solved the model using a small scale studied case with details by GAMS optimization software, and we will compare the obtained practical results with the studied case situation before being solved and will analyze the sensitivity in the following sections.

4-1- Case study

We have aimed to apply our numerical example on two successive stations located in a passenger and freight path in the Iran railway network with heavy traffic to validate the proposed mathematical model. The length of the railway line in Iran was 13348 Kilometers by the end of 2016 that 10459 Kilometers of which is the main track. Currently, Railway Company owns 910 locomotives, 23021 freight cars, and 2310 passenger cars which are working throughout the country transporting passengers and cargos. The total amount of cargos loaded by Iranian Railway Company was 36 million tons in 2016 and the passenger fleet of the railway company transported nearly 26 million passengers all over the country. Based on the Iran’s 20-year vision plan and until the 2025 horizon, the inter-city railway passenger and cargo transportation contribution should be raised from 4 to 18 percent and from 10 to 30 percent of the whole national transportation, respectively. In order to reach
these goals, improving the trains movement schedule and raising the freight and passenger transportation contribution in the shortest possible time using resources and existing infrastructures (using the maximum stations tracks) has always been considered as one of the main goals of the organization.

Thus, in order to reduce the transportation costs and according to the resources and infrastructures present in the country, we are after minimizing the time of delays caused in the train departure schedule in a selected line with heavy traffic. Since the blocking at stations and delays in the freight trains movement is more than passenger trains, the inter-city railway network between Mirjaveh (origin) and Bafgh (destination) in the city of Kerman consisting of a two-way single-track rail with mixed traffic (the path for passenger and freight trains movements) is chosen as the studied case in this paper which is highlighted with a red rectangle in the Iran’s map in figure 4.

Fig. 4. The inter-city railway network Mirjaveh-Bafgh in the Kerman region

The freight train movement is more than passenger train in this line. The schematic figure 5 shows the two chosen crowded stations from the Mirjaveh-Bafgh line. Based on the greater movement of freight trains relative to passenger trains in this line, two stations of Rayn-Hossein Abad in which the delays and train blocking probability is high are chosen.

Fig. 5. Schematic of two chosen crowded stations from the Mirjaveh-Bafgh line in the Kerman region
It should be noted that various factors causing delays and procrastination are identified at stations in the Mirjave(origin)-Bafgh(destination) line that four effective factors of which are presented in table 1.

Table 1. Four effective factors causing delays in the Mirjaveh-Bafgh line in the Kerman region in a certain time interval

<table>
<thead>
<tr>
<th>Row</th>
<th>Detailed delay</th>
<th>Count</th>
<th>Time(min)</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Prevent the arrival of trains to the station because of blocking event</td>
<td>8121</td>
<td>679648</td>
<td>33%</td>
</tr>
<tr>
<td>2</td>
<td>The impossibility of overtaking trains from each other inside the block</td>
<td>7756</td>
<td>421234</td>
<td>23%</td>
</tr>
<tr>
<td>3</td>
<td>Refueling of trains Passag-stops due to the priority of the passenger train to freight</td>
<td>1084</td>
<td>59478</td>
<td>9%</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>270</td>
<td>270250</td>
<td>15%</td>
</tr>
</tbody>
</table>

Table 1 shows that delays of OD and DO trains movements between stations on the Mirjaveh-Bafgh line are higher than allowed value because of two factors of trains blocking at stations and single-track line which prevents trains from overtaking each other in the selected line. For instance, the average of delays due to the blocking relative to the total planned time of an OD and DO trains movement between two stations of Rayn-Hossein Abad located on the Mirjaveh-Bafgh line in a six-month statistical interval in 2017 is as described in table 2.

Table 2. The average situation of delays of an OD and DO trains movement between two stations of Rayn and Hossein Abad located on the Mirjaveh-Bafgh line

<table>
<thead>
<tr>
<th>Month</th>
<th>Planned $ttod$ (hour)</th>
<th>Planned $ttdo$ (hour)</th>
<th>Actual $ttod$ (hour)</th>
<th>Actual $ttdo$ (hour)</th>
<th>OD train delay (hour)</th>
<th>DO train delay (hour)</th>
</tr>
</thead>
<tbody>
<tr>
<td>first</td>
<td>1.4</td>
<td>1.5</td>
<td>2.78</td>
<td>3.4</td>
<td>1.38</td>
<td>1.9</td>
</tr>
<tr>
<td>second</td>
<td>1.4</td>
<td>1.5</td>
<td>3.2</td>
<td>2.84</td>
<td>1.8</td>
<td>1.34</td>
</tr>
<tr>
<td>third</td>
<td>1.4</td>
<td>1.5</td>
<td>3.12</td>
<td>3.47</td>
<td>1.72</td>
<td>1.97</td>
</tr>
<tr>
<td>forth</td>
<td>1.4</td>
<td>1.5</td>
<td>2.5</td>
<td>3.92</td>
<td>1.1</td>
<td>2.42</td>
</tr>
<tr>
<td>fifth</td>
<td>1.4</td>
<td>1.5</td>
<td>2</td>
<td>2.6</td>
<td>0.6</td>
<td>1.1</td>
</tr>
<tr>
<td>sixth</td>
<td>1.4</td>
<td>1.5</td>
<td>1.9</td>
<td>2.9</td>
<td>0.5</td>
<td>1.4</td>
</tr>
</tbody>
</table>

As can be seen in Table 2, there is a great difference between planned $ttod$ and $ttdo$ values and values of actual $ttod$ and $ttdo$. In other words, delays of OD and DO trains movements between two stations due to the trains blocking, especially freight trains, have caused several hours of delays in trains running on Mirjaveh-Bafgh line. We are now going to use the proposed model to figure out in what condition trains blocking can be minimized at presumed stations and the time of $ttod$ and $ttdo$ can be improved. Based on the literature review, stated in the preceding section, the distribution of trains to a station is assumed to be exponential, and this assumption fits in exactly the terms of the modeling conditions of the Markova queue networks. The evidence and statistics on a rail network in Iran show that a train enters a station at a distance of 20 minutes. Therefore, before we solve the proposed model, we consider the variable $\lambda = 3$, in order to obtain the initial basis of the average hourly incomes from each OD and DO train movement.

In normal condition and by collecting real data in six months in 2017 from Rayn and Hossein Abad stations with $\lambda = 3$, it was found out that the average hourly incomes from each OD and DO
train movement are 150,000,000 and 140,000,000 Toman, respectively. Thus, the amount of constraint (31) in the proposed model is:

\[
\text{Income}_{od} + \text{Income}_{do} = 290,000,000\text{Toman}
\]

According to the collected data, the average daily income gained from OD and DO trains movements between Rayn and Hossein Abad stations is equal to 10,000,000,000 Toman upon which we have based our comparison of the results obtained by solving the studied case.

We have formulated the objective function in the proposed model in a way that it would be able to maximize the profit gained from OD and DO trains movement between one station and the next in a one-day period to meet the demand and reach the best train departure schedule where the minimum number of blocking occurs between Rayn and Hossein Abad stations.

4-2- Comparison of the case study before and after being solved by the proposed model

In this section, we will analyze and compare the OD and DO trains departure scheduling of the studied case before and after being solved by the proposed model. In the best case and according to $t_{tod} = 1.617$ and $t_{tdo} = 1.787$ obtained from solving the fourth problem, and current delays of OD and DO trains movement between two Rayn and Hossein Abad stations presented in table 2, the comparison of the delays associated with OD and DO trains movements between Rayn and Hossein Abad stations before and after solving the model is as presented in table 3:

<table>
<thead>
<tr>
<th>Table 3. The comparison of delays of OD and DO trains movements between Rayn and Hossein Abad stations (Mirjaveh-Bafgh line) before and after solving the model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Month</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>first</td>
</tr>
<tr>
<td>second</td>
</tr>
<tr>
<td>third</td>
</tr>
<tr>
<td>forth</td>
</tr>
<tr>
<td>fifth</td>
</tr>
<tr>
<td>sixth</td>
</tr>
</tbody>
</table>

As table 3 demonstrates, we could considerably reduce the delays of the OD and DO trains movements running between Rayn and Hossein Abad stations relative to the current situation by solving the proposed scheduling model. Our aim is to propose an optimal state for the presented model in the next section by analyzing the main parameters sensitivity of the studied case.

4-3- Sensitivity analysis of the studied case

According to the obtained results and examinations, by solving the problems in section 4 and the description of each case presented in this section, only the second, third and fourth problems will be opted at most, to go under sensitivity analysis and examination. We have aimed to study two important parameters, the first one is the change of the speed variable (the variations of the variable $\lambda$ which is dependent on the speed parameter of trains when entering the station) and the second one is the change of parameter $\mu_k$:
• **Analysis of the speed parameter** \((V_{ik} \text{ and } V_{jk})\)

Table 4 shows that because of low probability of blocking in the fourth problem, the average number of trains in the queue is lower in the fourth problem \((L_q = 0.475)\) in the long run, in other words, trains experience the least delay at stations and blocks in the fourth problem in the long run. As we know the rate of trains entrance to each station \((\lambda)\) is a function of passenger and freight trains speed parameter in the railway network which is very vital. So we want to perform the sensitivity analysis on the speed parameter in the second, third and fourth problems. The reduction in the speed parameter is not advised because it increases \(ttod\) and \(ttdo\) times and causes the profit indicator to reduce. Now we increase the speed of all freight and passenger trains by 10 and 20 percent, respectively, so we could observe the results obtained by solving the problems and also how far the speed parameter increase can affect the blocking reduction and the profit indicator. The obtained results are presented in tables 4 and 5.

<table>
<thead>
<tr>
<th>Problem</th>
<th>(\lambda)</th>
<th>(ttod)</th>
<th>(ttdo)</th>
<th>(\pi_{1b2})</th>
<th>(\pi_{2b2})</th>
<th>(L)</th>
<th>(L_q)</th>
<th>Max (Z) (Toman)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2.98</td>
<td>2.95</td>
<td>3.12</td>
<td>0.029</td>
<td>0.394</td>
<td>3.628</td>
<td>0.817</td>
<td>1000000000</td>
</tr>
<tr>
<td>3</td>
<td>2.915</td>
<td>1.95</td>
<td>2.12</td>
<td>0.064</td>
<td>0.28</td>
<td>3.198</td>
<td>0.625</td>
<td>1000000000</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>1.617</td>
<td>1.787</td>
<td>0.078</td>
<td>0.198</td>
<td>2.811</td>
<td>0.475</td>
<td>9097320000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem</th>
<th>(\lambda)</th>
<th>(ttod)</th>
<th>(ttdo)</th>
<th>(\pi_{1b2})</th>
<th>(\pi_{2b2})</th>
<th>(L)</th>
<th>(L_q)</th>
<th>Max (Z) (Toman)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>2.95</td>
<td>3.12</td>
<td>0.029</td>
<td>0.395</td>
<td>3.631</td>
<td>0.819</td>
<td>92944800000</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>1.95</td>
<td>2.12</td>
<td>0.063</td>
<td>0.286</td>
<td>3.222</td>
<td>0.635</td>
<td>95320800000</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>1.617</td>
<td>1.787</td>
<td>0.078</td>
<td>0.198</td>
<td>2.811</td>
<td>0.475</td>
<td>82765200000</td>
</tr>
</tbody>
</table>

According to tables 4 and 5, the speed parameter depends directly upon the variable \(\lambda\) as we expected or in other words, more trains can enter a station per hour if the speed parameter is increased. The blocking probabilities in the second and third problems become more intense when the trains’ speeds are increased by 10 units and values obtained from \(\pi_{2b2}\) in table 6 are also increased and there is no change in \(ttod\) and \(ttdo\) times, so the total profit and the profit indicator will not change. This is why we observed that in the second and third problems, before increasing the speed of trains, \(X\) was calculated in a way that the blocking probability would achieve a minimum possible value.

Now if we increase the trains’ speeds by 20 units, the \(\lambda\) value will reach the maximum possible value which is 3 as presented in table 5. The important point is that by increasing the speed parameter, the variable \(\lambda\) increases proportional to the speed parameter, but because of the constraint \(\lambda \leq 3\), increasing the speed will simply cause the increase of the total costs of delays associated with the OD and DO trains entrance to stations (per a day) and the increase of costs related to the train and railway depreciation. Thus, the situation becomes worse and we face the total cost decrease or the profit indicator decrease which are correctly shown by the model. Figure 6 shows the diagram of the range of the total profit variation for different speeds.
Fig. 6. Speed-profit diagram (the sensitivity analysis of the speed parameter)

So it can be suggested that the speed should be chosen according to the variation range of $\lambda$ in the selected case of the railway transportation industry, otherwise it causes the profit loss in the long run.

- **Analyzing the service providing rate parameter $\mu_k$**

As mentioned in section 4, the best trains departure scheduling is obtained in the fourth problem where the obtained $\lambda$ is equal to the assumed $\mu_k$. Therefore, we want to examine the $\mu_k$ variations in the fourth problem to see if the rise and fall in $\mu_k$ can lead to the enhancement of the fourth problem model. The obtained results are summarized in table 6.

**Table 6. The results obtained by solving the fourth problem (The sensitivity analysis of the service providing rate)**

<table>
<thead>
<tr>
<th>Problem</th>
<th>$\mu_k$</th>
<th>$\lambda$</th>
<th>$ttod$</th>
<th>$ttdo$</th>
<th>$\pi_{1b_2}$</th>
<th>$\pi_{2b_2}$</th>
<th>$Max \ Z$ (Toman)</th>
<th>$Z_{indicator}$ (Toman)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>1.95</td>
<td>2.12</td>
<td>0.063</td>
<td>0.286</td>
<td>8158920000</td>
</tr>
<tr>
<td>4</td>
<td>2.5</td>
<td>2.5</td>
<td>3</td>
<td>1.75</td>
<td>1.92</td>
<td>0.073</td>
<td>0.239</td>
<td>8233320000</td>
</tr>
<tr>
<td>4 (main)</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>1.617</td>
<td>1.787</td>
<td>0.078</td>
<td>0.198</td>
<td>9918120000</td>
</tr>
<tr>
<td>4</td>
<td>3.5</td>
<td>3.5</td>
<td>3</td>
<td>1.521</td>
<td>1.691</td>
<td>0.079</td>
<td>0.164</td>
<td>8301891428</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>6</td>
<td>3</td>
<td>1.283</td>
<td>1.453</td>
<td>0.061</td>
<td><strong>0.064</strong></td>
<td>8317320000</td>
</tr>
</tbody>
</table>

The service-providing rate is different in the fourth problem according to the results presented in table 6. As $\mu_k$ is decreased from 3 to 2, the values of $ttod$ and $ttdo$ increase and the profit indicator reduces. In other words, since we have reduced the number of trains able to receive a service at the station per hour (the speed of providing service), the total time of OD and DO trains movements is increased and the profit indicator reduces by the rise of the blocking probability at stations. Now contrariwise, the values of $ttod$ and $ttdo$ reduce because the service providing speed is increased when the $\mu_k$ value is increased from 3 to 6. So the speed of providing service increases and the values of $ttod$ and $ttdo$ reduce. Finally, profit indicator reduces instead of increasing when we slightly increase the $\mu_k$ value, because $\pi_{2b_2}$ value is still great and the blocking has not been vanished yet. But if the $\mu_k$ value is increased up to a point that enables the blocked station to provide services to the dwelling trains at its maximum speed, as it can be seen in table 8, the best condition and the maximum profit is obtained for the maximum service providing rate ($\mu_k = 6$) (when trains are provided with services within 10 minutes in a station) in the long run. Thus, the global solution obtained in this case, is suggested as the best solution for eliminating trains blocking at Rayn and Hossein Abad stations in which the profit indicator $Z_{indicator} = 3039956140$ is improved, and by
eliminating trains blocking, time delays resulted from OD and DO trains movements between these two stations tend to zero.

5- Conclusion
In this paper, a mixed-integer nonlinear programming model is proposed in order to plan trains schedule. So stations productivity capacity will increase using queuing theory approach and considering blocking to increase the railway capacity at stations of the inter-city railway network, the adequate distribution of passenger and freight trains, reduce trains stops at two successive crowded stations and the delays caused in trains’ movements along the line due to trains blocking at stations.

The results obtained by solving the problems show that, we have achieved the best scheduling with lowest delays for movement of OD and DO trains between two successive crowded stations in the studied case by the aid of the proposed model, considering the constraints of track number inside stations (dimensions of the presented problems), where the solution of problems yielded an optimal global result in a reasonable time.

Finally, the practicality of the proposed model was evaluated using a sensitivity analysis. The obtained results showed that in order to increase the profit gained from movement of OD and DO trains between two successive crowded stations considering the number of tracks at stations in the studied case, we were able to decrease the total OD and DO trains travel time and increase the profit indicator gained from OD and DO trains movement per hour by eliminating the blocking.

In the future works, a more rigorous analysis can be performed on the other factors affecting crowded stations which play as a bottleneck in the whole inter-city railway network system by developing the proposed model in larger dimensions and then identify and rank the critical points and side effective factors. The proposed queuing model pattern, on the other hand, can be implemented and simulated in other locations where the blocking occurs.

References


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Yang, L., Qi, J., Li, S., & Gao, Y. (2016). Collaborative optimization for train scheduling and train stop planning on high-speed railways. Omega, Vol. 64, pp. 57-76.

