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# Joint optimal inventory control and preventive maintenance policy with stochastic demand

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**Abstract**

Joint optimal inventory control and preventive maintenance is a rich area of academic research that is still in its infancy and has the potential to affect manufacturing systems' performance. Also, due to uncertainties in demand, maintenance and inventory loss are virtually unavoidable. Therefore, determining the optimal amount of inventory storage, the time to create an additional inventory for storage, and the time of maintenance operations is a concern of many manufacturers. In this paper, a joint optimization model has been developed. In which, for the proximity of reality, demand is considered as an uncertain parameter. The strategy is such that the production component is placed under maintenance as soon as it reaches the $m$ level or in the event of a malfunction earlier than $m$, stopped system and placed under maintenance and repairs. Inventory of $A$ period with level $h$ is created, which during maintenance operations, stochastic demand will be provided. Finally, a model for joint optimization of maintenance and inventory control with random failure is used that minimize the cost and create the maximum level of accessibility. A numerical study is conducted to show the effectiveness and applicability of the proposed integrated model. An accurate algorithm is provided to solve the model. The results show that the model is generally sensitive to the cost.

**Keywords:** Preventive maintenance, stochastic demand, inventory control, joint optimization

## 1-Introduction

Nowadays, modern systems are more complex but more reliable than ever. However, the performance of production systems is still under the influence of the unavoidable failure of machines that lead to a reduction in production. Equipment may fail during actual production processes and should be provided with their repair requirements. As a result, regular maintenance and preventive measures (PM should be implemented to eliminate production requirements on the equipment to prevent both unpredictable and deterministic interruptions (Chen, 2013, Guo and et al., 2013, Ghodratnama and et al., 2010).

The failure of devices and equipment is a non - deterministic resource in reducing production efficiency so that it is necessary to ensure that devices and equipment can be used at maximum access levels to produce units with maximum capacity. One of the most important ways to achieve this goal is to implement preventive maintenance policies. Of course, the creation and implementation of maintenance operations may lead to a complete stoppage of production units within a period of time.

To minimize the effect of production parameters on the overall performance of the production system, maintenance policies and production control are simultaneously considered. For this reason, the review of production and maintenance policies and production planning has become an important research area divided into three categories: (i) environmental; (ii) inventory; and (iii) quality control.

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That is why a maintenance policy can be effective when combined with a suitable inventory policy (Horenbeek and et al. 2013). In this area, other issues such as application, frequency, service continuity, and cost savings techniques should be considered (Lynch and et al., 2013). Panagiotidou (2014) proposed two policies for identical replacement parts to eliminate failure: continuous review policy and periodic review policy. These strategies are based on current optimization policies for the maintenance and maintenance of solid parts. Van Jaarsveld et al. (2011) also stressed how key elements influence the scope of maintenance strategy. One of the most important research topics in recent years can be referred to as the current optimization of production control and maintenance policies, most of which focus on optimizing production control and project management policies aimed at minimizing the projected cost per unit. Over the past decade, many preservation and maintenance studies have been considered simultaneously and have pointed to improving the overall performance of production systems by conventional optimization of production and maintenance control policies. Kaio and Osaki, (1978) also introduced a simultaneous optimization policy considering the costs of inventory. If the maintenance costs and inventory costs are not compensated with the system's earnings, the result will not be optimal, but the production will be improved when both costs are considered and compensated. In a case study of the engine production line, the simultaneous optimization reduced the maintenance cost by 53 % and made 6 % improvement in monthly production (Ilgin and Tunali, 1978). Van Horenbeek et al. (2013) combined maintenance and maintenance policies (based on the criteria) and inventory control policies (periodic review and continuous review) and classify them.

Liu et al. (2015) are proposed two scenarios for the multi- product system that one of the two scenarios was implemented at the end of the production period and the other at the beginning of the system. Chakraborty et al. (2013) investigated the joint effect of machine failure, maintenance, the threshold level of a being in an economic output model with a random value (EMQ), in which failure was a function of production. In most cases, a failure occurs in a full production period. Berthaut et al. (2011) proposed a single –product system in a single production unit, a preventive and production /store policy based on the modified block policy (MBRP), and the Hedging point policy (HPP). This simulation model was presented to simulate the dynamic and stochastic behavior of the generating unit under the joint MBRP/ HPP policy.

Maintenance models in the literature are divided into two categories: time-based, which includes alternative replacement and age-based switching (Wang, 2002); and conditions based on Alaswad and Xiang, (2017) and Olde Keizer and et al. (2017), which include periodic inspection models. They are replaced only with defective components. An inspecting policy, if financially profitable, Scarf and Cavalcante, (2012) replaces a time-based replacement policy, that is, regardless of the condition or condition of its components after a certain period of time. The classic inspection model is a time delay model (Christer1999), which was examined by Wang (2012) and developed by many others (Berrade and Scarf 2017). In the past decades, many studies have focused on integrating PM and improving product quality. Ayed et al. (2012) introduced a system with a failure rate of random they considered the randomness and tried to reach the level of service according to the amount of production they produce, in fact, they provided an optimal production plan with preventive maintenance of policies, which led to a reduction in the total cost and device failure.

Yeung et al. (2007) introduced the process called Marquee and optimized the system economically by using a process control graph with time-based maintenance policies. Chalbi et al. (2004) analyzed the integrated system with a fixed failure rate and failure rate based on the analytical approach and integrated / inventory policy. Using a mathematical model to minimize total annual production costs, they minimize the optimization of buffer inventory and periodic inventory, which should be based on preventive maintenance performance. Radhoui and et al. (2009) provided a quality-based model in which decisions were made on the type of maintenance measures after quality control and determining the failure rate of the products. In this model, in each production period, a buffer inventory is maintained to meet demand during the production interruption period.

Liao (2012) developed several other models in which the effect of poorly produced products was studied in an incomplete process; in fact, they have the least cost production policy in random-rate processes with minimal maintenance and preventive maintenance. Liu et al. (2015) also provided a production, inventory, and preventive model of a multi-dimensional production system. Setak and et al. (2019) developed a model for pricing and control the inventory of perishable products with

exponential demand. That research, sales profit was maximized by presenting a mathematical model to determine the price change points and the optimal price and order quantity for perishable products with an exponential and price- and time-dependent distributed demand. Minou and et al. (2017) considered the joint optimization of condition-based maintenance and spares planning for multi-component systems. They formulated their model as a Markov Decision Process, and minimize the long-run average cost per time unit. Ribeiro et al. (2008) jointly optimized the maintenance of a capacity-constrained resourced, its feed machine, and inlet buffer size with a mixed-integer linear programming model. Gan et al. (2015) focused on the joint optimization for maintenance, buffer, and spare parts for a production system with a genetic algorithm. The buffer inventory affects the inventory holding cost. To determine the significance of optimizing four variables related to maintenance, buffer, and spare parts simultaneously, comparisons were made among four-variable optimization, three-variable optimization, and two-variable optimization. The optimization results are different. When only two of them are optimized, the decision on the third variable may be improper, therefore leads to undesired optimization results. Hwang and Samat (2019) presented a review on joint optimization of maintenance with production planning and spare part inventory management.

Polotski, et al. (2019) considered two machines: one uses raw materials for manufacturing, while another utilizes end-of-life products returned from the market for remanufacturing. Machines are failure-prone, demand and return rates fluctuate in time reflecting market behavior due to economical, seasonal, and environmental changes. The system performance is characterized by a long-term discounted cost that integrates several partial costs (those of manufacturing, remanufacturing, disposal, holding costs in serviceable and return inventories). Liu et al. (2019) present an integrated decision model that coordinates predictive maintenance decisions based on prognostics information with a single-machine scheduling decision so that the total expected cost is minimized. In the integrated model, the health status and dummy age subjected to machine degradation are considered. Babaeimorad and et al. (2019) provided joint optimization of maintenance and production scheduling with considering the back of order and probabilistic demand. Their results showed that, in the presented model, the total cost and decision variables are highly sensitive to the inventory holding cost but not also for the occurred scenario.

The EOQ model assumes inventory situation with constant demand and delivery lead time which does not conform to reality because in most cases demands are uncertain and therefore require the development of models that can handle stochastic demand situation (Çetinkaya and Lee, 2000). Hsieh (2004) emphasized that the reason for keeping safety stock is an attempt to curb arbitrary variations in customer demands and delivery lead time. Ling Wang and et al. (2008), presented a condition-based order-replacement policy for a single-unit system, aiming to optimize the condition-based maintenance and the spare order management jointly. La Fata and Passannanti (2017), propose a model for the combined optimization of production/inventory control and PM policies to minimize the total expected cost per unit time. The model is formulated referring to a continuous production system characterized by a random deteriorating behavior so that the presence of a buffer is considered to ensure the supply of a continuous during interruptions of service caused by breakdowns or planned maintenance actions on the production system.

In this paper a model is developed that is a joint optimal inventory control and preventive maintenance policy in which an approximation of reality, demand is considered as a random parameter (Rezg et al., 2008). Since the failure in the production unit in just-in-time problem is one of the effective factors in system disturbance and productivity reduction, implementing preventive maintenance policies is necessary to maintain the efficient capacity of the system and ensure high-level access as a result. On the other hand, taking maintenance and repairing actions requires a complete stoppage of the production system. We should consider maintenance and production control policies simultaneously to minimize cessations rate. The issue of this paper is the unit of production with an incremental rate of failure, as soon as the $m$ period or the failure that preceded it has been stopped and under maintenance, an additional period of period $A$ with the level $h$ is created and during the periodic maintenance operations, random demand occurs. Finally, a mathematical model and a numerical approach are used to simultaneously obtain optimal values for the variables that use the least cost and work for access constraints. The first innovation of the paper is that instead of optimizing separately in the production system, the proposed model optimizes inventory control and maintenance and operations together. The second innovation, close to real demand is considered

probable. And the third innovation is to provide an accurate algorithm for solving the integrated model. A summary of the literature review and study gaps are given in table 1.

**Table1.** A summary of the literature review

| | Single unit or multiunit system | Maintenance policy | Considering the buffer | Consider uncertainty | Provide algorithms |
|---|---|---|---|---|---|
| N. Rezg et al (2008) | Single unit | Age-based | Yes | No | Yes |
| Wang et al (2008a) | Multiunit | Condition based | No | Yes | No |
| Nguyen and Bagajewicz (2009) | Multiunit | Age-based | No | No | No |
| Shuyuan Gan et al. (2015) | Multiunit | Periodic | Yes | No | No |
| Anis Mjirda et al. (2016) | multi-unit | Periodic | No | No | No |
| Xiaohong Zhang and Jianchao Zeng (2017) | multi-unit | Condition-based | No | Yes | No |
| Leila Jafari et al. | multi-unit | condition-based | No | No | No |
| Nabil Nahas and Mustapha Nourelfath | multi-unit | Periodic | Yes | No | Yes |
| Kaican Kang and Velusamy Subramaniam (2018) | Single unit | Periodic | No | No | No |
| This study | Single unit | Age-based | Yes | Yes | Yes |

## 2-Proposed model

This model is developed based on the Rezg et al. (2008). Consider a production system that includes a machine with a random failure rate. It has a demand value of $(d)$ that for the approximation of reality, $d$ is considered probable. The desired machine has a maximum production rate that is displayed with $U_{max}$. This system normally produces at the rate of d. In some periods, the machine produces at its highest power,$U_{max}$, to store inventory and avoid possible deficiencies during maintenance. And after storing the specified size, it resumes production at d rate. It is assumed that $U_{max}$ is greater than $d$. This production system will be under maintenance during specified periods of time $(m)$. Due to the complete stop of the machine during maintenance, the buffer is considered to meet the stochastic demand. The production system at any time period $(A)$, starts buffer storage so that can be used when the machine is stopped. The buffer storage period should be earlier than maintenance $(A < m)$. This system is capable of storing a certain amount of inventory. The maximum inventory that can be saved is shown with $(h)$. The maintenance policy includes the complete stopping of the machine as soon as it reaches the desired period or failure to perform maintenance. It is assumed that each time the maintenance is carried out, the machine returns to its original state. Figure (1) shows the structure of the production system. This structure has 4 phases, including the first phase; there is no inventory in stock. The product is following the stochastic demand, the second phase, starts at the production rate $U_{max}$ also responds to demand and value for the buffer storage. Phase 3 refers to the period when the level of buffer inventory is filled and the manufacturing system produces just as much demand. Phase 4 is the start of maintenance. The manufacturing system is completely stopped and there is only as much demand as consumption. Simultaneously provided model optimizes inventory control, and maintenance operations. In many previous studies, this concept has been used separately. In the scenarios presented in this paper, To obtain the appropriate time for maintenance and the amount of inventory that must be saved and will be available when it is starting to storage. Because the production system is completely stopped during the maintenance operations and considering the possibility of demand is not clear how much demand is there. Therefore, different scenarios should be considered in such a way that the lowest cost is applied to the system under any circumstances and a high level of availability is met. In this paper, the demand parameter is random. In previous studies, researchers considered demand at a fixed rate; however, demand faces many uncertainties in reality. So we consider the production system in 5 scenarios with demand

randomization as an assumption. The demand function is a uniform random variable, indicated in section 2-2. In formulas, as we consider the demand d randomly with uniform distribution in the range [a, b], we substitute d with its mathematical expectation in the following equations.

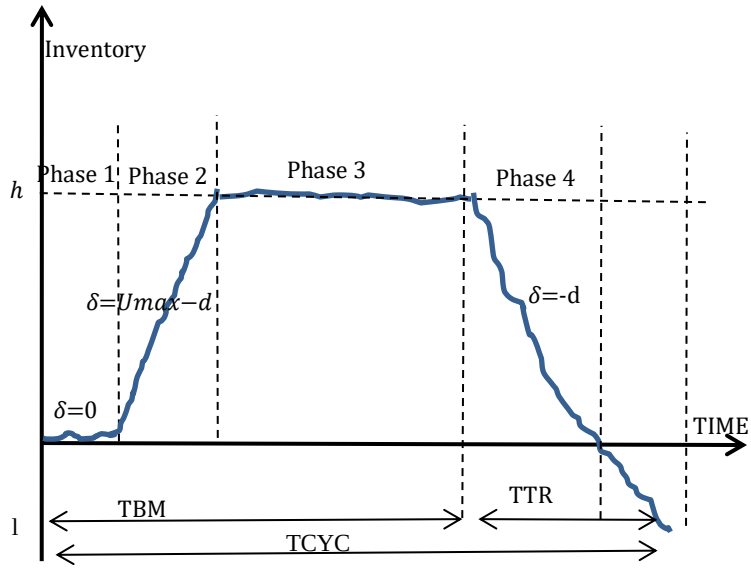


**Fig.1**.The inventory level evolution during a production cycle

## 2-1- Notations

### 2-1-1-parameters

| | |
|---|---|
| $C_s$ : | Holding cost of a product unit during a unit of time |
| $C_l$ : | Loss cost due to an unsatisfied demand of one produced item during a unit of time |
| $d$: | Demand with uniform distribution~ (a, b) |
| $\mu$ : | Machine average time to failure |
| $\varepsilon$: | Instant of failure of the machine |
| $U_{max}$ | A maximum production rate |

### 2-1-2-distribution functions

| | |
|---|---|
| $f(t)$: | Probability density function associated with the machine time to failure |
| $F(t)$: | Probability distribution function associated with the machine time to failure |
| $R(t)$: | Reliability function associated with the machine, $F(t)$: $1 - R(t)$ |
| $r(t)$: | Machine instantaneous failure rate function |
| $g_p(t)$: | Probability density function associated with the duration of a preventive maintenance action |
| $g_c(t)$: | Probability density function associated with the duration of a corrective maintenance action |

### 2-1-3-Decisions variables

| | |
|---|---|
| $A$: | Time to start storing safety stock |
| $m$: | Time to start preventive maintenance |
| $h$: | Maximum level of safety stock |
| $\partial(A,h,m)$: | Expected inventory cost |
| $\varphi(m)$: | maintenance cost |
| $CT(A,H,m)$: | The total average cost per time unit over an infinite horizon as a function of the three decision variables |

## 2-2- Calculate inventory control costs

The system's goal is to minimize costs by considering the level of availability. For values (A, h, m), there are 5 scenarios that each of these may occur. The following five scenarios include 1. The machine according to figure 2 continues and maintenance operations are completed before the end of the buffer, 2. The machine continues to maintenance and repair operations will end after the end of the buffer, and there will be a loss of inventory, 3. The machine will fail before reaching the maximum inventory, and maintenance operations will end before the end of the buffer, 4. The machine will fail before reaching the maximum inventory, and maintenance will end after the end of the buffer, 5. The machine will fail before starting buffer storage. The scenarios are as follows.

### 2-2-1-Scenario 1

In scenario 1, the machine produces as much as stochastic demand. Then, period A, starts to storage as much as h, so, continues its production at an initial rate until the period m. Maintenance operations before the end of the inventory end and does not face loss. Scenario 1 is shown in figure 2.

Assumptions: $TBM > A \ and \ (TBM - A)(U_{max} - d) \ \ and \ \ TTR \leq {}^{h}/_{d}$ (This phrase indicates that the level of buffer stock level reached h) and $TTR \leq {}^{h}/_{d}$ It shows that the system downtime period of the system is less than the consumption period of the buffer inventory. The cost of inventory ($L_{S_1}$(A, h)) given in equation (1) is equal to the surface below the graph in figure 2. Since the demand parameter is randomly assigned to the uniform distribution function [a,b], in equations(3 and 4), we substitute the demand variable with its mathematical expectation.

$$L_{S_1}(A,h) = (TTR + TBM - A)C_S - \frac{C_S[h^2 + dTTR^2U_{max} - d^2TTR^2]}{2[U_{max} - d]} \tag{1}$$

The average cost of inventory in scenario 1 is shown in equation (2).

$$E\left(L_{S_1}(A,h)\right) = (E(TTR) + E(TBM) - A)C_S - E\left(\frac{C_S[h^2 + (d)E(TTR^2)U_{max} - (d^2)E(TTR^2)]}{2[U_{max} - d]}\right) \tag{2}$$

$$E\left(\frac{C_S h^2}{2[U_{max} - d]}\right) = C_S h^2 \int_a^b \frac{1}{2(U_{max} - d)} f(d)\, dd = C_S h^2 \int_a^b \frac{1}{2(U_{max} - d)} \left(\frac{1}{b-a}\right) dd \tag{3}$$

$$E\left(C_S \frac{(d)E(TTR^2)(U_{max})}{2[U_{max} - d]}\right) = C_S(E(TTR^2)(U_{max})) \int_a^b \frac{d}{2(U_{max} - d)} \left(\frac{1}{b-a}\right) dd$$

$$E\left(C_S \frac{(d^2)E(TTR^2)}{2[U_{max} - d]}\right) = C_S(E(TTR^2)) \int_a^b \frac{d^2}{2(U_{max} - d)} \left(\frac{1}{b-a}\right) dd$$

$$ind\left(m \geq \frac{h}{(U_{max}-d)}+A\right)=\begin{cases}1 & if\ \ m \geq \frac{h}{(U_{max}-d)}+A \\ 0 & if\ \ m < \frac{h}{(U_{max}-d)}+A\end{cases} \tag{4}$$

The probability of scenario 1 is expressed in equation (5) and (6).

$$= ind\left(m \geq \frac{h}{(U_{max}-d)}+A\right) * P(\xi \geq \frac{h}{(U_{max}-d)}+A)G_d({}^{h}/_{d})P(S_1) \tag{5}$$

$$= ind\left(m \geq \frac{h}{(U_{max}-d)}+A\right) * R(\frac{h}{(U_{max}-d)}+A)G_d({}^{h}/_{d})P(S_1) \tag{6}$$

$$where\ G_d\left({}^{h}/_{d}\right)=\int_0^{h/d} g_d(u)=g_p(u)R(m)+g_c(u)F(m) \tag{7}$$

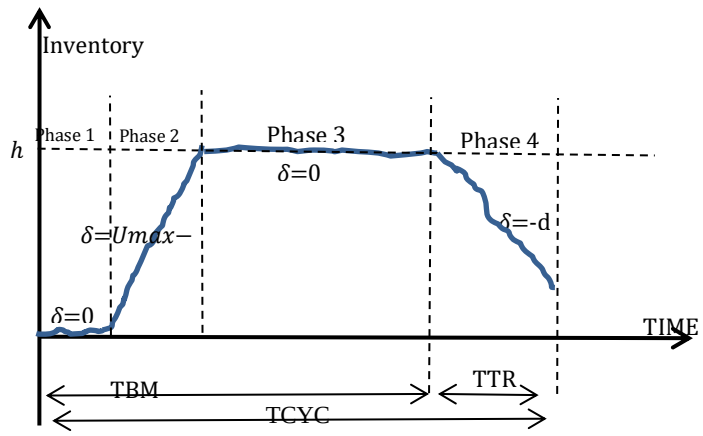


**Fig 2**. Scenario 1

**2-2-2-Scenario 2**

In scenario 2, the machine produces as much as stochastic demand. Then, from period A, starts to store as much as h, so, continues its production at an initial rate until the period m. Maintenance after the end of the inventory ends, and face loss. Scenario 2 is shown in figure3. Assumptions: $TBM > A\ \ and\ \ (TBM-A)(U_{max}-d)\ \ and\ \ TTR > {}^{h}/_{d}$The inventory cost $L_{S_2}(A)$ corresponding to scenario 2 is expressed in equation (8). This expression is obtained from the sum of inventory costs $C_S A_S$ and shortage cost $C_l N_l$. Where in $A_S$is equal to the surface below the graph in figure 3. $N_l$ Indicates the unsatisfied demand during the $d$ ($TTR\ h/d$) period.

$$L_{S_2}(A,h)=\frac{C_S(U_{max}-2d)}{2d(U_{max}-d)}h^2+[C_S(TBM-A)-C_l]h+C_l dTTR \tag{8}$$

The average cost of inventory in scenario 2 is shown in equation (9).

$$E(L_{S_2}(A,h))=E(\frac{C_S(U_{max}-2d)}{2d(U_{max}-d)})h^2+[C_S(E(TBM)-A)-C_l]h+C_l E(d)E(TTR) \tag{9}$$

$$E\left(\frac{C_S(U_{max}-2d)}{2d(U_{max}-d)}\right)=C_S\int_a^b \frac{(U_{max}-2d)}{2d(U_{max}-d)}f(d)dd \tag{10}$$

$$=C_S\int_a^b \frac{(U_{max}-2d)}{2d(U_{max}-d)}(\frac{1}{b-a})dd$$

$$ind\left(m \geq \frac{h}{(U_{max}-d)}+A\right)=\begin{cases}1 & if\ \ m \geq \frac{h}{(U_{max}-d)}+A \\ 0 & if\ \ m < \frac{h}{(U_{max}-d)}+A\end{cases} \tag{11}$$

The probability of scenario 2 is expressed in equation (12).

$$P(S_2)= ind\left(m \geq \frac{h}{(U_{max}-d)}+A\right) * P(\xi \geq \frac{h}{(U_{max}-d)}+A)(1-G_d\left({}^{h}/_{d}\right)) \tag{12}$$

$$P(S_1)= ind\left(m \geq \frac{h}{(U_{max}-d)}+A\right) * R(\frac{h}{(U_{max}-d)}+A)(1-G_d({}^{h}/_{d}))$$

$$G_d\left({}^{h}/_{d}\right)=\int_0^{{}^{h}/_{d}} g_d(u)=g_p(u)R(m)+g_c(u)F(m)$$

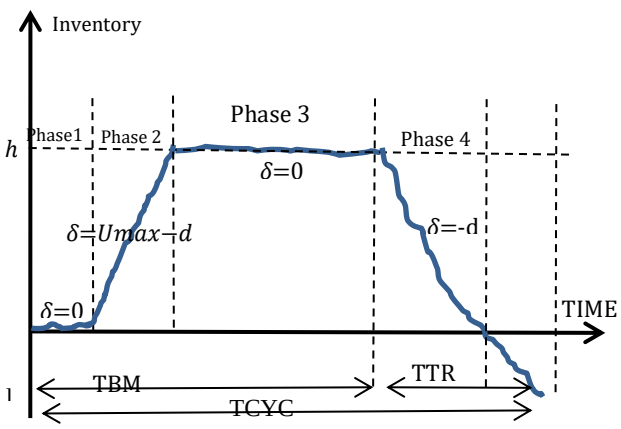


**Fig 3**. Scenario 2

**2-2-3-Scenario 3**

In scenario 3, after the period A and before the inventory is completed, the machine is damaged and maintenance operations begin. And the stock will be filled up to $S(TBM)$. Maintenance operations before the end of the inventory end and does not face loss. Scenario 3 is shown in figure 4.

Assumptions: $TBM > A \quad and \quad (TBM-A)(U_{max}-d) < h \quad and \quad TTR \leq {}^{S(TBM)}/_{d}$.The inventory cost $\mathbf{L_{s_3}}(\boldsymbol{A})$ corresponding to scenario 3 is expressed in equation (13). It is equal to the surface below the graph in figure 4.

$$L_{s_3}(A)=(TTR+TBM-A)C_s(STBM)-\frac{C_s[(STBM)^2+dTTR^2U_{max}-d^2TTR^2]}{2(U_{max}-d)} \tag{13}$$

With

$$(STBM)=(TBM-A)(U_{max}-d) \tag{14}$$

The average cost of inventory in scenario 3 is shown in equation (15).

$$E\left(L_{s_3}(A)\right)=(\mu_d+E(TBM)-A)C_s\left(E(STBM)\right)-C_s[\int_a^b \frac{((E(STBM))^2}{2(U_{max}-d)}dd + \int_a^b \frac{(d)E((TTR)^2)U_{max}}{2(U_{max}-d)}dd-\int_a^b \frac{(d^2)E(TTR)^2}{2(U_{max}-d)}dd \tag{15}$$

With

$$\left(\left(E(STBM)\right)\right) = (E(TBM) - A)(U_{max} - E((d)) \tag{16}$$
$$= \left(\int_0^m R(t)dt - A\right)(U_{max} - (\frac{a+b}{2}))$$

The probability of scenario 3 is expressed in equation (17).

$$P(S_3) = ind(m > A)P(\xi > A)$$
$$\times \left(1 - \frac{ind\left(m \geq \frac{h}{U_{max}-d} + A\right) P\left(\xi \leq \frac{h}{(U_{max}-d)} + A\right)}{P(\xi > A)}\right) G_d((E(STBM))/d) \tag{17}$$

$$P(S_3) = ind(m > A)R(A)(1 - \frac{ind\left(m \geq \frac{h}{U_{max}-d} + A\right) R\left(\frac{h}{(U_{max}-d)} + A\right)}{R(A)}) \tag{18}$$
$$\times G_d((E(STBM))/d$$

With

$$\frac{G_d\left(E(STBM)\right)}{d} = \int_0^{s(E(TBM))/d} g_d(u)\, du \tag{19}$$

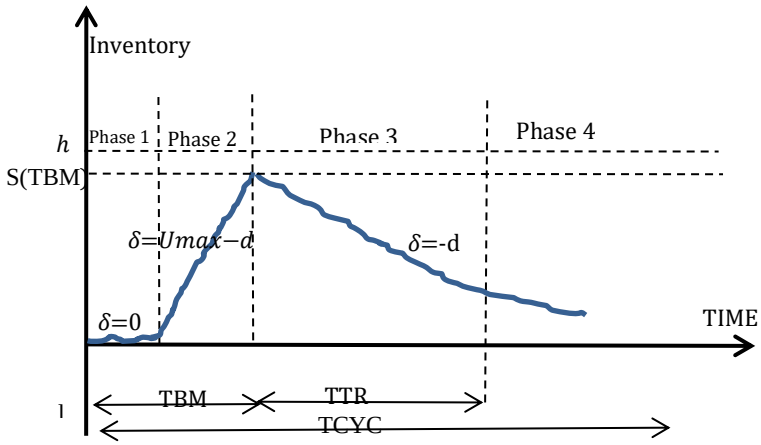


**Fig 4**. Scenario 3

**2-2-4-Scenario 4**

In scenario 4, after period A and before the inventory is completed, the machine is damaged and maintenance operations begin. And the stock will be filled up to $S(TBM)$. Maintenance operations after the end of the inventory end, and face loss. Scenario 4 is shown in figure 5. Assumptions: $TBM > A \;\; and \;\; (TBM - A)(U_{max} - d)\, h \;\; and \;\; TTR > \frac{S(TBM)}{d}$

The inventory cost $\mathbf{L_{s_4}}(\boldsymbol{A})$ corresponding to scenario 4 is expressed in equation (20):

$$L_{s_4}(A) = \frac{C_s(STBM)^2}{2d} + \left[\frac{C_s(TBM - A)}{2} - C_l\right](STBM)C_l dTTR \tag{20}$$

The average cost of inventory in scenario 4 is shown in equation (21). The probability of scenario 4 is expressed in equation (23).

$$E(L_{s_4}(A)) = \int_a^b \frac{C_s(sE(TBM))^2}{2d} + \left[\frac{C_s(E(TBM) - A)}{2} - C_l\right]\left(E(STBM)\right)C_l E(d)\mu_d. \tag{21}$$

With

$$\big(E(STBM)\big) = (E(TBM) - A)\big(U_{max} - E(d)\big) = \left(\int_0^m R(t)dt - A\right)(U_{max} - \frac{a+b}{2}) \tag{22}$$

$$P(S_4) = ind(m > A) \times R(A) \times \left(1 - \frac{ind\left(m \geq \frac{h}{U_{max}-d} + A\right) \times R\left(\frac{h}{(U_{max}-d)} + A\right)}{R(A)}\right) \times (1 - G_d\left(\frac{s\big(E(TBM)\big)}{d}\right) \tag{23}$$

With

$$\frac{G_d\big(E(STBM)\big)}{d} = \int_0^{(E(STBM))/d} g_d(u)\, du \tag{24}$$

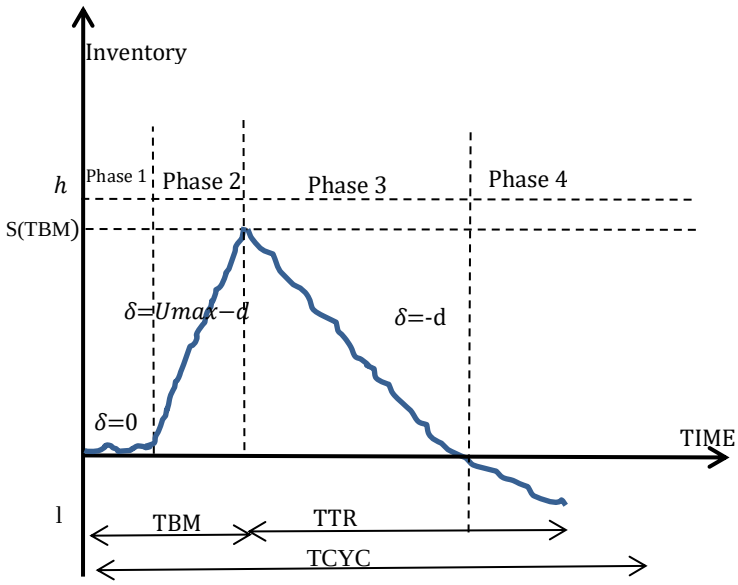


**Fig 5.** Scenario 4

**2-2-5- Scenario 5**

In scenario5, before period A, the machine is damaged and maintenance operations begin. Scenario 5 is shown in figure 6. Assumptions: $TBM < A$The inventory cost $\mathbf{L_{s_5}}(\boldsymbol{A})$ is restricted only to the incurred loss:

$$L_{s_5}(A) = C_l(dTTR) \tag{25}$$

The average cost of inventory in scenario 5 is shown in equation (26).

$$E\big(L_{s_5}\big) = C_l(E(d))(\mu_d) \tag{26}$$

The probability of scenario 5 is expressed in equation (27).

$$P(s_5) = \big(1 - ind(m > A)P(\xi > A)\big) = (1 - ind(m > A)R(A)) \tag{27}$$

Given the scenarios in the preceding sections, the sum of the probabilities of the scenarios must be equal to (1) according to equation (28).

$$\sum_{i=1}^{5} P(s_i) = 1 \qquad \forall(m, A, h) \tag{28}$$

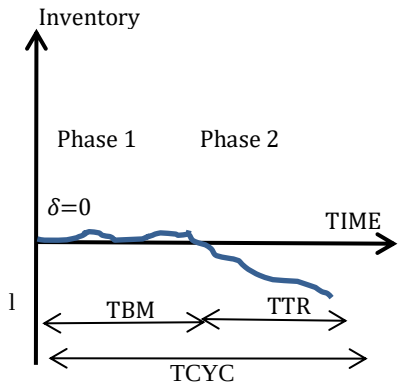


**Fig 6.** Scenario 5

## 2-3-The average total cost per time unit

The total average cost per time unit is the sum of the unitary costs related to maintenance and inventory control as they have been presented in this section. This total expected cost is expressed as a function of the three decision variables: $m, h$ and $A$:

$$CT(m, A, h) = \bar{\delta}(A, h, m) + \varphi(m) \tag{33}$$

$$CT(m, A, h) = \frac{\sum_{i=1}^{5} P(s_i) E\left(L_{s_i}\right) + C_{cm}\, F(m) + C_{pm} R(m)}{\int_0^m R(u)du + \mu_p\, R(m) + \mu_c F(m)} \tag{34}$$

## 2-4- Optimization

The objective consists of finding the optimal values of the decision variables that minimize the total average cost per time unit under the constraint of a minimum required stationary availability level $K$.

$$SA(m) = \frac{\int_0^m R(u)du}{\int_0^m R(u)du + \mu_p R(m) + \mu_c F(m)} \tag{35}$$

For systems with increasing failure rates (for which preventive maintenance is generally recommended), the stationary availability function is concave in m, which means that it has a unique maximum. Hence, obtained for a given situation, the following nonlinear optimization problem:

$$\min Z = CT(m, A, h)$$
$$St: SA(m) \geq K$$
$$(m, h, A) \in (R^* \times R^* \times N) \tag{36}$$

## 2-5- Description of the algorithm

A numerical algorithm is presented with an iterative method to determine the time interval between $[\mathrm{m}_1, \mathrm{m}_2]$, $[\mathrm{A}_1, \mathrm{A}_2]$, and the buffer inventory value $[\mathrm{h}_1, \mathrm{h}_2]$ to minimize the total cost is shown in equation (34) (Rezg and et al. 2008). Figure 7 shows the graphical process of solving, as the failure rate increases and the production system is concave, the values of $[\mathrm{m}_1, \mathrm{m}_2]$, $[\mathrm{A}_1, \mathrm{A}_2]$, $[\mathrm{h}_1, \mathrm{h}_2]$, are first determined. If there are values of the decision variables, the acceptable level of availability will be calculated. The maximum availability is introduced by k (see figure 7). In the next section numerical algorithm to find strategy is developed. $(A, h)$ are random and d to be stochastic. The algorithm is

implemented using $MATLAB$ software and the results are presented in numerical examples in tables 3 and 4.

**Case 1:** $K\epsilon[0, SA(+\infty)] \rightarrow [m_1, m_2] = [m_k, +\infty]$

**Case 2:** $K\epsilon[SA(+\infty), SA^*] \rightarrow [m_1, m_2] = [m_1, m_2]$

**Case 3:** $K\epsilon[SA^*, 1] \rightarrow [m_1, m_2]$ Not available

With:
SA* is maximum of availability level
And

$$SA(+\infty) = \lim_{m \to +\infty} SA(m) = \frac{\mu}{\mu + \mu_c} \tag{37}$$

In this paper, the inventory control optimization model and maintenance and repair operations are presented in a single-machine production system, taking into account the potential demand. There are 5 scenarios for solving that the total probability of 5 scenarios should be equal to 1.As shown in figure 7, we entered the initial value of the input data, and then calculated the maximum availability level. If the availability level is in the range $[SA^*. 1]$(case 3), there is no answer for $m_1. m_2$ If not, it means that the availability level is in the range $[SA(+\infty). SA^*]$ (case 2), the values of $m_1. m_2$are calculated. Since the present algorithm is of numerical type, we convert continuous space into discrete space, use different values of cost for different m, h, and A values, and calculate an optimal value.

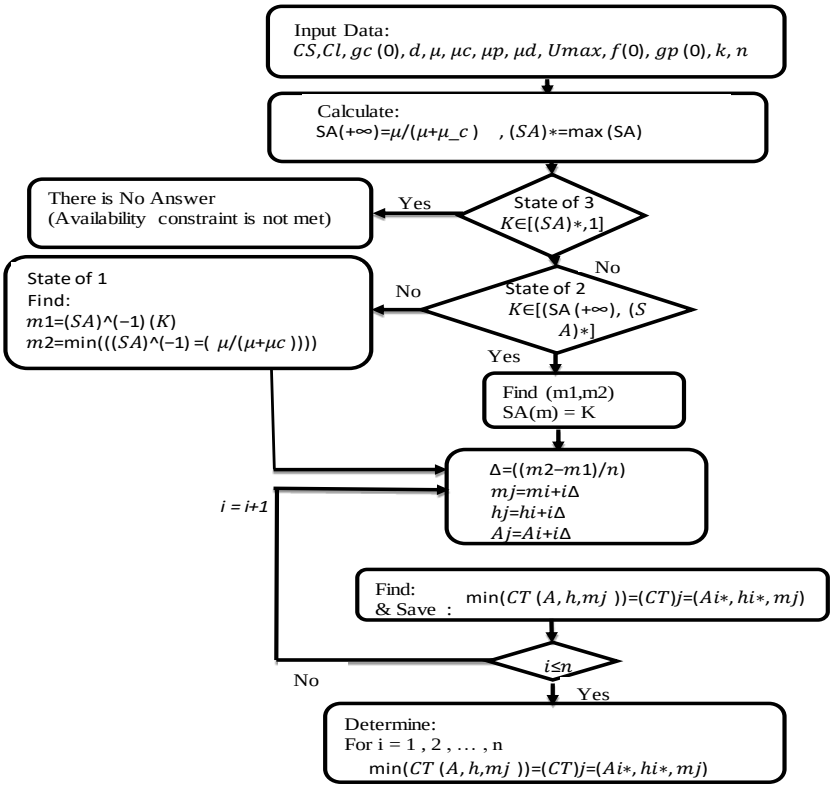


**Fig7.** Presented algorithm

## 3-Results and discussion

### 3-1-Numerical example

Consider a single machine manufacturing system with a random failure, and provides the amount

stochastic demand value of uniform (0.15, 0.90) unit/time unit. The ability to produce a machine, the maximum is 1 unit/ time unit. When is not in storage or defective, it produces the size of $d$ at other times $U_{max}$. It is assumed that $U_{max}$is greater than $d$. When there is a loss of machines being produced at a rate of $U_{max}$, the excess production amount is stored as a protective inventory till the time of maintenance, the machine is completely stopped responding to the demand. At the starting time the production rate is $U_{max}$. Based on the production policy, maintenance must be done during the time $m$ or when a failure occurs, depending on which one happens sooner. Historic data show that the failure time of machines in production systems follows the Weibull distribution; The average life of the production system is assumed to be 88.6 units of time, also, Time operation, maintenance of log-normal distribution with ($\mu_c$= 50, $\delta_c$ = 2), Distribution preventive maintenance $g_p(t)$ follows a lognormal distribution follow with ($\mu_p$ = 10, $\delta_c$ = 1.5). Costs include holding cost $C_s$= 2, loss cost $C_l$ = 250, preventive maintenance $C_{cm}$ = 2,000 and corrective maintenance action $C_{pm}$ = 300, Demand follow uniform distribution $d$ = uniform (0.15,0.90) unit/time unit, maximum production rate: $U_{max}$ = 1 unit/time unit, probability distribution function associated to the machine time to failure $F(t)$: Weibull distribution with (2,100) parameters, maximum level of accessibility: $K = 70\%$. A summary of the input data are given in table 2 and 3.Using the algorithm in figure 7, with $K = 70\%$ and $[\mathrm{m}_1, \mathrm{m}_2]$ = [37, 72], the values in table 4 are obtained for optimal strategies. The values of the decision variables are proportional to the optimal scenario and the probability of each scenario as shown in tables 4 and 5.

**Table 2.** Summary of input data

| | Distribution | Parameter | |
|---|---|---|---|
| Time operation maintenance: $g_c(t)$ | log-normal distribution | $\mu_c$ | 50 |
| | | $\delta_c$ | 2 |
| Distribution preventive maintenance: $\mathrm{g_p(t)}$ | log-normal distribution | $\mu_p$ | 10 |
| | | $\delta_c$ | 1.5 |
| Demand | uniform distribution | $d$ | (0.15,0.90) |
| Time To Failure: F(t) | weibull distribution | Parameters | (2,100) |

**Table 3. Summary of cost input data**

| | | | |
|---|---|---|---|
| Costs | holding cost | $\mathrm{C_s}$ | 2 |
| | loss cost | $\mathrm{C_l}$ | 250 |
| | preventive maintenance | $\mathrm{C_{cm}}$ | 2,000 |
| | corrective maintenance action | $\mathrm{C_{pm}}$ | 300 |

**Table 4.**The values of the decision variables are proportional to the optimal scenario with $\mathrm{C_s}$ = 2

| $m$*(time units) | $A$*(time units) | $h$* | $CT$* (monetary units/time unit) |
|---|---|---|---|
| 38 | 28.03 | 28.23 | 301.47 |

**Table5.** Optimal the probability of each scenario with $\mathrm{C_s}$= 2

| $p_{s_1}$= | $p_{s_2}$ | $p_{s_3}$ | $p_{s_4}$ | $p_{s_5}$ |
|---|---|---|---|---|
| 0.3333 | 0 | 0.6667 | 0 | 0 |

As shown in table 5, scenarios 1 and 3 are more probability to occur and the most possible of the scenario is almost 0.The most probable scenario is scenario 2, where the production system begins to stock buffers but before reaching level h in $m$=38, the machine crashes and starts maintenance. It is a

small probability ($p_{s_1}$ = 0.3333) to occur after the buffer level is completed and then preventive maintenance is made and a new cycle begins. Now reduce the cost of $C_s$ from 2 to 1. The results are shown in tables 6 and 7.

**Table 6.**The values of the decision variables are proportional to the optimal scenario with $C_s$ = 1

| $m$*(time units) | $A$*(time units) | $h$* | $CT$* (monetary units/time unit) |
|---|---|---|---|
| 41 | 27.59 | 11.81 | 287.27 |

**Table 7.**Optimal the probability of each scenario with $C_s$ = 1

| $p_{s_1}$= | $p_{s_2}$ | $p_{s_3}$ | $p_{s_4}$ | $p_{s_5}$ |
|---|---|---|---|---|
| 0.1538 | 0 | 0.8462 | 0 | 0 |

As shown in table 6, scenarios 1 and 3 are more probable to occur and the most possible of the scenario is almost 0. By reducing the cost from 1 to 2, the buffer stock time for the most probable scenario is reduced from 28.03 to 27.59. Scenario 3 is the most optimal strategy in which, before reaching the $h$ level, the machine is failed and maintenance begins and ends before the loss occurs and a new cycle begins. In this scenario buffer stock equals 11.81. While in the former case is equal to 28.23. The total cost is also reduced compared to the first case. Reducing maintenance costs from 2 to 1 has had an impact on maintenance operation times. In fact, in each replication, the best value is calculated for $A, h$, and scenario, and then it is observed which one has a lower cost. Finally, for minimum cost, the value of $A, h, m$, and scenario probability is calculated. As other factors are consistent, sensitivity analysis in figure 8 is shown to reduce inventory costs.

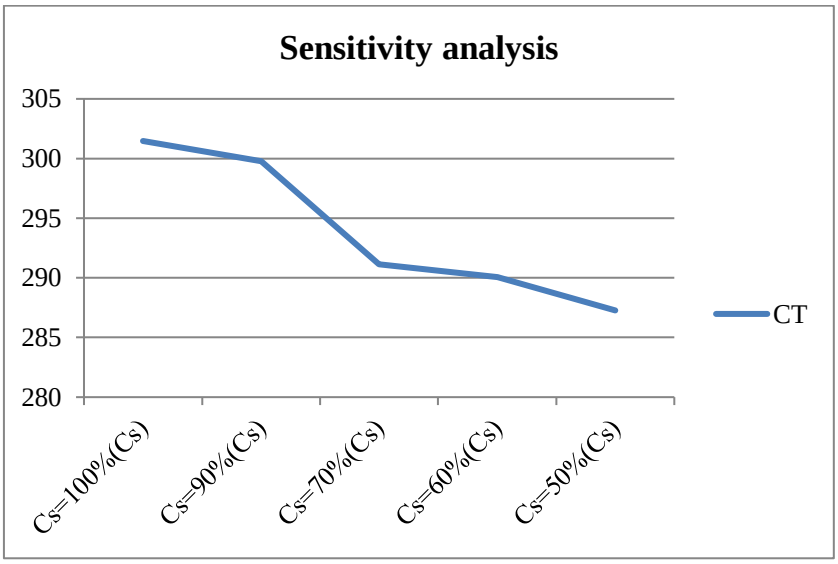


**Fig 8.**Sensitivity analysis

## 4-Conclusion

In this paper, a model for joint optimization of maintenance and inventory control with random failure is presented. The decision-maker is able, using a decision-maker, to find the decision variables by the desired quantities of production capacity, demand, maintenance and repair costs, and the distribution of machine failure, which minimizes the total cost. These decision variables are Time to create a protective inventory size and when the machine stops for maintenance operations.

Finally, a numerical example was used to examine the proposed solution approach. The model was solved using numerical example parameters by the proposed algorithm. The results show that the preventive maintenance policy contributes to a reduction in the overall incurred cost. The results show that the joint optimal policy is generally sensitive to the cost, In other words, by reducing the cost of inventory, the total cost is reduced. The production/maintenance of the manufacturing cell considered. Also, the results show the most probable scenario 3 is a scenario that is not a loss but failed machine before it reaches level $h$. The optimal value is $h$=11.81. For future studies, shortages can be considered as a back of order; other methods can be used to make demand possible, such as robust, etc. Instead of the production system of a single machine, several machines can be used.

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## Appendix A

Proof:

$$E(TBM) = \int_0^m R(t)dt \tag{A.1}$$

$$E(TTR) = \mu_d\ , \qquad \mu_d = \mu_p R(m) + \mu_c F(m) \tag{A.2}$$

$$d \sim U(a,b) \rightarrow\rightarrow\rightarrow E(d) = \frac{a+b}{2}, \qquad VAR(d) = \frac{(b-a)^2}{12}, \tag{A.3}$$

$$E(d^2) = \frac{(b-a)^2}{12} + (\frac{a+b}{2})^2 \tag{A.4}$$