

A credit mechanism in coordinating quality level, pricing and replenishment decisions with deteriorating items

Mahdi Ebrahimzadeh-Afrouzi¹, Alireza Aliahmadi^{1*}

¹ *School of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran*

m_ebrahimzade94@ind.iust.ac.ir, pe@iust.ac.ir

Abstract

In this paper, we present a two-level supply chain (SC) consisting of a single manufacturer that supplying one type of deteriorating product to a single retailer that the market demand rate for the product is time-varying and depends on two endogenous variables that include the retail price and product quality. The objective of this paper is to determine simultaneously pricing policy and ordering for the retailer, as well to determine the product quality optimizing strategy for the manufacturer. Firstly, the problem is formulated under a decentralized structure with a manufacturer-Stackelberg game where each member optimizes his/her decisions regardless of the others' profitability. Next, a centralized structure was presented and optimized based upon the whole SC profitability. Although the centralized model improves the quality level of the product and profitability of the entire SC, it may reduce the profitability of each SC members. Then, this paper is developed with an incentive scheme based on credit policy to coordinate this system. Moreover, Numerical examples and sensitivity analysis are presented to indicate the effectiveness of the contract. The results show that the credit contract can lead to perfect coordination, while the coordinated system is more robust than the centralized system. This paper extends the understanding of supply chain coordination in the context of deteriorating items that indisputably have a reciprocal relationship with market time-varying demand in many real-life cases.

Keywords: Supply chain coordination, pricing, quality optimizing, deteriorating product, credit contract

1-Introduction

During recent years, supply chain coordination (SCC) for deteriorating items has been playing a significant role in supply chain management (SCM) that has received considerable attention from practitioners and researchers. Generally, the quality or quantity of deteriorating items is reduced with time, resulting in declining value. This coordination is vital for some retail industries such as drugstores, fruit and grocery stores, dairy and meat shops and so on. Also, it is vital for other supply chain members of these retail industries such as their suppliers and manufactures.

*Corresponding author

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Using the right mechanism at the right time is one of the most important challenges of manufacturers. Similarly, this issue can be defined for other mentioned retail industries.

Specially, the topic is more interesting in places by which small shopping stores are much more than large super markets, shopping malls and chain stores. Degree of economic stability plays an important role in encouraging suppliers/retailers towards using a credit mechanism. In addition, transportation and storage of deteriorating items can cause damage to the environment. For instance, warehouse heating and cooling to ensure a constant temperature typically increase deterioration as time passes.

Given the growing importance of effective SCM, many researchers have focused on coordination issues between members at different levels of a supply chain (SC). In a supply chain, since a decision of one member affect the other members' decisions, which seems each member coordinates his decisions with other member decisions to obtain more profit for the entire supply chain (Cachon 2003). Therefore, a credit policy as a coordination strategy aims to encourage SC members to accept decisions that are optimal for the entire SC and as a result improve in the SC performance. Also, applying a coordination incentive scheme is needed to coordinate SC members to decide properly about replenishment, production, and distribution strategies and consequently satisfy their customers' demand (SeyedEsfahani, Biazaran et al. 2011). The market demand is usually affected by many factors such as the advertisement, service, stock, etc. Marketing literature supports the belief that market demand may indeed vary with the quality level of the product.

According to the importance of the above-discussed issues, the main contribution of the proposed coordination model is to coordinate the SC members' decisions with a deteriorating item under the time-varying demand rate simultaneously. This paper aims to respond to the following research questions.

- How do triple SC structures (i.e. decentralized, centralized and coordinated) can affect the optimality conditions of the whole SC and its members when the market demand is time-varying and sensitive to the retail price and product quality?
- What is the optimal amount of product quality degree and retail price under decentralized and centralized SC structures?
- Is it possible to implement a coordination scheme to persuade the whole SC members to shift them to the centralized optimal solutions?
- Does the proposed credit mechanism always lead to the achievement of whole SC profit in the centralized structure, with the maximum possible profit?

The rest of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 explains the methodology of this research. In section 4, the problem definition, assumptions, and notations are presented, in addition, mathematical models are proposed in three structures decentralized, centralized, and coordinated scenarios. Section 5 contains numerical examples and sensitivity analyses with discussions and finally, section 6 presents conclusions and potential future studies.

2-Literature review

Undoubtedly, price and product quality play main roles in marketing competition. Electron products, semiconductors, and Japan's automobiles occupy the Europe and U.S. market, due to higher quality and lower prices from the 1970s to 1990s (Banker, Khosla et al. 1998). In nowadays competitive life, price and quality are always very important, especially when the cooperation of product quality and pricing policies between supply chain members can influence supply chain competitiveness. In a decentralized supply chain, firms maximize their profits through cooperation and competition, in which firms work together to increase their share of market demand. The research on the impact of product quality and pricing on the market demand started from the marketing literature that focuses on substitutable products. Most early papers on the quality level decision of substitutable products suppose there are two competitive enterprises and marketing demands, which can be affected by price and product quality. (Moorthy 1988) is analyzed quality decisions of two identical firms competing on price and product quality with the non-initial investment cost. Nair and Narasimhan (2006) studied retail price, product

quality, and advertising investment decisions, in which firms compete by price and means of their goodwill that is formed by investments in advertising and product quality. Vörös (2006) considered the price and quality from the viewpoint of the process of product development. He solves the optimal control of price and quality in a continuous model. Xie, Yue et al. (2011) studied about product quality and price decisions in a risk-averse SC. Liu, Zhang et al. (2015), and Maiti and Giri (2015) analyzed the joint dynamic pricing and investment policy for perishable foods, in which the demand is price-quality dependent. Ye, Ma et al. (2016) analyzed quality-dependent price-only mechanisms and found out the selling price of anarchy in a competitive reverse SC set up. In our research, we consider pricing and product quality management policies along with one manufacturer and one retailer.

There are other policies which have been used in coordination contracts such as social responsibility (Panda 2014), safety stock (Chaharsooghi and Heydari 2010), lead time (Zhai, Zhong et al. 2017), rework process (Noori-Daryan, Taleizadeh et al. 2019), sales force incentives (Kovach, Atasu et al. 2018), etc. But, among all of them, the pricing and optimal quality is an important policy that needs to be coordinated; especially in the cases which market demand is sensitive to the selling price and the quality level of product. Therefore, another one of this paper's basic categories can be the coordination of pricing and product quality degree decisions. Major coordination mechanisms consist of, quantity-flexibility contract (Tsay 1999), revenue-sharing contract (Cachon and Lariviere 2005), credit period contract (Goyal 1985), sales rebate contract (Taylor 2002), and buy-back contract (Xiong, Chen et al. 2011, Tibrewala, Tibrewala et al. 2018). Recently, many researchers have paid attention to the credit mechanism to coordinate their developed inventory models. Goyal (1985) is one of the first researchers who named 'credit policy' in EOQ models as 'delay in payment mechanism'. Aggarwal and Jaggi (1995), Jamal, Sarker et al. (2000) applied ordering policy and credit payment mechanism for designing a two-echelon inventory model for perishable items. In another paper, Ho (2011) designed an integrated two-echelon inventory model under credit payment mechanism as an incentive approach in which the market demand rate is deterministic and depends on retail price and credit as well. In the mentioned studies, credit period is determined based on the buyer's situation. Du, Banerjee et al. (2013) have developed a model in which the optimal credit payment period is calculated from the whole supply chain perspective by ordering quantity. In contrast to the most mentioned studies, Heydari, Rastegar et al. (2017) provided a coordination model in which the customer's demand depends on credit and also the optimal credit payment period is determined through considering both the seller and buyer's situation to coordinate the decisions of supply chain members. In this regard, a two-part trade credit mechanism has been considered as another type of credit payment and has been used as a trade-off between discount and delay in payment in recent years (Ho, Ouyang et al. 2008, Gholamian and Ebrahimzadeh-Afruzi 2019).

None of the aforementioned studies considers deteriorating items in their analytical models and its influence on SC coordination. Coordinating SC systems with deteriorating items has an important influence because deterioration is an unavoidable reality. In recent years, many researchers have proposed different coordination strategies for integrated SC systems with deteriorating items. Deterioration is defined as the spoilage, damage, vaporization, dryness, etc., resulting in decrease of usefulness of the original material or product (Jaggi, Khanna et al. 2011, Panda, Saha et al. 2013, Wang, Teng et al. 2014). (Hsieh and Dye 2017) proposed a two-echelon SC model with deteriorating items. When the demand rate is dependent on selling price and the inventory has a deteriorating rate, they proposed a credit incentive option and cooperative investment contract to coordinate the system and derived certain conditions for attaining a win-win outcome.

From the above-mentioned review, deteriorating items that indisputably have a reciprocal relationship with market time-varying demand in many real-life cases was not incorporated into supply chain coordination management. This study aims to fill the above research gaps and contribute to the existing literature by proposing the concept of pricing and the quality of deteriorating items facing the time-varying demand in a coordinated structure. These variables make the chain more flexible and share the surplus profit between the retailer and manufacturer fairly.

3-Research methodology

In this paper, a two-level supply chain (SC) of one retailer and one manufacturer is investigated. The demand rate is time-varying and dependent on price and product quality. At the same time, we consider the problem of pricing and product quality decisions in a two-level assembly system, wherein a manufacturer produces and sells a final deteriorating item to the retailer. Consumers buy a product in consideration of not only its price but also its product quality level which is affected by the quality of each component. Under this demand structure, the market price of the final item is set by the retailer and the product quality is decided by the manufacturer. We describe three possible decision sequences and do a comparative analysis. The purpose of this paper is to analyze the effect of time-varying demand on the optimal decisions and profits of SC members. The mathematical models are developed under three models: (1) decentralized model, (2) centralized model, and (3) coordinated model. In the decentralized model, each SC member maximizes its profit and determines optimal decision variables without considering one central decision-maker or an integrated channel. In this case, the retailer and the manufacturer determine the optimal retail price and the optimal degree of product quality respectively, which are local optimal solutions from the entire SC viewpoint. In the centralized model, it is assumed that there is one channel as an integrated firm which simultaneously determines the optimal value of the decision variables to maximize the entire SC profit. Indeed, the profit function of the centralized model is equal to the sum of SC members' profit functions in the decentralized model. Although the centralized decisions will lead to more profit for the entire SC than decentralized model, a member may have no incentive to accept the centralized structure because its average profit in the centralized model may be less than that in the decentralized model. In this study, a credit policy is proposed as a motivated scheme to coordinate the activities of both SC members into one single framework. Therefore, a coordinated model is formulated based on delay in payment as credit period policy, which is a guaranteed situation for both SC members. Indeed, the coordinated model determines the optimal decisions to maximize the average profits of the entire SC and members in order to motivate them to participate. In the selected mechanism, the bargaining power of the SC members is considered to take advantage from surplus profit in the centralized decision, by profit sharing strategy. Finally, using sensitivity analyses, the efficiency of the proposed model is investigated in reducing the operational costs and increasing the profit and productivity.

4-Problem definition and mathematical Models

The major notations and assumptions are used throughout the entire paper. Additional assumptions and notations are listed when needed.

4-1- Assumptions

- This article considers a two-level SC which consists of a manufacturer supplying one type of deteriorating product to a retailer.
- The market demand rate $D(s, p, t)$ that the retailer faces is dependent on the retail price p and product quality s , and time t . It is assumed to be in the following multi-variable function: $D(s, p, t) = (\alpha - \beta p + \gamma s)f(t)$, $0 \leq t \leq T$ where a , β and $\gamma > 0$, β is the retail price sensitivity parameter, γ is the product quality sensitivity parameter and $\alpha - \beta p + \gamma s > 0$. There are advantages of this type of demand pattern can be seen in Maiti and Giri (2015) paper.
- The manufacturer follows the “lot-for-lot” policy, and he/she carries no inventory.
- The retailer replenishes its inventory when consumers have bought all the items.
- Shortages are not allowed to occur.
- The lead time is zero.

4-2- Notations

Parameters:

Q : Order quantity

c : Manufacturer's production cost per unit

w : Wholesale price per unit

$f(t)$: Time function that measures the efficacy of time on demand, $f(t) > 0$

$q(t)$: Retailer's inventory level at time t

q_r : Stock amount which carried by the retailer

θ : The product's Deterioration rate, $0 < \theta < 1$

h_{r1} : Unit inventory holding cost per unit time for the retailer

h_{r2} : Unit deteriorating cost per unit time for the retailer

τ : Quality cost coefficient per unit, $\tau > 0$

T : Replenishment cycle length

I_r : Retailer's interest rate (The rate which can be earned per \$ by the retailer)

I_m : Manufacturer's interest rate

π_r : Retailer's average profit

π_m : Manufacturer's average profit

π_{sc} : Supply chain average profit

Decision variables:

p : Retail price per unit (decision variable), $c < w < p$

s : Product quality degree (decision variable)

μ : Length of the credit period that the manufacturer offers to the retailer

In this paper, a two-level supply chain consisting of a manufacturer and a retailer with one type of deteriorating product is considered. The demand rate at the retailer's end is dependent on time, the retail price of retailer and quality level of product. At time $t = 0$ (the beginning of each replenishment cycle), the retailer orders Q units of a type of product, with a deteriorating rate, from the manufacturer. The retailer's inventory is depleting at a decreasing rate due to the multi-variable demand until the inventory becomes zero. Thus, we can describe the retailer's inventory level $q(t)$ by the following differential equation:

$$\begin{cases} \frac{dq(t)}{dt} = -(\alpha - \beta p + \gamma s)f(t) - \theta q(t). & 0 \leq t \leq T \\ q(0) = Q. & q(T) = 0 \end{cases} \quad (1)$$

By solving equation (1) and simplifying, we have:

$$q(t) = (\alpha - \beta p + \gamma s)e^{-\theta t} \int_t^T e^{-\theta s} f(s) ds \quad (2)$$

Hence, from equation (1) and equation (2), we get

$$Q = q(0) = (\alpha - \beta p + \gamma s) \int_0^T e^{\theta t} f(t) dt \quad (3)$$

And

$$q_r = \int_0^T q(t) dt = \frac{(\alpha - \beta p + \gamma s)}{\theta} \int_0^T (e^{\theta t} - 1) f(t) dt \quad (4)$$

From equation (3) and equation (4), we get the quantity of the product sold to customer by the retailer as

$$Q - \theta q_r = (\alpha - \beta p + \gamma s) \int_0^T f(t) dt \quad (5)$$

We normalize the replenishment cycle length T to 1.00, for easier calculation, as: $\vartheta_1 = \int_0^1 e^{\theta t} f(t) dt$, $\vartheta_2 = \frac{1}{\theta} \int_0^1 (e^{\theta t} - 1) f(t) dt$, $\vartheta_3 = \int_0^1 f(t) dt$ and $\vartheta_1, \vartheta_2, \vartheta_3 \geq 0$

4-3- Decentralized model

Under this system, members individually decide base on their profits and determine the price and quality of the product as the decision variables. Among the many possible uncooperative gaming assumptions, we will consider the popular ‘‘manufacturer-Stackelberg’’, in which the manufacturer is the Stackelberg leader who sets the product quality and the retailer is the follower who determines the corresponding retail price. Under such a setting, for any s set by the manufacturer, the retailer would decide a retail price p to maximize the following average profit.

The retailer determines selling price p to maximize the following average profit:

$$\Pi_r^{dc}(p) = p(Q - \theta q_r) - wQ - (h_{r1} + \theta h_{r2})q_r \quad (6)$$

In equation (6), the elements of the retailer’s profit function are sales revenue, purchasing cost and holding cost of inventory. Substituting equations (3) _ (5) into equation (6) and simplifying, we get

$$\Pi_r^{dc}(p) = [p\vartheta_3 - w\vartheta_1 - (h_{r1} + \theta h_{r2})\vartheta_2](\alpha - \beta p + \gamma s) \quad (7)$$

Since

$$\frac{d^2 \Pi_r^{dc}}{dp^2} = -2\beta\vartheta_3 < 0 \quad (8)$$

Thus, the optimal retail price p^* can be calculated at $d\Pi_r^{dc}/dp = 0$ for any given s -value as below:

$$p^{*dc} = \frac{1}{2} \left[\frac{\alpha + \gamma s}{\beta} + \frac{w\vartheta_1 + (h_{r1} + \theta h_{r2})\vartheta_2}{\vartheta_3} \right] \quad (9)$$

The elements of the manufacturer’s profit function are sales revenue, production cost and cost of quality investment. Under decentralized mode, the manufacturer’s profit in general form is expressed as:

$$\Pi_m^{dc}(s) = (w - c)Q - \frac{1}{2}\tau s^2 = (w - c)(\alpha - \beta p + \gamma s)\vartheta_1 - \frac{1}{2}\tau s^2 \quad (10)$$

Since, the manufacturer has to follow the retailer's decision and determines the quality of product s . The optimal retail price gives the manufacturer's average profit, by substituting equation (9) into equation (10), as follow

$$\Pi_m^{dc}(s) = \frac{1}{2} \left\{ (w - c) \vartheta_1 \left[\alpha + \gamma s - \frac{\beta}{\vartheta_3} (w \vartheta_1 + (h_{r1} + \theta h_{r2}) \vartheta_2) \right] - \tau s^2 \right\} \quad (11)$$

As the leader, the manufacturer will ultimately set the optimal product quality to maximize $\Pi_m^{dc}(s)$. From $d\Pi_m^{dc}/ds = 0$, we obtain a unique root as

$$s^{*dc} = \frac{\gamma \vartheta_1 (w - c)}{2\tau} \quad (12)$$

Because of, the manufacturer's profit function is concave in s as follows:

$$\frac{d^2 \Pi_m^{dc}}{ds^2} = -\tau < 0 \quad (13)$$

So s^{*dc} is the optimal product quality set by the manufacturer. This in turn gives

$$p^{*dc} = \frac{\alpha \vartheta_3 + \beta \vartheta_2 (h_{r1} + \theta h_{r2}) + w \vartheta_1 \beta + \frac{\gamma^2 \vartheta_1 \vartheta_3}{2\tau} (w - c)}{2\beta \vartheta_3} \quad (14)$$

Hence, the expected profit of the supply chain under decentralized system will be obtained based on the optimal values of the decision variables s^{*dc} and p^{*dc} as: $\Pi_{sc}^{dc} = \Pi_r^{dc}(p^{*dc}) + \Pi_m^{dc}(s^{*dc})$.

4-4- Centralized model

Under this model, one decision-maker as an integrated firm simultaneously determines the retail price and the quality of the product to maximize the entire SC profit. In this case, the decision variables are expected to be globally optimized. The profit function of the entire SC is equal to the sum of profit functions of the SC members in the decentralized system. The expected profit function of the entire SC can be formulated as:

$$\Pi_{sc}^c(p, s) = p(Q - \theta q_r) - cQ - (h_{r1} + \theta h_{r2})q_r - \frac{1}{2} \tau s^2 \quad (15)$$

Substituting ϑ_1 , ϑ_2 and ϑ_3 into equation (15), and simplifying, we get

$$\Pi_{sc}^c(p, s) = (p \vartheta_3 - c \vartheta_1 - (h_{r1} + \theta h_{r2}) \vartheta_2) (\alpha - \beta p + \gamma s) - \frac{1}{2} \tau s^2 \quad (16)$$

Proposition 1. Under the centralized system, the expected profit function of the supply chain $\Pi_{sc}^c(p, s)$ is jointly concave in p and s simultaneously if $\frac{\gamma^2 \vartheta_3}{2\beta \tau} < 1$, thus with this condition, the optimal values of the retail price p^{*c} and the quality of product s^{*c} , which maximize Π_{sc}^c , are

$$p^{*c} = \frac{\alpha \tau \vartheta_3 + (\beta \tau - \gamma^2 \vartheta_3) (c \vartheta_1 + (h_{r1} + \theta h_{r2}) \vartheta_2)}{\vartheta_3 (2\beta \tau - \gamma^2 \vartheta_3)} \quad (17)$$

$$s^{*c} = \frac{\gamma}{\tau} \left[\frac{\alpha \vartheta_3 - \beta (c \vartheta_1 + (h_{r1} + \theta h_{r2}) \vartheta_2)}{2\beta (1 - \frac{\gamma^2 \vartheta_3}{2\beta \tau})} \right] \quad (18)$$

Proof. To prove concavity, we take the second partial derivatives of Π_{SC}^c with respect to p and s . Therefore, we have $\frac{d^2\Pi_{SC}^c}{dp^2} = -2\beta\vartheta_3 < 0$, $\frac{d^2\Pi_{SC}^c}{ds^2} = -\tau < 0$ and $\frac{d^2\Pi_{SC}^c}{dpds} = \gamma\vartheta_3$. The Hessian matrix of $\Pi_{SC}^c(p, s)$ is

$$\begin{bmatrix} \frac{d^2\Pi_{SC}^c}{dp^2} & \frac{d^2\Pi_{SC}^c}{dpds} \\ \frac{d^2\Pi_{SC}^c}{dsdp} & \frac{d^2\Pi_{SC}^c}{ds^2} \end{bmatrix}. \quad (19)$$

With $\frac{\gamma^2\vartheta_3}{2\beta\tau} < 1$, from Eq. (19), we get $\frac{d^2\Pi_{SC}^c}{dp^2} \cdot \frac{d^2\Pi_{SC}^c}{ds^2} - \left(\frac{d^2\Pi_{SC}^c}{dpds}\right)^2 = \vartheta_3(2\beta\tau - \gamma^2\vartheta_3) > 0$. Since $\frac{d^2\Pi_{SC}^c}{dp^2} < 0$ and $\frac{d^2\Pi_{SC}^c}{dp^2} \cdot \frac{d^2\Pi_{SC}^c}{ds^2} - \left(\frac{d^2\Pi_{SC}^c}{dpds}\right)^2 > 0$, the Hessian matrix $\nabla^2\Pi_{SC}^c$ is negative. This implies that $\Pi_{SC}^c(p, s)$ is concave in p and e simultaneously.

The optimal values of the supply chain decision variables p and s can be calculated by solving $\frac{d\Pi_{SC}^c}{dp} = 0$, and $\frac{d\Pi_{SC}^c}{ds} = 0$ as follows:

$$p^{*c} = \frac{1}{2} \left[\frac{\alpha + \gamma s}{\beta} + \frac{c\vartheta_1 + (h_{r1} + \theta h_{r2})\vartheta_2}{\vartheta_3} \right] \quad (20)$$

$$s^{*c} = \frac{\gamma}{\tau} [p\vartheta_3 - c\vartheta_1 - (h_{r1} + \theta h_{r2})\vartheta_2] \quad (21)$$

Thus, we can simply derive Equations (17)- (18) by iterative solving the Equations (20)- (21).

It is expected that the whole SC profit of the centralized model will be higher than that of the decentralized model, $\Pi_{SC}^{c*} > \Pi_{SC}^{dc*}$, which is the case in this study. Therefore, the centralized model optimizes the expected profit of the whole SC, which may lead to a loss for the retailer ($\Pi_r^{c*} < \Pi_r^{dc*}$). Thus, he/she may not accept the global optimal decisions of the centralized system and may prefer to make decisions independently under the decentralized system. Thus, to incentivize the members to accept the centralized solutions, in the next section, a coordination scheme based upon the credit payment mechanism is formulated.

4-5- Coordination model

The main goal of any coordination model is to reach a coordinated channel, which is desirable for all supply chain members. In the proposed framework, the coordination mechanism is modeled as a coordination model in which the retail price and the quality of the manufacturer's product are decisions of the members in the investigated supply chain. Thus, changing the decision-making mode from decentralized (traditional) mode may cause profit loss to a member. Under coordination, the manufacturer asks the retailer to change his/her decisions so that the manufacturer can obtain higher profit. However, the retailer may not want to change his/her decisions because he/she is already optimal under the decentralized model. Therefore, in this credit payment policy, the manufacturer offers a credit period μ for delaying the payment to satisfy the retailer to change his/her decisions, whereby the retailer may advantage of the interests.

In this policy, the retailer can have some interest when the manufacturer offers him/her a credit period μ for delaying the payment. By adding an additional term which represents the earning from delaying the payment as the credit period to Eq. (7), the retailer's objective function can be calculated as:

$$\Pi_r^{co}(p^{*c}, \mu_{p^{*c}, s^{*c}}) = p^{*c}(Q - \theta q_r) - wQ - (h_{r1} + \theta h_{r2})q_r + wI_r\mu_{p^{*c}, s^{*c}}Q \quad (22)$$

By simplifying, we get

$$\Pi_r^{co}(p^{*c}, \mu) = [p^{*c}\vartheta_3 - w\vartheta_1(1 - I_r\mu_{p^{*c}, s^{*c}}) - (h_{r1} + \theta h_{r2})\vartheta_2](\alpha - \beta p^{*c} + \gamma s^{*c}) \quad (23)$$

On the other hand, the manufacturer has a lost investment opportunity during this delay period in payment by offering this credit mechanism to the retailer. Therefore, the manufacturer's expected profit function is presented as follows:

$$\Pi_m^{co}(s^{*c}, \mu_{p^{*c}, s^{*c}}) = (w - c)Q - \frac{1}{2}\tau s^{*c2} - wI_m\mu_{p^{*c}, s^{*c}}Q = [w(1 - I_m\mu_{p^{*c}, s^{*c}}) - c](\alpha - \beta p^{*c} + \gamma s^{*c})\vartheta_1 - \frac{1}{2}\tau s^{*c2} \quad (24)$$

Both the SC members must decision on the length of the delay in payment, μ . Thus, the acceptable conditions from both the supply chain members' viewpoints are found.

4-5-1- Conditions for members to participate

Obviously, the retailer accepts this proposed mechanism if his/her position does not get worse when he/she moves from decentralized decision-making mode to coordinated scheme. A condition which encourages the retailer to change his/her decisions from decentralized mode to coordinated scheme is as follows:

$$\Pi_r^{co}(p^{*c}, \mu_{p^{*c}, s^{*c}}) \geq \Pi_r^{dc}(p^{*dc}) \quad (25)$$

Substituting equations (7) and (24) into equation (25), a lower limit for the duration of the credit period is calculated from the retailer's viewpoint, which is, in fact, at least a grant that the manufacturer must provide to ensure that the retailer participates in this mechanism scheme:

$$[p^{*c}\vartheta_3 - w\vartheta_1(1 - I_r\mu_{p^{*c}, s^{*c}}) - (h_{r1} + \theta h_{r2})\vartheta_2](\alpha - \beta p^{*c} + \gamma s^{*c}) \geq [p^{*dc}\vartheta_3 - w\vartheta_1 - (h_{r1} + \theta h_{r2})\vartheta_2](\alpha - \beta p^{*dc} + \gamma s^{*dc}) \quad (26)$$

Simplifying equation (24), we get

$$\mu_{p^{*c}, s^{*c}}^{min} = \frac{1}{I_r} \left[\frac{\vartheta_3}{w\vartheta_1} (\eta p^{*dc} - p^{*c}) + \left(1 + \frac{(h_{r1} + \theta h_{r2})\vartheta_2}{w\vartheta_1} \right) (1 - \eta) \right] \quad (27)$$

Where $\eta = (\alpha - \beta p^{*dc} + \gamma s^{*dc}) / (\alpha - \beta p^{*c} + \gamma s^{*c})$ under this mechanism, the manufacturer can obtain more profit if the retailer is accepted to gain the same profit under decentralized mode which is possible by equation (25). However, the retailer may not be satisfied and may require a higher profit to participate in this mechanism. This additional profit is shifted to the retailer when the manufacturer is satisfied with the least amount of profit equal to that of the decentralized model. The necessary condition for ensuring the manufacturer's participation in this coordination scheme is as follows:

$$\Pi_m^{co}(s^{*c}, \mu_{p^{*c}, s^{*c}}) \geq \Pi_m^{dc}(s^{*dc}) \quad (28)$$

Substituting equation (11) into equation (28), an upper limit for the credit period interval is calculated from the manufacturer's perspective by equation (28), which is in fact the maximum grant that the manufacturer can pay the retailer.

$$[w(1 - I_m\mu_{p^{*c}, s^{*c}}) - c](\alpha - \beta p^{*c} + \gamma s^{*c})\vartheta_1 - \frac{1}{2}\tau s^{*c2} \geq (w - c)(\alpha - \beta p^{*dc} + \gamma s^{*dc})\vartheta_1 - \frac{1}{2}\tau s^{*dc2} \quad (29)$$

Simplifying equation (29), we get

$$\mu_{p^{*c}, s^{*c}}^{max} = \frac{1}{I_m} * \left[(1 - \eta) \left(1 - \frac{c}{w} \right) + \frac{\tau(s^{*dc2} - s^{*c2})}{2w\vartheta_1(\alpha - \beta p^{*c} + \gamma s^{*c})} \right] \quad (30)$$

4-5-2- Profit sharing strategy

If the interval $[\mu_{p^*c.s^*c}^{min}, \mu_{p^*c.s^*c}^{max}]$ is a non-empty interval, then supply chain coordination is achievable. Since $\mu \in [\mu_{p^*c.s^*c}^{min}, \mu_{p^*c.s^*c}^{max}]$, setting μ at any distance from $[\mu_{p^*c.s^*c}^{min}, \mu_{p^*c.s^*c}^{max}]$ leads to an increase in the members' profits as compared to the decentralized mode, and the amount of each member's share from this surplus profit depends on the bargaining power of each member against that of other members. Following Chaharsooghi and Heydari (2010), we assume that the bargaining powers of the manufacturer and the retailer are equal. Consequently, the optimal value of the credit period ($\mu_{p^*c.s^*c}^{sharing}$) is computed using mean of $\mu_{p^*c.s^*c}^{min}$ and $\mu_{p^*c.s^*c}^{max}$, as follows:

$$\mu_{p^*c.s^*c}^{sharing} = \frac{1}{2}(\mu_{p^*c.s^*c}^{min} + \mu_{p^*c.s^*c}^{max}). \quad (31)$$

5- Numerical examples and sensitivity analysis

In this section, datasets of numerical examples are conducted to investigate the performance of the proposed incentive scheme. The proposed model is calculated by using four test problems datasets. Parameters of the four test problems are shown in table 1.

Table 1. Three test problems datasets

#Test Problem	<u>TP. 1</u>	<u>TP. 2</u>	<u>TP. 3</u>	<u>TP. 4</u>
α	120	150	200	250
β	1.4	1.9	1.7	1.9
γ	1	1.3	1.2	1.5
w	25	27	35	42
c	8	9	12	16
θ	0.5	0.3	0.7	0.8
h_{r1}	9	14	15	19
h_{r2}	3	9	8	11
τ	5	7	8	10
$I_r\%$	18	15	20	15
$I_m\%$	14	15	20	20
$f(t) = e^{-0.96t}$				

Results of four test problems are presented in table 2. Due to the results obtained for the members' profits under different modes, four scenarios have occurred. Results obtained show that the centralized mode may cause a loss for the retailer although it increases the entire SC profit compared to the decentralized model. The retailer's profit and entire SC profit in the credit policy is increased compared to the centralized model in case the retailer's interest rate I_r is more than the manufacturer's interest rate I_m ; in such situations (see, e.g., test problems 1). Moreover, in test problem (4), the entire SC profit in the credit policy is lower than in the centralized mode because the retailer's interest rate I_r is less than the manufacturer's interest rate I_m . When the interest rate for both members is equal, the centralized mode and the coordinated mechanism lead to identical profits (see, e.g., test problems 2 and 3).

Table 2. Running model in the decentralized, centralized and coordinated models

	TP. 1	TP. 2	TP. 3	TP. 4
Decentralized SC				
p^{*dc}	62.01	59.55	89.31	101.64
s^{*dc}	2.72	2.44	3.03	3.6
π_r^{dc}	592.136	542.411	1015.31	873.081
π_m^{dc}	470.774	506.642	1012.56	1187.32
π_{sc}^{dc}	1062.91	1049.05	2027.87	2060.41
Centralized SC				
p^{*c}	52.13	49.76	74.07	83.23
s^{*c}	5.85	5.09	3.21	4.34
π_r^c	536.093	450.391	763.783	442.69
π_m^c	634.713	727.106	1536.96	2069.34
π_{sc}^c	1170.81	1177.5	2300.74	2512.03
Coordinated SC				
p^{*c}	52.13	49.76	74.07	83.23
s^{*c}	5.85	5.09	3.21	4.34
π_r^{co}	669.489	606.625	1151.75	988.641
π_m^{co}	530.961	570.873	1148.99	1341.4
π_{sc}^{co}	1200.45	1177.5	2300.74	2330.04
$\mu_{p^{*c},s^{*c}}^{min}$ (day)	107.2	182.4	191.1	299.6
$\mu_{p^{*c},s^{*c}}^{max}$ (day)	403.4	437.2	398.5	460.6
$\mu_{p^{*c},s^{*c}}^{sharing}$ (day)	255.3	309.8	294.8	380.1
Deviation	137.54	128.45	272.87	269.63
Improvement* (%)	12.93	12.24	13.45	13.08

* Improvement= (Deviation/ π_{sc}^{dc}) * 100%

As shown in table 2, the SC profitability and profits of both members in coordinated model is more than those in decentralized model; therefore, participating in the proposed model can be guaranteed. Thus, a set of sensitivity analysis on key parameters including the retailer's interest rate I_r , Coefficient of product quality γ , and quality cost coefficient τ is examined. Parameters for sensitivity analyses are taken from test problem 1.

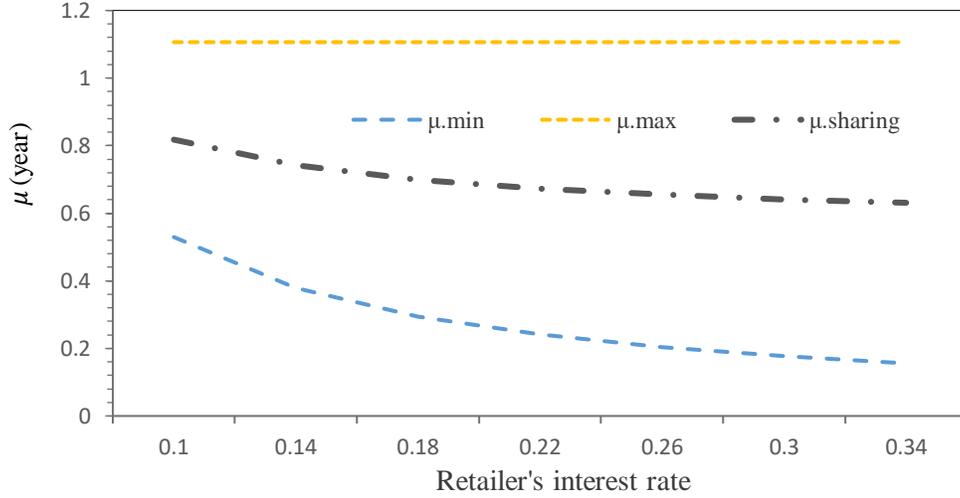


Fig. 1. The effect of retailer's interest rate on credit period

Figure 1 illustrates the conditioned where the proposed credit policy leads to the implementation of the coordinated scheme when I_r changes, because the supply chain is coordinated when μ_{p^*c,s^*c}^{\max} is always more than μ_{p^*c,s^*c}^{\min} . As shown, the implementation of the coordinated model is achievable under the credit policy when I_r moves from 0.1 to 0.34.

Figure 2 shows the SC profit changes of both members and the entire SC under the decentralized mode and the credit policy of coordination scheme when I_r changes. As can be seen, the profits of the entire SC will increase under the credit policy when I_r increases. Since I_r is 0.1 in our test, when $I_r < I_m$, the SC profit under centralized mode is more than that under credit policy but members will gain more profit under credit policy compared to the decentralized mode, and on the contrary, when $I_r > I_m$, they will gain more profit under credit policy compared to the centralized mode. Also, when $I_r = I_m$, the implementation of the coordination scheme under credit policy and centralized mode will result in the same profit for the members and the entire SC.

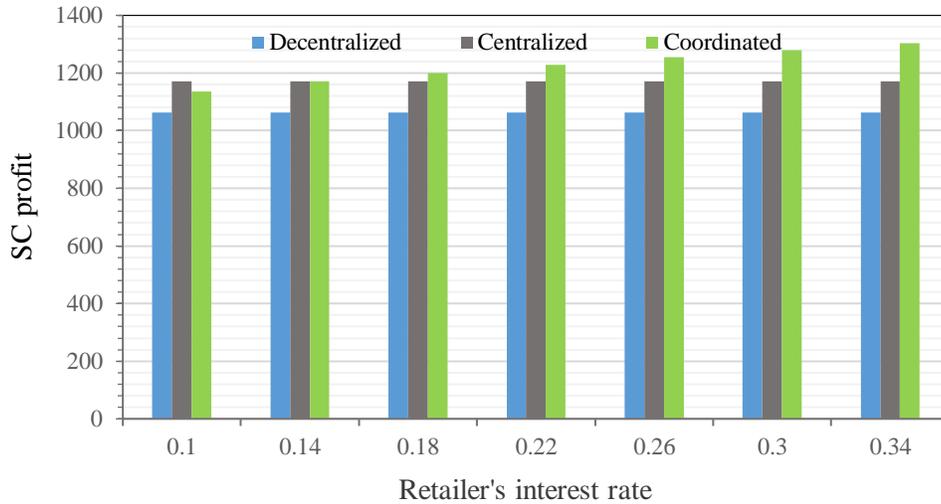


Fig. 2. The effect of retailer's interest rate on profit

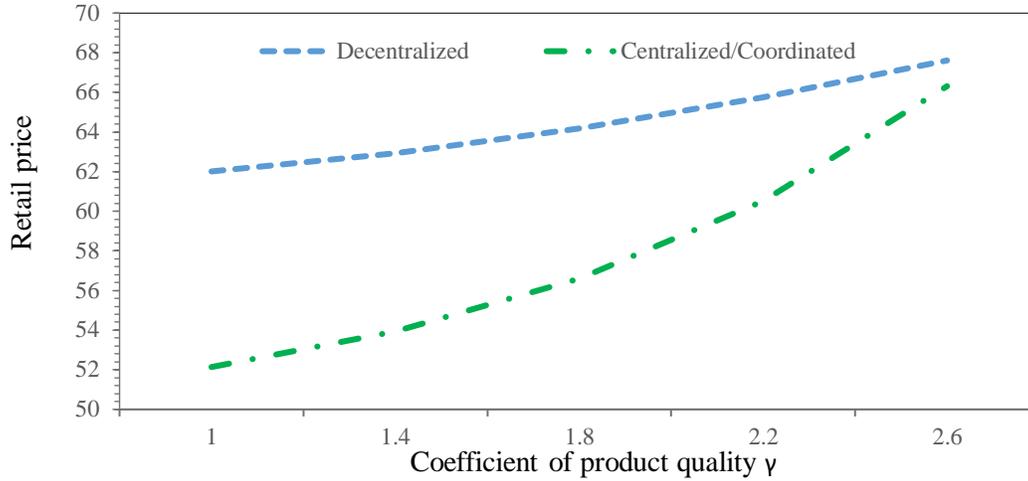


Fig. 3. Coefficient of product quality γ on the retailer's decision variable p

Figure 3 show that the retail price p increases under three different modes when an increase occurs in the coefficient of product quality γ . In all three models, the retailer price increases with an almost constant gradient when the coefficient of product quality γ increases, because the retailer increases the price of its product by increasing the effect of product quality on the demand function, But, from figure 3, we can find that the retail price decreases when we move from decentralized model to centralized/coordinated model.

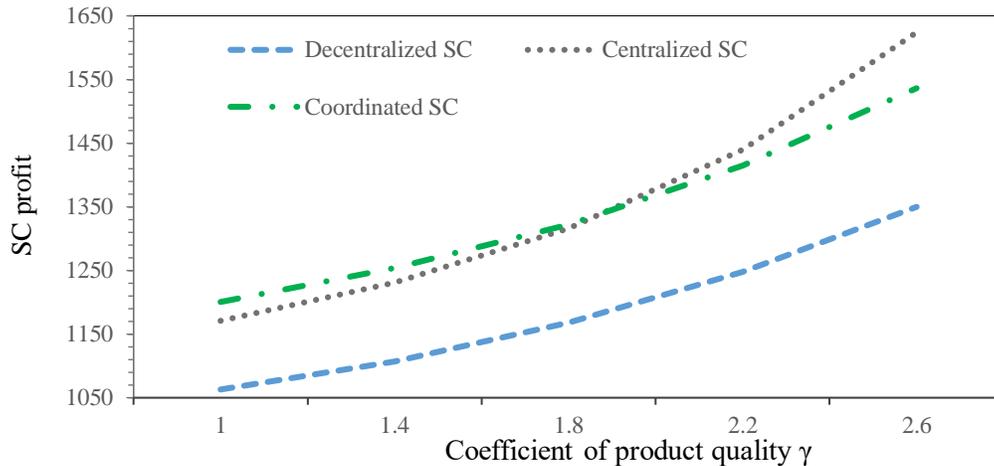


Fig. 4. Coefficient of product quality γ on the profit

In figure 4, it can be seen that the entire SC profit will be increased under three modes when the effect of product quality on the demand γ increases, because the SC share of market demand will be increased by increasing the coefficient of product quality γ . Indeed, we can find that the SC profit increases when we move from decentralized model to centralized and coordinated models. Moreover, the proposed credit policy considerably improves the SC profit compared to the decentralized model under small values of γ .

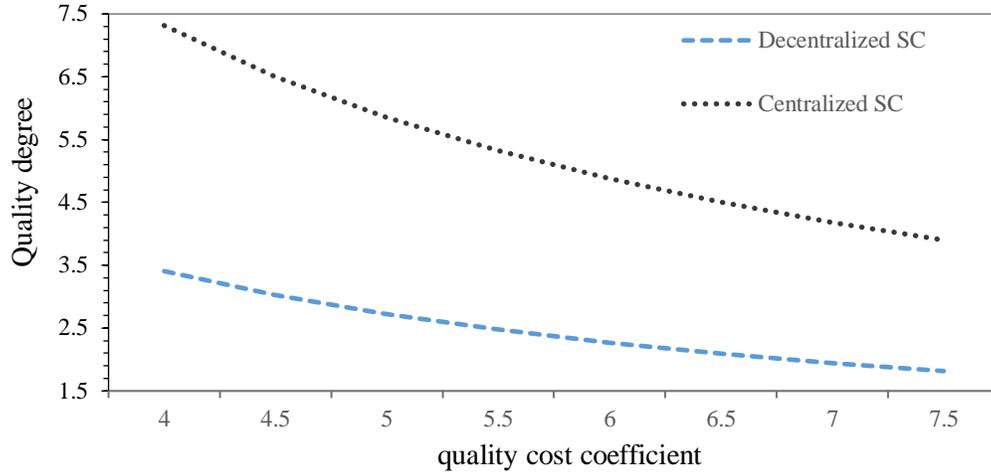


Fig. 5. The effect quality cost coefficient τ on the quality degree s .

Figure 5 indicates that the degree of the product quality will be decreased by increasing in the quality cost coefficient τ . As a result, to prevent the additional costs, the manufacturer reduces the degree of quality. In figure 6, it can be seen that the profit of the entire SC will be decreases under three models by increasing in the quality cost coefficient τ , because the product quality will clearly decreased according to figure 5 when the effect of product quality on the demand increases. Moreover, we can find that the entire SC profit increases when we move from decentralized model to centralized and coordinated models.

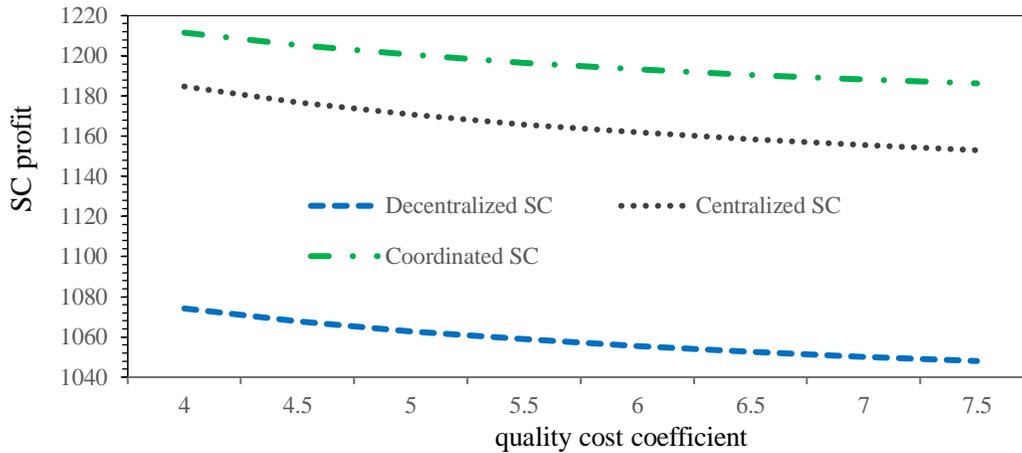


Fig. 6. The effect quality cost coefficient τ on the SC profit

6-Conclusion

In this research, mathematical models are developed for coordinated model in a two-level SC that consisting of a single manufacturer and a single retailer with one type of product with a deteriorating rate, and the demand rate for the product is time-varying and dependent on retail price and product quality. Firstly, the proposed inventory model is presented and compared in two various decision-making models; decentralized, and centralized models. Under the decentralized model, the retailer tries to obtain greater market share by planning optimal strategies on its decision variables. Additionally, inventory replenishment, pricing, and product quality optimizing decisions are made by the retailer and the manufacturer independently. Under the centralized model, decisions were optimized from the viewpoint of the entire supply chain. Then, one coordinated model is proposed to incorporate the supply chain

members by using the credit policy, which helps to increase the sales volume of SC and reduce operational costs. Indeed, the manufacturer tries to induce the retailer, by proposing an incentive mechanism, to optimize pricing and quality optimization decisions globally. In addition, a profit sharing strategy based on SC members bargaining power has been used to allocate the extra profit. The results showed that the supply chain can be coordinated perfectly by the mentioned mechanism and conditions for achieving a win-win outcome exist. Numerical examples and sensitivity analysis on key parameters are also examined to better understand the effect of this incentive scheme.

As for future research, this study can be extended to more SC networks, such as a multi-stage inventory model with several downstream and upstream members or companies. Indeed, this proposed model considers one player at each echelon; while we can consider multiple retailers or multiple suppliers in the model. Also, we can be considering elasticity of demand on the retailer's inventory level, credit time, and advertising. Moreover, the coordination incentive schemes can be considered such as discount quantity, buy-back, quantity-flexibility and sale rebate.

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