Competition of risk-averse and risk-neutral financial chains under government policy-making

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Abstract
This research analyzes the competition of two risk-neutral and risk-averse centralized financial chains (FCs) while the government regulates the market to prevent the disproportionately costly interest rates by eliminating unreasonable arbitration of transactions. Each FC consists of an investor and a broker, helping to fund the financial needs of the capital-constrained firms. Utilizing the Stackelberg game theory method, we formulate two-level and three-level optimization models for four potential scenarios and create an integrative structure for evaluating scenarios through the perspectives of both FCs’ risk orientations (i.e. risk-neutral and risk-averse) and two policies of the government (i.e., deregulation and regulation to mitigate the effect of arbitration). We found that under the government’s regulation policy, risk-averse FCs cause a lower amount of arbitration than risk-neutral FCs. We also realized that the increased volume of risk-free interest rate results in less arbitration. Results also demonstrate that the regulator can organize the competing FCs in the market by enforcing limits on interest rate and restricting costly interest rates by controlling the impacts of arbitration, which ensures a steady economy and encourages the funding of capital-constrained businesses.

Keywords Financial chain, government policy-making, risk-neutral, risk-averse, game theory

1- Introduction
The role of capital markets has been more critical since the financial crisis (2007-2009) and thus, as the part of that industry, financial intermediaries have played a crucial role (Woodford, 2010). There is an undeniable need for economies in contributing to developed financial structures and offer improved financing schemes to obtain an enhancement in GDP growth. Financing mechanisms with poor infrastructure permeate through whole markets and render them vulnerable to economic decline during the period (Ang, 2008).

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Trends in financing evidence reported by Ajello (2012) indicate that 35 percent of corporates’ funding needs are generated by capital markets. The financial crisis shows that major arbitrators will affect the market and take advantage of their arbitration activities. Price fluctuations and financial impacts push the economy through extreme depression and instability, while as a result of these shocks; the arbitrators make huge profits (Musolino, 2012).

The regulator will monitor the abuse of arbitrators from monetary transactions. Carfì and Musolino (2012) suggested a game theory model which limits the arbitrary activities without eliminating business income and to maintain a competitive stock market through the use of tax policies for company transactions. Therefore, capital institutions such as European nations should be free from the arbitrators’ attacks. Carfì and Musolino (2014) addressed a model when the tax is levied on risky financial activities with an introduction to game theory in fuel and currency marketplace. These studies resulted in an intervention framework that has two significant effects: restricting the behavior and numbers of arbitrators and sharing the income between economic agents and medians. Specifically, on the currency market, Della Posta (2016) stressed preventing the arbitration at first steps. Lam (2002) has developed a system to examine currency trading and policies of the government, as well as Qiao (2013), which examined the effect of government policies on funding decisions in China.

Abuse of financial activities by arbitrators could create problems in various financial chains (FCs), retailers, government agencies, and money-related procedures. For example, the recession that threatens the European economy’s bonds, suggests that arbitrary operations in the business and profit-making by arbitration influence the stock markets (Carfì & Musolino, 2012). In controlling financial uncertainty by interest rates, Hoffmann and Schnabl (2016) emphasized on economic stability in the developing markets and policy-makers’ objectives. The responsible and rational governments may have a leading task in regulating the conduct of the arbitrators to equilibrate the market and FCs, or in promoting the competitive industry through subsidy strategies (Xiao et al., 2017).

While some studies by Carfì and Musolino (2014), Hafezalkotob (2015), and Reza-Gharehbagh et al. (2019b) are related to the subject, wherein Carfì and Musolino (2014) established one game model for the competition of hedging and trading activities on the currency and oil sectors; Hafezalkotob (2015) analyzed competition among different green-conventional supply chains (SCs), in which the government follows different goals by financial regulation in product chains; and Reza-Gharehbagh et al. (2019b) investigated the competition of FCs under government regulation, to the best of the author’s knowledge, the principles of risk behavior in a rivalry between FCs are rarely investigated in the previous literature, although the significance of the FC participants’ arbitrary actions and their impact on high competition levels are discussed in the literature. Therefore, there is no research exploring the effect of government intervention strategies (i.e., regulation or deregulation) on the competitiveness between risk-averse and risk-neutral FCs to reduce the financial market volatility effects via controlling the interest rates.

To close this research gap, current paper provides a theoretical framework for the Stackelberg game and formulates several problems of optimization for the competition of two risk-averse and risk-neutral FCs under the government deregulation and regulation policies. Consequently, three major contributions of our study include: (i) the government acts as the leading decision-maker in the competitiveness of FCs in the financial market. The regulation or deregulation choices are coordinated with the decisions of the FCs in two-stage programming framework; (ii) the regulator pursues a policy of arbitration restriction; (iii) competing FCs have specific risk attitudes, including risk-averse and risk-neutral orientations, and the government has the right to enforce interest rate limits. Therefore, under government various policies, we evaluate the issue across four scenarios to control the financial market.

The rest of our study is arranged as follows: The previous literature is discussed in section 2. Section 3 outlines the notations and assumptions, and section 4 includes the structure of our model. Section 5 explains the comparison through graphical descriptions of scenarios and provides some strategic perspectives. Finally, section 6 ends with observations and provides guidelines for future study.
2- Literature review

The problem of financing an enterprise is a challenge distinguished by financial systems and economies. While policy-makers are attempting to handle the problem, scientists have researched the issue for possible causes. Financial markets in China as one of the most competitive and largest economies are growing along with government supports (Masood & Sergi, 2011).

Small and medium-sized businesses are applying for bank loans to provide their working capital and to finance their projects. Financial institutions call for guarantees of up to 60% of the debt value to handle the burden of repaying the loan by small and medium-sized enterprises, while they are new and unable to provide guarantees. Intermediaries take full advantage of the opportunities by offering a payment-based incentive of about 3 percent of the loan’s value and guaranteeing the redemption of loans to the bank. Furthermore, the cost of financing small and medium-sized enterprises has increased due to risky intermediary actions. Thus, high rates of debt financing in small and medium-sized businesses contribute to reduced productivity (Cheng, 2016). Li (2011) compared Chinese and Korean companies relying on financial intervention by policy-makers to offer financial benefits and control interest rates.

Our study is closely connected to different research bodies, including money transactions and governmental financial regulation, wherein the game theory paradigm is largely used. The first group of studies is related to an effort by corporate finance to link supply to the demand for capital, the so-called supply-need approach used for managing the investment and revenue (Zhao et al., 2015). Definition of “financial chains” that indicate credit and debt relationships forms socio-economic structures through different networks, including social relations and capital transfer (Sokol, 2017). Gao et al. (2018) investigated an online lending-based financing scheme in a SC using a Stackelberg game approach and found the financing platform’s interest rate as a decisive factor. Li and Wang (2020) studied the financing models including trade credit, bank loan, partial credit guarantee, and internet-based platforms in a SC with risk-averse and initial capital of capital-constrained member as the significant factors in financing a SC.

Risk management in the chains has become a common subject. Essentially, in the sense of the SC, where there are many causes of ambiguity, such as demand volatility and supply instability, the output of the SC would, therefore, be impacted and unpredictable. As a consequence, ambiguity arises and SC managers need to take ambiguity-based decisions (Chiu and Choi, 2013; Malekmohammadi & Makui, 2019; Dehghani Sadrabadi et al., 2019).

We are now discussing the governmental policies by financial participation, often established using a game-theoretical paradigm. Governments also use reward and disincentive programs to control companies on the market. Cooray (2011) studied two facets of the policy in 71 economies on two dimensions of money-related categories Additionally, Ang (2008), by government legislation in Malaysia, discussed two factors, namely financial and economic growth. Some researchers have historically used a game theory technique to study government regulation in SC. Hafezalkotob (2015) has established a Stackelberg game template with two green and normal SC competitions that are influenced by policies of the government. Furthermore, Hafezalkotob (2017) used a game-theoretic structure to examine eco-friendly and non-green SCs’ competition regarding government regulation. State interference in public service programs for social services is considered by Chen et al., (2015) and the competitiveness of green SCs with financial policies is discussed by Sheu (2011). Other academics also utilized a game approach to identify approaches for investing in competitive pressure (Morellec & Schurhoff, 2011; Arasteh, 2016). Some studies discuss the impact of a game-based strategy on policy-making (Hafezalkotob et al., 2018). Although there are some related studies on game approach consistency (Chu et al., 2018; Zutshi et al., 2018), we concentrated primarily on the research of governmental intervention strategies with the game theory system.

This study distinguishes from the aforementioned works wherein the risk behaviors in the financial chains, different government intervention approaches, and game-theoretical methodology are considered. We construct a theoretical framework for the Stackelberg game and propose two and three stage optimization problems from the perspectives of the FCs’ risk orientations (i.e., risk-neutral and risk-averse) and government deregulation and regulation policies.
3- Model notations and assumptions

The following section provides the notations and assumptions used throughout the paper. We assume that two rival FCs are selling partly replaceable financial products type \( i \) and \( j \) (i.e., \( i = 1, 2; j = 3-i \)). Table 1 represents notations of the models that have been developed.

<table>
<thead>
<tr>
<th>Notations</th>
<th>Descriptions</th>
</tr>
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<tbody>
<tr>
<td>( \alpha_i )</td>
<td>market demand baseline for the product ( i );</td>
</tr>
<tr>
<td>( R_i )</td>
<td>interest rate determined by FC ( i ), ( R_i \geq r_i &gt; 0 );</td>
</tr>
<tr>
<td>( \eta )</td>
<td>demand sensitivity to the corresponding FCs’ interest rate, ( 0 \leq \eta \leq 1 );</td>
</tr>
<tr>
<td>( \beta )</td>
<td>substitutability coefficient of two rival financial products, ( 0 \leq \beta \leq 1 );</td>
</tr>
<tr>
<td>( r_f )</td>
<td>risk-free interest rate;</td>
</tr>
<tr>
<td>( \lambda_i )</td>
<td>risk-aversion coefficient of the FC ( i );</td>
</tr>
<tr>
<td>( \sigma_i )</td>
<td>demand uncertainty coefficient related to financial product type ( i );</td>
</tr>
<tr>
<td>( \bar{R}_i )</td>
<td>interest rate upper limit determined by government on the financial product type ( i );</td>
</tr>
<tr>
<td>( Arb )</td>
<td>total arbitration amount;</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>arbitration impact of corresponding FC;</td>
</tr>
</tbody>
</table>

Also, we define vector \( R = (R_1, R_2) \) as FCs’ decision variables. Assumptions that describe analysis scope for model formulation are given by:

**Assumption 1.** Rival FCs supply financial products for consumer demand. Capital-constrained enterprises decide on the funding requirements and choose financial products according to the interest rate of each FC \( (R_i; i = 1, 2) \).

**Assumption 2.** As the leading player of the Stackelberg game, the government intervenes in the market by deciding on the interest rate limits of each FC. Depending on the risk attitude of each FC, it is presumed that there are two deterministic and stochastic demand mechanisms for financial products which are functions of interest rates \( (R_i; i = 1, 2) \) and market baseline. For each financial commodity, the deterministic consumer demand is as follows:

\[
D_i^{(n)}(R) = a_i - \eta R_i + \beta R_j, \quad i = 1, 2; \quad j = 3-i, \tag{1}
\]

where \( n = 1:2, i \neq j \). The superscript “\( n \)” is utilized to indicate the scenario number.

The stochastic market demand \( D_i^{(n)} \) related to each financial product is given by:

\[
\hat{D}_i^{(n)}(R) = \bar{a}_i - \eta R_i + \beta R_j, \quad i = 1, 2; \quad j = 3-i, \tag{2}
\]

Where \( n = 3:4, i \neq j \) and the market demand of product type \( i \) depends on stochastic market base for product \( i \), with mean \( \bar{a}_i > 0 \), and variance \( \sigma_i^2 \) and the interest rates of two rival FCs.

The proposed demand functions are equivalent to the demand functions assumed by previous studies such as FCs’ demand function by Reza-Gharehbagh et al., (2019b) and SCs’ demand function by Hafezalkotob (2017) and Xiao and Yang (2008).

**Assumption 3.** Our suggested Stackelberg game includes two participants: the government as a leading decision-maker and the rival FCs as followers. We consider the centralized structure of decision-making for competing FCs. When the system is centralized, the interest rates are determined concurrently by two FCs.

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The playing game framework of the leader and the subsequent followers could be solved by utilizing backward induction methodology.

4- Model formulation

Assume that two rival FCs in the financial market have financing solutions in funding requirements of capital-constrained companies. Each FC is composed of one investor and one intermediary. We presume that each FC has a particular degree of arbitration, while arbitrators aim to enhance their profits by setting higher interest rates and aroused volatilities. The government as the leading player intervenes in the market to determine interest rate limits and track the effects on the intermediary’s arbitrary operations. If the arbitration generates further uncertainty in the financial markets, the government imposes tighter administrative restrictions and diminishes the impacts. Figure 1 shows the model schematic using a diagram.

We discuss two main risk attitudes of centralized FCs including risk-neutral and risk-averse. The government can adopt different policies including deregulation (scenarios 1 and 3) and regulation to minimize the arbitrations caused by excessively high interest rates of intermediaries (scenarios 2 and 4). In figure 2, four cases show the contrast of FCs’ risk orientation and policy-making of the regulator.

Based on the backward induction methodology in each risk attitude of FCs, first, the Nash equilibrium problem of FCs’ competition is presented and then the government’s models are considered as the leading decision-maker (Reza-Gharehbagh et al., 2019a).

4-1- Model of risk-neutral FCs: Deterministic model

In this section, we present the model formulation for the competition of risk-neutral FCs under government deregulation and regulation policies.

4-1-1- Government deregulation policy (scenario 1)

At first, with focusing on risk-neutral FCs under government deregulation in scenario 1, we formulate the model for centralized chains. According to deterministic demand function (i.e., equation 1), the profit of FC is given as follows:
\[ \Pi_i^{(n)}(R) = (R_i - r_f) \left( a_i - \eta R_i + \beta R_j \right), \quad i = 1, 2; \quad j = 3 - i, \quad n = 1. \quad (3) \]

To maximize profits, FCs decide on the amount of interest rate in a competitive environment. We derive the following proposition from equation (3).

**Proposition 1.** The interest rate \( R_i \) of rival FCs with integrated structures in scenario 1 and the Nash equilibrium problem is given by:

\[ R_i^{(n)} = r_f + A_i^{(n)}, \quad (4) \]

Where \( A_i^{(n)} = \left[ a_i - (\eta - \beta) r_f \right] + \left( \beta / 2 \eta \right) \left[ a_j - (\eta - \beta) r_f \right] \); \( i = 1, 2, j = 3 - i, n = 1 \)

\( A_i^{(n)} \) is the risk-neutral FCs’ net marginal benefit according to the risk-free interest rate \( r_f \).

When \( A_i^{(n)} < 0 \), the FC\( i \) will recede and FC\( j \) attempts to monopolize the market.

All proofs are prepared in the Appendix.

From Proposition 1 and equations (1) and (3), it could be obtained that the optimum demand for FC\( i \) is as \( D_i^{(n)} = A_i^{(n)} \) and optimum profit of FC\( i \) is as \( \Pi_i^{(k)} = \left( A_i^{(n)} \right)^2 \). By the fact that the situation \( A_i^{(n)}, A_j^{(n)} > 0 \) could warranty the FCs’ profit, but that is not warranted the market will be persistent and safe. Thus, the minimum net income \( \Pi_i \) and \( \Pi_j \) are supposed for FCs to remain in a competing marketplace. The case can be presented as \( A_i^{(n)} \geq \sqrt{\Pi_i}, A_j^{(n)} \geq \sqrt{\Pi_j} \).

**4.1.2- Government regulation policy (scenario 2)**

We now focus on risk-neutral centralized FCs’ problems as the game follower under government regulation in scenario 2 and form the model. Based on the demand function (i.e., equation 1) and government regulatory policy, the profit of FCs are presented as follows:

\[ \Pi_i^{(n)}(\bar{R}) = (R_i - r_f) \left( a_i - \eta R_i + \beta R_j \right), \]

st:
\[ R_i \leq \bar{R}_i; \quad i = 1, 2, \quad j = 3 - i, \quad n = 3. \quad (5) \]

Rival FCs set their interest rates \( \bar{R}_i; i = 1, 2 \) in the rivalry market concerning the determined upper limit on the interest rate by the government \( \bar{R}_i \). Proposition 2 is obtained from equation (5).

**Proposition 2.** The interest rate \( R_i \) of FCs with centralized structures and equilibrium status for competition in scenario 2 is derived satisfying the below KKT conditions:

\[ a_i - \eta R_i + \beta R_j - \eta (R_i - f_i) - \tau_i = 0, \quad (6) \]

\[ \tau_i (\bar{R}_i - R_i) = 0, \quad (7) \]

\[ (\bar{R}_i - R_i) \geq 0, \quad (8) \]

\[ \tau_i \geq 0; \quad i = 1, 2, \quad j = 3 - i. \quad (9) \]

Corollary 1 is derived from Proposition 2.

**Corollary 1.** In regulation policy, the government can orchestrate the equilibrium interest rates by determining a threshold for the interest rate.

Next, we focus on the modeling of government problem as the game leader in scenario 2.
In the 2\textsuperscript{nd} scenario (i.e., \( n=2 \)), upper limit on the interest rate of each FC \( \bar{R}_i \) are levied by the regulator to reduce the arbitration impact of FCs’ activities; thus, the formulation of government problem is provided as follows:

\[
Min \ Arb^{(n)}(\mathbf{R, \bar{R}}) = \phi_i D_i^{(n)} + \phi_j D_j^{(n)} = \phi_i A_i^{(n)} + \phi_j A_j^{(n)},
\]

st:

\[
\prod_i^{(n)} = \left( A_i^{(n)} \right)^2 \geq \prod_i,
\]

\[
Max \ \prod_i^{(n)}(\mathbf{R, \bar{R}}) = (R_i - r_f) \left( a_i - \eta R_i + \beta R_j \right).
\]

st:

\[
R_i \leq \bar{R}_i; \quad i = 1,2, \quad j = 3-i, \quad n = 2.
\]

Competing FCs have particular arbitration impacts due to the intermediaries’ arbitrary actions to make higher profits; hence, the regulator attempts to decrease the total arbitration impacts in equation (10). The government problem (i.e., equation 10) restricted to (i) inequalities relevant to the restriction of individual rationality which demonstrate the situation that FCs prefer to stay on the market to meet financial requirements of the capital-constrained firms; otherwise, they prefer not to take part in providing the market needs; (ii) the profit-maximizing problem of FCs as the game followers.

The extended model of government problem regarding the KKT conditions (6)-(9), is as follows:

\[
Min \ Arb^{(n)}(\mathbf{R, \bar{R}}) = \phi_i D_i^{(n)} + \phi_j D_j^{(n)} = \phi_i A_i^{(n)} + \phi_j A_j^{(n)},
\]

St:

\[
\prod_i^{(n)} = \left( A_i^{(n)} \right)^2 \geq \prod_i,
\]

\[
a_i - \eta R_i + \beta R_j - \eta \left( R_i - r_f \right) - \tau_i = 0,
\]

\[
\tau_i \left( \bar{R}_i - R_i \right) = 0,
\]

\[
\left( \bar{R}_i - R_i \right) \geq 0
\]

\[
\tau_j \geq 0; \quad i = 1,2, \quad j = 3-i, \quad n = 2.
\]

The function (11) and constraints (12)-(16) form a nonlinear problem. While it is not broadly applicable to find the optimal solution analytically, a number of methods (e.g., interior level and active set) may be used to solve this problem numerically (Bazaraa et al., 2013). Section 5 provides a detailed empirical analysis.

**4-2- Model of risk-averse FCs: Stochastic model**

In this section, we formulate and present the competition models of risk-averse FCs under government deregulation and regulation policies.

**4-2-1- Government deregulation policy (scenario 3)**

At first, with focusing on risk-averse FCs under government deregulation in scenario 3, we formulate the model for centralized chains. One agent’s risk attitude towards market volatility plays a major role in his decisions such as selling, buying, and investment in business. For instance, businesses hold on with more cash since the management’s risk aversion is still above normal due to the severe strike from prior years’ bankruptcies. Economists believed that the inversion of the economy occurred precisely when the spending on business investment was much smaller than expected if the risk aversion enhanced. A risk-averse agent aims to increase its utility rather than the profit it expects (Xiao and Yang, 2008). To obtain the utility function of each FC and according to stochastic demand function (i.e., equation 2), the profit function of each FC is given as follows:
\[ \hat{\Pi}_{i}^{(n)}(R) = (R_i - r_f)\hat{D}_i, \quad i = 1, 2; \quad j = 3 - i, \quad n = 3. \] (17)

Taking the expectation of equation (17), we obtain the FCs’ expected profit function as follows:

\[ E\left(\hat{\Pi}_{i}^{(n)}(R)\right) = (R_i - r_f)\left(\hat{a}_i - \eta R_i + \beta R_j\right), \quad i = 1, 2; \quad j = 3 - i, \quad n = 3. \] (18)

The variance of the FCs’ profit function is as follows:

\[ \text{Var}\left(\hat{\Pi}_{i}^{(n)}(R)\right) = \left(R_i - r_f\right)^2 \text{Var}\left(\hat{a}_i\right) = \left(R_i - r_f\right)^2 \sigma_i^2; \quad i = 1, 2; \quad j = 3 - i, \quad n = 3. \] (19)

The utility function of FCs regarding each FC’s expected profit in equation (18) and variance in equation (19) is derived as follows:

\[ U_i(R) = E\left(\hat{\Pi}_{i}^{(n)}(R)\right) - \lambda \text{Var}\left(\hat{\Pi}_{i}^{(n)}(R)\right) \]
\[ = (R_i - r_f)\left(\hat{a}_i - \eta R_i + \beta R_j\right) - \lambda \left(R_i - r_f\right)^2 \sigma_i^2; \quad i = 1, 2; \quad j = 3 - i, \quad n = 3. \] (20)

A risk-averse FC aims to enhance its utility and specifically, the utility of each FC is an increasing function of its expected profit but a decreasing function of the volatility and sensitivity to risk. To maximize their utilities, FCs decide on the amount of interest rate in a competitive environment. We derive the following proposition from equation (20).

**Proposition 3.** The interest rate \( R_i \) of rival FCs with integrated structures in scenario 3 and the Nash equilibrium problem, is given by:

\[ \bar{R}_i^{(n)} = r_f + \bar{B}_i^{(n)}, \] (21)

where \( \bar{B}_i^{(n)} = \frac{2(\eta + \lambda_j \sigma_j^2)\left[\hat{a}_i - (\eta - \beta)r_f\right] + \beta\left[\hat{a}_j - (\eta - \beta)r_f\right]}{4(\eta + \lambda_j \sigma_j^2)(\eta + \lambda_j \sigma_j^2) - \beta}, \quad i = 1, 2, j = 3 - i, \quad n = 3. \)

\( \bar{B}_i^{(n)} \) is the risk-averse FCs’ net marginal benefit according to the risk-free interest rate \( r_f \).

When \( \bar{B}_i^{(n)} < 0 \), the FC \( i \) will recede and FC \( j \) attempts to monopolize the market. By the fact that the situation \( \bar{R}_i^{(n)}, \bar{B}_j^{(n)} > 0 \) could warranty the FCs’ profit, but that is not warranted the market will be persistent and safe. Thus, the minimum net utility \( U_i \) and \( U_j \) are supposed for FCs to remain in a competing marketplace. The case can be presented as \( \bar{B}_i^{(n)} \neq \sqrt{U_j}, \bar{B}_j^{(n)} \neq \sqrt{U_i} \).

### 4-2-2- Government regulation policy (scenario 4)

We now focus on risk-averse centralized FCs’ problems as the game follower under government regulation in scenario 4 and form the model. Based on the demand function (i.e., equation 2), each FC’s utility function (i.e., equation 20), and government regulatory policy, the utility of FCs is presented as follows:

\[ U_i^{(n)}(R) = (R_i - r_f)\left(\hat{a}_i - \eta R_i + \beta R_j\right) - \lambda_i \left(R_i - r_f\right)^2 \sigma_i^2, \]

st:

\[ R_i \leq \bar{R}_i; \quad i = 1, 2; \quad j = 3 - i, \quad n = 4. \] (22)

Rival FCs set their interest rates \( R_i; i = 1, 2 \) in the rivalry market concerning the determined upper limit on the interest rate by the government \( \bar{R}_i \). Proposition 4 is obtained from equation (22).
Proposition 4. The interest rate \( R_i \) of FCs with centralized structures and equilibrium status for competition in scenario 4 is derived satisfying the KKT conditions below:

\[
\begin{align*}
\alpha_i - \eta R_i + \beta R_j - \eta \left( R_i - f_j \right) - 2\lambda_i \left( R_i - r_f \right) \sigma_i^2 - \tau_i &= 0, \\
\tau_i \left( \bar{R}_i - R_i \right) &= 0, \\
\left( \bar{R}_i - R_i \right) &\geq 0, \\
\tau_i &\geq 0; \quad i = 1, 2, \quad j = 3 - i.
\end{align*}
\]

Next, we focus on the modeling of government problem as the game leader in scenario 4.

In the 4th scenario \((n=4)\), limits on the interest rate of each FC \( R_i \) are imposed by the regulator to reduce the arbitration impact of FCs’ activities; thus, the formulation of government problem is provided as follows:

\[
\begin{align*}
\text{Min} \quad & Arb^{(n)}(R, \bar{R}) = \phi_i D^{(n)}_i + \phi_j D^{(n)}_j, \\
\text{st:} \\
& U_i^{(n)} \geq U_i, \\
& \text{Max} \quad U_i^{(n)}(R, \bar{R}) = \left( R_i - r_f \right) \left( \bar{a}_i - \eta R_i + \beta R_j \right) - \lambda_i \left( R_i - r_f \right) \sigma_i^2. \\
& \text{st:} \\
& R_i \leq \bar{R}_i; \quad i = 1, 2, \quad j = 3 - i, \quad n = 4.
\end{align*}
\]

Competing FCs have particular arbitration impacts due to the intermediary’s arbitrary actions to make higher profits; hence, the regulator attempts to decrease the total arbitration impacts in equation (27). The government problem (i.e., equation 27) restricted to (i) inequalities relevant to the restriction of individual rationality which demonstrate the situation that FCs prefer to stay on the market to meet financial requirements of the markets; otherwise, they prefer not to take part in providing the market needs; (ii) the profit-maximizing problem of FCs as the game followers.

The extended model of government problem regarding the KKT conditions (23)-(26), is as follows:

\[
\begin{align*}
\text{Min} \quad & Arb^{(n)}(R, \bar{R}) = \phi_i D^{(n)}_i + \phi_j D^{(n)}_j, \\
\text{st:} \\
& U_i^{(n)} \geq U_i, \\
& \alpha_i - \eta R_i + \beta R_j - \eta \left( R_i - r_f \right) - 2\lambda_i \left( R_i - r_f \right) \sigma_i^2 - \tau_i = 0, \\
& \tau_i \left( \bar{R}_i - R_i \right) = 0, \\
& \left( \bar{R}_i - R_i \right) \geq 0, \\
& \tau_i \geq 0; \quad i = 1, 2, \quad j = 3 - i.
\end{align*}
\]

The function (28) and constraints (29)-(33) form a nonlinear problem. As discussed in section 4.1.2, it is not applicable to find the optimal solution analytically and we provide a detailed empirical analysis in section 5.

5- Numerical analysis

We now analyze and discuss our models numerically to demonstrate the impact of policy-making on the economy and the FCs’ competition. This agenda as a tool for leaders to organize economic development is a crucial topic for economists to research and analyze. Properly, Iran’s economy is
strongly driven by fiscal policies of the government, as well as income and GDP that is also impacted by budget shortages and inflation rate. Iran’s central bank as a regulatory agency has the authority to decide on fiscal policies and to track interest rates on the money markets, funding companies, and enterprises, channel trading conduct, and deal with funding problems (Chitsaz et al., 2017).

Due to the financial recession and economic condition of the region, Iranian companies and small and medium-sized firms have faced budgetary difficulties leading to reduced access to the financing capital. Financial market inflation has resulted in excessive interest rates that have exacerbated high debt-borrowing costs and borrowing difficulties for businesses (IMF Research, 2017). As a policy-maker, the central bank is responsible for governing on financial issues and taking proper action to arrange the supply and demand of the industry. The central bank will turn to the regulatory capacity when enforcing a monetary policy or indirectly influence the conditions of the money market as an issuer of high-powered currency (Central Bank Study, 2017).

Within these terms, policy-makers will use their power to implement and utilize policies to control the sector and promote investment. Such targets are closely linked to those that we described as impacts of governmental policies in previous sections (IMF Report, 2017). To get the optimum variables, initial parameter values are displayed as

\[ \bar{\alpha}_i = 5, \bar{\alpha}_j = 3, r_f = 10\% \prod_{FC} = 0, U_{FC} = 0, \eta = 0.4, \beta = 0.2, \varphi_i = 0.9, \varphi_j = 0.5. \]

Figures (3)-(4) demonstrate FCs’ optimal profits in scenario 2 and FCs’ optimal utilities in scenario 4 under various degrees of risk-free interest rate and financial product’s substitutability coefficient. As shown in figure 3, by increasing amount of risk-free interest rate, both risk-averse FCs’ optimal profit in scenario 2 and risk-neutral FCs’ optimal utility in scenario 4 are decreasing amount of risk-free interest rate. As we suppose a higher market baseline for financial product 1, the profit and utility of FC\(_1\) are higher than those for FC\(_2\). In figure (3.b), risk-averse FCs obtain lower utilities than risk-neutral FCs in scenario 2 (see figure 3.a); this occurs because of considering risk-factors for risk-averse FCs in scenario 4. As a result, we realize that those financial service providers behaving as risk-averse agents determine their interest rates close to risk-free interest rates and earn lower utilities.

![Image](image_url)

**Fig. 3.** The optimal profits and utilities of FCs versus risk-free interest rate

In figure (4.a), the optimal profits of two rival FCs in scenario 2 (i.e., the FCs are risk-neutral), are increasing amount of product’s substitutability coefficient. From this result, it is figured out that the higher sensitivity of the product’s demand to rival product’s interest rate encourages the corresponding FC to enhance its market demand and obtains higher profits. While in scenario 4 in figure (4.b), the risk-averse FC makes less risk by increasing amount of substitutability coefficient and obtains smaller utilities. Decreasing amount of corresponding FCs’ utility shows that when
\[ \beta \geq 0.8 \text{ the rival FC could overtake the market and make higher profits.} \]

\[ \Pi^{(2)*}_{FC_1} \quad \Pi^{(2)*}_{FC_2} \]

\[ U^{(4)*}_{FC_1} \quad U^{(4)*}_{FC_2} \]

Figure 4. The optimal profits and utilities of FCs versus coefficient (\( \beta \))

Figure 5 demonstrates the optimal arbitration amount regarding the different levels of risk-free interest rate and substitutability coefficient of financial products. As shown in figure (5.a), an increasing amount of risk-free interest rate yields a decreasing amount of arbitrations, wherein the comparison between FCs’ behavior in scenario 4 and scenario 2 shows that risk-averse FCs are affected more than risk-neutral FCs. As a result, from the viewpoint of optimal minimized arbitrations amount, risk-averse FCs cause lower arbitration amount than risk-neutral FCs, especially when \( r_f \geq 10\% \). We can realize that scenario 4 is the best scenario regarding the government intervention goal in reducing the impact of arbitrary actions. Figure (5.b) illustrates the enhancing amount of arbitrations by an increased level of coefficient (\( \beta \)). It shows that the higher substitutability coefficient of two rival products yields in higher inclination of FCs to obtaining profits and as the result more arbitration effects. When \( \beta \geq 0.5 \), risk attitudes of FCs yield equal arbitrations amount; thus, in higher levels of substitutability coefficient, risk-averse and risk-neutral FCs have the same effects.

\[ Arb^{(4)*} \quad Arb^{(2)*} \]

\[ Arb^{(4)*} \quad Arb^{(2)*} \]

Figure 5. The optimal arbitrations amount versus risk-free interest rate and coefficient (\( \beta \))

Now, we focus on the risk-factors of scenario 4 to investigate the impact of risk items on the optimal upper limits of interest rate determined by the government. As shown in figure(6.a), when the FCs are more risk-averse, the government levies limits on the FCs’ interest rate to decrease the excessively high interest rates. This occurs due to the risk behavior of the FCs in which risk-averse
FCs have fewer arbitration impacts in the financial market. As a result, the smaller amount of interest rate limit determined by government yields in lower interest rates in the financial market; thus, capital-constrained firms as the customer of financial products are the beneficiaries of this situation. Figure (6.b) presents that higher uncertainty in financial products’ demand yields in more strict regulation by the government to reduce the interest rate. In this regard, the government makes policies to decrease the arbitrations in the financial market when the situation is uncertain.

Fig. (6.a): \( \bar{R}_i^{(4)} \) versus \( \lambda_i \)

Fig. (6.b): \( \bar{R}_i^{(4)} \) versus \( \sigma_i \)

**Fig. 6.** The optimal government limit on interest rate versus risk-factors

The results of the established models can be used by FC managers and policy-makers in decision-making procedures. Potential insights produced in this study are provided as follows:

- The government regulation by enforcing interest rate caps prevents overly high amounts of arbitration, which results in a restriction on extremely high interest rates;
- Government regulation on capital markets affects the earnings of FCs and the shareholders and promotes a stable market;
- Risk-averse FCs determine smaller interest rates for financial products and as a result, cause a lower amount of arbitrations in the financial market;
- With an increased amount of risk-free interest rate, FCs attempt to stabilize their interest rates to attain a higher market share.

**6- Conclusion**

In this research, the competition between two risk-neutral and risk-averse centralized FCs was analyzed while the government makes policies on the financial system to reduce the arbitration effects by controlling costly interest rates and facilitate the funding needs of capital-constrained firms. We proposed that two FCs compete to achieve higher benefits by arbitrary behaviors. The government as a leading actor may implement policies of deregulation and regulation to impose limits on the operation of the financial sector and control the impacts of arbitration that are caused by intermediaries’ financial transactions. Besides, four scenarios were analyzed using a Stackelberg game theory approach and proposed optimization models based on the risk preferences of FCs (i.e. risk-neutral and risk-averse) and government policies (deregulation and regulation).

The findings indicate that the strict interest rate limits determined by policy-makers would reduce the effect of arbitration on the financial marketplace, affect the earnings of FCs and their representatives, and preserve a healthy competitive market. Outputs also show that with an increased amount of risk-free interest rate, FCs attempt to stabilize their interest rates to attain higher market share; thus, a lower amount of arbitrations will arise and affect the financial market. We find that the risk-averse FCs determine smaller interest rates for financial products and as a result cause a lower amount of arbitrations in the financial market.
Other possible extensions have been identified in this study. First, this research discusses the role of government in mitigating the impacts of arbitrary decisions whereas other policy intervention strategies will be an important area for research. Second, the further extension of our analysis is to consider the model within asymmetric situations of information that we discussed in the symmetric situation of the information. Third, it may be useful to extend the concept with more than two FCs and with complicated structures.

References


Appendix

Proof of Proposition 1. The competing FCs’ profit function $\Pi_i$ is concave on $R_i$ based on the Eq. $\frac{\partial^2 \Pi_i}{\partial R_i^2} = -2[\eta - \beta(\beta/2\eta)]$. Therefore, the optimum point in $\pi_i$ yields the optimum interest rate:

$\frac{\partial \Pi_i}{\partial R_i} = -2\eta R_i + \beta R_j + a_i + \eta r_f = 0 , \ i \neq j.$

(A.1)

By solving the equations $\frac{\partial \Pi_i}{\partial R_i} = 0, \ \frac{\partial \Pi_j}{\partial R_j} = 0$ obtain $R_i^*$ and $R_j^*$. □

Proof of Proposition 2. The profit-function $\Pi_i$ on $R_i$ is concave according to the equation $\frac{\partial^2 \Pi_i}{\partial R_i^2} = -2[\eta - \beta(\beta/2\eta)]$ and is constrained to $R_i \leq \bar{R}_i$. Thus, the optimal $R_i$ can be obtained by solving the KKT conditions. □

Proof of Proposition 3. In equation $\frac{\partial^2 U_i}{\partial R_i^2} = -2[\eta - \beta(\beta/2\eta)] - \lambda_i \left(R_i - r_f\right)^2 \sigma_i^2$ we know the FCs’ utility function is concave. The optimum interest rates are obtained as follows:

$\frac{\partial U_i}{\partial R_i} = a_i - \eta R_i + \beta R_j - \eta \left(R_i - r_f\right) - 2\lambda_i \left(R_i - r_f\right)^2 \sigma_i^2 = 0, \ i \neq j.$

(A.2)

Proof of Proposition 4. The utility-function $U_i$ on $R_i$ is concave according to the equation $\frac{\partial^2 U_i}{\partial R_i^2} = -2[\eta - \beta(\beta/2\eta)] - \lambda_i \left(R_i - r_f\right)^2 \sigma_i^2$ and is constrained to $R_i \leq \bar{R}_i$. Thus, the optimal $R_i$ can be obtained by solving the KKT conditions. □