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Impact of government's policies on competition of two closed-loop and regular supply chains

Ashkan Hafezalkotob[1*], Tina Hadi[2]

[1] *Industrial Engineering College, Islamic Azad University, South Tehran Branch, Tehran, Iran.*

[2]*Knowledge Engineering and Decision Sciences Department, KharazmiUniversity, Tehran, Iran.*

a_hafez@azad.ac.ir, tina.hadi66@gmail.com

Abstract

With progressing technologies and new features of production, new products compete with older ones in markets. Also new products initiate contest with olden ones and this process repeats in different productions life time for several times. In this situation recycling the olden products seems to be significant for supply chains. Governments often levy special tariffs for these products as a control tool which aims to incentive production recovery. In the real world, government purposes financial incentive plans for recoverable productions, and also punitive plans for unrecoverable products. This paper tries to model the competition of a closed-loop supply chain and an ordinary supply chain using a game theory approach. In next step, the effects of persuasive and punitive governmental plans are modeled. Optimal retail and wholesale prices of the products are found in two chains. Numerical examples including sensitivity analysis of some key parameters will compare the results between different models of this study.

Keywords: Game theory; competition; closed-loop supply chain; government intervention.

1- Introduction

Nowadays, competition is evolved among supply chains (SCs). For example, Google and Samsung both have constituted a supply chain that competes with the existing supply chain consisting of Microsoft and Nokia. The decisions of members in one SC affect the decisions of members in its rival supply chain. In this paper the competition between two supply chains under government financial intervention (environmental protection, revenue seeking on and revenue seeking policies) is studied.

Green supply chain management (GSCM) criteria have been used to control flow of information and material. Researchers have applied GSCM criteria to expand a green strategy. (Vachon and Klassen, 2008). In a GSCM, manufacturer and customers work together to reduce the environmental effects of processes. That is, the supplier and the manufacturer both set some goals in order to limit the environmental effect with the help of GSCM approach. (Tseng et al., 2009; Bowen et al., 2001).Hjaila et al. (2015) have studied a scenario that is proposed for the optimization of coordinated decentralized multi-site multi-product supply chains in a competitive environment with a Stackelberg game model.Tian et al. (2013) have investigated the green supply chain management process' to simulate a case study of Chinese automotive manufacturing industry. The relationships of stakeholders such as government and consumers are analyzed through evolutionary game theory.

*Corresponding author.

Due to global environmental concerns, reverse supply chain becomes extremely important. Many firms design products that can be reused. Furthermore, Japanese and American governments use green legislation and financial instruments, to extent the product life cycles and promote recycling the products (Robeson, 1992; Fleischmann et al., 2002). In this paper the competition between two supply chains under government financial intervention (environmental protection, revenue seeking on and revenue seeking policies) is investigated.

The remainder of this paper is organized as follows. A review is provided in Section 2 while the notations and assumptions underlying these models are discussed in Section 3. Section 3 also presents model formulation. Section 4 provides some numerical results. Finally, the paper concludes in Section 5 with some suggestions for future works in this field.

2- Literature Review

This paper is closely related to closed-loop supply chains, green supply chain management, government intervention and game theory. Therefore, let us first review the literature on closed-loop supply chains. Qiaolun et al. (2007) have introduced a closed-loop supply chain with a supplier, retailer, and recycler. In the reverse supply chain, the recycler collects products with different scenarios, so that the rate of return for the used products is related to the consumption of customer. In a forward supply chain the optimal wholesale and retail prices are determined. Savaskan and Van Wassenhove (2006) have studied a closed-loop supply chain consisting one supplier and two retailers in order to study different scenarios of collection and recycling. Wu (2012) has studied the competition between closed-loop and regular supply chain. This article investigates the competition under two factors of duty and price. Those supply chains employ one retailer to present their products to a market. Wie and Zhao (2011) have considered two retailers and one supplier on a closed-loop supply chain. Retailers compete with each other under ambiguity of demands. Wei and Zhao (2011) have concentrated on fuzzy closed-loop supply chain with competition of retailers. The cost of collection, the demand of costumers and the cost of rework are considered as fuzzy numbers. They have determined the optimal price of wholesale price, the optimal retail prices and the rate of rework in centralized and decentralized scenarios.

There are a few papers on green supply chain management related to this paper. Green supply chain management involves customization and coordination of the flow of information and materials between the manufacturers and the customers. Vachon and Klassen (2008) have studied environmental collaboration within organizational interactions of supply chain members. The obtained results have shown that the upstream practices are related to the process-based performance, whereas the downstream collaboration is associated to the product-based performance. Zhu et al. (2008) found that Chinese firms tend having greater environmental awareness than the other firms because they are under pressure of green regulations of European countries. Zamarripa et al. (2013) have improved the tactical decision-making of a supply chain under an uncertain competition scenario. They evaluated different optimization criteria with the help of game theory.

Chou and Chang (2008) have introduced several green supply chain management strategies and their results suggest that the decision-maker incorporates the risks of individual suppliers into the final decision making process for selection of alternative suppliers. Tseng et al. (2009) have studied selection of the manufacturers with an analytical network process. Chan and Kumar (2007) have concentrated on several important criteria of manufacturer to expand the system for selection of the manufacturer.

The consequence of global pollution, environmental concern is growing that the key members of supply chain and reverse supply chain should be managed to maximize the profit and manage the product life cycle. Robeson (1992) has studied the governmental financial tools. Green legislation was used in Europe, Japan, and North America to promote the awareness of extended producer responsibility. Advanced recycling fees (ARFs) and government subsidies are important to encourage manufacturer to recycle their products. Shinkuma (2003) has considered the relation between ARF and unit price of product. Sheu and Chen (2012) have studied the effect of financial intervention of government on reverse supply chains' competition. The costs of financial intervention of government are compared with the cost of non-intervention of government. The results show that the government should adopt green taxation and subsidization to ensure that the green profit attributed to the green production was non-negative. Hong et al. (2014) have considered a model consisting of the firms and recyclers as the followers and an environmental protection agency as a leader. The firms and recyclers tried to maximize their profit functions. The fees subsidizing recyclers are paid to maximize the social welfare in a closed-loop supply chain.

Several studies on the game theory investigate the influence of government financial instrument related to this paper. Governments often use motivation and penalties to exert positive and negative effects, respectively (Sheu, 2011; Sheu and Chen, 2012). Murphy (2000) has analyzed the government's policies on achievement of green supply chain management from different points of view. In a similar manner, Tsireme et al. (2012) have investigated some cases of the environmental legislations and their role in managers' determination to adopt GSCM actions.

Many scientists have conducted research on the game theory to study the impact of government financial involvements on green supply chain. Jin and Mei (2012) have studied a game model between government and suppliers in the green supply chain. Zhou and Zhang (2007) have focused on the game theoretical model to recognize the interaction between a green supply chain and government. Sheu (2012) has investigated the impact of government financial intervention on competition among green supply chains. Sheu (2011) has studied the game theoretical model between producers and reverse-logistics suppliers' under financial interventions of government using a Nash bargaining model. Zhang and Liu (2013) have analyzed coordination mechanism in a three level green supply chain when government influences the bargaining power of its members to increase the operation of green supply chain.Yu(2012) established a theoretical game model for competition of fashion supply chains in the case of differentiated products with the inclusion of environmental concerns. Hafezalkotob (2015) and Hafezalkotob et al. (2015) considered competition of one regular supply chain and one green supply chain under intervention of the government.

This paper is also closely related to that of Sheu and Chen (2012) for investigation of the impact of government financial intervention. They have considered the competition among the green supply chains. In this paper, the effect of government financial intervention is evaluated on the competition among closed loop and regular supply chains under two scenarios. In the first scenario, the closed-loop supply chain consists of one supplier, one retailer, and one recycler. In the second scenario, the closed-loop supply chain consists of one supplier and one recycler-retailer as a one firm. Government takes two strategies: 1.environmental protection and revenue seeking 2. revenue seeking policies of government.

3- Notation and Problem Formulation

The competition between two supply chains, namely regular and closed loop, is investigated in this study under two different scenarios. In the first scenario, the regular supply chain includes a supplier and a retailer, while the closed loop one involves a supplier, a retailer and a recycler. The supplier sells his product with wholesale price to the retailer and the retailer also delivers the same product with the retail price to customers. In return, the third part adopts to collection of used products and sends them to the supplier for recycling. In the second scenario the competition is between a regular supply chain (i.e. supplier and retailer) and a closed loop supply chain (i.e. supplier and retailer which acts also as the collector). The supplier sells his product with the whole sale price, while the retailer delivers the same product with the retail price. In return, the retailer who also plays the collection role collects the products and sends them to the supplier for recycling. Meanwhile, the government sets tariffs to control the market and protect the environment. In real world, the government adopts its financial policies by defining subsidies or taxes.

The tariff parameter is considered to be variable and the problem is solved for both of these scenarios in terms of two policies which will be adopted by the government. The policies which are adopted by the government are listed below:

1. Revenue seeking policy:
   $Max\ GNR\ (T_r, T_c\ )$ (1)
   The government aims to maximize its income by setting this policy.
2. Environmental protection and revenue seeking policy:

$Max\ U\ (T_r, T_c)$=$GNR\ (T_r, T_c)$-$\lambda EIS\ (T_r, T_c)$ (2)

Where,

$EIS\ (T_r, T_c\ )= \theta_1 D_1 + \theta_2 D_2$

By following this policy, the government intends to increase its net income in addition to mitigate the adverse environmental effects. λ is indicative of the government's approach toward environmental effects. A stackelberg model is utilized in this section to solve the problem and to find optimal values of the governmental tariffs. Then, both of the scenarios will be solved in terms of two governmental policies taking into account stackelberg model (game theory) and using a backward approach.

This section introduces the notation and formulation used in competition of two supply chains under two proposed scenarios. All variables and assumptions underlying the proposed models will be stated.

3-1- Notation

Parameters:

a_i the potential size of market for product i , $i = 1, 2$;

b the fixed cost parameter of collection;

$c_{m_i}$the production cost of supply chain *i* per unit product, $i = 1, 2$;

$c_{r_2}$the cost of reproduction of supply chain 2 per unit product;

$\varphi_i$the fixed parameter of tax ratio, ($0 \leq \varphi_i \leq 1$) , *i*=1,2;

d the substitution rate of two products (0$\leq d \leq 1$);

$p_{b_2}$the retail price of a unit product from recycler to supplier in closed loop supply chain;

$p_{c_2}$the collection cost per unit product by recycler in closed loop supply chain ($p_{b_2} \geq p_{c_2}$);

λ_i the environmental risk aversion coefficient of government, $\lambda_i \geq 0$. The larger the coefficient λ_i, the higher the environmental protection tendency of government will be;

θ_r environmental impacts of regular products;

θ_g environmental impacts of closed-loop products;

$D_i$the product demand for supply chain $i, i = 1, 2$

Decision variables:

$p_i$the retail price of product of supply chain i , $i = 1, 2$;

$\tau_2$the rate of collection of used product by recycler of supply chain 2;

$T_i$the tariff imposed by government for supply chain i , *i*=1,2;

$w_i$the wholesale prices of product manufactured by supply chain *i*, *i*=1,2.

Decision functions:

$\Pi_{R_i}$the profit of retailer in supply chain i , $i = 1, 2$;

$\Pi_{t_i}$the profit of recycler in supply chain *i*, $i = 1, 2$;

$\Pi_{M_i}$the profit of supplier in supply chain *i*, $i = 1, 2$;

GNR the government's net revenue obtained from tax and subsidy on supply chains' products;

EIS the total environmental impacts of supply chains' products;

U the government's net revenue obtained from tax and subsidy on supply chains' products and the total environmental impacts of supply chains' products.

3-2- Assumptions

The proposed models are established upon the following assumptions:

1. It is assumed that the consumer demand functions for retailer i, i = 1, 2 are a linear function of the two retailers' prices. The linear demand functions can be written as D_i=a_i- $p_i + dp_j; (i, j = 1,2; i \neq j)$ (Wei and Zhao, 2011).

2. The amount of the supply is the same as demand.

3. The products of two suppliers are differentiated and each supplier sells products to customers through his retailers.

4. All companies have common knowledge about substitution rates, demand density, taxes, and other parameter values.

5. Each member of supply chains has rational behavior and seek to maximize his profit.

6.The θ_r and θ_g represent environmental impacts of regular and green products, respectively. For generalization purpose of the model, the environmental impacts of products are not restricted to a specific factor. The environmental impacts are industry-dependent and they can embrace air, soil, or water pollutions. They may also be measured by various factors such as reversible index of products, reproduction or recycling index, annual exhausting CO_2 or NO_x of products, or even a combination of different factors (Hafezalkotob, 2015).

7. Based on the article from Wei and Zhao (2013), the total cost needed to collect the used product $c(\tau)$ is shown as c(τ)=bτ^2+$p_c\tau D_i$.

Where b.τ^2 is the constant part of the cost and $p_c.\tau.D_i$ is its variable part. τD_i is the total number of used products which are collected. The model is examined under two following scenarios with respect to the above mentioned explanations.

3-3- Scenario No1

Via this scenario the competition occurs between a regular and a closed loop supply chains. The closed loop supply chain includes a supplier, a retailer and a recycler, while the regular supply chain incorporates one retailer and one supplier. The government is placed at a higher level and tries to balance the market and meets his requirements by following the mentioned policies and by properly setting the tariffs T_1 and T_2. Fig.1 illustrates the game framework between two supply chains in the first scenario.

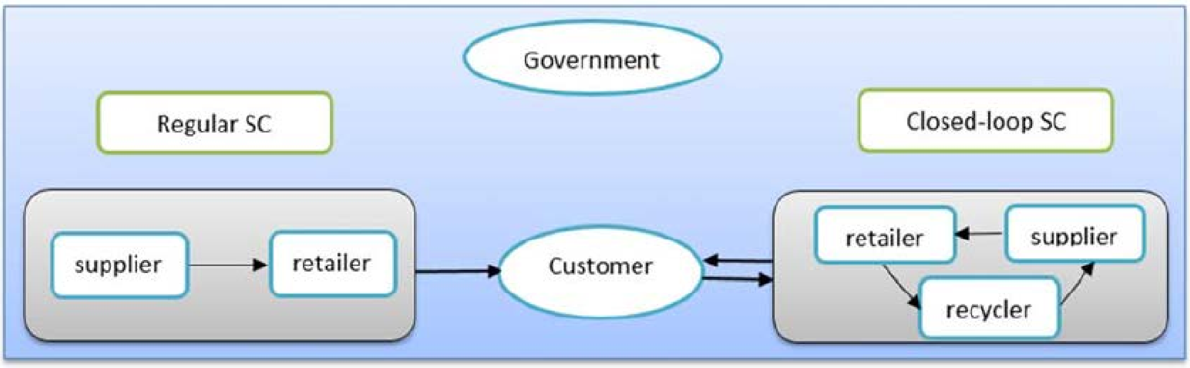

Figure 1. Competition between regular supply chain and closed-loop supply chain under government financial intervention in the first scenario

Using the following equation,

$c(\tau_I)=b.\tau^2{}_{+}p_{c}.\tau D_i$ (3)

The profit function of each member can be defined as below:

$\max_{p_1} \Pi_{R_1} = (p_i - w_i - \varphi_i T_i)D_i$ i=1,2. (4)

$\max_{w_1} \Pi_{S_1}=(w_1 - c_1\text{-}(1-\varphi_1)T_1)D_1.$ (5)

$\max_{W_2} \Pi_{S_2}=(w_2 - (1-\tau_2)c_{m_2} - \left(c_{r_2} + p_{b_2}\right)\tau_2 - (1-\varphi_2)T_2).D_2.$ (6)

$\max_{\tau_2} \Pi_{\tau_2} = \left(p_{b_2} - p_{c_2}\right)\tau_2 D_2 - b{\tau_2}^2.$ (7)

Solution of this problem is described below with respect to Stackelberg model:

Step (1): First of all, the government as the game's main leader, specifies appropriate T_1 and T_2 to maximize his profit function.

Step (2): Suppliers as the followers, define the optimal values of w_i^* that maximize their profit function, considering the tariff.

Step (3): Considering the wholesale price and the government's tariff, each retailer as the follower, determines the optimal value of p_i^*.

Step (4): The recycler acting as the game follower, indicates the return rate of collection of used product according to the wholesale price, retail price and tariff.

Now, the problem can be solved by backward approach. Proposition (1) can determine the optimum rate of collection of used product.

Proposition 1. *If the profit function of a recycler is a concave function, the return rate of the used products will be calculated from first derivative conditions of the recycler function as below:*

$\tau_2^*(p_1,p_2,T_1,T_2) = M\text{-}\mu P_2 + d\mu P_1,$ (8)

where $M = \frac{(p_{b_2}-p_{c_2})(a_2)}{2b}$, $\mu = \frac{(p_{b_2}-p_{c_2})}{2b}$.

Taking into account concavity of the profit function and the retail sell, Proposition 2 calculates the amount of the optimal retail sells.

Proposition 2. *If the profit function of a retailer is a concave function, the optimal retail price will be calculated from the first derivative conditions of the retailer functions as following:*

$p_1^*(w_1,w_2,T_1,T_2)=N_1 + f_2\varphi_2 T_2 + f_1\varphi_1 T_1 + f_1 w_1 + f_2 w_2$ (9)

$p_2^*(w_1,w_2,T_1,T_2)=N_2 + f_1\varphi_2 T_2 + f_2\varphi_1 T_1 + f_2 w_1 + f_1 w_2$ (10)

where, $N_i = \frac{2a_i + da_j}{4-d^2}(i,j=1,2)$,$f_1 = \frac{2}{4-d^2}$and $f_2 = \frac{d}{4-d^2}$.

By replacing equations (8) to (10) in the profit functions of the suppliers, the profit functions of the producer can be presented as below:

$\max_{w_1} \Pi_{S_1}=(w_1 - c_1(1-\varphi_1)T_1).(a_1 - N_1 + dN_2 + (df_2 - f_1)\varphi_1 T_1 + (df_1 - f_2)\varphi_2 T_2) +$
$(df_2 - f_1)w_1 + (df_1 - f_2)w_2)$ (11)

$\max_{\tau_2} \Pi_{\tau_2} ==(w_2 - \big(1 - M - \mu(N_1 + f_2\varphi_2 T_2 + f_1\varphi_1 T_1 + f_1 w_1 + f_2 w_2) - d\mu(N_2 + f_1\varphi_2 T_2 +$
$f_2\varphi_1 T_1 + f_2 w_1 + f_1 w_2)\big) - \left(c_{r_2} + p_{b_2}\right)\big(M - \mu(N_1 + f_2\varphi_2 T_2 + f_1\varphi_1 T_1 + f_1 w_1 + f_2 w_2) -$

$d\mu(N_2 + f_1\varphi_2 T_2 + f_2\varphi_1 T_1 + f_2 w_1 + f_1 w_2)\big) - (1-\varphi_2)T_2)((a_2 - N_2 + dN_1 + (df_1 - f_2)\varphi_1 T_1 + (df_2 - f_1)\varphi_2 T_2) + (df_1 - f_2)w_1 + (df_2 - f_1)w_2)$ (12)

If $\frac{(d^2-2)(2+(p_{b_2}-p_{c_2})(2-d^2)\left((4-d^2)^2 b^2-(c_{r_2}+p_{b_2})\right))}{b(4-d^2)} < 0$, then the profit function of the suppliers will be a concave function, with the optimal values of the wholesale price being obtained by establishing the first derivative conditions of equations (11) to (12) as w_1^* and w_2^*.

Proposition 3. *If* $\frac{(d^2-2)(2+(p_{b_2}-p_{c_2})(2-d^2)\left((4-d^2)^2 b^2-(c_{r_2}+p_{b_2})\right))}{b(4-d^2)} < 0$, *the optimal values of* w_1^* and w_2^* *are calculated as below.*

The linear wholesale price variables can be written as below. e_i , o_i and $g_i (i = 1,2)$

are calculated by a mathematical software.

$w_1^*(T_1, T_2) = e_1 + o_1 T_1 + g_1 T_2$ (13)

$w_2^*(T_1, T_2) = e_2 + o_2 T_1 + g_2 T_2$ (14)

By replacing the optimal values of w_i^*, p_i^* and (i=1,2) in the profit functions of the government, these functions are calculated as follows under two following policies which are chosen by the government:

a) Revenue seeking policy

In this case the government intends to maximize his profit.

Max GNR $(T_1, T_2) = T_1 D_1 + T_2 D_2 =$

$$(T_1)\left[\frac{2a_1 + da_2 + d\varphi_2 T_2 + \varphi_1(d^2-2)T_1 + (d^2-2)(e_1 + f_1 T_1 + g_1 T_2) + d(e_1 + f_2 T_1 + g_2 T_2)}{4-d^2}\right]$$
$$+(T_2)\left[\frac{(2a_2 + da_1 + d\varphi_1 T_1 + \varphi_2(d^2-2)T_2) + (d^2-2)(e_2 + f_2.T_1 + g_2.T_2) + d(e_1 + f_1.T_1 + g_1.T_2)}{4-d^2}\right]$$

(15)

If the equation (16) holds, the profit function of the government will be a concave function and the optimal values of T_1 and T_2 can be obtained from Proposition 4 .

$(2\varphi_1(d^2-2) + 2(d^2-2)f_1 + 2df_2)(2\varphi_2(d^2-2) + 2dg_1 + 2g_2(d^2-2) > (f_2(d^2-2) + 2\varphi_2 + (d^2-2)g_1 + dg_2 + d\varphi_1 + f_1 d)^2$ (16)

Proposition 4. *If equation (16) holds, the optimal values of* T_1 and T_2 *can be found by establishment of the first derivative conditions of equation (15).*

b) Environmental protection and revenue seeking policy

In this case, the government decides to maximize his profit and protect the environment. The profit function of the government is described as below:

Max $U(T_r, T_c) = GNR\ (T_r, T_c) - \lambda EIS(T_r, T_c)$ =

$$(T_1 - \lambda\theta_1)\left[\frac{2a_1 + da_2 + d\varphi_2 T_2 + \varphi_1(d^2-2)T_1 + (d^2-2)(e_1 + f_1 T_1 + g_1 T_2) + d(e_1 + f_2 T_1 + g_2 T_2)}{4-d^2}\right]$$
$$+(T_2 - \lambda\theta_2)\left[\frac{(2a_2 + da_1 + d\varphi_1 T_1 + \varphi_2(d^2-2)T_2) + (d^2-2)(e_2 + f_2.T_1 + g_2.T_2) + d(e_1 + f_1.T_1 + g_1.T_2)}{4-d^2}\right]$$ (17)

obtained by establishing the first derivative conditions of the optimal values of T_1 and T_2 from Proposition (5).

Proposition 5. *If equation (17) holds, the optimal values of* T_1 and T_2 *can be obtained by meeting the first derivative conditions.*

3.4. Problem Statement in Scenario 2

As discussed earlier, the competition takes place between a regular supply chain and a closed-loop one. In this case the closed-loop supply chain includes a supplier and a retailer who is the collector as well, but the regular supply chain involves a retailer and a supplier.

The government acts as the leader at a higher level, which determines the proper tariffs in accordance with his policies by selecting various policies (i.e. income-oriented and environment protection). Figure 2 illustrates the game framework between two supply chains in the second scenario.

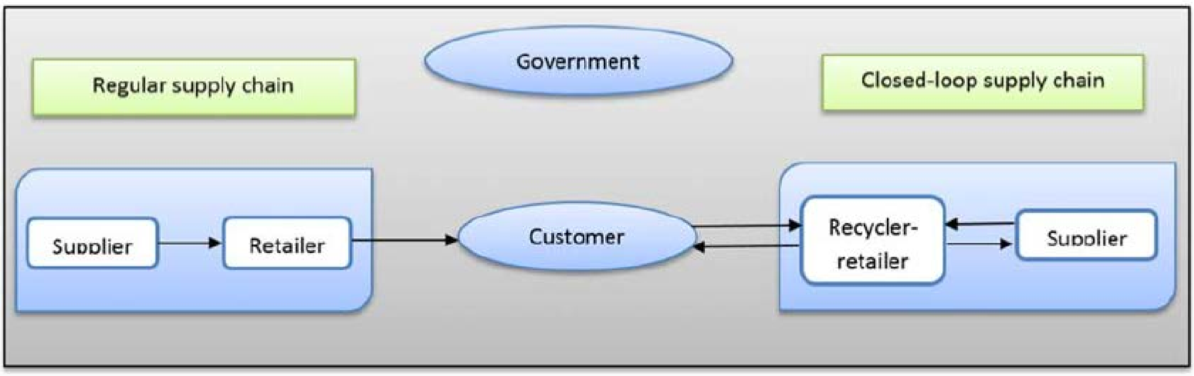

Figure 2. Competition between regular supply chain and closed-loop supply chain under government fanatical intervention in the second scenario

The objective function of the retailer in regular supply chain can be written as follows:

$$\max_{p_1} \Pi_{R_1} = (p_i - w_i - \varphi_i T_i)D_i \; i=1,2. \quad (18)$$

The objective function of supplier in regular supply chain can be written as below:

$$\max_{w_1} \Pi_{S_1}=(w_1 - c_1 - (1-\varphi_1)T_1)D_1. \quad (19)$$

Similar to Wie and Zhao (2013), the objective function of the recycler-retailer in the closed-loop supply chain can be formulated as follows:

$$\max_{RR_2} \Pi_{R_2,\tau_2}=(P_2 - w_2 - \varphi_2 T_2)D_2 + (p_{b_2} - p_{c_2})\tau_2 D_2 - b{\tau_2}^2. \quad (20)$$

The objective function of supplier in the closed-loop supply chain can be stated as:

$$\max_{w_2} \Pi_{S_2}=(w_2 - (1-\tau_2)c_{m_2} - (c_{r_2} + p_{b_2})\tau_2 - (1-\varphi_2)T_2)D_2. \quad (21)$$

This problem is solved with respect to Stackelberg model and the following steps can be followed to address the optimal values:

Step (1): First of all, the government as the main leader specifies the optimal tariffs to maximize his profit function according to the adopted policy.

Step (2): The suppliers determine the optimal values of $\mathrm{w_i}$ that maximize their profit function considering the defined tariffs.

Step (3): The retailer of regular supply chain and the retailer-recycler addresses the return rate of products as well as the retail price as the game follower.

For this purpose, a backward approach is utilized considering the optimal tariff rate and considering the defined response functions from suppliers, retailers and retailer-recycler. Noteworthy is considering the competition of the supply chain members outside the chain and also taking the government as the main leader.

Proposition 6. *If 4b>$\left(p_{b_2}-p_{c_2}\right)$ holds, then the optimal retail prices and the return rate collection of used products are calculated as below.*

$p_1^c(w_1,w_2,T_1,T_2)=k_1+l_1T_2+s_1T_1+\gamma_1w_1+v_1w_2$ (22)

$p_2^c(w_1,w_2,T_1,T_2)=k_2+l_2T_2+s_2T_1+\gamma_2w_1+v_2w_2$ (23)

$\tau_2^c(w_1,w_2,T_1,T_2)=k_3+l_3T_2+s_3T_1+\gamma_3w_1+v_3w_2$ (24)

where,

$k_1=\frac{\left(-(p_{b_2}-p_{c_2})^2+4b\right)(a_1+da_2)+2bda_2}{(p_{b_2}-p_{c_2})^2(d^2-2)-2b(d^2-4)}$ and $k_2=\frac{\left(-(p_{b_2}-p_{c_2})^2+2b\right)(2a_2+da_1)}{(p_{b_2}-p_{c_2})^2(d^2-2)-2b(d^2-4)}$ and

$k_3=\frac{(p_{b_2}-p_{c_2})(2a_2+da_1)}{(p_{b_2}-p_{c_2})^2(d^2-2)-2b(d^2-4)}$ and $\gamma_1=\frac{\left(-(p_{b_2}-p_{c_2})^2+4b\right)}{(p_{b_2}-p_{c_2})^2(d^2-2)-2b(d^2-4)}$ and

$\gamma_2=\frac{\left(-(p_{b_2}-p_{c_2})^2+2b\right)}{(p_{b_2}-p_{c_2})^2(d^2-2)-2b(d^2-4)}$ and $\gamma_3=\frac{(p_{b_2}-p_{c_2})d}{(p_{b_2}-p_{c_2})^2(d^2-2)-2b(d^2-4)}$ and

$v_1=\frac{2bd}{(p_{b_2}-p_{c_2})^2(d^2-2)-2b(d^2-4)}$ and $v_2=\frac{2b}{(p_{b_2}-p_{c_2})^2(d^2-2)-2b(d^2-4)}$ and

$v_3=\frac{(p_{b_2}-p_{c_2})^2d^2-2(p_{b_2}-p_{c_2})}{(p_{b_2}-p_{c_2})^2(d^2-2)-2b(d^2-4)}$ and $s_1=\frac{(-(p_{b_2}-p_{c_2})+4b)\varphi_1}{(p_{b_2}-p_{c_2})^2(d^2-2)-2b(d^2-4)}$ and

$s_2=\frac{\left(-(p_{b_2}-p_{c_2})^2+2b\right)d\varphi_1}{(p_{b_2}-p_{c_2})^2(d^2-2)-2b(d^2-4)}$ and $s_3=\frac{(p_{b_2}-p_{c_2})d\varphi_1}{(p_{b_2}-p_{c_2})^2(d^2-2)-2b(d^2-4)}$ and

$l_1=\frac{2bd\varphi_2}{(p_{b_2}-p_{c_2})^2(d^2-2)-2b(d^2-4)}$ and $l_2=\frac{2b\varphi_2}{(p_{b_2}-p_{c_2})^2(d^2-2)-2b(d^2-4)}$ and

$l_3=\frac{\left((p_{b_2}-p_{c_2})d^2\right)-2(p_{b_2}-p_{c_2})\varphi_1}{(p_{b_2}-p_{c_2})^2(d^2-2)-2b(d^2-4)}$.

By replacing equations (22) to (24) in the profit functions, the following results are obtained.

$\max_{w_2}\Pi_{S_2}=(w_2-((1-(k_3+\gamma_3w_1+v_3w_2+s_3T_1+l_3T_2))c_{m_2})-\left(c_{r_2}+p_{b_2}\right)(k_3+\gamma_3w_1+v_3w_2+s_3T_1+l_3T_2)-(1-\varphi_1)T_2)(a_2-k_2+dk_1+(-\gamma_2+d\gamma_1)w_1+(-v_2+dv_1)w_2+(-s_2+ds_1)T_1+(-l_2+dl_1)T_2$ (25)

$\max_{w_1}\Pi_{S_1}=(w_1-C_1-(1-\varphi_1)T_1)(a_1-k_1+dk_2+(-\gamma_1+d\gamma_2)w_1+(-v_1+dv_2)w_2+(-s_1+ds_2)T_1+(-l_1+dl_2)T_2$ (26)

If $\left(p_{b_2}-p_{c_2}\right)^2(1-d)$< (4b-2bd) holds, the profit function of the first producer will be concave on w_1, and if $(-v_2+dv_1)(1+v_3c_{m_2}-v_3)\left(c_{r_2}+p_{b_2}\right)<0$ holds, the profit function of the second producer will become concave on w_2, and the optimal values of w_i^c(i=1,2) are obtained from Proposition (7).

Proposition7.*If*$(-v_2+dv_1)\left(1+v_3c_{m_2}-v_3\right)\left(c_{r_2}+p_{b_2}\right)<0$ *and* $\left(p_{b_2}-p_{c_2}\right)^2(1-d)$<(4b-2bd) *holds, the optimal values of* $\mathrm{w}_\mathrm{i}^\mathrm{c}$(i=1,2) *can be found as follows:*

We can write the linear wholesale price variables as the following.

(x_i , y_i and $z_i(i=1,2)$ are calculated by a mathematical software).

$w_1^c(T_1,T_2)=x_1+z_1T_2+y_1T_1$ (27)

$w_2^c(T_1,T_2)=x_2+z_2T_2+y_2T_1$ (28)

By replacing the optimal values of w_i^c, p_i^c(i=1,2) and τ_2^c in profit functions of the government under two scenarios adopted by the government, these profit functions can be calculated as below and the optimal values can be obtained accordingly.
a) Revenue seeking policy:
The government tries to increase his income following this policy.
Max GNR(T_1, T_2)=T_1D_1+T_2D_2 =

$$(T_1)\left[\frac{\begin{array}{c}(p_{b_2}-p_{c_2})^2(a_1-da_2)+8bda_2+\varphi_1T_1\left(-(p_{b_2}-p_{c_2})^2d^2+(p_{b_2}-p_{c_2})^2+(4b+2bd^2)\right)\\+(x_1+z_1T_2+y_1T_1)\left((p_{b_2}-p_{c_2})^2(1-d)-2b(2-d)\right)\end{array}}{(-2+d^2)(p_{b_2}-p_{c_2})^2-2b(d^2-4)}\right] \quad (29)$$

The profit function of the government is a concave function and by establishing the first derivative conditions for the profit function of the government, the optimal values of T_1 and T_2 are obtained from Proposition (8).

Proposition 8. *If equation*
$(2(p_{b_2}-p_{c_2})^2(d^2\varphi_1+\varphi_2)+2\varphi_1(4b+2bd^2))(p_{b_2}-p_{c_2})^2(\varphi_2-d\varphi_2)+2\varphi_2(-2b+4bd)+\varphi_2(-2b+4bd^2))>0$
holds, the profit function of the government is a concave function.

b) Environment Protection and Income-oriented Policies:
In this case, the government decides to maximize his profit and protect the environment, so that his profit function is found as below:

Max U(T_r, T_c)=*GNR* (T_r, T_c)-λ*EIS*(T_r, T_c) =

$$(T_1-\lambda\theta_1)\left[\frac{\begin{array}{c}(p_{b_2}-p_{c_2})^2(a_1-da_2)+8bda_2+\varphi_1T_1\left(-(p_{b_2}-p_{c_2})^2d^2+(p_{b_2}-p_{c_2})^2+(4b+2bd^2)\right)\\+(x_1+z_1T_2+y_1T_1)\left((p_{b_2}-p_{c_2})^2(1-d)-2b(2-d)\right)\end{array}}{(-2+d^2)(p_{b_2}-p_{c_2})^2-2b(d^2-4)}\right]$$

$$+(T_2-\lambda\theta_2)\left[\frac{\begin{array}{c}2bda_1+4ba_2(-1+3d^2)+\left(12(p_{b_2}-p_{c_2})^2-12d(p_{b_2}-p_{c_2})^2-4b\varphi_2+8bd\varphi_2-24b+24bd^2\right)T_2\\+(x_1+z_1T_2+y_1T_1)\left((1-d)(p_{b_2}-p_{c_2})^2+4bd-2b\right)+(x_2+z_2T_2+y_2T_1)(-2b+2b(d^2))\end{array}}{(-2+d^2)(p_{b_2}-p_{c_2})^2-2b(d^2-4)}\right] \quad (30)$$

Proposition 9. *If equation* $(2(p_{b_2}-p_{c_2})^2(d^2\varphi_1+\varphi_2)+\varphi_1(4b+2bd^2))*2(p_{b_2}-p_{c_2})^2(\varphi_2-d\varphi_2)+2\varphi_2(-2b+4bd)+\varphi_2(-2b+4bd^2))>0$ *holds, the profit function of the government is a concave function and the optimal values of* T_1^C *and* T_2^C *are obtained by considering the first derivative conditions.*

4- Numerical Example

The problem under study is examined in terms of a numerical example. For this purpose, the following parameters are used:
$\varphi_i=0.5, a_i=1, d=0.5, p_{b_2}=2, p_{c_2}=1, c_{m_2}=1, c_{r_2}=1, (i=1,2)$

Two supply chains start to produce the same product with equal quality and potential market size. Table.1 lists different values of the profit function for the members in the first scenario.

Table 1. Calculation of tariff parameter in the first scenario

T1	*T2*	w_1^*	w_2^*	p_1^*	p_2^*	τ_2^*	π_{w1}^*	π_{w2}^*	π_{p1}^*	π_{p2}^*	$\pi_{\tau 2}^*$	*GNR*	*U*
2.06	4.01	1.66	1.28	2.94	4.37	0.55	0.07	2.99	0.18	1.20	0.52	4.90	-
14.10	16.95	3.92	3.34	8.49	10.03	-0.89	7.98	8.81	-2.02	3.18	1.28	-	4.35

By choosing an income-oriented policy by the government, the wholesale price of the regular supply chain is greater than that of the closed-loop supply chain. Although the supplier's profit is still smaller and the retail price of the regular supply chain is again smaller than that of the closed-loop one, the retailer's profit in the closed-loop supply chain is smaller in spite of its higher price. In such a case, the set of the tariff values for the closed-loop supply chain are almost twice of the regular supply chain, because the government intends to acquire as much profit as possible.

By adoption of profit-oriented and environment protection policies by the government, the wholesale price in the regular supply chain becomes greater than that of the close-loop supply chain. Nevertheless, the supplier's profit in the regular supply chain is smaller. However, the retail price of the regular supply chain is below than the closed-loop supply chain, with the retailer's profit of the closed-loop supply chain being smaller. The retailer's profit from the regular supply chain is negative here. It means that she/he will lose something, which is expectable considering the policies adopted by the government. In this case, where the government aims to get his profit and protect the environment at the same time, the set of tariff values for the closed-loop supply chain were a little greater than those of the regular supply chain. This time, the contribution of government is not greater than the previous case, while the recycler's profit is also increased.

When the government decides to follow the income-oriented and environment protection policies, all the wholesale prices and retail prices rise, and the profit function of all the members grow except for the retailer of the first supply chain.

Table 2. Different values of profit function for members in the second scenario

T1	*T2*	w_1^c	w_2^c	p_1^c	p_2^c	τ_2^c	π_{w1}^c	π_{w2}^c	π_{p1}^c	π_{RR2}^c	*GNR*	*U*	*T1*
-2.37	1.78	9.28	2.03	5.60	4.21	1.29	2.82	1.07	-15.91	5.03	-2.71	-	- 2.37
17.5	28.8	37	5.82	28.3	19.5	-0.7	193.8	10.8	305.9	1.47	-	25.2	17.5

By adoption of the profit-oriented policy by the government, the wholesale price in the regular supply chain is greater than that of the closed-loop, while the supplier's profit is greater in the regular SC as well. Moreover, the retail price of the regular supply chain is greater than that of the closed-loop. However, the retailer's profit in the regular supply chain is smaller despite its higher prices, so that she/he will lose something. In this case, since the only objective of the government is to acquire his own profit, the set of tariff values of the closed-loop supply chain are almost twice of the regular one, and the government allocates some subsidies to this regular supply chain.

Following the profit-oriented and environment protection policies by the government, the wholesale price in the supply chain is still more than the closed-loop supply chain, with the supplier's profit being greater in the regular supply chain. Furthermore, the retailer price of the regular supply chain is greater than that of the closed-loop, with the retailer's profit of the closed-loop being greater than that of the retailer-recycler. In this case, the government prefers to acquire his profit and to protect the environment as well. The set of tariff values for the closed-loop supply chain are a bit greater than those of the open loop supply chain. Here, the contribution of the government is somehow smaller than the previous case. The profit of the government decreases in the second scenario through both approaches (and the government has even lost in some instances). This indicates that the cooperation between the retailer and the recycler was not in favor of the government. Total contribution of the retailer and the recycler in the first scenario is smaller than the retailer-recycler profit, which indicates providing some benefits for them both.

Once the government selects profit-oriented and environment protection policies, all the retail and wholesale prices will increase and the profit functions of all members grow, except for that of retailer-recycler and that of government. As can be observed from Table 3, increasing the value of λ causes an increase in the profit function of the government. This indicates that the environment protection considerations are in favor of the government to acquire profit.

Table 3. Effect of λ parameter on profit function of government in first scenario

Lambda	*U*
5	41
10	148
20	361
40	787
50	1000
70	1426

As can be seen in Table 4 increasing the value of λ parameter, reduces the profit of the government in the second scenario. This is indicative of this fact that paying attention to the environmental issues is disadvantageous for the government in this scenario.

Table 4. Effect of λ parameter on profit function of government in second scenario

Lambda	*U*
5	-25
10	-59
20	-127
40	-262
50	-329
70	-465

5- Conclusion

This research provides a competitive model for competition of two supply chains, namely closed-loop and regular, under intervention of the government. By the means of the games theory concepts, the optimal values of wholesale, retail sale, collection of used products parameter and tariff can be obtained for each of the supply chains in order to model these supply chains in terms of Stackelberg game. It is shown that the profit of the chain members is completely influenced by the government as the main leader of the game. Moreover, by selecting various scenarios, the government can influence the profit of each of the chain members, and can persuade the regular supply chain to withdraw.

At the end, one of the potential subjects for being developed in the future can be eliminating the dominant assumptions of the current work. Some of the possible suggestions for future works are considering the costs of inventory, maintenance, ordering or shortage in the modeling. Meanwhile, linear dependence of the demand to the stock price is an over simplification approach. By considering the dependence of the demand to other factors such as quality, alternative and complementary goods, and costs for advertisement and marketing, much conformation between the model outputs and reality will appear. Solution and modeling of the game in such situation is rather difficult, though much realistic in practice.

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References

Bowen, F. E., Cousins, P. D., Lamming, R. C., &Farukt, A. C. (2001). The role of supply management capabilities in green supply. *Production and operations management*, *10*(2), 174-189.

Chan, F. T., & Kumar, N. (2007). Global supplier development considering risk factors using fuzzy extended AHP-based approach. *Omega*, *35*(4), 417-431.

Chou, S. Y., & Chang, Y. H. (2008). A decision support system for supplier selection based on a strategy-aligned fuzzy SMART approach. *Expert systems with applications*, *34*(4), 2241-2253.

Hafezalkotob, A. (2015). Competition of two green and regular supply chains under environmental protection and revenue seeking policies of government. *Computers & Industrial Engineering*, *82*, 103-114.

Hafezalkotob, A., Alavi, A., & Makui, A. (2015). Government financial intervention in green and regular supply chains: Multi-level game theory approach. *International Journal of Management Science and Engineering Management*, (ahead-of-printd), 1-11.

Hjaila, K.,Puigjaner, L,. Espuna A.(2015). Scenario-Based Price Negotiations vs. Game Theory in the Optimization of Coordinated Supply Chains. *Computer Aided Chemical Engineering*, Volume 37, 2015, Pages 1859-1864

Hong, I. H., Lee, Y. T., & Chang, P. Y. (2014). Socially optimal and fund-balanced advanced recycling fees and subsidies in a competitive forward and reverse supply chain. *Resources, Conservation and Recycling*, 82, 75-85.

Jin, C., & Mei, L. (2012). Game Analysis of Multi-Strategy between Government and Suppliers in Green Supply Chain. In *Green Communications and Networks* (pp. 185-191). Springer Netherlands.

Murphy, J. (2000). Ecological modernisation. *Geoforum*, *31*(1), 1-8.

Nagurney, A., & Yu, M. (2012). Sustainable fashion supply chain management under oligopolistic competition and brand differentiation. *International Journal of Production Economics*, 135(2), 532–540.

Qiaolun, G., Jianhua, J., &Tiegang, G. (2008). Pricing management for a closed-loop supply chain. *Journal of Revenue & Pricing Management*, 7(1), 45-60.

Robeson, J. F. (1994). Logistics *Handbook*. Simon and Schuster.

Savaskan, R. C., & Van Wassenhove, L. N. (2006). Reverse channel design: the case of competing retailers. *Management Science*, *52*(1), 1-14.

Sheu, J. B. (2011). Bargaining framework for competitive green supply chains under governmental financial intervention. *Transportation Research Part E: Logistics and Transportation Review*, *47*(5), 573-592.

Sheu, J. B., & Chen, Y. J. (2012). Impact of government financial intervention on competition among green supply chains. *International Journal of Production Economics*, *138*(1), 201-213.

Shinkuma, T. (2003). On the second-best policy of household's waste recycling. *Environmental and Resource Economics*, *24*(1), 77-95.

Tian, Yihui, KannanGovindan, and Qinghua Zhu. A system dynamics model based on evolutionary game theory for green supply chain management diffusion among Chinese manufacturers. *Journal of Cleaner Production* 80 (2014): 96-105.

Tseng, M. L., Chiang, J. H., &Lan, L. W. (2009). Selection of optimal supplier in supply chain management strategy with analytic network process and choquet integral. *Computers & Industrial Engineering*, *57*(1), 330-340.

Tsireme, A. I., Nikolaou, E. I., Georgantzis, N., &Tsagarakis, K. P. (2012). The influence of environmental policy on the decisions of managers to adopt G-SCM practices. *Clean technologies and environmental policy*, *14*(5), 953-964.

Vachon, S., &Klassen, R. D. (2008). Environmental management and manufacturing performance: the role of collaboration in the supply chain. *International journal of production economics*, *111*(2), 299-315.

Wei, J., & Zhao, J. (2011). Pricing decisions with retail competition in a fuzzy closed-loop supply chain. *Expert Systems with Applications*, *38*(9), 11209-11216.

Wu, C. H. (2012). Price and service competition between new and remanufactured products in a two-echelon supply chain. *International Journal of Production Economics*, *140*(1), 496-507.

Zamarripa, Miguel A., et al. Mathematical programming and game theory optimization-based tool for supply chain planning in cooperative/competitive environments. Chemical Engineering Research and Design 91.8 (2013): 1588-1600.

Zhang, C. T., & Liu, L. P. (2013). Research on coordination mechanism in three-level green supply chain under non-cooperative game. *Applied Mathematical Modelling*, 37(5), 3369–3379.

Zhou, M., & Zhang, S. W. (2007). Three players game analysis between government, coal and electricity power. *Coal Economic Research*, (4), 33-34.

Zhu, Q., Sarkis, J., Cordeiro, J. J., & Lai, K. H. (2008). Firm-level correlates of emergent green supply chain management practices in the Chinese context. *Omega*, *36*(4), 577-591.

Appendix A.

Proof of Proposition 1. The objective functions of recycler is a concave function on τ_2.

$\frac{\partial^2 \pi_{\tau_2}}{\partial \tau_2^2}$=-2b<0

According to equation (7), recycler 2 determines the rate of collection used product τ_2.
The unique τ_2 that maximizes the profit functions of recycler is obtained from the first order conditions of the objective function (7).

$\frac{\partial \pi_{\tau_2}}{\partial \tau_2}=0,$

$\frac{\partial \pi_{\tau_2}}{\partial \tau_2}=\left(p_{b_2}-p_{c_2}\right).(a_2\text{-}p_2+dp_1)-2b\tau_2=0.$

Recycler (2) determines the optimal rate of collection used product. The optimal rate of collection used product is given below:
$\tau_2^*(p_1,p_2,T_1,T_2)=M\text{-}\mu P_2+d\mu P_1,$

where $M=\frac{(p_{b_2}-p_{c_2})(a_2)}{2b}, \mu=\frac{(p_{b_2}-p_{c_2})}{2b}.$

Proof of Proposition 2. The objective function of retailer one is a concave function on p_1 and the objective function of retailer two is a concave function on p_2.

$\frac{\partial^2 \pi_{p_1}}{\partial p_1^2}$=-2<0, $\frac{\partial^2 \pi_{p_2}}{\partial p_2^2}$=-2<0.

According to equation (1), the retailer i (i=1, 2) jointly determines the retail price p_i .The objective functions of retailers are concave functions.
The unique p_i (i=1, 2) that maximizes the profit functions of retailer are obtained from the first order conditions of the objective function (4).
$\frac{\partial \pi_{R_1}}{\partial P_1}=0, \frac{\partial \pi_{R_2}}{\partial P_2}=0,$
$\frac{\partial \pi_{R_1}}{\partial P_1}=(a_1\text{-}p_1+dp_2)-(p_1-w_1-\varphi_1 T_1)=0, \frac{\partial \pi_{R_2}}{\partial P_2}=(a_2\text{-}p_2+dp_1)-(p_2-w_2-\varphi_2 T_2)=0.$

Retailer i (i=1,2) jointly determines the retail price p_i .The optimal retail prices are
$p_1^*(w_1,w_2,T_1,T_2)=N_1+f_2.\varphi_2.T_2+f_1.\varphi_1.T_1+f_1w_1+f_2w_2 p_2^*(w_1,w_2,T_1,T_2)=N_2+f_1.\varphi_2.T_2+f_2.\varphi_1.T_1+f_2w_1+f_1w_2$

where, $N_i=\frac{2a_i+da_j}{4-d^2}$ (I,j=1,2) , $f_1=\frac{2}{4-d^2}$ and $f_2=\frac{d}{4-d^2}$.

Proof of Proposition 3. The objective function of supplier 1 is a concave function on w_1.

. If $(1\text{-}\rho_1\mu+d\mu\rho_2)>0$, then the objective function of supplier 2 would be a concave function on w_2.
$\frac{\partial^2 \pi_{w_1}}{\partial w_1^2}=(-2\rho_1+2d\rho_2)<0, \frac{\partial^2 \pi_{w_2}}{\partial w_2^2}=(-2\rho_1+2d\rho_2)(1-\rho_1\mu+d\mu\rho_2)<0.$

According to equations (5) and (6), supplier i (i=1, 2) jointly determines wholesale prices w_i.
A unique w_i (i=1, 2) that maximizes the profit functions of suppliers is obtained as follows:
$\frac{\partial \pi_{M_1}}{\partial w_1}=0, \frac{\partial \pi_{M_2}}{\partial w_2}=0,$

$\frac{\partial \pi_{M_1}}{\partial w_1}=D_1+(-\rho_1+d\rho_2).(w_1-c_1\text{-}(1-\varphi_1).T_1)=0, \frac{\partial \pi_{M_2}}{\partial w_2}=(1+c_{m_2}(-\mu\rho_1+\mu d\rho_2)-\left(c_{r_2}+p_{b_2}\right)(-\mu\rho_1+\mu d\rho_2)).D_2+(-\rho_1+d\rho_2).(w_2-(1-\tau_2).c_{m_2}-\left(c_{r_2}+p_{b_2}\right).\tau_2-(1-\varphi_2).T_2)=0.$

$w_1^*(T_1,T_2)$=$e_1+o_1T_1+g_1T_2$
$w_2^*(T_1,T_2)$=$e_2+o_2T_1+g_2T_2$

Recycler i (i=1, 2) jointly determines the optimal wholesale price w_i.
Proof of proposition 4. If Equation (*) holds, The Hessian matrix of objective function from the government is a concave function on (T_1,T_2) if and only if the Hessian matrix H_i is defined negatively.

Max GNR (T_1,T_2)=T_1D_1+T_2D_2 =

$$(T_1)\left[\frac{2a_1+da_2+d\varphi_2T_2+\varphi_1(d^2-2)T_1+(d^2-2)(e_1+f_1T_1+g_1T_2)+d(e_1+f_2T_1+g_2T_2)}{4-d^2}\right]$$

$$+(T_2)\left[\frac{(2a_2+da_1+d\varphi_1T_1+\varphi_2(d^2-2)T_2)+(d^2-2)(e_2+f_2.T_1+g_2.T_2)+d(e_1+f_1.T_1+g_1.T_2)}{4-d^2}\right]$$

The Hessian matrix of *GNR*(T_1,T_2) is
H_i =

$$\begin{bmatrix}\frac{(2\varphi_1(d^2-2)+2(d^2-2)f_1+2df_2)}{4-d^2} & \frac{2\varphi_2+f_2(d^2-2)+(d^2-2)g_1+d\varphi_1+dg}{4-d^2}\\ \frac{2\varphi_2+f_2(d^2-2)+(d^2-2)g_1+d\varphi_1+dg_2+f_1d}{4-d^2} & \frac{(2\varphi_1(d^2-2)+2(d^2-2)g_2+2dg_1}{4-d^2}\end{bmatrix}$$

$$(2\varphi_2+f_2(d^2-2)+(d^2-2)g_1+d\varphi_1+dg_2+f_1d)^2<(2\varphi_1(d^2-2)+2(d^2-2)f_1+2df_2)*(2\varphi_1(d^2-2)+2(d^2-2)g_2+2dg_1) \quad (*)$$

The optimal values of T_1^* and T_2^* will be found from establishment of the first derivative conditions of Equation (15).
Proof of Proposition 5. If equation (*) holds, The Hessian matrix of objective function from the government is a concave function on (T_1,T_2) if and only if the Hessian matrix H_i is defined negatively.

Max U(T_r,T_c)=*GNR* (T_r,T_c)-λ*EIS*(T_r,T_c) =

$$(T_1-\lambda\theta_1)\left[\frac{2a_1+da_2+d\varphi_2T_2+\varphi_1(d^2-2)T_1+(d^2-2)(e_1+f_1T_1+g_1T_2)+d(e_1+f_2T_1+g_2T_2)}{4-d^2}\right]$$

$$+(T_2-\lambda\theta_2)\left[\frac{(2a_2+da_1+d\varphi_1T_1+\varphi_2(d^2-2)T_2)+(d^2-2)(e_2+f_2.T_1+g_2.T_2)+d(e_1+f_1.T_1+g_1.T_2)}{4-d^2}\right]$$

The Hessian matrix of $U(T_1,T_2)$ is
H_i =

$$\begin{bmatrix}\frac{(2\varphi_1(d^2-2)+2(d^2-2)f_1+2df_2)}{4-d^2} & \frac{2\varphi_2+f_2(d^2-2)+(d^2-2)g_1+d\varphi_1+dg_2+f_1d}{4-d^2}\\ \frac{2\varphi_2+f_2(d^2-2)+(d^2-2)g_1+d\varphi_1+dg_2+f_1d}{4-d^2} & \frac{(2\varphi_1(d^2-2)+2(d^2-2)g_2+2dg_1)}{4-d^2}\end{bmatrix}$$

$$(2\varphi_2+f_2(d^2-2)+(d^2-2)g_1+d\varphi_1+dg_2+f_1d)^2<(2\varphi_1(d^2-2)+2(d^2-2)f_1+2df_2)*(2\varphi_1(d^2-2)+2(d^2-2)g_2+2dg_1) \quad (*)$$

The optimal values of T_1^* and T_2^* will be found from establishment of the first derivative conditions of equation (17).

Proof of Proposition 6. According to equations (16) and(18) ,the retailer and the recycler-retailer jointly determine the retail price p_i and τ_2.
The Hessian matrix of objective function from the recycler-retailer is a concave function on ((p_2, τ_2) if and only if the Hessian matrix $H_i$is defined negatively.

$$H_i = \begin{bmatrix} -2 & -(p_{b_2} - p_{c_2}) \\ -(p_{b_2} - p_{c_2}) & -2b \end{bmatrix}$$

If 4b$>(p_{b_2} - p_{c_2})^2$, then the objective function of recycler-retailer is a concave function on (p_2, τ_2).

The definition of $H_i$is given below:

$|H_i| = 4b - (p_{b_2} - p_{c_2})^2$.

If $4b > (p_{b_2} - p_{c_2})^2$, then the objective function of recycler-retailer should be a concave function on (p_2, τ_2).

The objective function of retailer one is a concave function onp_1. The optimal retail prices and the optimal rate of collection used product are

$\frac{\partial^2 \pi_{p_1}}{\partial p_1^2} = -2 < 0.$

The unique p_i (i=1, 2) and , τ_2 that maximize the profit functions of the retailer and the recycler-retailer are obtained as follows:

$\frac{\partial \pi_{R_1}}{\partial P_1} = 0, \frac{\partial \pi_{RR_2}}{\partial P_2} = 0, \frac{\partial \pi_{R_2 \tau_2}}{\partial \tau_2} = 0,$

$\frac{\partial \pi_{R_1}}{\partial P_1} = (a_1 - p_1 + dp_2) - (p_1 - w_1 - \varphi_1 T_1) = 0, \frac{\partial \pi_{RR_2}}{\partial P_2} = (a_2 - p_2 + dp_1) - (p_2 - w_2 - \varphi_2 T_2) - \tau_2(p_{b_2} - p_{c_2}) = 0, \ \frac{\partial \pi_{R_2 \tau_2}}{\partial \tau_2} = (p_{b_2} - p_{c_2})(a_2 - p_2 + dp_1) - 2b\tau_2 = 0.$

The retailer and the recycler-retailer determine retail price p_i (i=1, 2) and τ_2 together with each other.

$p_1^c(w_1, w_2, T_1, T_2) = k_1 + l_1 T_2 + s_1 T_1 + \gamma_1 w_1 + v_1 w_2$

$p_2^c(w_1, w_2, T_1, T_2) = k_2 + l_2 T_2 + s_2 T_1 + \gamma_2 w_1 + v_2 w_2$

$\tau_2^c(w_1, w_2, T_1, T_2) = k_3 + l_3 T_2 + s_3 T_1 + \gamma_3 w_1 + v_3 w_2$

Where,

$k_1 = \frac{\left(-(p_{b_2}-p_{c_2})^2+4b\right)(a_1+da_2)+2bda_2}{(p_{b_2}-p_{c_2})^2(d^2-2)-2b(d^2-4)}$ and $k_2 = \frac{\left(-(p_{b_2}-p_{c_2})^2+2b\right)(2a_2+da_1)}{(p_{b_2}-p_{c_2})^2(d^2-2)-2b(d^2-4)}$ and

$k_3 = \frac{(p_{b_2}-p_{c_2})(2a_2+da_1)}{(p_{b_2}-p_{c_2})^2(d^2-2)-2b(d^2-4)}$ and $\gamma_1 = \frac{\left(-(p_{b_2}-p_{c_2})^2+4b\right)}{(p_{b_2}-p_{c_2})^2(d^2-2)-2b(d^2-4)}$ and

$\gamma_2 = \frac{\left(-(p_{b_2}-p_{c_2})^2+2b\right)}{(p_{b_2}-p_{c_2})^2(d^2-2)-2b(d^2-4)}$ and $\gamma_3 = \frac{(p_{b_2}-p_{c_2})d}{(p_{b_2}-p_{c_2})^2(d^2-2)-2b(d^2-4)}$ and

$v_1 = \frac{2bd}{(p_{b_2}-p_{c_2})^2(d^2-2)-2b(d^2-4)}$ and $v_2 = \frac{2b}{(p_{b_2}-p_{c_2})^2(d^2-2)-2b(d^2-4)}$ and

$v_3 = \frac{(p_{b_2}-p_{c_2})^2 d^2-2(p_{b_2}-p_{c_2})}{(p_{b_2}-p_{c_2})^2(d^2-2)-2b(d^2-4)}$ and $s_1 = \frac{(-(p_{b_2}-p_{c_2})+4b)\varphi_1}{(p_{b_2}-p_{c_2})^2(d^2-2)-2b(d^2-4)}$ and

$s_2 = \frac{\left(-(p_{b_2}-p_{c_2})^2+2b\right)d\varphi_1}{(p_{b_2}-p_{c_2})^2(d^2-2)-2b(d^2-4)}$ and $s_3 = \frac{(p_{b_2}-p_{c_2})d\varphi_1}{(p_{b_2}-p_{c_2})^2(d^2-2)-2b(d^2-4)}$ and

$l_1 = \frac{2bd\varphi_2}{(p_{b_2}-p_{c_2})^2(d^2-2)-2b(d^2-4)}$ and $l_2 = \frac{2b\varphi_2}{(p_{b_2}-p_{c_2})^2(d^2-2)-2b(d^2-4)}$ and

$l_3 = \frac{\left((p_{b_2}-p_{c_2})d^2\right)-2(p_{b_2}-p_{c_2})\varphi_1}{(p_{b_2}-p_{c_2})^2(d^2-2)-2b(d^2-4)}$.

Proof of Proposition 7. According to equations (25) and (26), the supplier i (i=1, 2) jointly determine the wholesale prices w_i.If $((p_{b2}-p_{c2})^2(1-d)) < (4b-2bd)$,then $\gamma_1 > d\gamma_2$ *and* the objective function of supplier 1 is a concave function on w_1. If$(-\vartheta_2+d\vartheta_1)$ $(1+\vartheta_3 cm_2-\vartheta_3(c_{r_2}+p_{b_2}))$ <0, then the objective function of supplier 2 is obtained as a concave function on w_2.

$\frac{\partial^2\pi_{w_1}}{\partial w_1^2}=2(-\gamma_1+d\gamma_2)$,$\frac{\partial^2\pi_{w_2}}{\partial w_2^2}$=$2(-\vartheta_2+d\vartheta_1)$ $(1+\vartheta_3 cm_2-\vartheta_3(c_{r_2}+p_{b_2}))$.

The unique w_i (i=1,2) that maximizes the profit functions of the suppliers are obtained as follows:

$\frac{\partial\pi_{M_1}}{\partial w_1}$=0, $\frac{\partial\pi_{M_2}}{\partial w_2}$=0,

$\frac{\partial\pi_{M_1}}{\partial w_1}$=$(w_1-c_1$-$(1-\varphi_1).T_1)$ $(-\gamma_1+d\gamma_2)$+D_1=0,

$\frac{\partial\pi_{M_2}}{\partial w_2}$=$(1+\vartheta_3 cm_2-\vartheta_3(c_{r_2}+p_{b_2})).D_2$+$(-\vartheta_2+d\vartheta_1)$.$(w_2-(1-\tau_2).c_{m_2}-(c_{r_2}+p_{b_2}).\tau_2-(1-\varphi_2).T_2)$ =0.

Supplier i (i=1, 2) jointly determines the wholesale prices of w_i^c.

$w_1^c(T_1,T_2)$=$x_1+z_1T_2+y_1T_1$

$w_2^c(T_1,T_2)$=$x_2+z_2T_2+y_2T_1$

Proof of Proposition 8. If equation

$(2(p_{b_2}-p_{c_2})^2(d^2\varphi_1+\varphi_2)+\varphi_1(4b+2bd^2))*2(p_{b_2}-p_{c_2})^2(\varphi_2-d\varphi_2)+2\varphi_2(-2b+4bd)+\varphi_2(-2b+4bd^2)) > 0$ holds, The Hessian matrix of objective function from the government is a concave function on (T_1,T_2) if and only if the Hessian matrix $H_i$is defined negatively.

MaxGNR(T_1,T_2)=T_1D_1+T_2D_2 =

$$(T_1)\left[\frac{\begin{gathered}(p_{b_2}-p_{c_2})^2(a_1-da_2)+8bda_2+\varphi_1T_1\left(-(p_{b_2}-p_{c_2})^2d^2+(p_{b_2}-p_{c_2})^2+(4b+2bd^2)\right)\\+(x_1+z_1T_2+y_1T_1)\left((p_{b_2}-p_{c_2})^2(1-d)-2b(2-d)\right)\end{gathered}}{(-2+d^2)(p_{b_2}-p_{c_2})^2-2b(d^2-4)}\right]$$

$$+(T_2)\left[\frac{\begin{gathered}2bda_1+4ba_2(-1+3d^2)+\left(12(p_{b_2}-p_{c_2})^2-12d(p_{b_2}-p_{c_2})^2-4b\varphi_2+8bd\varphi_2-24b+\right.\\+(x_1+z_1T_2+y_1T_1)\left((1-d)(p_{b_2}-p_{c_2})^2+4bd-2b\right)+(x_2+z_2T_2+y_2T_1)(-2b+2\end{gathered}}{(-2+d^2)(p_{b_2}-p_{c_2})^2-2b(d^2-4)}\right.$$

The Hessian matrix of *GNR*(T_1,T_2) is

H_i =

$$\begin{bmatrix}\frac{(2(p_{b_2}-p_{c_2})^2(d^2+\varphi_2+\varphi_1)+\varphi_1(4b+2bd^2))}{((-2+d^2)(p_{b_2}-p_{c_2})^2-2b(d^2-4))} & 0\\ 0 & \frac{(2(p_{b_2}-p_{c_2})^2(\varphi_1+d\varphi_1)+3\varphi_1(4bd-2b))}{((-2+d^2)(p_{b_2}-p_{c_2})^2-2b(d^2-4))}\end{bmatrix}$$

$$|H_i| = \left(2(p_{b_2} - p_{c_2})^2 (d^2 + \varphi_2 + \varphi_1) + \varphi_1(4b + 2bd^2)\right) * (2(p_{b_2} - p_{c_2})^2 (\varphi_1 + d\varphi_1) + 3\varphi_1(4bd - 2b)) > 0$$

The optimal values of T_i^c(I=1,2) will be found from establishing the first derivative conditions of equation (29).

Proof of Proposition 9. If equation

$(2(p_{b_2} - p_{c_2})^2 (d^2\varphi_1 + \varphi_2) + \varphi_1(4b + 2bd^2)) * 2(p_{b_2} - p_{c_2})^2 (\varphi_2 - d\varphi_2) + 2\varphi_2(-2b + 4bd) + \varphi_2(-2b + 4bd^2)) > 0$

holds, The Hessian matrix of objective function from the government is a concave function on (T_1, T_2) if and only if the Hessian matrix H_i is defined negatively.

Max $U(T_r, T_c) = GNR\,(T_r, T_c) - \lambda EIS(T_r, T_c) =$

$$(T_1 - \lambda\theta_1)\left[\frac{(p_{b_2}-p_{c_2})^2(a_1-da_2)+8bda_2+\varphi_1 T_1\left(-(p_{b_2}-p_{c_2})^2 d^2+(p_{b_2}-p_{c_2})^2+(4b+2bd^2)\right) + (x_1+z_1T_2+y_1T_1)\left((p_{b_2}-p_{c_2})^2(1-d)-2b(2-d)\right)}{(-2+d^2)(p_{b_2}-p_{c_2})^2-2b(d^2-4)}\right]$$

$$+(T_2 - \lambda\theta_2)\left[\frac{2bda_1 + 4ba_2(-1+3d^2) + \left(12(p_{b_2} - p_{c_2})^2 - 12d(p_{b_2} - p_{c_2})^2 - 4b\varphi_2 + 8bd\varphi_2 - 24b + 24bd^2\right)T_2 + (x_1 + z_1T_2 + y_1T_1)\left((1-d)(p_{b_2} - p_{c_2})^2 + 4bd - 2b\right) + (x_2 + z_2T_2 + y_2T_1)(-2b + 2b(d^2))}{(-2 + d^2)(p_{b_2} - p_{c_2})^2 - 2b(d^2 - 4)}\right]$$

Hessian matrix of $U\,(T_1, T_2)$ is

$$H_i = \begin{bmatrix} \frac{(2(p_{b_2} - p_{c_2})^2 (d^2 + \varphi_2 + \varphi_1) + \varphi_1(4b + 2bd^2))}{((-2 + d^2)(p_{b_2} - p_{c_2})^2 - 2b(d^2 - 4))} & 0 \\ 0 & \frac{(2(p_{b_2} - p_{c_2})^2 (\varphi_1 + d\varphi_1) + 3\varphi_1(4bd - 2b))}{((-2 + d^2)(p_{b_2} - p_{c_2})^2 - 2b(d^2 - 4))} \end{bmatrix}$$

$$|H_i| = \left(2(p_{b_2} - p_{c_2})^2 (d^2 + \varphi_2 + \varphi_1) + \varphi_1(4b + 2bd^2)\right) * (2(p_{b_2} - p_{c_2})^2 (\varphi_1 + d\varphi_1) + 3\varphi_1(4bd - 2b)) > 0$$

The optimal values of T_i^c*(i=1,2)*will be found from establishment of the first derivative conditions of equation (30).